

CRITICAL SPEED OF SHAFTS

All rotating shaft, even in the absence of external load, deflect during rotation. The combined weight of a shaft and wheel can cause deflection that will create resonant vibration at certain speeds, known as Critical Speed.

The magnitude of deflection depends upon the followings :-

- (a) stiffness of the shaft and its support
- (b) total mass of shaft and attached parts
- (c) unbalance of the mass with respect to the axis of rotation
- (d) the amount of damping in the system

Therefore, the calculation of critical speed for fan shaft is necessary.

Critical Speed Equation (N_c)

There are two methods used to calculate critical speed, Rayleigh-Ritz and Dunkerley Equation. Both the Rayleigh-Ritz and Dunkerley equation are approximations to the first natural frequency of vibration, which is assumed to be nearly equal to the critical speed of rotation.

In general, the Rayleigh-Ritz equation overestimates and the Dunkerley equation underestimates the natural frequency.

The equation illustrated below is the Rayleigh-Ritz equation, good practice

suggests that the maximum operation speed should not exceed 75% of the critical speed.

$$\text{Critical speed, } N_c = \frac{30}{\pi} \sqrt{\frac{g}{\delta_{st}}}$$

where :

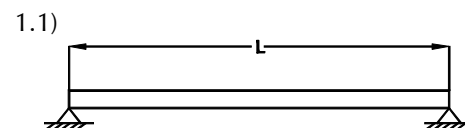
- g = gravity acceleration (9.81 m/s^2)
- δ_{st} = total maximum static deflection

Critical speed depends upon the magnitude or location of the load or load carried by the shaft, the length of the shaft, its diameter and the kind of bearing support.

Total Maximum Static Deflection (δ_{st})

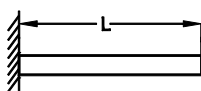
The maximum static deflection, δ_{st} , is obtained by adding both the maximum static deflection of the rotating shaft and the load.

- (1) Maximum static deflection on shaft (δ_{stl})



$$\delta_{stl} = \frac{5wL^3}{384EI}$$

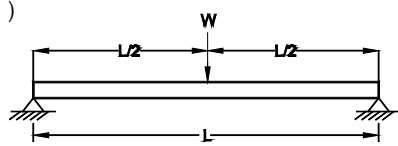
1.2)



$$\delta_{st1} = \frac{wL^3}{8EI}$$

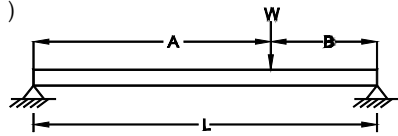
(2) Maximum static deflection on load only (δ_{st2})

2.1)



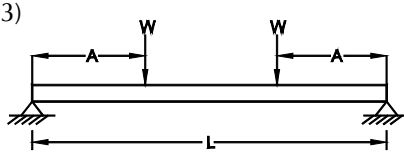
$$\delta_{st2} = \frac{WL^3}{48EI}$$

2.2)



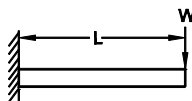
$$\delta_{st2} = \frac{WB(L^2 - B^2)^{3/2}}{9\sqrt{3}EIL}$$

2.3)



$$\delta_{st2} = \frac{WA(3L^2 - 4A^2)}{24EI}$$

2.4)



$$\delta_{st2} = \frac{WL^3}{3EI}$$

where :

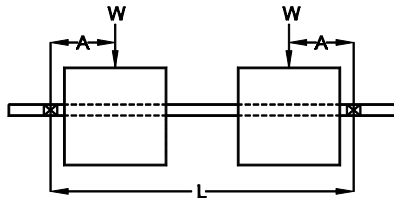
w = weight of shaft, kg
W = weight of wheel, kg
E = modulus of elasticity, kg/m²
for shaft C40 = 200 x 10⁸ kg/m²
I = moment of inertia = $\pi D^4/64$, m⁴
L = length of shaft, m

Shaft Diameter D (mm)	Moment of inertia I (m ⁴)	Weight per metre (kg/m)
20	7.85 x 10 ⁻⁹	2.47
25	19.17 x 10 ⁻⁹	3.85
30	39.76 x 10 ⁻⁹	5.51
35	73.66 x 10 ⁻⁹	7.99
40	125.66 x 10 ⁻⁹	9.87
45	201.29 x 10 ⁻⁹	13.00
50	306.79 x 10 ⁻⁹	15.40
55	449.18 x 10 ⁻⁹	18.70
60	636.17 x 10 ⁻⁹	22.20
70	1178.59 x 10 ⁻⁹	30.20

Table I

Example 1

Given the following specifications, find the critical speed.



