Analysis of Transmit-Receive Diversity in Rayleigh Fading

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Abstract — **We analyze the error performance of a wireless communication system employing transmit-receive diversity in Rayleigh fading. By focussing on the complex Gaussian statistics of the independent and identically distributed entries of the channel matrix, we derive a formula for the characteristic function (c.f.) of the maximum output signal-to-noise ratio (SNR). We use this c.f. to obtain a closed-form expression of the symbol error probability (SEP) for coherent binary keying. An approximate expression for the SEP when the average SNR per branch is large is also obtained. The method can be easily extended to obtain the SEP of M-ary modulation schemes.**

I. INTRODUCTION

With the rapid increase in the number of wireless services, more and more wireless communication systems may require diversity at the transmitter in addition to diversity at the receiver to combat the severe effects of fading. This has motivated the study of transmit diversity along with receive diversity. Different transmit diversity techniques, like delay transmit diversity [1, 2] and space transmit diversity [3, 4, 5], have been proposed, but these are based on objectives other than maximizing the signal-to-noise ratio (SNR).

In [6], maximum ratio transmission, which is based on transmit-receive space diversity, has been studied under the assumption that the optimum complex weight vector at the receiver, which maximizes the output SNR, has all entries with the same modulus since the entries of the complex channel matrix are statistically identical. An approximate expression for the symbol error probability (SEP) for binary phase-shift keying (BPSK) when the average SNR per branch is large and the channel matrix has independent complex Gaussian entries, which corresponds to Rayleigh fading, has also been obtained.

In this paper, we analyze transmit-receive diversity in Rayleigh fading by focussing on the complex Gaussian statistics of the independent and identically distributed (i.i.d.) entries of the channel matrix. Without making any assumption on the structure of the optimum complex weight vector at the receiver, we derive a formula for the characteristic function (c.f.) of the maximum output SNR as a finite linear combination of elementary gamma c.f.s. We use this c.f. to obtain a closed-form expression of the SEP for coherent binary keying. An approximate expression for the SEP when the average SNR per branch is large is also obtained. The method can be easily extended to obtain the SEP of M-ary modulation schemes. We present plots of the SEP versus the average SNR per diversity branch to see the effect of different transmit and receive diversity orders on the error performance.

II. THE CHANNEL MODEL

Consider a transmit-receive diversity system employing N antennas for transmission and L antennas for reception. Thus there are NL diversity branches. After sampling at the symbol interval the Lx 1 complex signal vector received at the L antennas is given by

$$r = \sqrt{cP_s}Hw + \boldsymbol{n} ; \tag{1}$$

where c is the transmitted symbol satisfying $|c| = 1$, $\boldsymbol{w}$ the JVx1 transmit weight vector, H the L x N channel matrix, P_s the signal power at each receiving antenna, and **n** the *Lxl* additive noise vector. We assume the noise to be temporally and spatially white with mean zero and a multivariate complex circular Gaussian distribution. Owing to circularity of n, we have $E[nn^T] = 0$, where E[-] denotes the expectation operator and $(-)^T$ denotes the transpose operator. Since the noise is uncorrelated between the diversity branches, we have

$$E[nn^H] = \sigma^2 I_L , \tag{2}$$

where $(-)^H$ denotes the Hermitian operator, and *IL* is the L x L identity matrix.

The channel matrix H can be written as

$$H=[H_{i,j}]i,jL,N_{=1}, \tag{3}$$

where *Hi,j* is the channel coefficient from the *jth* transmitting antenna to the ith receiving antenna. We consider *Rayleigh fading,* in which the channel coefficients $H_{i,j}$, $i = 1,\ldots, L$, $j = 1,\ldots, N$ are i.i.d. complex circular Gaussian random variables, each with a $CN(0,1)$ distribution, implying

$$\mathbf{E}[H_{i,j}] = 0, \quad \mathbf{E}\left[H_{i,j}^2\right] = 0, \quad \mathbf{E}\left[|H_{i,j}|^2\right] = 1.$$

III. THE MAXIMUM EIGENVALUE PROBLEM

The transmit weight vector *w,* which we choose to be a unit vector, can be expressed as

$$w = \frac{}{\|H^H y\|}, \tag{4}$$

where y is the weight vector at the receiver, and $\|\cdot\|$ is the Euclidean norm. Substituting (4) in (1), we get

$$r = c\sqrt{P_s}\boldsymbol{H}\frac{H^H y}{|\backslash H^H y\backslash\ |} + n. \tag{5}$$

