



Addis Ababa University

Addis Ababa Institute of Technology (AAiT)

School of Electrical and Computer Engineering

Traction Power Consumption Analysis to Investigate Freight Train  
Operational Speed

In the case of Ethio-Djibouti Railway Corridor

Awash~Sirba Kunkur~Sebaka Line Subsection

By

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Addis Ababa University

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## DECLARATION

I, the undersigned, declare that this thesis is my original work, and has not been presented for a degree in this or any other university, and all sources of materials used for the thesis have been fully acknowledged in their respective points.

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## Abstract

This thesis report presents traction power consumption analysis on the existing Ethio-Djibouti electrified railway network, particularly along Awash~Sirba Kunkur~Sebaka Line Subsection. The result of the analysis is recommendation of possible freight train operational speed in the line subsection and the corridor in general to make the subsection occupancy time as short as possible increasing the frequency of services.

In this report, Newton's Second law of Motion is used for train dynamics modeling. Analytical approach is used to evaluate the model as it provides a quick way to analyze train dynamic behaviors within accepted level of dynamic assumptions. Moreover, the corresponding power quality problems created due to the moving characteristic of the train are evaluated.

Train dynamics analysis results show that the traction current required increases continuously and the maximum possible operational speed at the highest gradient is calculated to be 72.4kmph which is less than the freight design speed. Power consumption analysis results show, current drawn from Awash Substation are calculated to be 252A, 431A and 653A and from Sirba Kunkur 604A. Similarly, for a train running between Sirba Kunkur Substation and Sebaka Section Post current values are 238A, 384A and 374A. In both cases the first two current values are during acceleration and notch-up periods in principle which have to be the maximum values. However, even though there is an increase in current demand, the maximum requirements are 653A for 2.724min and 604A for 12.018min which are less than the 848A for 20min, the corridor traction network capacity [8] [9] [31]. The minimum catenary voltage and the maximum traction substation voltage drop are calculated to be 21.6074kV and 2.5926kV which are within the standard limits of 20kV [8] [9] [31] and 6.0kV, see Appendix C Table 2, respectively.

Generally, the above train dynamics evaluation and the corresponding power consumption analysis results show that the traction power supply system is under safe condition and could provide the required power to drive the train at the maximum targeted speed of 72.4kmph. The analysis of the train speed profile results with average train speed of 65.86kmph which improves the proposed average train speed 54kmph [9].

**Key Terms:** Train Speed, Train Dynamics, Power Consumption, Traction Modeling, Power Quality Evaluation, Train Resistance Derivation and Fourier series

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## Glossary of Terms / Abbreviations and Nomenclatures

Term	Explanation / Meaning / Definition
$a$	— — — — — <i>complex operator = <math>1\angle 120^\circ</math></i>
A	— — — — — <i>train resistance component independent of speed</i>
AC	— — — — — <i>Alternating Current</i>
AREMA	— — — <i>American Railway Engineering and Maintenance of way Association</i>
$a_t$	— — — — — <i>train instantaneous tangential acceleration at curves</i>
B	— — — — — <i>train resistance coefficient that varies linearly with speed</i>
BD	— — — — — <i>the two transformer feeding sections balance degree</i>
$B_e(v)$	— — — — <i>braking force</i>
C	— — — — — <i>train resistance coefficient that varies with square of the speed</i>
CCECC	— — — <i>China Civil Engineering Construction Cooperation</i>
CDE	— — — — <i>Chemin de Fer Djibouti – Ethiopien</i> <i>French version for Djibouti – Ethiopia old Railway line</i>
CREC	— — — — <i>China Railway Engineering Corporation (Group Limited)</i>
D	— — — — — <i>gradient (%o or %)</i>
DC	— — — — — <i>Direct Current</i>
DMU	— — — — <i>Diesel Multiple Unit</i>
$E_a$	— — — — — <i>track super elevation</i>
$E_c$	— — — — — <i>unbalanced track super elevation</i>
$E_{xy}$	— — — — <i>transformer secondary voltage (<math>xy = ab, ac, bc</math>)</i>
$E_{acc1}$	— — — — <i>tain energy consumption during acceleration</i>
$E_{totch-up}$	— — <i>train energy consumption during notch – up</i>
$E_{const,spd}$	— — <i>tain energy consumption during free – running</i>
$E_{grade}$	— — — <i>energy consumption to overcome grade resistance</i>
$E_{tr}$	— — — — <i>energy consumption to overcome train resistance</i>
EMU	— — — — <i>Electrical Multiple Unit</i>
ERC	— — — — <i>Ethiopian Railway Corporation</i>
$E_t$	— — — — <i>Voltage Source (linear traction load model)</i>
$F_t$	— — — — <i>traction tangential force at curves</i>

$F_{net\ 1,2,3,4,5}$  – net force(1,2,3,4,  
 = train acceleration, notchup, constant speed, coasting, breaking )

$h$  – – – – track gauge (1435mm for standard gauge)

$h^{th}$  – – – – order of harmonic components

$HD$  – – – – Harmonic Distortion

$HP$  – – – – horse power

$HSR$  – – – – High Speed Rail/Railway

$HST$  – – – – High Speed Train

$Hz$  – – – Hertz (unit of frequency)

$I$  – – – – train RMS current

$I_{max}$  – – – maximum train current

$I_y$  – – – – utility grid side phase current ( $y = A, B, C$ )

$I_x$  – – – – utility grid side line – to – line RMS current ( $x = AB, BC, CA$ )

$I_z = I_{traction}$  – – – Load side RMS current ( $z = bc, ac, ab$ ) or traction current

$X^{(2)}, X^{(1)}$  and  $X^{(0)}$  – – negative, positive and zero phase sequence( $X = V$  or  $I$ )respectively

$j$  – – – – complex operator =  $1 \angle 90^\circ$

$kmph$  – – – – kilometer per hour

$K_T$  – – – – transformer ratio =  $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{132}{27.5} = 4.8$

$kV$  – – – – kilo volt

$kWh$  – – – – kilo watt hour

$L_{Li}$  – – – – traction network inductance at the point of contact from substation

$LRT$  – – – – Light Tail Transit

$LV, MV$  and  $HV$  – low, medium and high voltage respectively

$LUF$  – – – – Line Utilization Factor

$M$  – – – – train mass (ton)

$M_e$  – – – – train mass (ton) with rotational effect

$MJ$  – – – – mega joule

$mph$  – – – – miles per hour

$mps$  – – – – meter per second

$MVA$  – – – – mega volt ampere

$MVA_{rac}$  – – – – mega volt ampere reactive  
 $NRN$  – – – – National Railway Network  
 $NRNE$  – – – – National Railway Network of Ethiopia  
 $OCL$  – – – – Overhead Contact Line  
 $OCS$  – – – – Overhead Contact System  
 $P_{acc1}$  – – – – traction power consumption during acceleration  
 $P_{totch-up}$  – – – – traction power consumption during notch – up  
 $P_{const,spd}$  – – – – traction power consumption during free – running  
 $P_{grade}$  – – – – traction power consumption to overcome grade resistance  
 $P_{tr}$  – – – – traction power consumption to overcome train resistance  
 $P_{rated}$  – – – – traction rated power consumption  
 $PCC$  – – – – Point of Common Coupling  
 $pf$  – – – – power factor  
 $PWM$  – – – – Pulse Width Modulated converter  
 $Q_{traction}$  – traction reactive power ( $MVA_{rac}$ )  
 $R_e(v)$  – – – – train equivalent resistance  
 $R_s(v)$  – – – – starting resistance  
 $R_r(v)$  – – – – running resistance  
 $R_g(i)$  – – – – grade resistance  
 $R_c(\gamma)$  – – – – curve resistance  
 $R_t$  – – – – tunnel resistance  
 $R_{Li}$  – – – – traction network resistance at the point of contact from substation  
 $S_{acc1}$ , – – – – train running distance during acceleration  
 $S_{totch-up}$  – – – – train running distance during notch – up  
 $S_{const,spd}$  – – – – train running distance during free running  
 $S_{coast}$  – – – – train running distance during coasting  
 $S_{br}$  – – – – train running distance during breaking  
 $S_j$  – – – – the transformer secondary winding capacity (MVA) ( $j = \text{winding}$ )  
 $S_x$  – – – – utility grid side phase complex powers ( $x = A, B, C$ )  
 $S_k$  – – – – primary (utility grid) side short circuit capacity (MVA)  
 $S_L$  – – – – the line capacity (MVA)

$S_R$  – – – – – *the maximum capacity utilization*  
 $S_T$  – – – – – *traction transformer rating (MVA)*  
 $S_{traction}$  – – – *Complex power drawn by load or powering train =  $P_{train} + jQ_{train}$*   
 $STS$  – – – – – *Single Track Section*  
 $SVC$  – – – – – *Static Var Compensator*  
 $t_{acc1}$  – – – – *train running time during acceleration*  
 $t_{notch-up}$  – – – *train running time during notch – up*  
 $t_{const,spd}$  – – – *train running time during free – running*  
 $t_{coast}$  – – – *train running time during coasting*  
 $t_{br}$  – – – – *train running time during breaking*  
 $TDD$  – – – *Total Demand Distortion*  
 $TES$  – – – *Traction Electrification System*  
 $TE(v)$  – – *tractive effort*  
 $TE_{max}$  – – *maximum tractive effort*  
 $THD$  – – – *Total Harmonics Distortion*  
 $ton$  – – – – *tonnage (traction mass unit)*  
 $TUF$  – – – – *Transformer Utilization Factor*  
 $U_k\%$  – – – *traction transformer short circuit voltage (generally 10.5%)*  
 $UBF$  – – – *Unbalance Factor*  
 $UIC$  – – – – *Union International des Chemins de Fer*  
                     *French version for 'International Union of Railways'*  
 $U_{Li}$  – – – – *catenary voltages at  $i^{th}$  section of the line*  
 $U_z$  – – – – *Load side RMS voltages ( $z = bc, ac, ab$ )*  
 $U_{2N}$  – – – *transformer no – load voltage*  
 $V$  – – – – *train RMS voltage*  
 $V_y$  – – – – *utility grid side phase voltage ( $y = A, B, C$ )*  
 $V_x$  – – – – *utility grid side line – to – line RMS voltages ( $x = AB, BC, CA$ )*  
 $v_{max}$  – – – – *maximum train speed*  
 $v_{max,c}$  – – – *maximum allowable curve speed*  
 $v_t$  – – – – – *speed limit due to train makeup/formation*  
 $v_r$  – – – – – *curvature speed limit*

$v_g$  – – – – – *downgrade speed limit*  
 $v_o$  – – – – – *switch/turnout speed limit*  
 $v_w$  – – – – – *speed limit due to track strength*  
 $v_s$  – – – – – *speed limit due to blocking signal*  
 $v_{br}$  – – – – – *train break speed*  
 $V_s$  – – – – *source voltage referred to load side*  
 $V_{grid}$  – – – – *grid voltage*  
 $Z_s$  – – – – *source impedance referred to load side*  
 $Z_t$  – – – – *Series Impedance (linear traction load model)*  
 $Z_T = X_T$  – – – – *transformer impedance referred to load side*  
 $\psi$  – – – – – *voltage drop angle*  
 $\alpha_1$  – – – – *train starting acceleration*  
 $\alpha_2$  – – – – *train notch – up acceleration*  
 $\beta_1$  – – – – *train coasting deceleration*  
 $\beta_2$  – – – – *train breaking deceleration*  
 $\theta_L$  – – – – *traction power factor angle*  
 $\theta_{Li}$  – – – – *power factor angle at  $i^{th}$  section of the line*  
 $\theta$  – – – – *load angle at transformer secondary*  
 $\omega_m$  – – – – *motor speed*  
 $\gamma_G$  – – – – *overall gear ratio*  
 $\eta_t$  – – – – *torque transmission efficiency*  
 $\eta$  – – – – *system efficiency*  
 $\psi_u$  – – – – *unbalance phase shift angle*

Symbol	Unit	Explanation / Meaning / Definition
$\alpha$	$kmphps$	acceleration
$\beta$	$kmphps$	deceleration
$\tau$	$kNm$	torque
$\tau_{max}$	$kNm$	maximum torque
$\gamma$	$m$	curve radius
$B_e(v)$	$N$	braking force
$R$	$kN$	train resistance
$g$	$mpsps$	gravitational acceleration (meter per second per second)
$M_e$	$kg$	train equivalent mass
$v$	$mph$ or $kmph$ or $mps$	train instantaneous speed
$P$	$MW$	real (active) power
$P_{sh}$	$kW$ or $MW$	motor shaft power
$Q$	$MVA_{rac}$	reactive power
$S$	$MVA$	complex power
$TE(v)$	$N$ or $kN$	tractive effort
$t$	$s$	time
$W_t$	$N$ or $kN$	train weight

# Chapter One

## Introduction

### 1.1 Introduction

A transport system can be appreciated as the backbone of modern societies and involves private car, bus, truck, waterway, airway, railway, etc. transportation. Of the aforementioned transport systems rail transport service is the accustomed mode of mass and bulk transport which is a reality nearly two centuries ago, i.e., since early 1800s [1]. Particularly, electrified railroad transport system is an admired alternative compared to other technology options-steam and diesel- mainly due to its salient features; namely: energy efficient, high power to mass ratio (kW/tonne), higher speed capability, advantageous at high traffic situation, [1] easily controlled and integrated due to technological advances in electrical energy manipulation, environmentally friendly with low emission rate of carbon dioxide (CO<sub>2</sub>) per passenger km or per ton of freight per km transported, etc.