**Model : KAT 15/15 S2
with 2-bearings**

Diameter of shaft, $D = 40 \text{ mm}$
 Weight of wheel, $W = 7.5 \text{ kg}$
 Shaft length, $L = 1.37 \text{ m}$
 Length, $A = 0.205 \text{ m}$
 Moment of inertia, $I = 125.66 \times 10^{-9} \text{ m}^4$
 Modulus of Elasticity, $E = 200 \times 10^8 \text{ kg/m}^2$
 (C40)
 Shaft weight, $w = 1.37 \times 9.87$
 $= 13.52 \text{ kg}$ — refer to Table I

(a) Deflection from shaft weight only (δ_{st1})

$$\delta_{st1} = \frac{5wL^3}{384EI} \text{ ————— refer to Fig. 1.1}$$

$$= \frac{5(13.52)(1.37)^3}{384(200 \times 10^8)(125.66 \times 10^{-9})}$$

$$= 0.00018 \text{ m}$$

(b) Deflection from load only (δ_{st2})

$$\delta_{st2} = \frac{WA(3L^2 - 4A^2)}{24EI} \text{ —refer to Fig. 2.3}$$

$$= \frac{7.5(0.205)[3(1.37)^2 - 4(0.205)^2]}{24(200 \times 10^8)(125.66 \times 10^{-9})}$$

$$= 0.000139 \text{ m}$$

(b) Total maximum static deflection (δ_{st})

$$\delta_{st} = \delta_{st1} + \delta_{st2}$$

$$= 0.00018 + 0.000139$$

$$= 0.000319 \text{ m}$$

(d) Critical Speed (N_c)

$$N_c = \frac{30}{\pi} \sqrt{\frac{g}{\delta_{st}}}$$

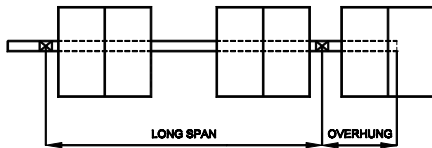
$$= \frac{30}{\pi} \sqrt{\frac{9.81}{0.000319}}$$

$$= 1675 \text{ rpm}$$

Safety factor 25%, therefore max.
 operation speed = 1675×0.75
 $= 1256 \text{ rpm}$

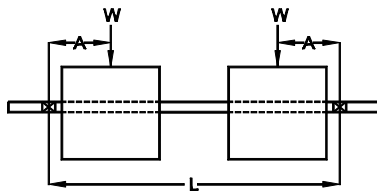
Example 2

To check critical speed for KAT 12/12 S3 with 2-bearing, one side of the bearing overhung.



Diameter of shaft, D = 35 mm
 Weight of wheel, W = 5.4 kg
 Moment of inertia, I = $73.66 \times 10^{-9} \text{ m}^4$
 Modulus of Elasticity, E = $200 \times 10^8 \text{ kg/m}^2$ (C40)

Check Critical Speed For Long Span



Length, A = 0.197 m
 Length, L = 1.114 m
 Shaft weight, w = 8.9 kg

(a) Deflection from shaft weight (δ_{st1})

$$\delta_{st1} = \frac{5wL^3}{384EI}$$

$$= \frac{5(8.9)(1.114)^3}{384(200 \times 10^8)(73.66 \times 10^{-9})}$$

$$= 0.000109 \text{ m}$$

(b) Deflection from load only (δ_{st2})

$$\delta_{st2} = \frac{WA(3L^2 - 4A^2)}{24EI}$$

$$= \frac{5.4(0.197)[3(1.114)^2 - 4(0.197)^2]}{24(200 \times 10^8)(73.66 \times 10^{-9})}$$

$$= 0.000107 \text{ m}$$

(c) Total maximum static deflection (δ_{st})

$$\delta_{st} = \delta_{st1} + \delta_{st2}$$

$$= 0.000109 + 0.000107$$

$$= 0.000216 \text{ m}$$

(d) Critical speed for long span (N_c)

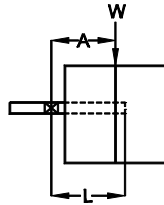
$$N_c = \frac{30}{\pi} \sqrt{\frac{g}{\delta_{st}}}$$

$$= \frac{30}{\pi} \sqrt{\frac{9.81}{0.000216}}$$

$$= 2035 \text{ rpm}$$

Safety Factor 25%, therefore max.
 operation speed = $2035 \text{ rpm} \times 0.75$
 = 1526 rpm

Check Critical Speed For Overhung



Length, A = 0.5215
 Length, L = 0.534 m
 shaft weight, w = 4.27 kg

(a) Deflection from shaft weight only (δ_{st1})

$$\begin{aligned}\delta_{st1} &= \frac{wL^3}{8EI} \\ &= \frac{4.27(0.534)^3}{8(200 \times 10^8)(73.66 \times 10^{-9})} \\ &= 0.000055 \text{ m}\end{aligned}$$

(b) Deflection from load only (δ_{st2})

$$\begin{aligned}\delta_{st2} &= \frac{WA^3}{3EI} \\ &= \frac{5.4(0.5215)^3}{3(200 \times 10^8)(73.66 \times 10^{-9})} \\ &= 0.000173 \text{ m}\end{aligned}$$

(b) Total maximum static deflection (δ_{st})

$$\begin{aligned}\delta_{st} &= \delta_{st1} + \delta_{st2} \\ &= 0.000055 + 0.000173 \\ &= 0.000228 \text{ m}\end{aligned}$$

(d) Critical Speed at overhung (N_c)

$$\begin{aligned}N_c &= \frac{30}{\pi} \sqrt{\frac{g}{\delta_{st}}} \\ &= \frac{30}{\pi} \sqrt{\frac{9.81}{0.000228}} \\ &= 1980 \text{ rpm}\end{aligned}$$

Safety factor 25%, max. operation speed = $1980 \times 0.75 = 1485 \text{ rpm}$

Conclusion

Long Span

Critical Speed = 2035 rpm
 Max. operation speed = 1526 rpm

Overhung

Critical Speed = 1980 rpm
 Max. operation speed = 1485 rpm

Therefore, the max. operation speed for this KAT 12/12 S3 should be according to the overhung, ie. whichever lesser, which is = 1485 rpm