The decision variable for detecting the symbol c is obtained by taking the dot product of y and r, resulting in

$$\boldsymbol{y}^H \boldsymbol{r} = c\sqrt{P_s}\|\boldsymbol{H}^H \boldsymbol{y}\| + y^H \boldsymbol{n}. \tag{6}$$

The output SNR, conditioned on the channel matrix *H,* is given by

$$7 = \frac{Ps\backslash\backslash H^H y\ \|^2}{\mathrm{E}\|\boldsymbol{y}^H n\|^2} = \frac{P_s}{\sigma^2}\frac{\boldsymbol{y}^H}{\|\boldsymbol{y}\|}\boldsymbol{H}\boldsymbol{H}^H\frac{\mathrm{y}}{\mathrm{llyll'}} \tag{7}$$

where |f denotes the *average SNR per branch.* In order to minimize the symbol error rate, we have to maximize γ with respect to y. It is clear from (7) that this maximization problem is the same as finding the squared-L2 norm of the matrix H^H, or alternatively, that of the matrix *H.* This squared-L2 norm is the maximum eigenvalue of the Hermitian matrix HH^H, which we denote as Λ_{max}, and the eigenvector of HH^H corresponding to Λ_{max} is the optimum weight vector y_{max} which maximizes γ to yield *γmax.* Thus

$$\gamma_{max} = \max_{\boldsymbol{y}} \frac{P_s}{\sigma^2}\frac{\backslash\backslash H^H y f}{\|\boldsymbol{y}\|^2} = \frac{P_s}{\sigma^2}\Lambda_{max}. \tag{8}$$

For *coherent reception of binary signals,* the symbol error probability, conditioned on γ_{max}, is given by

$$P_{e|\gamma_{max}} = Q\left(\sqrt{2g\gamma_{max}}\right), \tag{9a}$$

where

$$9 \triangleq \frac{1\ \text{- e}}{2}, \tag{9b}$$

and e is the correlation coefficient between the two signaling waveforms. If *C0* and *c1* are the two values of the symbol c, then

$$\text{e} = 3?\{_{\text{Cl}}\text{cS}\}, \tag{9c}$$

where SR{-} denotes the real-part operator, and (•)* denotes the complex conjugate. Using Craig's formula of the Q-function [7], (9a) can be rewritten as

$$P_{p}\backslash_{\gamma_{max}} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left\{-\frac{g\gamma max}{\sin^2\theta}\right\} d\theta. \tag{10}$$

The average SEP is then given by [8]

$$P_e = \frac{l}{\text{Tl'}}\int_{Jo}^{\frac{\pi}{2}} \Psi_{\gamma_{max}}\left(-\frac{9}{\sin^2\theta}\right) d\theta, \tag{11}$$

where ^~(max(j^) = E *[exp(jiij^max)]* denotes the c.f. of γ_{max}.

We focus on finding the c.f. of γ_{max}, which will result in a closed-form expression for the SEP.

IV. SEP FOR COHERENT BINARY KEYING

Let *hi* denote the ith column vector of the channel matrix *H,* which implies

$$H = [h1, h2, \cdots, hN]. \tag{12}$$

The Hermitian matrix *V* is defined as

$$\boldsymbol{V} \triangleq \boldsymbol{H}\boldsymbol{H}^H = \sum_{i=1}^{N} \boldsymbol{h}_i \boldsymbol{h}_i^H. \tag{13}$$

The *L x 1* vectors *h1,..., hN* are i.i.d. complex Gaussian random vectors each distributed as $CN(0, IL)$.

