Power-Efficient Non-coherent Space-Time Constellations

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We consider a non-coherent communication system with M transmit and N receive antennas in a block Rayleigh flat fading channel with coherence time of T symbol intervals. We use the following complex baseband notation

$$X = SH + W, \qquad (1)$$

where S is the $T \times M$ transmitted matrix, X is the $T \times N$ received matrix, and H and W are the unknown (to both transmitter and receiver) $M \times N$ and $T \times N$ matrices of the i.i.d. fading coefficients and additive noise terms from $\mathcal{CN}(0,1)$. The transmitted symbols are also assumed to be power constrained, $\sum_{t=1}^{T} \sum_{m=1}^{M} \mathbb{E}\left\{|s_{tm}|^2\right\} = TP$.

It is known [1] that at high SNR, or when the coherence interval, T, is much larger than the number of transmit antennas, M, capacity can be achieved by using a constellation of unitary matrices (i.e., with orthonormal columns). However, at low SNR or for small values of T (e.g., $T = M$), the unitary constellations lose their optimality. In [2], we have shown that a design criterion based on maximizing the minimum Kullback-Leibler (KL) distance between the conditional distributions can result in a significant performance improvement for single transmit antenna constellations at low SNR. In this work, we extend the single antenna results of [2] to the case of multiple transmit antennas. Assuming that each signal matrix is a scalar multiple of a unitary matrix (i.e., $S_i^H S_i = d_i I_M$), the KL distance of [2] reduces to

$$
\begin{aligned}
\mathcal{D}(p_i \| p_j) &= M \left[\frac{1+d_i}{1+d_j} - \ln\left(\frac{1+d_i}{1+d_j}\right) - 1 \right] \\
&+ \frac{d_i d_j}{1+d_j} d_E^2(W_{S_i}, W_{S_j}),
\end{aligned}
\qquad (2)
$$

where $d_E^2(W_{S_i}, W_{S_j})$ is the square *Euclidean distance* or the *chordal distance* between the two subspaces W_{S_i} and W_{S_j} spanned by columns of S_i and S_j, respectively. In (2), the first term gives the distance between points from the same M-dimensional subspace of the T-dimensional space, whereas the second term gives the distance between two constellation points which have the same power and represent two different M-dimensional subspaces. In general, the overall distance is greater than or equal to either of these parts. Recalling that the unitary constellations are designed to maximize the *Euclidean distance* between subspaces, the above partitioning of the KL distance suggests partitioning the signal space into subsets of unitary constellations, and using only intra-subset and inter-subset KL distances in the maximin problem.

In this work, as an example, we use the systematic unitary designs of [3] as the constituent subsets of the
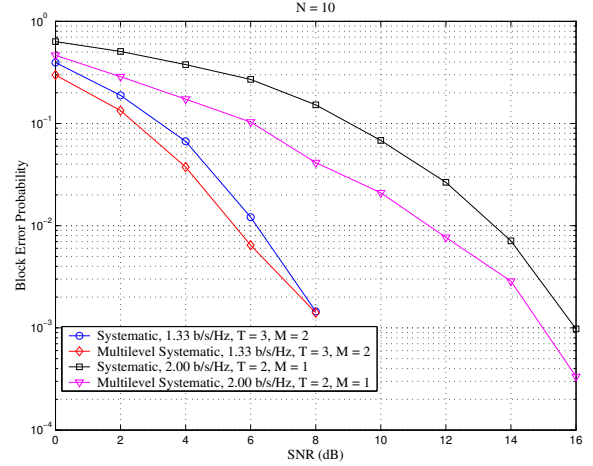

Fig. 1: Performance comparison of one and two-antenna systematic constellations of [4] and their multilevel versions.

multilevel constellations. Figure 1 compares the performances of the unitary constellations of [3] and their multilevel versions in two different scenarios. We observe that the multilevel unitary constellation can provide up to 3dB gain over its corresponding one-level unitary constellation at low SNR. We should emphasize that the performance of the multilevel constellation greatly depends on its constituent unitary subsets, and using more efficient unitary constellations (e.g., the optimal packings in $G(T, M)$) in its levels, will result in even better performance. Even if multiple receive antennas are not available, similar results can be observed in the coded system, when the transmitted signal is encoded over several coherence intervals using an outer code. We also notice that as SNR increases, the two curves become closer, confirming the high SNR optimality of the unitary constellations.

References

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