

Analyzing Selfish Topology Control in Multi-Radio Multi-Channel Multi-Hop Wireless Networks

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Abstract—Typically, topology control is perceived as a per-node transmit power control process that achieves certain network-level objectives. We take an alternative approach of controlling the topology of a network purely by assigning channels to multiple radio interfaces on nodes. Specifically, we exploit the synergy between topology control and channel allocation to reduce the overall interference in multi-radio multi-channel wireless ad hoc networks.

We formulate channel assignment as a non-cooperative game, with nodes selecting low interference channels while maintaining some degree of network connectivity. This game is shown to be a potential game, which ensures the existence of, and convergence to, a Nash Equilibrium (NE). Next, we evaluate the performance of NE topologies with respect to interference and connectivity objectives. By quantifying the impact of channel availability on interference performance, we illuminate the tradeoff between interference reduction that can be achieved by distributing interference over multiple channels and the cost of having additional channels. Finally, we study the spectral occupancy of steady state topologies, and show that despite the non-cooperative behavior, the NE topologies achieve load balancing.

I. INTRODUCTION

Wireless ad hoc networks are emerging as a low cost yet powerful tool for communication. These networks, where a set of fixed nodes may operate on multiple channels over a band of spectrum and communicate with each other in a multi-hop manner, are expected to provide orders of magnitude improvement in the overall network capacity [1]. Proposed ad hoc networking solutions are typically based on IEEE 802.11 a/b/g and 802.16 standards (or some combination of the two). Utilizing multiple non-overlapping channels offered in the 2.4GHz and 5GHz bands (permitted by the 802.11 PHY specification, for instance) for simultaneous operation effectively reduces channel contention (and therefore the co-channel interference), enhances spectrum reuse, and significantly improves overall network capacity. Thus, it seems natural to consider and analyze wireless systems where nodes are equipped with multiple radio interfaces, each having the capability to operate on multiple channels simultaneously. The focus of this work is on the game-theoretic analysis of such multi-radio, multi-channel, multi-hop systems. Specifically, we examine the synergy between topology control and channel allocation to minimize the overall interference in multi-hop wireless networks.

This material is based upon work supported by the National Science Foundation under Grant No. 0448131.

Channel assignment has a direct impact on network connectivity and interference, and therefore on the topology of a network [2], [3]. With this viewpoint, we examine the problem of channel allocation and Topology Control (TC) jointly: a TC problem that minimizes some measure of interference subject to network connectivity may be viewed as a channel assignment problem that assigns channels to radio interfaces taking the network-wide interference and connectivity goals into account. We cast the channel assignment problem as a non-cooperative game and analyze the topologies that emerge in steady state NE with respect to its interference performance.

The problem of channel assignment has been widely studied with different emphases. Marina *et al.* [2] were among the first to cast channel assignment as a TC problem. When the connectivity of a network is fixed and there are fewer number of channels than needed, [2], [4] present heuristics to minimize the interference in the network. A similar multi-channel network architecture and topology design philosophy is adopted in [5]. Self-stabilizing distributed channel assignment algorithm for minimizing interference and improving throughput of wireless mesh networks have been proposed elsewhere, e.g. [6]. In the context of dynamic spectrum access and cognitive radio networks, channel assignment has been considered for improving spectrum utilization, e.g. [7], [8]. Distributed channel selection has been examined using game theory, e.g. in [8], but the authors assume that every radio interferes meaningfully with every other radio, regardless of their spatial separation. This model of interference applies only for single-hop networks.

Our work is related to all the aforementioned studies in that we too address the problem of interference minimization through channel assignment. Yet, our formulation and framework is significantly different, e.g., from [3], [7]. The objective function of our optimization construct differs from that of [2], [4], [5]. Unlike these studies, our model does not enforce connectivity as a constraint and isolates some portion of the network if that improves interference in the rest of the network. While [6], [8] adopt a game-theoretic approach similar to ours, they analyze the problem from a radio viewpoint to improve link level performance and do not consider the network connectivity aspect explicitly. Additionally, unlike [8], our interference model accounts for multi-hop networks.