When the transmit diversity order is no less than the receive diversity order, that is, $N \geq L$, *V* has a Wishart distribution, and its *L* eigenvalues Λ1,..., ΛL, which are real and positive with probability one, have the joint probability density function (p.d.f.) [9]

$$f_{\Lambda_1,\ldots,\Lambda_L}(\lambda_1,\ldots,\lambda_L) = \frac{\left[\prod_{i=1}^{L}\lambda_i\right]^{N-L}\exp\left(-\sum_{i=1}^{L}\lambda_i\right)}{L!\left[\prod_{i=1}^{L}(L-i)!(N-i)!\right]} \times \left[\prod_{1\leq i<j\leq L}(\lambda_i-\lambda_j)^2\right], \tag{14}$$

$$\lambda_1,\ldots,\lambda_L > 0.$$

On the other hand, when the transmit diversity order is less than the receive diversity order, that is, $N < L$, *V* has a pseudo-Wishart distribution; *N* of its eigenvalues, denoted as Λ1,..., *ΛN,* are real and positive with probability one, and the remaining *L — N* of the eigenvalues, denoted as ΛN+1, ..., ΛL, are zero, that is

$$\Lambda_{N+1} = \ldots = \Lambda_L = 0. \tag{15}$$

The joint p.d.f. of Λ1,..., *ΛN* is given by

$$f_{\Lambda_1,\ldots,\Lambda_N}(\lambda_1,\ldots,\lambda_N) = \frac{\left[\prod_{i=1}^{N}\right]^{L-N}\exp\left(-\sum_{i=1}^{N}\lambda_i\right)}{N!\left[\prod_{i=1}^{N}(N-i)!(L-i)!\right]} \times \left[\prod_{1\leq i<j\leq N}(\lambda_i-\Lambda_j)^2\right], \tag{16}$$

$$\lambda_1,\ldots,\lambda_N > 0,$$

which has the same form as the p.d.f. (14) for $N \geq L$ with L and N exchanged.

Without loss of generality, we consider the case when $N \geq L$.

The cumulative distribution function of $\Lambda max = \max(\Lambda 1, \ldots, \Lambda_L)$ is obtained from (14) as

$$F_{\Lambda_{max}}(u) = \int_0^u \cdots \int_0^u \frac{\left[\prod_{i=1}^{L} \lambda_i\right]^{N-L} \exp\left(-\sum_{i=1}^{L} \lambda_i\right)}{L! \left[\prod_{i=1}^{L} (L-i)!(N-i)!\right]} \times [A(Ai, \ldots, A_L)]^2 dAi \cdots dA_L, (17)$$

where A(Ai,..., λL) denotes the Vandermonde determinant of λ1, • • •, λL.

We consider the evaluation of the integral I(u) given by

$$I(u) = L! \left[\prod_{i=1}^{L} (L-i)!(N-i)!\right] F_{\Lambda_{max}}(u)$$
$$= \int_0^u \cdots \int_0^u \prod_{j=1}^{L} \lambda_j^{N-L} \exp(-\lambda_j) \times [A(Ai, \ldots, A_L)]^2 dAi \cdots dA_L. \quad (18)$$

By making use of the fact that the integrand of I(u) is symmetric in λ1,..., λL, it can be shown after some algebra that we can express *I(u)* in terms of the determinant of a matrix function whose elements are integrals. Thus

$$\mathcal{I}(u) = L! \det(\boldsymbol{S}(u)), \quad (19a)$$

where the element in the *kth* row and the lth column of S(U) is given by

$$\begin{aligned} (S(u))_{k,l} &= S_{k,l}(u) \\ &= \int_0^u \lambda_k^{N-L} \exp(-\lambda_k) \lambda_k^{k+l-2} d\lambda_k \\ &= \int_0^u x^{N-L+k+l-2} \exp(-x) dx \\ &= \Gamma(N-L+k+l-1, u), \end{aligned} \quad (19b)$$

the incomplete gamma function $\Gamma(k+1, u)$ for $k = 0, 1, 2, \ldots$ and $u > 0$ having the representation

$$\begin{aligned} \Gamma(k+1, u) &= \int_0^u x^k \exp(-x) dx \\ &= k! \left[1 - e^{-u} \sum_{m=0}^{k} \frac{u^m}{m!}\right]. \end{aligned} \quad (20)$$

Note that *S(u)* is an $L \times L$ Hankel matrix.