Electrified railway systems are very complex systems in which a variety of components (or subsystems) cooperate to realize the transport service for which the system has been designed, see Figure 1.1[2]. The performance of these kinds of systems strictly depends on the capability of the subsystems to be correctly integrated [2]. The performance measure include: improved subsystem and/or combined system level efficiencies, low life time costs, reliability, maintainability, failure rate, mean time to fail, mean time to repair in case of failure, etc. at subsystem level and appreciated quality of services- measured in delays, comfort and safety-, the ability of the train to run at or near the design speed over the track designed for etc. at system level. Moreover, the contribution of each subsystem varies from each other for the effectiveness of the railway system in general. Some of the performance measures stated above are design parameters which require engineering skill and judgments for their selection.

As a design parameter train speed selection is limited by many factors such as infrastructure cost, rolling stock types, proposed track to be designed-single or multiple track and space between tracks for multiple track design, selected rail type and strength, safety and security levels expected to be maintained by the signaling, communication and traffic control systems; operation mode to be followed after commissioning, transportation requirements, traction power available from the supply side, traction electrification technology option, traction substation rating, spacing and location,

power supply mode, overhead system structure arrangements, overhead conductor type, geographical feature of the corridor, whether condition, service type -freight or passenger, mixed use or dedicated line services, urban transit or enter-city service-, socio-economic importance of the corridor, traffic congestion, awareness of the society found along the line, etc.

But a line constrained by the above general engineering requirements and designed for a given speed limit is also challenged by many technical and non-technical requirements physically existing over the route. Some of these requirements include: requirements from train formation, curvature, downgrade, switch, track strength, blocking signal, and weather, etc. [3]. As a result a train traveling along the rail line must obey, in addition to equation of motion, all these speed limits at every location of the line [3]. Consequently, the maximal allowable speed at a particular location is the minimum of all the speed limits [3]:

$$v \leq v_{max} = \min_{in\ kmph} \{v_t, v_r, v_g, v_o, v_w, v_s\} \quad (1.1)$$

All the above factors, except speed limit due to train makeup/formation which exists at any time and any place, hence, must be obeyed, tend to decrease train design speed along the route at each section they are found and they need infrastructural rearrangement, modification or complete replacement to reduce their corresponding effects. Similarly, train makeup/formation speed limit could be overcome only by rolling stock performance upgrading or replacing the old by new ones. Furthermore, there are also a lot of bottlenecks which result in train speed reduction some mainly being expressed in equation of motion, see Eq. (2.12), and others such as location of passing loops for single track, over takings for heterogeneous trains operating on double-track lines, location of stations, etc. individually affecting train running speed. From those expressed in equation of motion the dominant factors include: train wheel-rail and train-aerodynamic interactions, train internal rotational inertia, up-grade resistance which try to negate the operational speeds and those off-equation of motion are pantograph-contact wire interaction nature, electrical current signal propagation speed through contact wire i.e. train speed has to be less than signal speed-yet this is for high speed trains, etc.

Not to lessen the effects of all the above speed limiters, mechanical power available at the shaft of the driving motors, which is directly proportional to the product of the current drawn from the supply main and the voltage across motor terminals, is the main speed determinant of a train. Hence, for a train to run near its design speed the traction network need to supply it with the required

traction power to initiate, accelerate and maintain motion during powering mode of operation. Moreover, train dynamics analysis indicates that train's power requirement is directly proportional with the cube of the maximum speed, "power rule" [4].

The quality and reliability from the supply main side and distribution from the traction network side determine the electrical power available at the induction motor air-gap; the mechanical power at the motor shaft-the rotational drive force in the form of torque; translational force in the form of tractive effort at the wheel-rail contact point in turn which is the actual cause of train movement provide that effective slip exists at the contact point. Even though the tractive effort required at the start is maximum and reduces as time goes on, the traction network should supply the traction motors at the maximum possible constant power to maintain further motion.

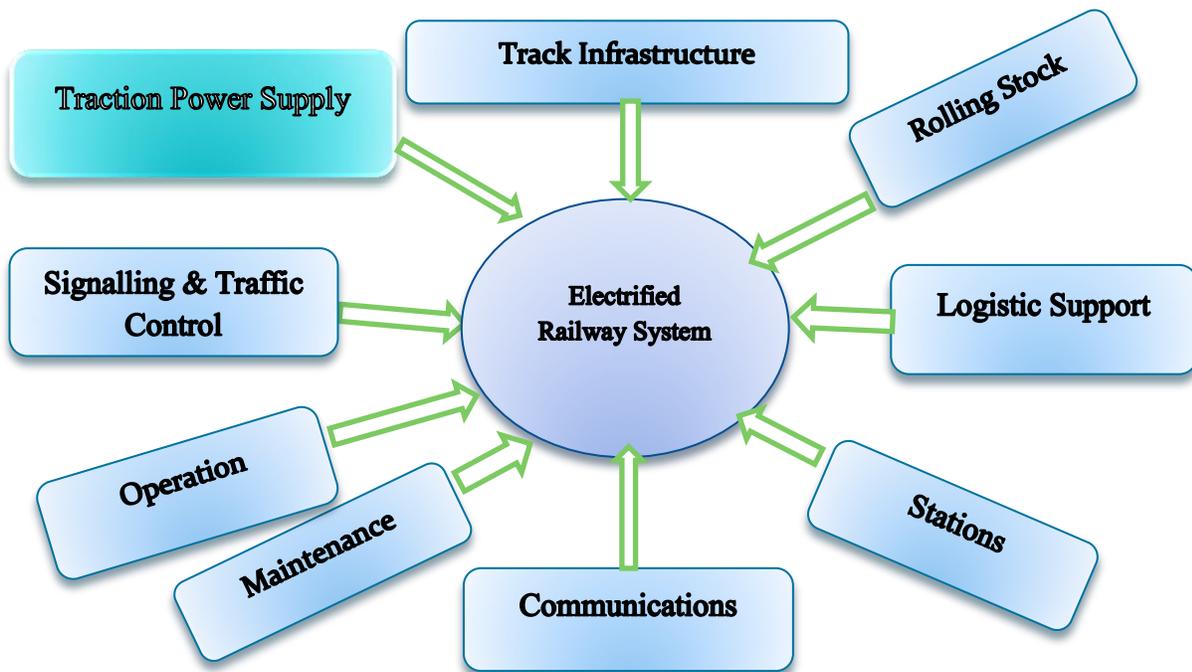


Figure 1.1 Electrified Railway Subsystems

## 1.2 Research Background

Transportation is a large and important part of the economy and the need for transportation is increasing continuously. Railway transport, in particular, is the main artery for mass mobility especially, in a developing country like Ethiopia, its contribution towards the economic growth is immense as other modes of transport i.e. roadways, airways, inland waterways etc. are not available at reasonable rates and in sizable quantity to cope up with the quantum of traffic which needs well organized operation management system. Railway operation planning is a complex task comprising important fields such as capacity analysis, scheduling, rescheduling, timetable stability analysis, traffic control, and simulation and energy consumption [5].

The planning process is limited by the existing infrastructure components such as the number of track in operation per direction, rolling stock and all railway subsystems interacting to complete the system operation. The customary tracking system is to use either single or double track system. The track selection process is based on investment cost and required transport capacity along the corridor. Investment cost wise, adding a track to the system involves adding additional subsystems such as signals, power supply infrastructure etc., thus the price is generally in the region of 60-70% additional costs [6]. Similarly, removing a track reduces this cost, which is why single track operation might be a great alternative to double track [6]. However, as single track railway subsystems are much more interdependent than double track they require careful planning of the passing loops, based on the infrastructure, the rolling stock performance and the timetable [6], challenging the engineering selection process.

In addition, operating trains over single track system might be limited due to the operational constraints involved such as low capacity, low frequency of services and flexibility, but the potential cost saving in infrastructure costs can outweigh these constraints [6]. Other constraints related to timetable robustness; rolling stock performance and homogeneity; reliability and punctuality; headway time and/or distance; loss of running time due to speed restrictions on the diverging track in passing loop; distance between passing stations and their optimum location; station design-i.e. capacity of a multiple track section or platforms; bidirectional traffic; mixed operation-i.e. speed difference among trains using the track are some of the bottlenecks for the competitiveness of the single track railway transport system with other transport modes, particularly with road transport which is the main railroad competent.

The competitiveness factors, for better rolling stock utilization, are related to a rail service that is fast, frequent, comfortable, reliable and not too expensive. Yet, all these factors are determined by how much train are operated close to the rolling stock as well as infrastructure design speed. On single-track line with only ordinary crossing stations, each crossing implies longer running time which propagates between crossing trains reducing reliability; constraining capacity and thereby also frequency of services [5]. Although it is not advisable to run trains at their design speed for the whole life time due to infrastructure components deterioration, trains are always required to make up delays they made at passing stations. This delay make up at crossings and/or over takings, for opposing and following trains respectively, always requires additional energy from the supplying station in each track section, particularly to maintain acceleration as maximum as possible to save running time. Moreover, due to abrupt or continuous change of the route gradient power required from the supply system may increase abruptly or continuously. Hence, the geographic feature of the track line is another energy determinant.

Being an economic choice of construction the railway system selected for Ethiopian National Railway Network is a single track, single phase world standard 25kV, 50Hz electrified railway system. Yet, in the future, Ethiopian Railway Corporation needs to standardize the entire preliminary design criterion because trends show that one way of keeping railroad investment costs as low as possible is through standardization [7].

The trunk railway line underway along Ethio-Djibouti corridor is designed to operate trains at speeds of 120kmph and 80kmph for passenger and freight respectively with freight transport as the main part [8]. But technical documents prepared so far disclose that the operational speeds are limited to '72kmph and 54kmph' [9] which are 60% and 67.5% of their respective design speed respectively. From this result, on average, trains reach Addis Ababa/Sebeta from Djibouti, a distance of 752km, with in 10.44hr and 13.93hr respectively with zero buffer time at loading-unloading stations. If we assume low percentage buffer time such as 10%, trains will not return with in 24hr affecting frequency of service. Hence, train speed wise, it is not possible to say that the Ethiopian Railroad transportation system is effective, yet this has to be approved through practical experience rather than this theoretical conclusions and the forthcoming analytical presentations.

Besides the speed differences herein, recalling all the facts outlined above single-track line railway operation by itself is challenging which makes average train speed reduction and loss of punctuality

inevitable. However, through coordinated studies of railway operation and planning processes, such as energy consumption studies proposed in this research report, it is possible to keep heterogeneous train operational speeds such as Ethio-Djibouti corridor near its design speeds. Hence, in order to keep the intended momentum that the railway transport system takes part in the socio-economic throughput of the country and to move the traffic efficiently and punctually, it is necessary that the operational speeds of the freight rail transport shall be augmented or increased.

### 1.3 Problem Statement

The instantaneous power on the feeder line at the point of contact is the prominent and direct measure of train instantaneous speed; hence, what is the maximum continuous speed, within the design speeds, could be attained by the existing traction power supply infrastructure provide that the effects of other speed determinants are assumed to be known?

### 1.4 Objectives

#### 1.4.1 General Objective

The general objective of this research is to analyze traction power consumption in the Ethio-Djibouti railway network, particularly at Awash~Sirba Kunkur~Sebaka Line Subsection, in order to investigate freight train operational speeds and finally present documented findings that will serve as reference for future rail operation planning process studies.

#### 1.4.2 Specific Objectives

- Calculate the real gap between design and operational speeds;
- Identify causes of speed reduction;
- Analyze train dynamics and estimate train movement pattern in Awash~Sirba Kunkur~Mieso Line Section;
- Calculate the power demand in Awash~Sirba Kunkur~Sebaka Line Subsection;
- Traction network analysis and line characteristic investigation;
- Analyze the effects of traction load on the utility grid

### 1.5 Methodology

This research report is based on information and literature reviews found in research papers, journals, books and on the world wide web (internet). In the report detail analytical calculations on train dynamics and power consumption have been done. All calculations are presented with

formulae. Where there is insufficient information available, reasonable assumptions have been quantitatively done for the calculations. Some simplifications are made to the calculations where needed. Analytical results, generally, on due research and report preparation are presented graphically and in tabular formation.