This work provides a rich framework for performing topology control by balancing the network interference and network

connectivity objectives purely through channel assignment. If there are fewer number of available channels than necessary, nodes “switch off” some of their radios by not assigning any channels to the corresponding links, causing the topologies to be a lot sparser. We evaluate the performance of NE topologies with respect to network connectivity and interference minimization goals, and examine the tradeoff between the two objectives. Having more channels for a given level of network connectivity naturally leads to lower interference topologies. Likewise, for a given number of channels, supporting a larger portion of the network on those channels results in increased levels of interference in the network. Furthermore, despite the non-cooperative node behavior, the number of radios on each available channel are evenly distributed, suggesting the load (interference) balancing effect of NE. These form the core contributions of our work.

II. SYSTEM MODEL AND ASSUMPTIONS

A. Network and Interference Model

Consider a multi-hop network formed by a set of nodes $N = \{1, 2, \dots\}$. Each node i may be equipped with multiple transceiver radios. Let k_i be the number of radios on node i ; the j^{th} radio on node i is indexed by r_i^j . All radios on nodes can transmit omnidirectionally at a fixed common power level, meaning that power control is not allowed in our model. All transmissions are unicast and each transmitter is capable of communicating with only one other neighboring receiver on a single channel at any given time instant. Limitations on the number of radios on each node may also necessitate each radio to communicate with multiple neighboring radios on a single channel using time-sharing techniques such as Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) or Time Division Multiple Access (TDMA). Given the power levels of all radios, the induced network is commonly modeled by a communication graph $G = (N, E)$ over N . The set E contains all feasible directional links e_{ij} between nodes i and j ; the feasibility is dictated by node power levels. Each directed link e_{ij} corresponds to the communication between a single transmitter interface on node i and a single receiver interface on node j . Additionally, links may be shared by radios; a single radio may be assigned to multiple links e_{ij} , e_{ik} and so on.

In addition to the communication graph G , we also have an interference graph G_I that specifies the set of transmissions that can potentially interfere with each other if those transmissions occur simultaneously on the same channel. Transmissions from one node may interfere with transmissions from every other node in the network (as in a physical network model), or with only a subset of those transmissions (as in a protocol model) [9]. We model the conflicts in the network by a weighted undirected graph $G_I = (N, E_I, W)$. Here E_I represents the set of edges between all pairs of conflicting nodes. The weights $w_{ij} \in W$ of edges $e_{ij} \in E_I$ specifies interference contribution of node i in the total interference level perceived by node j . Typically, these weights are associated with interference powers. Our model of conflict graph

works because all nodes transmit at the same power level. We assume that the channel gains are symmetric, which gives rise to symmetric edge weights, *i.e.* $w_{ij} = w_{ji} \forall i, j \in N$. Note that while it is common to consider interference terms only from the strongest interferers as in the protocol model, our model is general enough to work with any weighted undirected conflict graph (that is not necessarily complete) where W is symmetric. In general, some entries of W may be 0 while others non-zero. Obviously, a link weight of 0 in G_I is equivalent to that link being absent from G_I . For our purpose, we leave the form of G_I unspecified in our model, except that we require it to be symmetric. In some sense, W may be considered exogenous, and our model works with any symmetric G_I .

We assume that radios can access multiple channels but can only operate on a single channel at a time both while transmitting as well as while receiving. This multi-channel capability extends across the entire spectrum that can be sensed. The sensed spectrum is divided into orthogonal channels. The transmitter and receiver interfaces of a link must be tuned to the same channel for meaningful communication to take place. It is possible for the forward and reverse links (e_{ij} and e_{ji}) to be on different channels, thus allowing for full-duplex mode operation. A full-duplex topology is formed out of G when radios in the network are assigned to non-conflicting channels. In some sense, our multi-channel network may be viewed as a series of overlaid single channel “sub-networks”.