By careful examination of the entries of matrix *S(u)*, it can be shown from (17), (18), and (19) that

$$\begin{aligned} f_{\Lambda_{max}}(u) &= \frac{1}{\left[\prod_{i=1}^{L} (L-i)!(N-i)!\right]} \frac{d}{du} \det(\boldsymbol{S}(u)) \\ &= \frac{1}{\left[W^{L} -i)\backslash(N - i)!\right]} \\ &\quad \times \sum e^{-iu} \sum_{m=N-L}^{(N+L)i-(i+1)i} c_{i,m} u^m, \end{aligned} \quad (21)$$

where $c_{i,m}$ is the coefficient of $e^{-iu} u^m$. This can be written as a finite *linear combination of elementary gamma p.d.f.s, each with parameter m* + 1 *and mean* $\frac{m+1}{i}$, as

$$f_{\Lambda_{max}}(u) = \sum_{i=1}^{L} \sum_{m=N-L}^{(N+L)i-(i+1)i} d_{i,m} \frac{i^{m+1} u^m e^{-iu}}{m!}, \quad (22)$$

$u > 0$,

where

$$d_{i,m} = \frac{m! c_{i,m}}{i^{m+1} \left[\prod_{i=1}^{L} (L-i)!(N-i)!\right]}. \quad (23)$$

By applying in (22) the result

$$\int_0^\infty e^{\jmath\omega v} e^{-av} v^k dv = \frac{k!}{(a-\jmath\omega)^{k+1}}, \quad (24)$$
$$a > 0, \quad k = 0, 1, 2, \ldots, \quad \jmath = \sqrt{-1},$$

and noting from (8) that $\Psi_{\gamma_{max}}(jw) = \Psi_{\Lambda_{max}}$ \ju%z), the c.f. of γ_{max} is given by

$$\Psi_{\gamma_{max}}(\jmath\omega) = \sum_{i=1}^{L} \sum_{m=N-L}^{(N+L)i-(i+1)i} \frac{d_{i,m}}{\left(1 - \frac{\jmath\omega P_s}{i\sigma^2}\right)^{m+1}}. \quad (25)$$

Thus (25) is a *formula for the c.f. of* γ_{max} *as a finite linear combination of elementary gamma c.f.s.*

By combining (11) and (25), the SEP for coherent reception of binary signals is expressed as

$$P_e = \sum_{i=1}^{L} \sum_{m=N-L}^{(N+L)i-(i+1)i} \frac{d_{i,m}}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{\frac{gP_s}{i\sigma^2} + \sin^2\theta}\right)^{m+1} d\theta. \quad (26)$$

It is known from [10] (equation (77)) that

$$\begin{aligned} &\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{a + \sin^2\theta}\right)^{m+1} d\theta \\ &= \frac{1}{2} - \frac{1}{2\sqrt{1+a^{-1}}} \sum_{l=0}^{m} \binom{2l}{l} \frac{1}{4^l (1+a)^l}, \end{aligned} \quad (27)$$
$$a > 0, \quad m = 0, 1, 2, \ldots$$

Substituting (27) in (26), we get

$$\mathrm{ft} = \mathrm{IE} \sum_{i=1}^{L} \sum_{m=N-L}^{(N+L)i-(i+1)i} \ast\!\!\gg\bullet$$

$$\times \left[1 - \frac{1}{\sqrt{1+\frac{i\sigma^2}{gP_s}}} \sum_{l=0}^{m} \binom{2l}{l} \frac{1}{4^l\left(1+\frac{gP_s}{i\sigma^2}\right)^l} \right]. \quad (28)$$

Thus (28) is a *closed-form expression of the SEP for coherent binary keying when* $N \geq L$. For different values of N and L, the coefficients $d_{i,m}$ can be easily computed by employing curve-fitting on the plot of J^ det $(S(u))$ versus u and stored in lookup tables. Two such lookup tables of $d_{i,m}$ for $N = 3$, $L = 2$ and $N = 5$, $L = 3$ constitute Table 1. Note that in both the lookup tables, the summation of $d_{i,m}$ over M and i is unity, as expected.

When $N < L$, the p.d.f. $f_{\Lambda_{max}}(u)$ is given by (21) with L and N exchanged, such that the element in the kth row and the lth column of the $N \times N$ Hankel matrix $S(u)$ is expressed as

$$\begin{aligned}(S(u))_{k,l} &= S_{k,l}(u)\\ &= \Gamma(L-N+k+l-1,u).\end{aligned} \quad (29)$$

As a result, the SEP has the same expression as (28) with L and N exchanged.