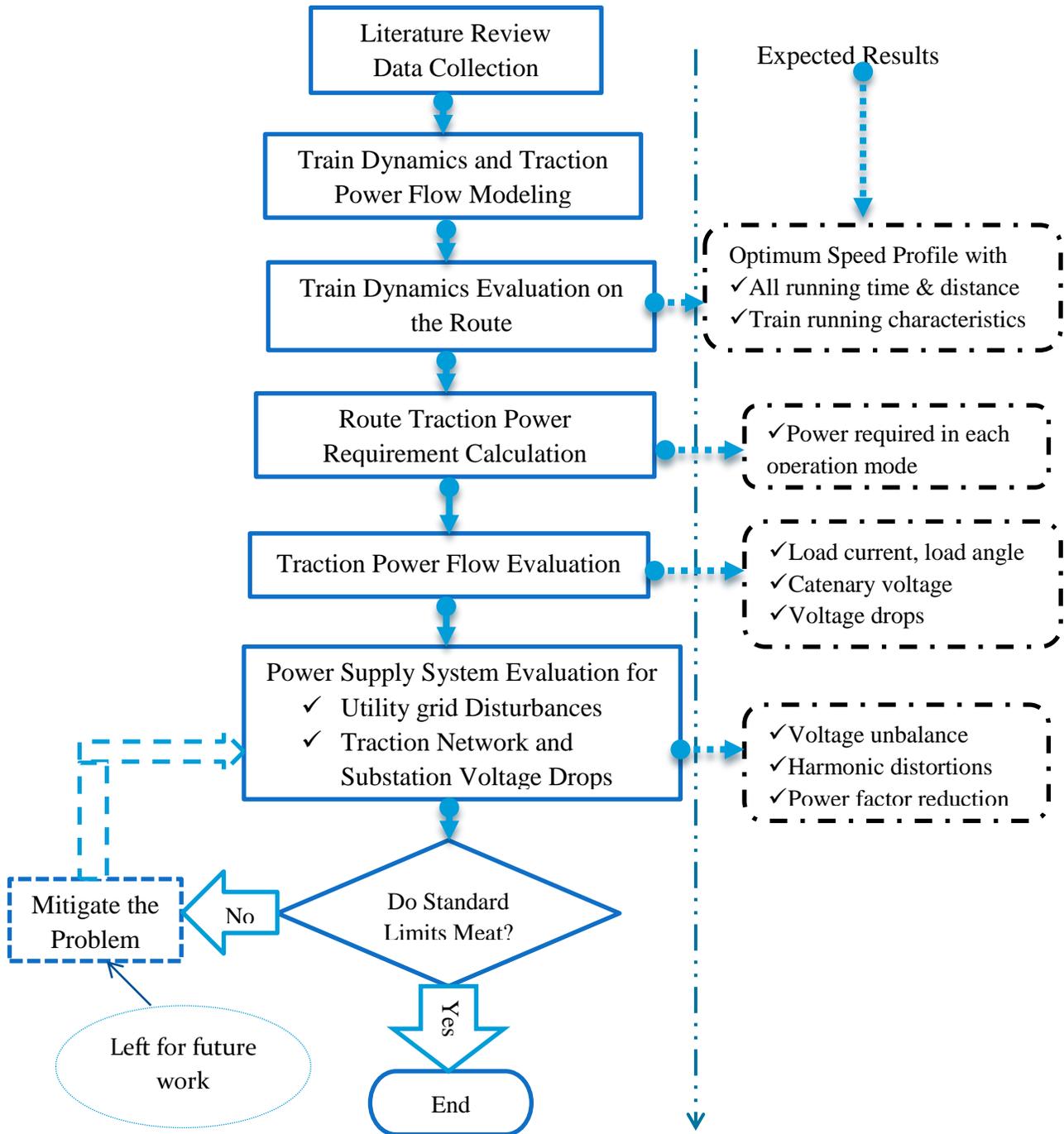


Figure 1.2 Summary of Methodology and Research Road Map

## 1.6 Related Work

Traction power supply analysis results and trends indicate that the main problems associated with electrified railway operation are power quality problems created due to the railway system, in general, on the utility grid. These power quality problems are the results of single-phase load characteristics of the train, traction power supply connection scheme, curvilinear path of the train, train loading, power supply configuration, etc. and mainly expressed in power factor reduction, voltage unbalance and harmonic distortions [23] [28] [29]. Moreover, train speed changes over curvilinear path of the train are the prominent cause of train instantaneous power fluctuations which results in current drawn by the load to be random. Besides, to maintain a targeted speed value during operation the current required from the supply system is also path dependent. To mitigate the power quality problems, different power quality problem studies have been done at different point of the electrified railway system network and studies done at the point of common coupling are the noticeable one, as in [23].

In literature review [23], traction system power quality issues, voltage unbalance and harmonic distortion for Ethio-Djibouti Railway Power Supply System, at the point of common coupling and mitigation methods have been studied. In the study, an external balancing method with booster transformer arrangement was proposed on the existing traction power supply system to improve the voltage unbalance and telecommunication disturbance and a multi-section harmonic filter was also designed to reduce higher order harmonic waves. Moreover, modeling and simulation of the traction system were performed using MATLAB/SIMULINK. Simulation results were compared with IEEE and other standards.

As depicted in the report, the voltage unbalance factor (UBF) 3.04% and total harmonic distortion (THD) of 7.75% which actually violates the IEEE limits, see Appendix B, 2% and 5% respectively were measured at the point of common coupling. Hence, by applying the proposed range of remedies, power quality improvements have been achieved which reduced UBF to less than 1% and THD lower than 0.5%. As a result, I, the writer of this report, accepted all the results of [23] and assumed that traction power quality problems that obviously result from the random current requirement of the train in the analysis of train dynamics done in this research could be mitigated by the design methods proposed and analyzed in the stated study.

## 1.7 Study Area and Scope

The physical scope of this thesis work is along Awash~Sirba Kunkur~Sebaka line subsection of the Ethio-Djibouti corridor. The line subsection is feed by two three phase Vv-connected traction transformer substations located at Awash, 11.9km to Mieso direction and at Sirba Kunkur, 22.38km to Sebata and 11km to Mieso directions. Besides supplying the line section, Awash Substation is designed as intermediate station and Sirba Kunkur as passing station. A neutral sections or section posts are installed at Awashit and Sebaka to isolate the two feeding sections.

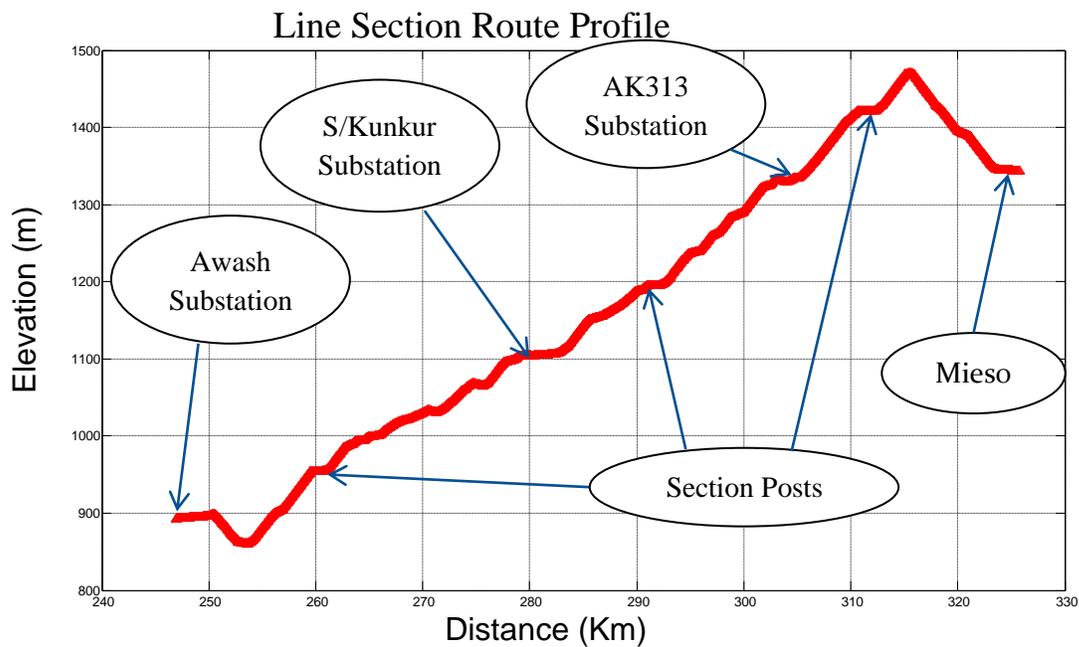


Figure 1.3 Line Section under Investigation, Route Profile

Moreover, this thesis report principally focuses on the traction energy consumption analysis for freight trains so that they could be operated near their design speed in the line section, the corridor in general, to make the section occupancy time short increasing the frequency of services. The line section is part of the corridor to be most congested in the future mid-to-long term plans since it is the immediate path to the port.

Recalling the engineering speed limit requirements, since the maximum speed considered in this thesis report is the line design speeds; speed limits due to train formation, curve, down grade, track strength, block signal are not the problem and it is assumed that the infrastructure underway is perfect to support them. Yet, speed limits due to the requirements of equation of motion are the target problem of this study; i.e. train speed limits caused by train dynamics and the energy

requirements to make up this speed loss. Henceforth, after a brief recall of the fundamental concepts of traction power supply subsystem and its relation with train speed, the contribution it has on maintaining train-track design speeds is dealt. Moreover, traction subsystem performance evaluation mainly focusing on power quality determinants is analyzed.

## 1.8 Thesis Organization

This thesis is organized into five chapters with each chapter explaining in detail about the research. The first chapter presents the theoretical background of the research, research problem, objectives, methodology, and related work in the area; particularly on Ethio-Djibouti corridor and finally the physical and conceptual scope of the thesis. The second chapter generally covers review of railway fundamentals and train dynamics. This chapter presents the details about the fundamental concepts in railway speed definition and standard classification. Moreover, train dynamics equations describing the relationship among train speed, train dynamics constraints and power consumption are presented.

In chapter three, following the discussions on railway electrification system technology options, traction power system modeling, including the utility grid side, load side and supply side current and voltage relations are presented. Furthermore, traction power flow modeling, the prominent power quality problems in railway electrification system in general and finally the Ethiopian railway power supply plan are discussed.

In the fourth chapter, using route information and rolling stock data, the target operational speed within design speed is calculated to be 72.4kmph. Based on the theoretical concepts developed, actual route information; in the line subsection, and the corridor rolling stock data traction power required, during powering operation, to attain the targeted speed and maintain it as much as possible is calculated. In addition, train running time and distance in each regime of train speed profile are calculated. At the end of the chapter, power quality problems created at the point of common coupling (PCC) when operating trains at the targeted speed are calculated compared with the standard limits and possible mitigation techniques are proposed.

In the final part, chapter five, the concluding remarks that summarize the research results and gives future work recommendations on subjects related to the thesis are presented.

## Chapter Two

### A Review of Railway Fundamentals and Train Dynamics

#### 2.1 Railway Fundamentals

##### 2.1.1 Definition

**Transport** or **transportation** is the movement of people, animals and goods from one location to another using road, water, air, rail, cable, pipelines and space modes; supported by fixed installation and terminal infrastructure [10]. Each mode may have different forms designed for different purposes, but transport something. Moreover, a given item or person to be transported may take a combination of two or more of these modes one after another to get in to its destination, in which case it is called a multimodal transport system. Although it is hard to find all of them along one origin-destination, when two or more of these modes exist nearby to provide same origin-destination services it is the merit of the user to select the mode that satisfies ones need. These transport needs may be travel time, transport cost, comfort, security, etc. or their combination.

**Rail transport**, in particular, refers to the land transport of people or goods along guided paths called railways; consisting of two parallel rail tracks at a fixed distance (gauge) apart, usually made of steel and mounted upon cross beams called ties or sleepers.

##### 2.1.2 Railway Transport Features

- The path taken by the train is determined by the mechanical guidance system of wheel and rail, and this can be changed only at points/switches.
- The steel-wheel contact surface has a relatively poor braking response on the steel rail, but there is a relatively high running speed.

##### 2.1.3 Railway Transport Principles and Procedures

The very basic principles; how a railway works, are the same throughout the world. However, operating procedures differ significantly between different countries or regions of the world. These differences are not only in some detail but also in the fundamental terms, definitions, and procedures; therefore, a lot of railway education and training centers concentrate on their national rules [11]. Despite the differences between the operating procedures of various national railway systems, there are three basic operating philosophies in running a railway that have influenced railway operation worldwide, namely: British, German and North American Operating Procedures

[11]. Some countries follow one of these three philosophies in a quite pure form, while other countries use a specific mix of several systems, or added national peculiarities. The railways of these three countries have achieved a very high level of harmonization [11].

## 2.2 Railway Speed

The mechanical energy, output power at the motor shaft which is used to drive the train along its path, in the form of effective tractive effort that exist at the contact surface between wheel and rail actually initiates train acceleration, if it is nearly about 5% more than the dead weight of the train [1]. This is the basic principle of mechanics for any type of motion, Newton's Second Law of Motion. This motion is prevailed by the change of position from time to time relative to the starting state; stand still or in motion. This relative change of position in time, one of the measures of the effectiveness of the transport mode, is called **speed**. In the field of transport in general and on the railway in particular, it is associated with many adjectives; maximum, instantaneous, average, operational, optimum, conditioned, design, line, service, practically applicable, and commercial speeds [12].

Starting from its early age, rail systems have been identified and grouped by different features such as motive power - steam, diesel and electric drive; type of services as passenger and freight; in terms of physical features as adhesion railway and railway based on the principle of the magnetic levitation, generally referred to as maglev; mileage as urban transit and inter city; etc. Likewise, differing by level of ranges for passenger and freight trains, since the appearance of the "bullet train" in Japan in the mid-twentieth century [12], speed has been in use to classify rail system as conventional speed and high speed railways. Yet, classifications related to speed don't have a clear cut point and it is adapted to each case and each country.