Let \mathcal{C} be the aggregate set of available orthogonal channels in the network. We suppose that $k_i < |\mathcal{C}|$ and distinct channels are assigned to links corresponding to different radio interfaces on a node to benefit from channel diversity. Owing to limitations in number of orthogonal channels that are available, typically $\sum_{i \in N} k_i > |\mathcal{C}|$, which gives rise to potential conflicts and channel sharing. We further note that while channel diversity must be employed to minimize multiple access interference, it is also important to maintain network connectivity as part of a topology control process. To avoid network partitioning, neighboring nodes must always share some common channels. Using a common control channel is one possible mechanism for assuring network connectivity and manageability in the face of multi-channel, multi-radio operation.

We assume that at the pre-specified power levels, the underlying “physical” topology G is connected. However, some links in the physical topology may be unrealized if radios at the two ends of the links are on different channels. Assigning channels to links in the topology, subject to channel availability, induces a “logical” topology $G_c = (N, E_c) \subseteq G$. The channel-assigned topology G_c contains edges $e_{ij} \in E_c$ between i and j on channel c , if $e_{ij} \in E$ is assigned channel c .

B. Game-Theoretic Model

We formulate the multi-radio channel selection in the context of topology control as a non-cooperative game. Each node

has some traffic for all other nodes, which necessitates nodes to communicate with each other over a set of channels.

Channel assignment is done on a per-link basis where two ends of a link must be tuned to the same channel for any meaningful communication to take place. Technically, transmitter-receiver pairs coordinate on which channel to communicate because interference is receiver-centric. Such coordination gives rise to two possibilities: the transmit channel is unavailable at the receive side, in which case the transmitter either determines a different channel or does not assign any channel to the corresponding link until a channel becomes available; or, the transmit channel is available at the receiver, in which case the transmitter decides whether or not the selected channel is an appropriate one by examining the level of interference on that channel. In any event, because the transmitter sides make decisions based on channel availability or interference estimates from receivers, and assign appropriate channels to links, we treat the transmit nodes as the primary decision-makers. Thus, each transmitting node in the network is a player in the game that assigns channels to each of its outgoing links.

Each node determines a channel assignment for its links and selects a vector of channels $\mathbf{c}_i = (c_i^1, \dots, c_i^{k_i})$ from its action set $A_i = \mathcal{C}$; we denote the channel assigned to outgoing links from node i by c_i^j . For simplicity, we enumerate the channels available for each link in a set $\mathcal{C}_i^j = \{0, 1, 2, \dots\}$. Again, for ease of exposition, $c_i^j = 0$ means that no channel is assigned to the outgoing link from r_i^j . When no channel is allocated to a link, the link is disabled, in the sense that it does not exist in G_c .

Each node determines appropriate channels to select for transmission by considering the level of interference perceived on those channels. Using the weighted conflict graph G_1 , each node evaluates the total interference on a given channel by summing up the interference contributions from all nodes sharing the same channel. The interference contribution terms are denoted by the weights of the conflict graph. Note that, it is sufficient for each node to know the sum of all interference weights instead of the individual contributions from each interferer, which may be difficult to measure precisely.

Based on the channel-assigned graph G_c and conflict graph G_1 , each node evaluates the total interference cost χ_i : the sum total of interference weights (obtained from W) over all interferers of i over all channels c_i^j that i is operating on. In a multi-hop network, each node i is able to communicate with their immediate neighbors on the same channels as those of i , and with other nodes (≥ 2 hops away) which may be on different (non-zero) channels than those i is operating on. The benefit in being able to communicate with other nodes is captured by f_i , which specifies the number of nodes i is able to reach (directly or over multiple hops) over all channels. Given these objectives, the utility of each node is given by:

$$u_i(\mathbf{c}) = \begin{cases} \alpha_i f_i(\mathbf{c}) - \chi_i(\mathbf{c}) & \text{if } \mathbf{c}_i \neq \mathbf{0}; \\ -\infty & \text{if } \mathbf{c}_i = \mathbf{0}. \end{cases} \quad (1)$$

Because we are studying the impact of channel availability

on the topology outcomes, we impose the condition that nodes must communicate on at least one channel. The term α_i is a constant that specifies the relative preferences of nodes: improving network connectivity vis-a-vis selecting low interference channels.