To obtain a closed-form expression for P_e *which is valid for both* $N \geq L$ *and* $N < L$, *replace* $N-L$ *by* $|N-L|$ *in the summation over* m *and* L *by* $\min(N, L)$ *in the summation over* i *in (28).*

We observe from (19b) and (26) that for any pair (N,L), the SEP depends O N N + L, $|N - L|$, and $\min(N, L)$. As a result, *the SEP is symmetric in N and L*. It is also to be noted that the SEP does not depend on NL alone. For example, the SEP for $(N, L) = (4, 3)$ will be different from that for $(N, L) = (6, 2)$.

When the average SNR per branch is large, that is, σs ^> 1, we can consider, in the SEP expression, only the ΣM = $|N - L|$ term in the summation over M and only the $i = \min(N, L)$ term in the summation over i. The p.d.f. of Λ_{max} then approximates to a *gamma p.d.f. with parameter* $|N - L|+1$ *and mean* $\frac{|N-L|+1}{\ldots}$, which gives rise to the approximate SEP expression

$$Pe\Big|_{\frac{P_s}{\sigma^2}\gg 1} \approx \frac{1}{2}\binom{2|N-L|+2}{|N-L|+1} \times \left\{\left[\frac{4g}{\min(N,L)}\right]\left[\frac{P_s}{\sigma^2}\right]\right\}^{-(|N-L|+1)}. \quad (30)$$

Thus the SEP decreases inversely with the $(|N-L|+1)$th power of the average SNR per branch.

The result (30) is different from that given by equation (25) of [6] where the SEP decreases inversely with the (NL)th power of the average SNR per branch. The reason is as follows. In [6], it has been assumed that each complex entry of the $L \times 1$ optimum weight vector y_{max} at the receiver has the same modulus since the entries of the channel matrix H are statistically identical. In the case of Rayleigh fading with i.i.d. complex Gaussian entries of H, the lower bound on γ_{max} has a *gamma distribution with parameter NL and mean* $N\sigma P2s$, and when the average SNR per branch is large, this gives rise to the approximate SEP expression (25) in [6], which can be written using our notations as

$$P_e\Big|_{\frac{P_s}{\sigma^2}\gg 1} \approx \binom{2NL-1}{NL}\left\{\left[\frac{4g}{L}\right]\left[\frac{P_s}{\sigma^2}\right]\right\}^{-NL}. \quad (31)$$

On the other hand, we consider here the p.d.f. of Λ_{max}, which, for Rayleigh fading with i.i.d. complex Gaussian entries of H, is a *weighted sum of gamma p.d.f.s* and is given by (22). When $\frac{P}{\sigma^2} \gg 1$, the p.d.f. of $\gamma_{max} = \sim$^$i\Lambda_{max}$ approximates to a *gamma p.d.f with parameter* $|N - L| + 1$ *and mean* $\left(\frac{|N-L|+1}{\min(N,L)}\right)\frac{P}{\sigma^2}$, resulting in the approximate SEP expression (30).

V. EXTENSION TO M-ARY MODULATION SCHEMES

In the case of M-ary modulation schemes such as M-PSK, M-QAM, and M-AM, the SEP for coherent reception depends directly on the integral $P(\Theta, g_M)$ given by

$$P(\Theta,g_M) = \frac{1}{\pi}\int_0^{\Theta} \Psi_{\gamma_{max}}\left(-\frac{g_M}{\sin^2\theta}\right) d\theta, \quad (32)$$

where g_M is a parameter which depends on M and the

$(N,L) = (3,2)$	$i = 1$	$i = 2$
$m = 1$	3.00	-0.75
$m = 2$	-4.00	-0.25
$m = 3$	3.00	0.00
$m = 4$		0.00

$(N, L) = (5, 3)$	$i = 1$	$i = 2$	$i = 3$
$m = 2$	10.5462	-0.1936	-0.3332
$m = 3$	-31.1751	0.1409	-0.3668
$m = 4$	49.3378	0.4666	-0.2786
$m = 5$	-40.8351	0.0420	-0.1833
$m = 6$	15.2272	-0.6937	-0.0922
$m = 7$		-0.0030	-0.0352
$m = 8$		-0.5626	-0.0108
$m = 9$		0.0005	-0.0029
$m = 10$		-0.0002	-0.0006
$m = 11$			-0.0001
$m = 12$			0.0000