Although this is the case, at the moment, the definition of high speed railway which is recognized in several places around the world is set by the European Union (EU) Directive 96/84/EC and supported by International Union of Railways (UIC-French acronym for Union Internationale des Chemins de Fer) [6]. Principally, the EU Directive defines HSR as: transport service where the infrastructure and rolling stock allow for the services of 250kmph or more and an extract of the Directive is given underneath [6]:

## 1) Infrastructure

- a) The infrastructure of the trans-European High-Speed system shall be the trans-European transport network identified in Article 129C of the Treaty:
  - Those built specially for High Speed travel;
  - Those specially upgraded for High Speed travel. They may include connecting lines in particular junctions of new lines upgraded for High Speed with town center stations located on them, on which speed must take account of local conditions.
- b) High Speed lines shall comprise
  - Specially built high speed lines equipped for speeds generally equal to or greater than 250kmph;
  - Specially upgraded high speed lines equipped for speeds of order of 200kmph;
  - Specially upgraded high speed lines which have special features as a result of topographical relief or town planning constraints, on which the speed must be adopted to each case.

## 2) Rolling Stock: The High Speed advanced technology trains shall be designed in such a way to guarantee safe, uninterrupted travel:

- At a speed of at least 250kmph on lines specially built for High Speed, while enabling speeds of over 300kmph to be reached in appropriate circumstances;
- At speed of the order 200kmph on existing lines which have been or are specially upgraded;
- At the highest possible speed on the other lines.

## 3) Compatibility of Infrastructure and Rolling Stock

- High Speed train services presuppose excellent compatibility between the characteristics of the infrastructure and those of rolling stock. Performance levels, safety, quality of services and cost depend upon that compatibility.

Generalizing the extract of the Directive, all speed ranges below high speed levels given so far are classified as **conventional railway speeds**. In any case, the speed around 200-250 km/h is considered to be the threshold between conventional and high speed rail in EU. Furthermore, it is easily conceivable that the above general definition given by EU Directive 96/84/EC concerns only passenger train services and it is not the practical case for freight transport. High-speed rail freight has to be defined by speed as well as other characteristics, like type of cargo, operating principles

and vehicle concepts [13]. However, when looking at speed, 160 km/h can be set as a lower limit for the demarcation of high-speed rail freight, while the business idea of high speeds rail freight is defined as faster than by truck cheaper than by air [13]. Though this high-speed rail freight development necessities to look at it as a system comprising of not only the rolling stock but also terminals, loading units, trans-loading techniques, train operation and operational coordination with other train traffic [13].

### 2.3 Train Speed Records

The average speed of a train over its track is mostly less than the line speed, except some rush time and places, due to infrastructural degradation throughout its life time. Especially, in freight train operation the case is prominent. Even though this, there are speed records in their respective services, passenger and freight. Until 1960s' passenger train speeds greater than 200kmph remained only to test speed. Since 1964, after Shinkansen-"bullet train" in Japan with speed of more than 210kmph have been launched, history marked the emergence of concrete ideas about high speed and the competition among nations all over the world, especially in Europe, started to flourish more than ever in the field. Figure 2.1 gives speed records since then until 2009 [14]; of course it is from different countries and on different corridors.

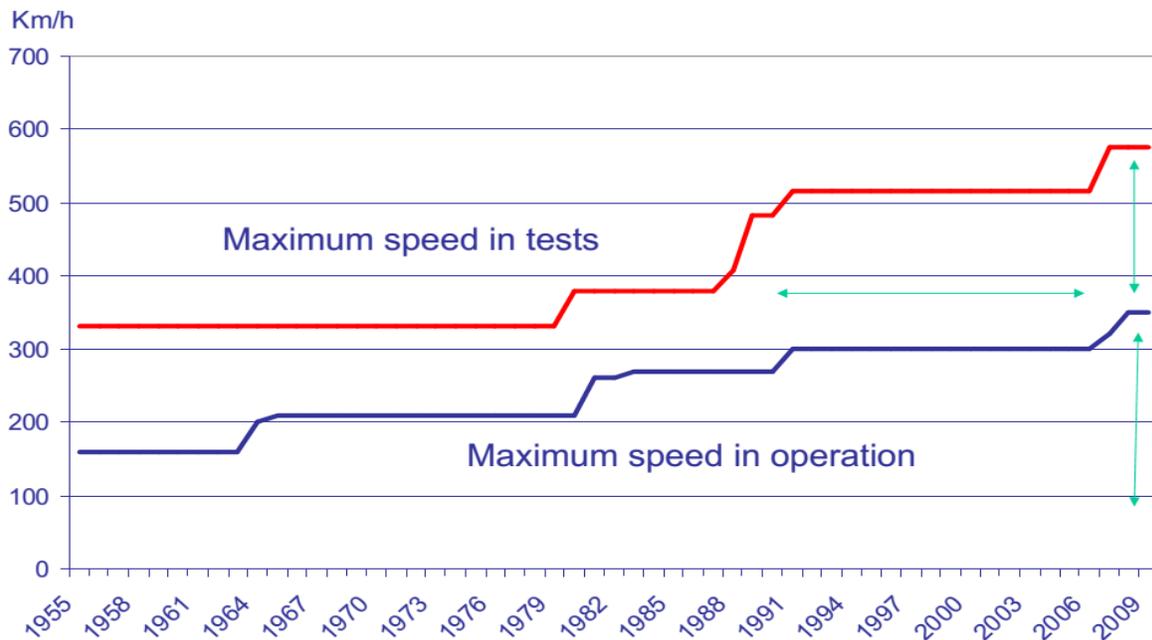


Figure 2.1 Evolution of Maximum Test and Operational Speeds for Passenger Trains [14]

In freight train speed definitions the business idea over weights the train speed. Even related to train speed, the reference speed for the definition defers from country to the country. Hence, there are a lot of definitions, similar to passenger trains, for high speed freight demarcation. The French TGV postal trains are recorded to run at speeds up to 270 km/h since the 1980-ies [13]. The table below gives freight train speed records in three European Countries.

Table 2.1 Freight Train Speed Records in France, Germany and Denmark

Country	Train Name	Speed Record (kmph)	Year
France	TGV	270	1981
	LGV	160	1997
	MGV (Messageries à Grande Vitesse)	200	1998
Germany	ICGE (Inter-Cargo-Express): new line	160	1991
	ICGE - on upgraded line	140	1991
	ICGE: Hbillss-y 307 bogie container wagons	213 (test)	1991
	ICGE with special high-speed container wagons	160	2000
Denmark	Danish Mail Trains	140	1997
	Two-axle wagons-second-class mail	120	1997

#### 2.4 Significance of Speed in the Railway Network Design

In the early days, because speed of competing transport modes such as horses and water transport was not very fast, construction financial considerations but not train speed was seen as a major factor in the railway network route design and selection process. However, as railway transportation became a more vital need for society, the need to increase the speed of trains became more and more important opening the way for improvements in railway practices and engineering. A number of factors, such as rolling stock design and track design ultimately influence the maximum speed of a train. Today's high speed railway routes are specifically designed for the speeds expected of the rolling stock.

When new rail lines are built to connect to inland ports and/or cross country transport services, the design speed of trains governs all the geometric design including radii and super elevation height from the outset [16]. It also controls the geometric layout of overhead contact lines for the electrified system [16]. On existing rail tracks where radii and super elevation are small and might not be up to

current design standards, an analysis should be conducted to ensure contact line design can support the existing tracks. Otherwise, the existing tracks need to be upgraded and reconstructed before contact lines are built [15]. Moreover, the following are some of the electrified railway route characteristics determined by train design speed or line speed:

- ✓ Electrification system technology selection; traction substation components rating; power supply modes; OCS conductor type, structure and formation, dropper length (minimum lengths 300mm for  $\leq 120\text{kmph}$  and 500mm for  $>120\text{kmph}$ ), space between droppers;
- ✓ Rate and amount of current flowing in the contact wire: The amount of current flowing in the railway network at stand still and at the maximum train speed; which determines the interaction between pantographs and contact wire.
- ✓ Signal location, signaling type and communication between on-board to way side;
- ✓ Safety components and safety level;
- ✓ Grade crossing type, number of crossing and level crossing technology installed
- ✓ Turn out location, operation principle and operating devices
- ✓ Braking type, brake action time and driver response factor
- ✓ Rail strength, distance between sleepers and between lines in multiple track rail system;
- ✓ Operation mode: completely segregated, mixed high speed system, fully mixed system, mixed conventional system; see Figure 2.2 HSR system operation;
- ✓ Determines rail transport price and affects price models and modal selection process etc.

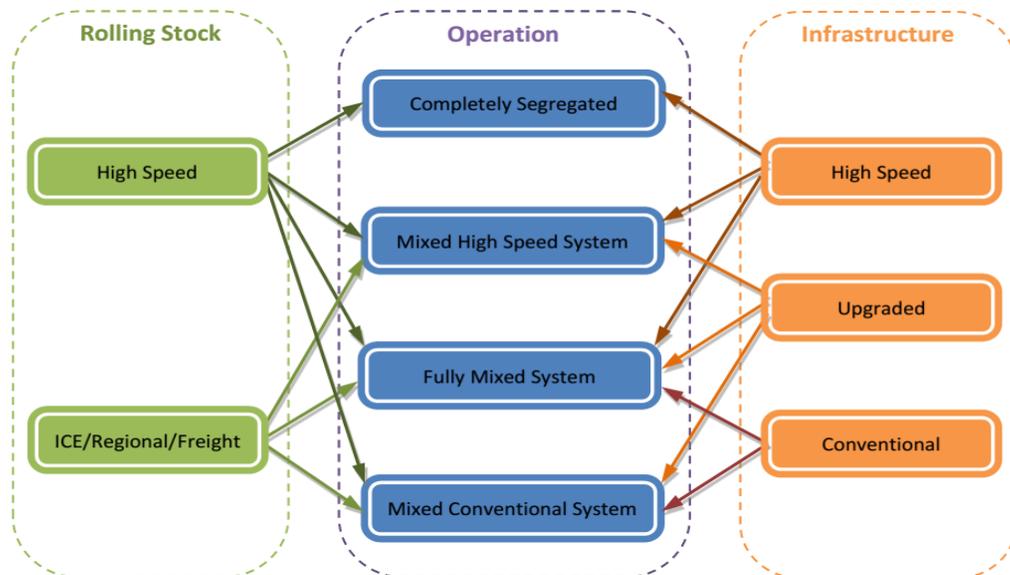


Figure 2.2 Types of High Speed Train Operation [13]

## 2.5 Railway Speed and Power Consumption

Railway transport mode, electrified railway in particular, is the most accustomed mode for mass and bulk transport due to its peculiar characteristics-“green” high-capacity transport mode that is fast and convenient, land-saving, energy-saving, environmental-friendly, and safe. Railroad researches and trends suggest that the energy efficiency gains are due to technological advances, operational efficiencies, and increased net to gross weight ratio compared to steam and diesel trains [16]. There are two ideas relating to the links between energy and the railway operation, basically related to train speeds, which are very widespread, even among many engineers and railway experts [4]:

- a) The first idea, which is called the “power rule” or “cube rule”, is that the power of the train would increase with the cube of its speed.
- b) The second idea, which is called the “energy rule” or “square rule”, indicates that energy consumption would increase in proportion to the square of the speed. This rule would correspond to a consumption induced solely by quadratic resistances, such as those of an aerodynamic nature.

These two ideas mark railway operation effectiveness’s to be directly linked to train speeds. In addition, energy consumption per passenger and/or freight-ton transported per km or efficiency, operational efficiencies expressed in train operational speed are good service level indicators. Transport agencies use different service level indicators in order to evaluate the level of services they provide; from service user and service provider sides. From service user side include: safety, comfort, transport time, cost of transport, etc. and from service provider side, in addition to the above with somewhat different purposes, include: gross ton-km or net ton-kilometer (cargo weight transported times distance transported—also reported as net ton-kilometers - NTK) for freight transportation and passenger trips or passenger-km (rail travel distance time number of passenger transported) for passenger transportation [17], transport delay and delay propagation, network congestion, capacity utilization, net revenue gained, social trust developed, socio-economic throughput, etc. However, all here above mentioned service level indicators, from both sides, are influenced by other factors such as technology, operation procedures and principles, integration among railway network sub-systems, etc. directly or indirectly.

For a given origin-destination (O-D) safety and comfort levels, transport costs, gross or net-ton and number of passenger per trip, and O-D distance are constant. However, time spent during transportation, by definition inversely proportion with speed, is a relatively conditioned service level indicator. The very basic principle in mode selection is ‘the time spent has to be minimal’.

Every transport system consumes energy from some favorable energy source(s) to perform the transportation work. Mathematically, this consumed energy is directly proportional to time spent doing the intended work, see Eq. (2.3b), which indicates that reduction in transportation time reduces energy consumption. Similarly, Eq. (2.5) shows energy consumed during transport work is directly proportional with the distance covered by the transportation system. As a result, increasing speed minimizes the time during which the vehicle stays connected to intake power reducing energy consumption. But, from the two energy rules outlined above an increase in speed increases energy consumption while reduces travel time. In any case, energy requirement is a design criterion for a specified type of services; see Table (2.5), and for a specified design speed. Furthermore, in addition to the mathematical descriptions in (2.3b) and (2.5), the energy requirement of a train movement is a function of train speed, weight, acceleration and resistances which determine the running distance and time in the line section [15].