III. GAME-THEORETIC ANALYSIS

In this section, we analyze the dynamics of the game with utilities given by (1). Both terms of (1) are interactive terms which together determine the course of the game. When the second term (the interference term) dominates (for instance, when $\alpha_i = 0 \forall i$) each node selects channels so as to minimize χ_i . In pursuit of minimizing local interference, such a channel assignment may increase the interference observed by some other node. The following lemma claims otherwise, from a single radio perspective. We omit the proof of this and all other subsequent results owing to space constraints; we refer the readers to Chapter 7 in [10].

Lemma 1: If the interference χ_i^j of a radio r_i^j increases (or decreases) by some constant δ owing to a channel switch, the aggregate interference of all radios affected by this switch also increases (or decreases) by δ .

We can extend the result of Lemma 1, which analyzes the impact of channel assignment on a per-radio basis, to consider the impact on a per-node basis. Note that, if nodes reduce their interference cost χ_i , they may increase the per-node interference cost χ_j of some other nodes j individually.

Lemma 2: If the interference cost χ_i increases (or decreases) by some constant δ upon channel selection by i , the aggregate interference cost of all nodes affected by this assignment also increases (or decreases) by δ .

The above lemma shows that while the interference reducing channel selections by a node may increase the interference cost observed by other nodes individually, the aggregate interference level (from a network viewpoint) still improves; this suggests a self-stabilizing effect of channel assignment.

If there are enough orthogonal channels available (say, the number of interferers is zero for each node), nodes can maximize their utility by utilizing all their radios. On the contrary, when α_i is low and if all available channels are shared, nodes can improve their utility by not allocating channels to any link, except one with least channel interference. Also, note that assigning the same channel across multiple radios on a node always gives a lower payoff than assigning it to a single radio; this justifies our assumption of assigning distinct channels across radio interfaces on a node.

Using the previous two lemmas, we show that for extreme values of α_i , the channel assignment game becomes a potential game. For a treatise on potential games, see [11].

Theorem 3: For $\alpha_i = 0 \forall i$, the game $\Gamma = \langle N, A, \{u_i\} \rangle$ with payoffs given by (1) is an Exact Potential Game (EPG). The Exact Potential Function (EPF) is given by

$$\mathcal{P}(\mathbf{c}) = -\frac{1}{2} \sum_{i \in N} \chi_i(\mathbf{c}) \quad (2)$$

It is clear that the value of α and number of channels available determine how nodes utilize the channels and how many radios are assigned non-zero channels. The following theorem considers the other extreme case when $\alpha_i \geq \chi_i^{\max}$. Under this scenario, we first show that the original topology G can never be partitioned by channel assignment. (We henceforth denote the value of α that preserves network connectivity of G_c by α_{\max} .)

Lemma 4: When $\alpha_i \geq \chi_i^{\max} \forall i$, then starting from any initial network for which G is connected, every NE achieves connectivity of G_c .

Using this lemma, we next show that for $\alpha_i \geq \chi_i^{\max}$ the corresponding channel assignment game is also an EPG.

Theorem 5: For $\alpha_i \geq \chi_i^{\max} \forall i$, the game $\Gamma = \langle N, A, \{u_i\} \rangle$ with payoffs given by (1) is an EPG. The EPF is given by

$$\mathcal{P}(\mathbf{c}) = \frac{1}{2} \sum_{i \in N} (\alpha_i f_i(\mathbf{c}) - \chi_i(\mathbf{c})) \quad (3)$$

Potential games ensure that at least one NE exists for the game. It is fairly obvious that the channel assignment game admits many NE; depending on the order in which nodes update, different NE topologies will emerge. For both games considered above, the potential maximizing NE minimizes the total interference in the network.

Theorem 6: For $\alpha_i = 0 \forall i$, the potential maximizing NE minimizes $\sum_i \chi_i$, whereas for $\alpha_i = \alpha_{\max}$, the potential maximizing NE minimizes $\sum_i \chi_i$ while maintaining network connectivity of G_c .