Tab. 1: Enumeration of coefficients $d_{i,m}$ for $(N,L) = (3,2)$ and $(N,L) = (5,3)$

modulation scheme, and γ_{max} and Λ_{max} are governed by

$$\gamma_{max} = \frac{P_{s,av}}{\sigma^2}\Lambda_{max},$$

Ps,av being the signal power averaged over all *M* signaling waveforms. Substituting (25) in (32), and replacing P_s by *Ps,av* in the resulting expression, we get, for $N \geq L$,

$$P(\Theta, g_M) = \sum_{i=1}^{L} \sum_{m=N-L}^{(N+L)i-(i+1)i} d_{i,m}\mathcal{J}_{i,m}(\Theta, g_M), \qquad (33a)$$

where the integral $\mathcal{J}_{i,m}(\Theta, g_M)$ given by

$$\mathcal{J}_{i,m}(\Theta, g_M) = \frac{1}{\pi}\int_0^{\Theta}\left(\frac{\sin^2\theta}{\frac{g_M P_{s,av}}{i\sigma^2} + \sin^2\theta}\right)^{m+1} d\theta \qquad (33b)$$

can be expressed in closed-form by equation (77) of [10].

VI. NUMERICAL RESULTS

Plots of the SEP P_e versus the average SNR per branch s2 for BPSK with different values of (N, *L)* are shown in Fig. 1. The coefficients $d_{i,m}$ for each *(N,L)* have been calculated by curve-fitting, and these have been used to compute the SEP using (28). We see from the plots that increase in N for a given *L* or increase in *L* for a given TV improves the performance. The plots also show clearly that the SEP does not depend on *NL* alone. The performance with *(N,L)* = (6,2) in Fig. 1(a) is better than that with (N, *L)* = (4, 3) in Fig. 1(b), although *NL* = 12 in both cases. Similarly *(N,L)* = (8,2) in Fig. 1(a) performs better than *(N,L)* = (4,4) in Fig. 1(b). On the other hand, the performance with (N, *L)* = (2, 2) in Fig. 1(a) is inferior to that with (N, *L)* = (4,1) in Fig. 1(b). Therefore, if we have to keep *NL* fixed, then it is desirable to have $|N - L|$ as large as possible. This is consistent with the analytical result (30) for high average SNR per branch which indicates that the SEP decreases exponentially with increase of $|N - L|$.

VII. CONCLUSION

We have analyzed the error performance of a transmit-receive space diversity system in Rayleigh fading. The channel matrix *H* of such a system has i.i.d. complex Gaussian entries. The weight vector at the receiver must be chosen to maximize the output SNR, and this results in finding the maximum eigenvalue of HH^H, which is proportional to the maximum output SNR. From the joint p.d.f. of the eigenvalues of HH^H, we derive a formula for the c.f. of the maximum output SNR as a linear combination of a finite number of elementary gamma c.f.s. We use this c.f. to obtain a closed-form expression of the SEP.

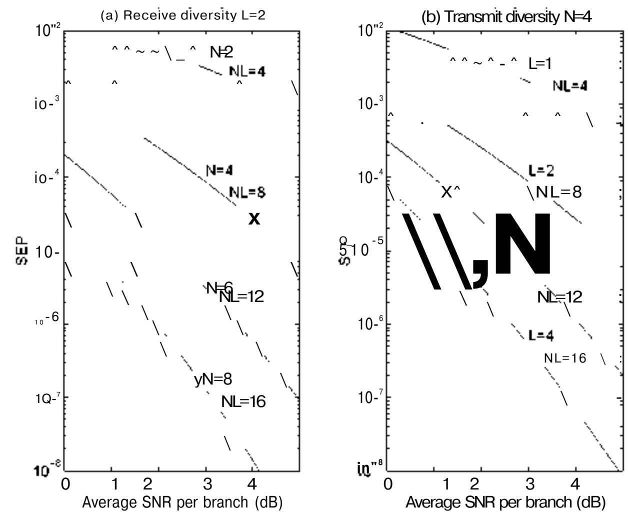

Fig. 1. Plots of P_e (SEP) versus $10\log_{10}\bar{\Gamma}$ (average SNR per branch in dB) for BPSK with different values of (N, L): (a) when $L = 2$ and N varies, (b) when N = 4 and L varies

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