Being the most important feature of railway operational efficiency determinant, train speed varies routinely as a result of changes in speed limits, planned stops and traffic control and hence, the corresponding consumed energy accounts for energy consumption during acceleration, traffic control delays and steady speeds [16]. As to train weight, it (the net weight) varies from line section to line section due to loading and unloading goods and passengers at planned stops. Similarly, resistance which is determined by locomotive characteristics, route alignment and instantaneous train speed is also different from line section to section. To have the general overview of train speed and the corresponding energy consumption it is necessary to analyze train dynamics over the actual path of the train. Hence, in the subsequent sections we will see train dynamics governed by equation of motion.

## 2.6 Train Dynamics Modeling

Train dynamics (i.e. the changes of train position, velocity, and acceleration with respect to the change of time) analysis is a basic planning task in railway operation under different circumstances [3]. During the trip between two successive stations by alternating powering, coasting, and braking operation modes train dynamics has four operation regimes; namely: acceleration, constant speed, deceleration and braking [3].

Train movement in each regime, could be written using Newton's Second Law of Motion, instantaneous power, horse power or torque equations [16]. For analysis purpose, although each equation has its own merits, due to the complexity of train dynamics, Newton's Second Law of motion, which denotes force as tractive effort, is the best choice since it is easily used to compare tractive effort with the drag forces and get the net force acting on the train [16]. The analytical formula which is familiar to describe this relation indicates that tractive effort is inversely proportional to the speed of the vehicle and limited by the available power at a given speed and the adhesion between the wheels and the track when the train starts from standstill [16], see Eq. (2.6) through Eq. (2.11). Besides the above generalizations, train movement is constrained by speed limits which regulate the maximal speed that must be obeyed on the course of movement for safety reasons and guided by particular operation objectives such as energy saving, reduction in harmonics, etc. [3].

However, in practical train dynamics analysis and curve estimation there are many physical constraints that need to be identified. These constraining factors are either static confined to the route such as curvature, downgrade, switch, track strength, blocking signal, grade crossing, - that provide line section speed restrictions [3], grade resistance, flange resistance, tunnel resistance, distance between station, etc. or dynamic, varying depending on the current train location such as available tractive effort or drawbar, aerodynamic drag forces, wheel-rail contact nature, weather, etc. Moreover, train technical specifications which are general to all locations such as maximum tractive effort or motor torque, gear ratio, transmission efficiency, rolling stock speed, motor output power, tonnage—determines starting resistance, acceleration, deceleration, and braking force - are other inputs that need due consideration for precise analytical analysis.

Putting all these together there are different approaches used for train dynamics analysis purposes. They are either analytical, statistical [16] or computer based [3]. The third, a programming language based simulation model, is the most accurate approach. The first two even though they are not as

such precise as the third one due to the complexity of train dynamics, are widely used as they provide good insight and being a benchmark to the third approach. In this report the first approach, analytical method, is used.

### 2.6.1 Resistance Forces and Equations

Train resistance consists of all the forces that oppose the motion of train [1]. It is mainly due to internal and external interaction of train and train parts; due to gradient and curves along the path of the train. The internal factors are; friction at the axle bearings and guides; bogie-pivots; friction at the motor bearings and gear and the external factors include; friction between wheel and rail; flange-friction; resistance due to temporary deflection of track; aerodynamic drag [1].

Train resistance is estimated by using general Davis Equation expressed as in Eq. (2.1), measured in Newton per ton [16]. Others such as curve resistance, grade resistance, tunnel resistance [17], starting and running (dynamic) frictional resistances, in Newton, representing the additional work needed to overcome the wheel-rail friction at curves, grades, train starting and on due motion respectively are calculated when required determined by incidental feature of the line section under investigation. These resistances being added to the train resistances give the total train resistance. There are a lot of analytical relations (empirical formulas) which have been in use to estimate train resistances. The best admired formulae for train resistance evaluation are Davis's Equation, Modified Davis Equation and the Canadian National (CN) Equation that differ from each other by the regression coefficients (A, B and C) evaluated using fitting test data, see Table 2.2 for the evaluation of the empirical formula by different approaches.

$$R = A + Bv + Cv^2 \quad (2.1)$$

Table 2.2 Total Train Resistance Approximations by Country or Company

Resistance	Country	Approximations by Country	Feature	Remark	
Train Resistance	AREMA [18]	$R_u = 0.6 + \frac{20}{W} + 0.01v + K \frac{v^2}{W * n}$	$R_u$ =Resistance [lb./ton] W =weight per axle in ton K=drag coefficient; $v = speed [mph]$	K= 0.07 conventional equipment K=0.0935 for containers K= 0.16 for trailers	
	Japan [18]	$R = 8.202 + 0.10656v + 0.01193v^2$	R=Resistance [kN]	Shinkansen Series 200 HST	
	Canada	$R = 1.5 + 18 \frac{n}{W} + 0.03v + Ca \frac{v^2}{10^4 W}$	R=Resistance [lb./ton];n=number of axles W=total loco or car weight in ton a=vehicle frontal area; $v = speed [mph]$	C=24 for leading loco C=5.5 for trailing power unit C=5.0 for mixture of freight car	
	Indian Railway		$R [kg] = 0.65W + 13n + 0.01Wv + 0.52Wv^2$	W[ton]=locomotive weight n=number of axles	For locomotives $R[N] = 9.81 * R[kg]$
			$r \left[ \frac{kg}{ton} \right] = 0.87 + 0.0103v + 0.000056v^2$	For loaded box wagon	$R[N] = 9.81 * r * W$ W=gross weight [ton] $v = speed [kmph]$
			$r \left[ \frac{kg}{ton} \right] = 1.425 + 0.0054v + 0.0025v^2$	For coaching stock	
		$r \left[ \frac{kg}{ton} \right] = 1.517 + 0.0103v + 0.00495v^2$	For empty box wagon		
France		$r \left[ \frac{kg}{ton} \right] = 1.5 \sqrt{\frac{10}{t}} + \frac{v^2}{120*t}$	t=axle load in tonne		
Curve	Japan [18]	$r_c = 0.01 \frac{k}{R_c}$	$r_c$ =resistance [kN/ton] $R_c$ = horizontal curve radius [m] k= dimensionless (Train char/tics)	k=500-1200	
	[20]	$R_c[N] = r_c W$	$r_c = \frac{k}{\gamma}$ ; $\gamma$ =curve radius	k=505-800; China :k=700	
Grade		$R_g [N] = \pm(Mg \sin \theta) = \pm WgD$	$D = Gradient (\% 0r\%00)$	( $\pm$ ) up grade or down grade	
Frictional Forces		$F_{sf} = Wg\mu_s = \text{Static frictional force [kN]}$	$\mu_s = 0.3$	$\mu_s$ =static friction coefficient	
		$F_{df}[kN] = Wg\mu_d$ ; Daynamic $F_f$	$\mu_d$ =dynamic friction coefficient;	$\mu_d = \frac{7.5}{44 + 3.6v} + 0.161$	

## 2.6.2 Tractive Effort and Horse-Power

Tractive effort is the total train force required from traction drive system to overcome total train resistances and provide sufficient acceleration for the train along its curvilinear path hence, the train does the transportation work. Newton's Second Law of motion tells us that it is the change in kinetic energy of a moving object used to do a useful work. This could be shown by taking the derivative of kinetic energy of a moving object with respect to time, (t).

By definition  $E(\text{kinetic energy}) = \frac{mv^2}{2}$ ; differentiating E with respect to time (t) we obtain

$$\frac{dE}{dt} = \frac{d \frac{mv^2}{2}}{dt} = \frac{1}{2} m 2v \frac{dv}{dt} = mv \frac{dv}{dt} \quad (2.2)$$

But from Newton's Second Law of motion  $F = am = m \frac{dv}{dt}$  so that

$$\frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v} = P \quad (2.3a)$$

$$E = \int_1^2 \mathbf{F} \cdot \mathbf{v} dt = \int_1^2 P dt \quad (2.3b)$$

If we multiply equation (2.3a) by 'dt' we obtain the differential change in kinetic energy 'dT' which is the force "dot" the differential distance moved.

$$dE = \mathbf{F} \cdot d\mathbf{s} \quad (2.4)$$

If we now integrate equation (2.4), we get the total energy consumed along the line section

$$\Delta E = \Delta W = \Delta T = \int_1^2 \mathbf{F} \cdot d\mathbf{s} \quad (2.5)$$

Equation (2.5) means that if an object is moving, in any way, under the influence of a force, moving in some kind of curved path, then the change in kinetic energy when it goes from one point to another along the curve is equal to the integral of the component of the force along the curve times the differential displacement ds. This integral is also called the *work done by the force on the object*. For one dimension motions the dot is ignored. In equation (2.3a) the vectors' dot product ( $\mathbf{F} \cdot \mathbf{v}$ ) is the power being delivered to the object by the force. Hence, by theorem, *the rate of change of kinetic energy of an object is equal to the power expended by the force acting on it*. This is the basic equation governing the relationship among train dynamics physical quantities; namely: power, speed and force called tractive effort.

From equations (2.3a) through (2.5) it is necessary to note that the small time interval ‘ $dt$ ’ and differential distance ‘ $ds$ ’ are very important quantities in train dynamics analysis and the corresponding energy consumption estimation. Indeed, due to dependency problems these are the only two possible quantities that could be obtained analytically from train dynamic equations and are dealt with in the subsequent sections of this study.

At this point before we discuss about the relationship among speed, tractive effort, power and energy consumption it is necessary to have an insight about *torque-speed characteristics of electric motors* since this curve is the basis for train dynamics examination. The characteristic curve- tractive effort curve -has three operation regions Figure (2.3); namely: constant torque region, constant power region and motor type/characteristic region with the speed on the x-axis and the force on the y-axis. In each region there are salient equations describing the relationship among tractive effort, torque, motor current, speed and power.

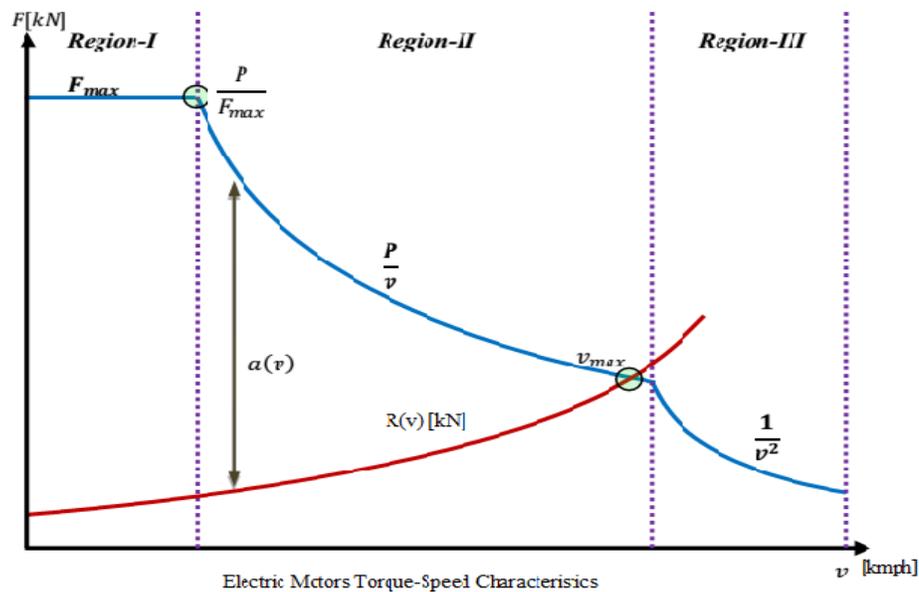


Figure 2.3 Typical Tractive Efforts versus Speed Curve [13]

The first region is limited by two factors; the motoring and the adhesion limits. Since tractive effort is related to the power divided by speed, at low speeds the engine would in theory produce extreme value, overloading the motor which eventually would breakdown. Yet, the adhesion limit will allow only a certain amount of force to be transferred between the wheel and the rail before it starts to slip (accelerating) or slide (braking).

$$\tau_{\max} \propto I_{\max(\text{starting})} = \text{constant} \quad (2.6)$$

$$TE = \eta_t \gamma_G \frac{\tau_{\max}}{R} = \text{constant}(\text{max.}) \quad (2.7)$$

$$P = \omega_m \tau_{\max} \quad (2.8)$$

$$P = \frac{v}{R} \tau_{\max} ; 0 \leq v \leq v_{\text{base}} \quad (2.9)$$

In the second region the motor is running with maximum power and the force available is therefore limited to the general formula of mechanical power:

$$TE * v = P = \eta * VI = \text{constant} \quad (2.10)$$

$$TE(v) = \frac{P}{v} = \eta \frac{VI}{v} ; v_{\text{base}} < v \leq v_{\text{max}} = v_{\text{balance}} \quad (2.11)$$

In the third region, the motor approaches its maximum limitations and the current and the flux is reduced at a greater rate to avoid motor excitation. To simplify further calculations done in this report, it is assumed that the given rolling stock will reach its maximum speed within region two, thus region three will be left out of the calculations. In Figure 2.3 the total train resistance is also plotted as it has a major impact on the performance of the rolling stock, which is the sum of all forces opposing to the driving movement. All the above facts are summarized in tractive effort and resistive effects as shown in Figure 2.4 using the force-mass block diagram. The equation of motion describing, extracted from force-mass block diagram, train dynamics [3] is given as in Eq. (2.12).