The above theorems may not generalize for arbitrary values of α . While this does not preclude the existence of NE for the channel assignment game, the game may or may not possess an NE. For these cases, we examine the existence of NE through simulations. For the cases where our algorithm converges to a NE, we evaluate the NE topologies with respect of interference and connectivity performance in the following section. If nodes adopt a selfish algorithm to improve their performance, the algorithm is guaranteed to converge to some NE. We propose a better-response-based channel selection algorithm that is simple to implement, and evaluate the performance of NE topologies that emerge in steady state.

IV. SIMULATION RESULTS

To determine the efficacy of our model, we develop a simulation consisting of $|N|$ nodes placed according to a uniform random distribution within a unit square. The power thresholds required to close a link between nodes i and j were assumed to be equal to $d^2(i, j)$ (we choose a path loss exponent of 2, although our basic conclusions remain the same for other channel attenuation factors as well), where d is the euclidean distance metric. The initial node transmit power level was chosen using the formula from [12] (and adjusting the value for finite networks), such that the induced network was 1-connected with 85% probability. We consider only the connected instances of G in our simulations (meaning that there exists a path from every node to every other node in the

network). Each node has a fixed number of radios capable of operating on different channels.

The connectivity graph G was transformed to a weighted undirected conflict graph G_1 that is derived from G . In our simulation setup, conflicting pairs are chosen according to the distance-2 interference [13]: conflicting radios include both one and two hop neighbors in the undirected graph G . The links weights in the conflict graph are determined using the free-space propagation model. Thus, weights associated with nodes in the conflict graph are proportional to the channel gain between them and therefore are a decreasing function of the corresponding inter-nodal separation (with path loss factor of 2). Our conflict model is reasonable both from an implementation and interference point of view, as it only requires radios to communicate with their bidirectionally connected radios and only makes conflict neighbors those radios that would cause meaningful interference.

Each node is a selfish player in the channel assignment game, selecting channels that improves its utility. The channel selection algorithm that nodes adopt is based on a random better response strategy. All nodes initialize their channel selections to the default non-zero channel before adapting their channel selections. Each node in the network is assigned a random backoff within a fixed window. The backoff periods induce an ordering that represents a random permutation. When the backoff ends, nodes randomly select an action $\mathbf{c}_i^{(k)}$ in every round $k = 0, 1, 2, \dots$, from the set:

$$\mathbf{c}_i^{(k+1)} \in \left\{ \mathbf{c}_i \in A_i \mid u_i(\mathbf{c}_i, \mathbf{c}_{-i}^{(k)}) > u_i(\mathbf{c}_i^{(k)}) \right\}, \forall i \in N \quad (4)$$

When no such improving action exists, nodes revert to their previous action.

To study the impact of channel availability on the topologies that emerge in steady state, we vary the number of channels available in the network, keeping the number of radios on each node fixed. Each network node is equipped with four radios, and for a given set of available channels, we evaluate the total interference in the steady state NE topology, and average it over 1000 different scenarios, with nodes randomly placed at different locations in each case. In each case, we let our selfish algorithm run for a sufficiently long time to closely approximate the NE; we use a termination criteria that the payoffs of every node must change from one round to the next by less than 0.1%. Figure 1 illustrates the interference performance of the steady state topologies; we use a sufficiently large α value (α_{\max}) to examine the case where network connectivity of G_c was to be supported by channel assignment. As expected, we observe that with increasing channel availability, channels are shared by fewer interfering transmissions, causing the aggregate interference in the network to decrease. By quantifying the interference in multi-channel networks, Figure 1 illuminates the interference reduction that can be achieved by utilizing orthogonal channels and by distributing interference over multiple channels. This result is particularly important when making the design decision on the optimal number channels to use, by examining the tradeoff between

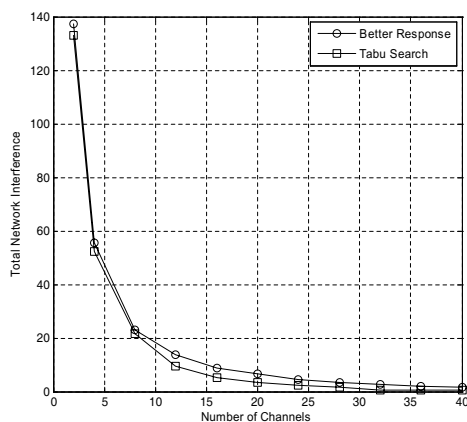


Fig. 1. Illustrating the impact of number of available channels on the total interference in NE topologies and Tabu search based topologies of a 25 node network (4 radios per node).

performance gains achieved and the cost of having additional channels.