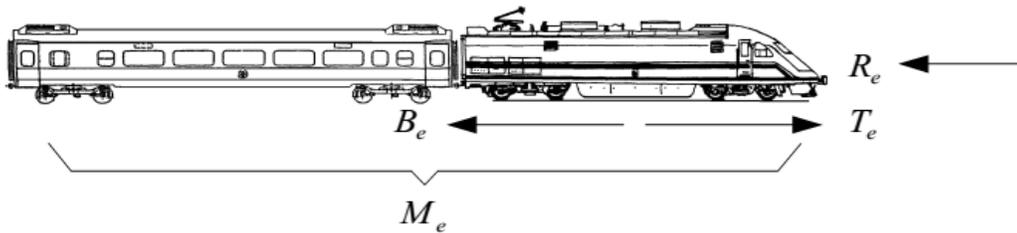


Figure 2.4 Train Force-Mass Block Diagrams [3]

The general equation of motion

$$M_e \frac{dv}{dt} = TE(v) - R_e(v, i, \gamma) - B_e(v) = F_{\text{net}}(v) \quad (2.12)$$

Where  $R_e(v, i, r) = R_s(v) + R_r(v) + R_g(i) + R_c(\gamma) + R_t = \text{total resistance}(N)$

Normally, the dynamic mass is 5-15% of the static train mass and in this report 10% is assumed.

$$M_e = \rho M ; \rho: \text{rotating mass factor} = 1.1$$

Since  $v[\frac{m}{s}] = \frac{ds}{dt}$ , the equation of motion in Eq. (2.12) is a second-order ordinary differential equation (ODE) of distance (s) with respect to time (t). A direct method to solve such an ODE is to integrate it twice over time. But this does not work for Eq. (2.12). The difficulty arises from the fact that  $F_{net}$  is not an explicit function of time. It is velocity dependent. Hence, to solve it analytically, the independent variable must be changed from t to v. To perform this change of variable let us do the following rearrangements.

$$M_e \frac{dv}{dt} = F_{net}(v) \quad (2.13)$$

$$dt = M_e \frac{dv}{F_{net}(v)} \quad (2.14)$$

Integrating and changing ‘dv’ from kmph to mps gives:

$$t[s] = \frac{M_e}{3.6} \int \frac{dv}{F_{net}(v)} \quad (2.15)$$

By definition,  $ds = vdt = vdv/F$  and we obtain

$$s[m] = \int ds = \frac{M_e}{(3.6)^2} \int \frac{v}{F_{net}(v)} dv \quad (2.16)$$

Both equations (2.15) and (2.16) take ‘v’ as an-independent variable for their corresponding integration. Therefore, they can be used to solve train running time and running distance, for different **target speeds**. Indeed, these quantities, t[s] and s[m], are used to estimate the amount of energy consumed in the line section, particularly, in acceleration and free running regimes when train is expected to be connected to the overhead catenary system’s (OCS) contact wire.

In Eq. (2.12), the tractive effort and braking force can be controlled by train driver or ATC (automatic train control) computer for the three train driving modes, as shown in Table 2.3 [3]. The net force acting on the train in the two acceleration regions, free running and coasting, and braking regimes is reduced as in equations (2.17a), (2.17b), (2.17c) and (2.17d) respectively.

$$F_{net,1}(v) = TE_{max} - R_e(v, D, \gamma) \quad (2.17a)$$

$$F_{net,2}(v) = T_e(v) - R_e(v, D, \gamma) \quad (2.17b)$$

$$F_{net,3,4}(v) = -R_e(v, D, \gamma) \quad (2.17c)$$

$$F_{net,5}(v) = -R_e(v, D, \gamma) - B_e(v) \quad (2.17d)$$

Table 2.3 Train Driving Modes [3]

Train operating mode		Tractive effort condition	
		$TE > 0$	$TE = 0$
Braking force condition	$B_e = 0$	Powering mode	Coasting mode
	$B_e > 0$	—	Braking mode

### 2.6.3 Typical Train Speed versus Time Curve

Effective railway operation requires continuously monitoring the state, position/location of train and geographical nature of the route along the course of motion. The instantaneous train states include direction of motion, running speed, location, etc. It is customary to express train movement/dynamics graphically for easy of train control and driver's assistance in the course motion. This train dynamics graph, which describes the actual movement of the train between two successive stoppages [1], is called train speed profile. It is a two-dimensional description of velocity-time, velocity-distance or distance-time [1] used to record the train dynamics along its journey. Train speed profiles are extensively used in railway system operation and research applications such as train performance calculation, journey time estimation, energy consumption evaluation, capacity analysis, train scheduling, new route planning, old route upgrading, etc. [3]. Atypical train speed profile has five successive periods of movement [1] see Figure 2.5. The speed-time curve, duration of movement in each segment, varies for different types of services depending on the requirement of the movement in the route, by the way of

- Acceleration and retardation;
- Maximum speed attained;
- Free-running period;
- Coasting and
- Breaking.

Table 2.4 Typical Characteristic Parameters for Different Train Service Types [1]

Service Type	Main line (passenger)	Main line (goods)	Main line (parcel)
Acceleration (kmphs)	0.5-1	0.2-0.4	0.3-0.5
Retardation (kmphs)	1.5-2	0.3-0.5	0.3-0.5
Maximum speed (kmph)	100-160	60-100	80-100
Free-running distance	Large	large	large

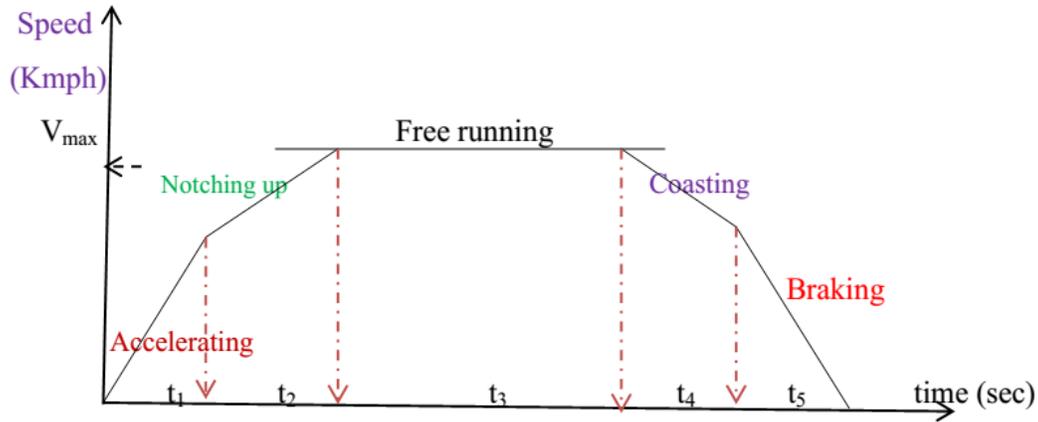


Figure 2.5 Typical Train Speed Profile

## 2.7 Train Speed Limits

In addition to the equation of motion, train dynamics are also regulated by speed limits due to train formation, curvature, downgrade, switch, track strength, blocking signal, weather, and temporary construction, etc. As a result a train traveling along the rail line must obey all these speed limits at every location of the line [3]. Consequently, the maximal allowable speed at a particular location is the minimum of all speed limits [3]:

$$v \leq v_{max,C} = \min_{in\ kmph} \{v_t, v_r, v_g, v_o, v_w, v_s\} \quad (2.18)$$

$v$  = train speed

$v_{max,C}$  = maximum allowable curve speed

$v_t$  = speed limit due to train makeup/formation

$v_r$  = curvature speed limit

$v_g$  = downgrade speed limit

$v_o$  = switch/turnout speed limit

$v_w$  = speed limit due to track strength

$v_s$  = speed limit due to blocking signal

Speed limit due to train makeup/formation must be obeyed at any time and any places, whereas those caused by curvature, downgrade and track strength may vary at different locations [3]. As to switch speed limit and signal speed regulation, they are often omitted in single train simulation and operation planning processes [3].

### 2.7.1 Maximum Curve Speed Calculation

When a train rounds a curve, it has a tendency to want to travel in a straight direction and the track must resist this movement, and force the train to turn following the track path [36]. Since rail tracks force the train to travel along the curve, the net force from centrifugal force,  $F_c$ , acting on the train and the friction between wheels and tracks produce the tangential motion [36]. The maximum speed of the train at the curves taking line curvature into account is calculated as shown here [36].

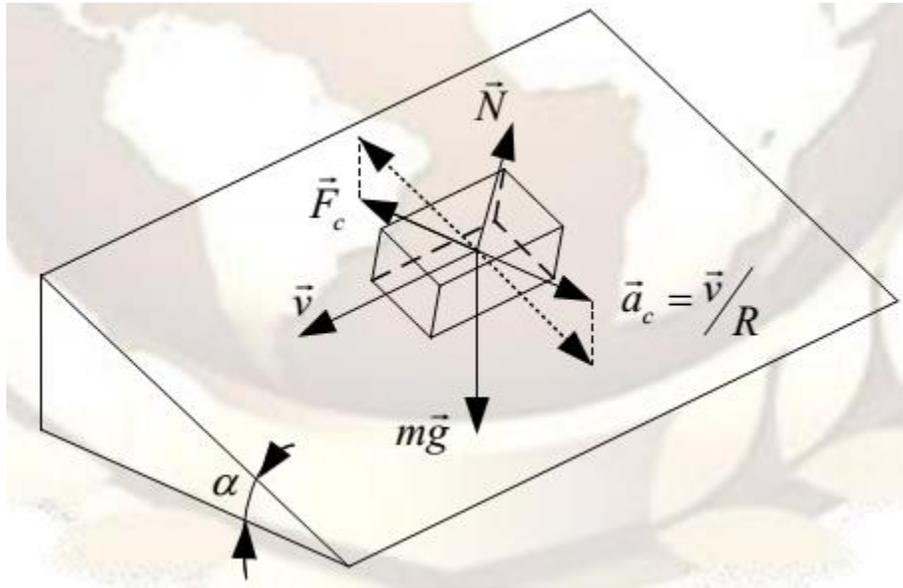


Figure 2.6 Force Diagram of the Train in an Inclined Plane

$$F_t = W_e a_t = F_c \cos \alpha - W_e g \sin \alpha \quad (2.19)$$

Since the super elevation angle ( $\alpha$ ) is usually a small value, we can make the following simplifications

$$\begin{aligned} \sin \alpha_{\alpha \rightarrow 0} &= \alpha = \frac{h}{d}; \quad \cos \alpha_{\alpha \rightarrow 0} = 1 \\ W a_t &= W \frac{v^2}{\gamma} - W g \frac{h}{d} \\ \frac{v^2}{\gamma} &= a_t + g \frac{h}{d} \\ v &= \sqrt{\gamma \left( a_t + g \frac{h}{d} \right)} \quad \left[ \frac{m}{s} \right] \end{aligned} \quad (2.20)$$

The above equation needs the instantaneous train tangential acceleration which is difficult to obtain analytically, yet, there is another approach to find the speed limit at curves [20]

$$v = \sqrt{\frac{Eg\gamma}{h}} \quad \left[\frac{m}{s}\right] \quad (2.21)$$

Where:

$E = E_a$  (track super elevation) +  $E_c$  (unbalanced super elevation)

$g$  = acceleration due to gravity

$\gamma$  = radius of curve

$h$  = track gauge (1435mm for standard gauge)

Track super-elevation typically will not be more than 6 inches (150mm) and unbalanced super-elevation (Cant Deficiency) is typically restricted to the following default values: conventional freight – 0 inches (0mm) and conventional passenger – 3 inches (75mm) [20].

## 2.7.2 Other Train Speed Limiters

The loss of running time due to speed restrictions on the diverging track (switches or turnouts) located at passing loops or at the entrance and exit of multiple sections, signal points, section posts and tunnel can potentially be very high. Indeed these are design issues that could be mitigated by selection of appropriate design methods and equipment during installation. Similar to speed limits at curves there are different analytical relations used to determine the maximum allowable speeds at all the limit points. In this research speed limits at the above points are assumed to be insignificant.

## 2.8 Traction Power Consumption

The power demand for any vehicle driven by an engine increases with increase in speed, weight, rate of acceleration and reacting drag forces. This consumed power goes to traction motors to accelerated train sets and overcome all the resistances on the course of motion. Although there are several other power consumers on board equipment such as heating, air condition, lights, ticket machines and electrical doors; trackside equipment such as traction substation apparatuses, data communication center apparatuses, wayside communication components, etc. still the main power consumer are traction motors, especially during acceleration. In train operation, energy required during free-running or constant speed regime is low compared to acceleration regime and must balance all the drag forces at the line section to keep the maximum speed attained at the

end of acceleration. During coasting and braking power required by the train traction system is nil except power required by the auxiliary systems. In addition to the above general case power consumption variation, traction power demand depends on the kind of train service requirements such as light rails, commuter trains, high speed trains, freight trains and the typical power demand for each of the classified railway traction systems is presented in Table 2.5 [2].

Table 2.5 Rail System Power Demand by Types of Services

Rail Systems	Power Demand
Light Rail	<1MW
Commuter trains	≈3-4 MW
High Speed Inter-city rails	≈4-6 MW
Very Fast Commuter Trains (TGV)	≈8-10 MW
Freight Trains(in Europe)	≈6-10 MW
Freight Trains(in USA)	> 18-24MW

## Chapter Three

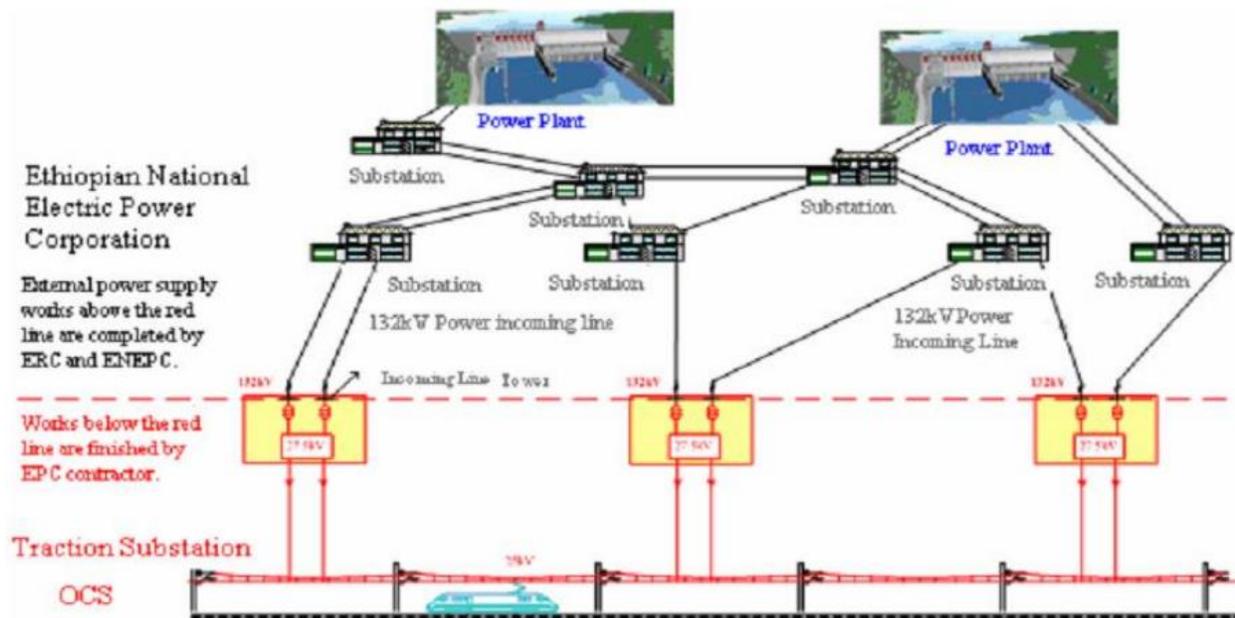
### Railway Electrification System Design

Any transportation mode needs a source of energy to drive its vehicles carrying their passengers or goods from one place to another [21]. In rail transport mode there are three types of energy sources, namely; steam, diesel and electricity to provide the driving forces. The first two, which use coal and fossil fuel as a direct or indirect energy sources that are quite scarce in many countries, have long history marking the chronological order of steam followed by diesel age. In electrified railway system transportation, the motive power or driving force is obtained from electrical motors supplied from national grid through a system of electrical traction network. As existing alternatives any nation/state is available with these three choices and a combination of diesel and electricity some times; it is the responsibility of the policy makers and fund rises to choose the best alternative based on the specific desire they want to accomplish. The choice mostly based on technology, comfort, reliability, safety, construction and life time costs, the overall system efficiency, environmental policy related issues and others which are basically corridor based. However, steam powered railways are almost extinct today.

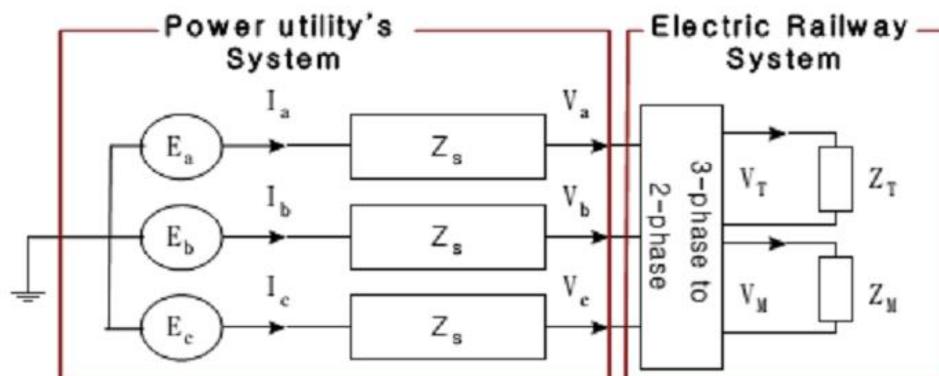
Having been the best proved propulsion system to provide traction power and the need of higher-speed, more luxurious and more reliable services, electricity has been the first choice and widely applied to modernize most of the railway transport systems across the world for several decades. Moreover, the diversion of the choice to electric traction is mainly due to its peculiar characteristics; namely: the cheapest method of traction, reduced cost of maintenance, reduced starting time compared to thermal motor railway traction, high starting torque with excellent dynamics, the possibility of regenerative braking that enables the traction system to feedback electrical energy to the national electric grid for direct further usage or to store it to use it later.

Hence, to commission, operate and exploit its merits traction electrification system studies are inevitable. In railway electrification system design, the complete power supply system study has two inter-dependent system studies: high voltage transmission system or power grid side study and traction network side study [22] see Figure 3.1(a) [33] for the case of Ethiopia. For the former, the complete power system route comprises of power generation, transformation, transmission, distribution at the power grid side and needs concrete system description, presentation and analysis with in accepted level of accuracies. In the latter case, a conditioned

power supply system study determined by the selected electrification technology option(s) is required. The overall traction power supply system study consists of two parts, in general, traction power supply system; which includes traction transformers, auto-transformers or boost transformers, switching gear devices, station protection devices, control and monitoring equipment etc. all installed and commissioned at traction substations or somewhere else along the route for power conditioning purposes; and traction power distribution network comprising of traction power distribution, feeding, return and protection systems, sectioning devices, traction power compensation system, etc. As the main railway electrification system design, most traction power supply system studies concentrate on these two parts.



(a) The Complete Traction Power Supply System Configuration



(b) Equivalent circuit model for power supply system in electric railway

Figure 3.1 Traction Power Supply System Configuration and the Equivalent Circuit [26]

### 3.1 Utility Grid Side System Study

The traction power substations will receive electrical power from the local utility company, hydro one, at relatively high voltage of 230 kV or 110kV or 132kV or one available, but has to be reliable. For economic reasons, the traction power substations should receive power directly from the power utility high voltage substations or transmission lines located in close proximity to the rail corridors. Connections to the utility high voltage system is required to ensure optimal supply reliability and to limit voltage flicker, phase unbalance, and harmonic distortion that may result from the addition of the highly fluctuating, single-phase, and non-sinusoidal traction loads. Hence, it is desirable to supply each traction power substation by two high voltage lines that are electrically as independent of each other as possible.

### 3.2 Railway Electrification Technology Options

Traction Electrification System (TES) provides electrical power to the trains by means of the traction power supply system, traction power distribution system, and traction power return system through pantograph or third rail. The traction power supply system delivers power to the distribution system. Trains collect their propulsion power from the distribution system by means of pantographs or third rail and return the power to the substations via the rails and the traction power return system. The pantograph collects current by sliding and friction. This electrical energy reaches train's motors, where it is to be changed to useful mechanical energy and does the transportation work; being conditioned by vast amount of electronic components, Figure 3.2.

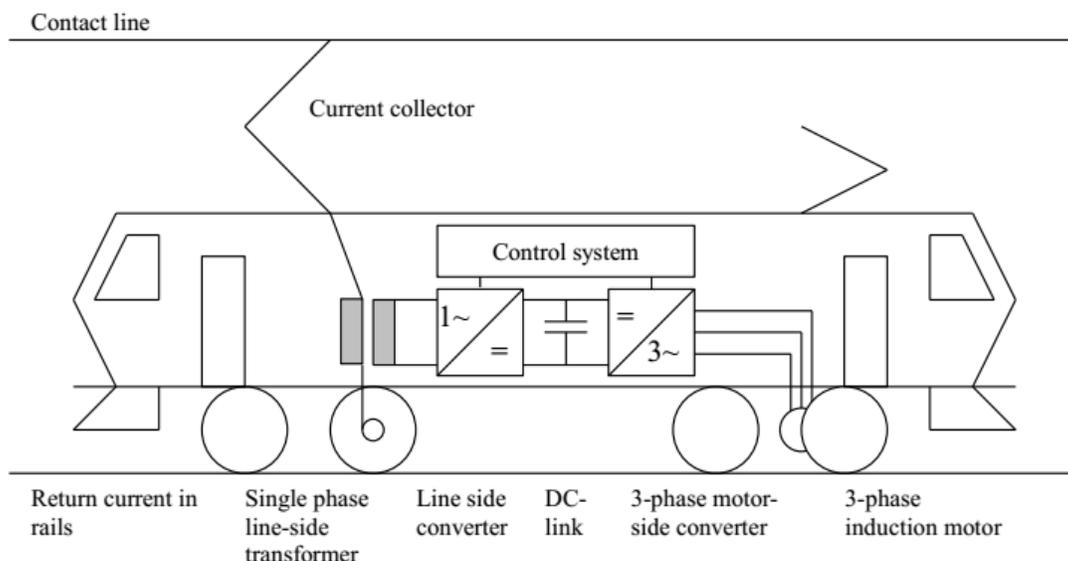


Figure 3.2 Electric Locomotive Block Diagram and its Basic Components [32]

The distribution system consists of overhead contact line (OCL) system structure with section posts, positioning devices, compensating devices, etc. The OCS (Overhead Contact System-system of electrification network designed and commissioned by railway agencies) serves two main purposes; first it distributes the power to the electric trains and second it establishes the electrical connection between the stationary power supply and the moving traction load.

Following the above general case, to supply traction power to moving trains, railway electrification technology options include DC electrification systems, AC electrification systems at commercial frequency or at non-commercial frequency, combination of AC and DC electrification systems and alternative system technologies and enhancements [24]. The fundamental difference between the two systems is that, i.e. DC and AC, in DC systems each substation consists of transformers and rectifiers which condition the power to relatively low voltage suitable for direct use by the vehicle propulsion equipment; while in AC powered systems electrical power is supplied to traction motors from traction substations directly, without rectification, at relatively high voltage necessitating further transformation on-board of the rolling stock for the voltage to be suitable for use by the vehicle propulsion equipment [24].

Historically, voltage levels from 50 V up to 50 kV have been used to propel railroad vehicles around the world [16]. Today, there are six voltage levels that are standardized according to IEC 60850 and these are also the most common ones used in all railway, tramway and subway systems which are the nominal voltages in the system [16]. In addition non-standard voltage levels such as 50kV and 2x25 kV, 50 Hz are also available options proven to be practical in high capacity railway lines. The low voltage DC system is used for light rails usually supplied at 750 Volts (metro transit) and for commuter and intercity trains usually supplied at 3000 Volts.

Table 3.1 Standardized Voltage Levels [16]

DC Voltage				AC Voltage	
600V	750V	1500V	3000V	15kV, 16.7Hz	25kV, 50/60Hz

Practically, technology selection from any one of these alternatives depends on different criterion underlying that the selected technology being proven to be technically feasible, economically viable, and compatible with the reference case infrastructure and the required transport service level [24].

Moreover, electrification system technology selection i.e. AC or DC standard is determined by: power loss, voltage drop reduction; installed equipment amount and cost; local laws and regulations; design, construction, maintenance and operation or life time costs; line capacity; transmission distance; network complexity; voltage conversion level, etc.

### 3.3 Traction Substations

Traction substations, in electrified railway systems, are used to step-down the high voltages, 66kV, 110kV, 132kV, 220kV, 230kV, etc. from the utility side to the voltage level required by the selected electrification system technology option, mostly 12.5kV, 15kV, 25kV, and 50kV at industrial or non-industrial frequencies and/or condition the stepped-down voltage to the DC voltages as per the electrification system requirements. The 25kV electrification at industrial frequency is recognized as world standard traction electrification system. Traction substations are commissioned at regular spacing determined by the traction power demand, train consistent size, train operation characteristics and the electrification system design [24].

The main component of traction substation is traction transformer. Each feeder substation typically consists of two equal rated power transformers; where one is in action and another is in hot standby. Each transformer should be rated to be capable of handling the entire substation load and to allow for continuous system feeding in the event of outage of one of the utility feeders, transformer, or other item of high voltage equipment [24]. In 25kV standard voltage electrification system, the feeding lines are connected to traction transformers through 27.5kV bus-bars in the substations and to overhead contact system of network outside the substation to the left and right side arms deliberately.