To get a feel for how our algorithm compares with a cooperative algorithm, we compare the average interference performance of NE topologies that result from our better response algorithm with Subramanian's centralized Tabu search approach [4] for the 25 node network. As observed in Figure 1, both algorithms perform comparably when the number of channels are low because of the relatively small search space. At higher channel availability, Tabu search outperforms by a small margin. Minimizing interference through multi-channel assignment can be mapped to a graph coloring problem with additional constraints on the number of interfaces and number of channels available. This problem is known to be NP-hard [4]. For this reason, we compare the performance of our algorithm with the global optimum (obtained using depth first search) for a smaller sized 10 node network. Although in the worst case an NE topology can perform arbitrarily poorly, from Figure 2 we observe that on average its total interference is less than 10% of the optimum for a 10 node network.

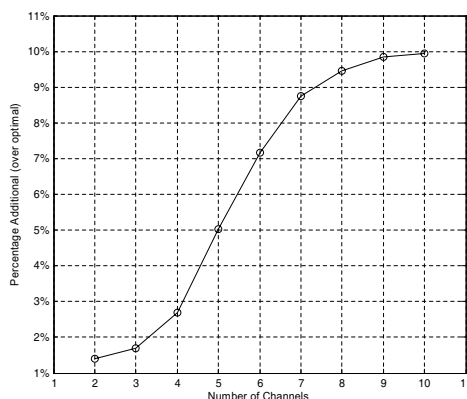


Fig. 2. Average additional interference as compared to optimal in a 10 node network (2 radios per node).

For the same set of scenarios used in determining the net-

work interference above, we examine the interference dependency on the benefit factor α . The term α indicates the relative preference between low interference and high connectivity. Higher values of α indicate that nodes prefer to maintain connectivity with greater number of nodes. Because we don't have the potential game results for $\alpha \in (0, \alpha_{\max})$, we first check whether or not the better response algorithm converges. We choose values of $\alpha = 0.1\alpha_{\max}$ and $0.5\alpha_{\max}$ and for each α , we examine 1000 randomly generated topologies of a 10 node network. In every case, we observed convergence within 5 – 10 iterations. With this knowledge of convergence, we then evaluate the connectivity of the NE topologies across various α values for a 25 node network. Figure 3 examines the connectivity of the resulting NE topologies as a function of α . For each steady state topology, we evaluate its connectivity fraction: the fraction of nodes belonging to the largest connected component of the network. For low values of α , certain links in the network are not assigned any channel and thus are disabled, so as to reduce interference. Thus, for low values α , the topologies become sparsely connected. Higher values of α indicate greater network connectivity even if supporting these connections over different channels come at the cost of high interference levels. Figure 4 validates this fact, and shows that, for a given number of available channels, accommodating more transmissions on various channels naturally leads to increased levels of co-channel interference.

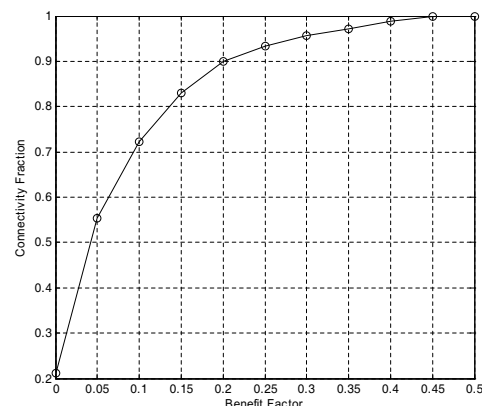


Fig. 3. Variation in network connectivity with benefit factor (fraction of α_{\max}) for a 25 node (4 radios per node), 25 channel network.

To make a meaningful assessment of how radios share channels, we evaluate the spectral occupancy of the steady state topologies. Unlike in the above studies, we assign equal weights to the contributions from all interferers. Thus, the level of interference observed on each channel is equal to the total number of interferers sharing that channel. We fix the number of available channels at 25, and determine a typical spectral occupancy profile of NE topologies for the scenarios where network connectivity is to be supported ($\alpha = \alpha_{\max}$) by channel assignment. Interestingly, we observe that some radios are not assigned any channel in NE. This is because each node's strategy is to minimize interference while just about ensuring network connectivity. The resulting minimum

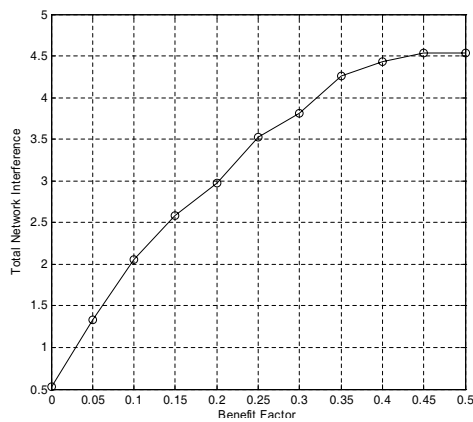


Fig. 4. Variation in network interference with benefit factor (fraction of α_{\max}) for a 25 node (4 radios per node), 25 channel network.

interference topologies are quite sparse; hence, nodes do not need to utilize all their radios to ensure network connectivity. We also observe that radios share channels fairly evenly across channels, suggesting a load balancing effect. In NE, nodes tend to minimize their interference number by utilizing all available channels and selecting channels with minimum number of interferers.

Figure 5 illustrates the spectral occupancy performance in an expected sense, averaged over 1000 randomly generated topologies. We observe that the load balancing trend holds in general, with every non-zero channel supporting 1-4% of radios in the network in equilibrium. As an extension of this result, we plan to show in future that this load balancing trend also holds for unequal weights (w_{ij} 's) and the resultant NE achieves interference balancing. Utilizing all channels evenly indicates efficient channel reuse, therefore such channel strategies are expected to perform well in improving throughput performance of the network.

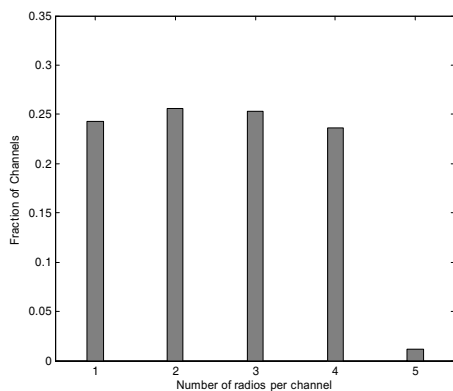


Fig. 5. Average spectral occupancy in a 25 node network (4 radios per node) with 25 channels.

V. CONCLUSION

We analyze the problem of interference minimization through multi-channel allocation in non-cooperative networks

using game theory. We show how channel assignment in multi-hop networks can be viewed as a topology control problem with the goal of minimizing aggregate interference while maintaining some degree of network connectivity. Nodes selfishly select the best channel to improve their own performance. In some cases, nodes may choose not to assign any channel to links, thus disabling links in the channel-assigned topology. We establish the stability of the selfish channel selection dynamic process.

We analyze the NE topologies with respect to interference and connectivity performance. With increasing channel availability, a larger portion of the network can be supported, thus leading to more connected topologies. In addition, the total network interference decreases with increasing number of available channels in order to support a connected network. Finally, our selfish algorithm achieves load balancing: radios on each non-zero channel in steady state are almost evenly distributed. Note that our utility function is general enough to allow NE topologies to be partitioned if needed; α_i can however be adjusted to model the connectivity preferences appropriately.

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