The rating of the traction transformers is calculated on the basis of primary voltage available across it and current it draws from this voltage;  $\text{Rated VA} = (\text{Line Voltage across} \times \text{Line Current through})$  [25]. For this type of traction load, the Line Voltage is regarded as the “Driving Voltage” to which the current to the system will lead or lag, according to the type of voltage. Metering on the primary side depends on the concepts of the connection of the load from the available voltage across the primary side [25]. Load connection is a design issue which is selected to mitigate some power system problems that may encounter during operation.

Technically speaking traction loads are single phase loads which injects undesirable harmonics and large negative sequence component (about half of the positive phase sequence component) to the utility grid due to their non-inevitable characteristics -i.e. - non-linear, non-sinusoidal, and non-symmetric and non-continuity [26]. Hence, the voltage across and currents through the feeding lines are not linear resulting in continuous pantograph voltage variation which directly affects the performance of electric traction equipment on-board a train and also power quality, stability and security of a whole power feeding section [21]. As a result, this has to be controlled and mitigated by some means at the design and installation level. The simplest mitigation methods are selection of appropriate voltage transformation equipment at the point of common coupling (PCC); traction transformers, and better voltage and current conditioning equipment on-board for further processing, IGBTs.

Mostly, for railway electrification purpose traction transformers with special connection-i.e. single-phase, single-phase Vee-Vee (V/V), three-phase Vee-Vee (V/V), Wye-Delta, YNd11, three-phase/two-phase balancing connection (Scott, Le-Black, and Wood-bridge), impedance-matching balance connection; are selected on the basis of electrical performance, physical profile of the network and economic issues and limit the level of power quality problems improving the efficiency and power factor of the utility grid [26]. Each connection scheme has different impact on the power system but also different investment, operation and maintenance costs [27].

In traction power system studies and analysis, traction transformer modeling and evaluation, especially, recalling voltage unbalance and the effects of harmonics and negative phase sequence currents, protection methods analysis, traction transformer and transmission line utilization factors analysis, etc. [26] are inevitable. Besides understanding the general and specific ideas and principles behind each transformer type and connection scheme, these studies have enabled scholars of the field to evaluate the effectiveness of the railway power supply system for the specific purpose the supply system is designed for [26]. Moreover, different expressions are used to relate the current through and voltage across traction transformers during the evaluation process. In this research report Equations ((3.1) through (3.5)) [16] are used to investigated the effects of unbalance loading and harmonic distortions. Indeed, these equations are also used to evaluate system utilization expressed in Line Utilization Factor (LUF) and Transformer Utilization Factor [16]. Analysis results are presented in their respective sections.

$$V(t) = V_o + \sqrt{2} \sum_{h \neq 0}^{\infty} V_h \sin(h\omega t + \alpha_h) \quad (3.1)$$

$$I(t) = I_o + \sqrt{2} \sum_{h \neq 0}^{\infty} I_h \sin(h\omega t + \beta_h) \quad (3.2)$$

$$S = VI^* \quad (3.3)$$

$$\begin{cases} |I_R| \leq |I_L| \\ BD_1 = \frac{|I_R|}{|I_L|}, 0 \leq BD_1 \leq 1 \end{cases} \quad (3.4)$$

$$\begin{cases} |I_L| \leq |I_R| \\ BD_2 = \frac{|I_L|}{|I_R|}, 0 \leq BD_2 \leq 1 \end{cases} \quad (3.5)$$

Where V and I denote the effective (RMS) phase voltage and current respectively,  $BD_1$  and  $BD_2$  are balance degree at both secondary sides of substation and S is the apparent power at the single phase system [26]. Due to identical analysis results for both balance degrees the following investigations are done only for  $BD_1$  equals BD.

### 3.3.1 Single Phase Connection of Traction Transformers

The single phase traction transformer connection employs certain combination of any two of the three phases of the utility grid. In the combination careful phase rotation is required to maintain load balance on the three phases of the primary side. The transformer secondary supplies the left-right arms equally at the same phase. Figure 3.3 below shows single phase traction transformer connection scheme and the equivalent circuit. The output voltage, primary and secondary currents relationships can be shown as:

$$U_{bc,L} = U_{bc,R} = \frac{1}{K_T} V_{BC} = \sqrt{3} \frac{1}{K_T} V_p e^{j0} = E e^{j0} \quad (3.6)$$

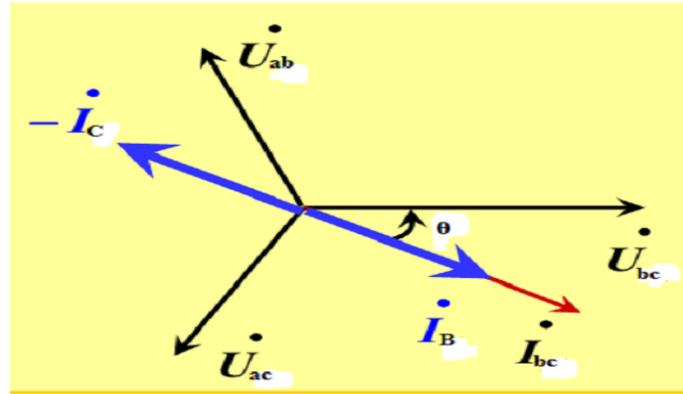
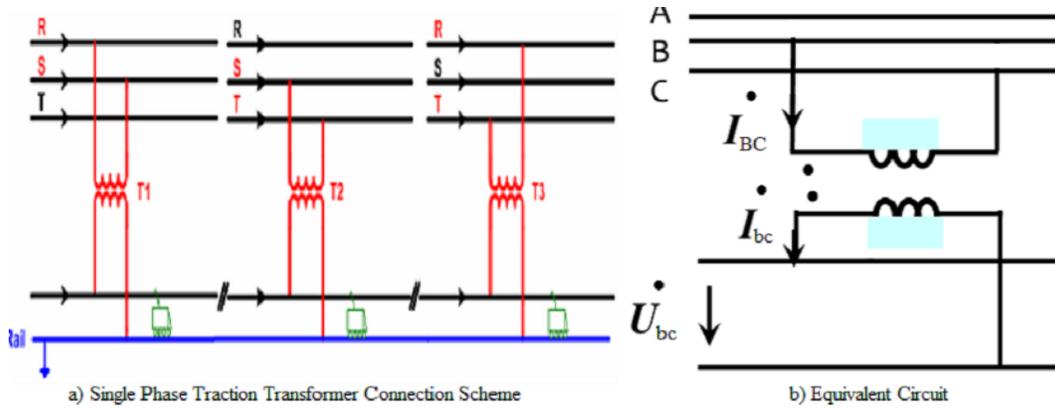
$$I_A = 0 \quad (3.7a)$$

$$I_B = \frac{I_{bc}}{K_T} = \frac{1}{K_T} (1 + BD) I e^{-j\theta} \quad (3.7b)$$

$$I_C = -I_B \quad (3.7c)$$

$$I_L = I e^{-j\theta} \quad (3.8a)$$

$$I_R = BD I e^{-j\theta} \quad (3.8b)$$



c) Phasor Diagram

Figure 3.3 Single Phase Traction Transformer Connection Scheme, Equivalent Circuit and Phasor Diagram

Similarly, the apparent powers of the three phases of the utility grid ( $S_A$ ,  $S_B$  and  $S_C$ ), the maximum capacity utilization ( $S_R$ ), and the transformer winding capacity ( $S_1$  and  $S_2$ ), the transformer capacity ( $S_T$ ) and the line capacity ( $S_L$ ) can be computed respectively as follows:

$$S_A = 0 \quad (3.9a)$$

$$S_B = \frac{(1 + BD)EI}{\sqrt{3}} e^{j(\theta - \frac{\pi}{6})} \quad (3.9b)$$

$$S_C = \frac{(1 + BD)EI}{\sqrt{3}} e^{j(\theta + \frac{\pi}{6})} \quad (3.9c)$$

$$S_R = |S_A + S_B + S_C| = (1 + BD)EI \quad (3.10)$$

$$S_1 = S_2 = EIe^{j\theta} \quad (3.11)$$

$$S_T = |S_1| + |S_2| = 2EI \quad (3.12)$$

$$S_L = |3V_B I_B^*| = 2\sqrt{3}EI \quad (3.13)$$

### 3.3.2 Three Phase Vv Connection of Traction Transformers

In three phases Vv-connection scheme the primary side is the utility side balanced three phase electrical; utility grid side, power source while the secondary side is a combination of two single phase transformer arrangements. Moreover, the phasor diagram shows that the secondary side voltages,  $U_{bc}$  and  $U_{ac}$  are in phase with the primary side line voltages  $V_{BC}$  and  $V_{AC}$  respectively but  $60^\circ$  out-of phase from each other.

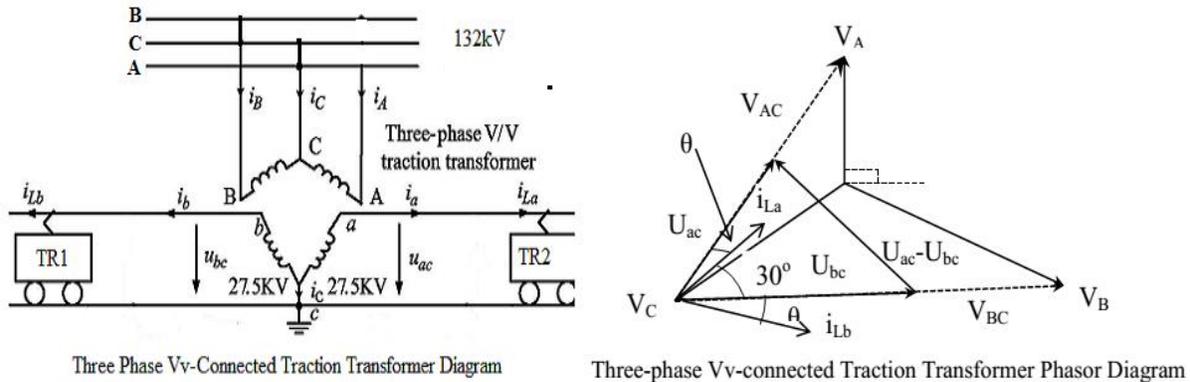


Figure 3.4 Three Phase Vv-Connected Traction Transformer Connection Scheme

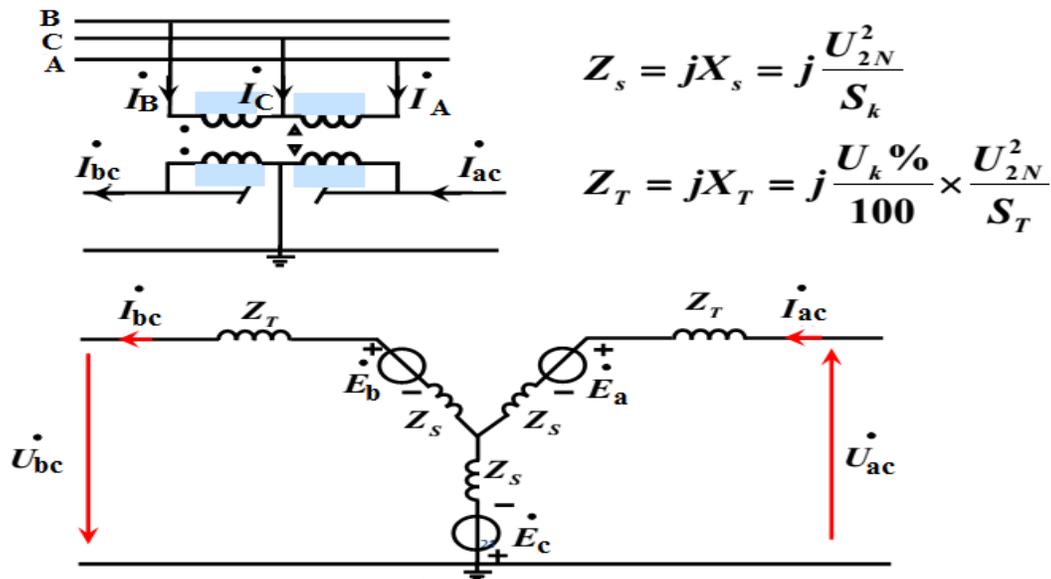


Figure 3.5 Three Phase Vv-connected Traction Transformer Equivalent Circuit

Where:  $S_k$  = power system (primary side) short circuit capacity (MVA)

$S_T$  = traction transformer capacity (MVA)

$U_k\%$  = short circuit voltage (generally 10.5%)

Hence, output voltage, primary and secondary currents, taking  $V_{BC}$  as reference voltage, can be calculated using the following equations:

$$U_{bc} = \frac{1}{K_T} V_{BC} = \sqrt{3} \frac{1}{K_T} V_p e^{j0} = U_{bc} e^{j0} \quad (3.14a)$$

$$U_{ac} = \frac{1}{K_T} V_{AC} = \sqrt{3} \frac{1}{K_T} V_p e^{j\frac{\pi}{3}} = U_{bc} e^{j\frac{\pi}{3}} \quad (3.14b)$$

$$i_{La} = \frac{U_{ac}}{Z_L} e^{j(\frac{\pi}{3}-\theta)} \quad (3.15a)$$

$$i_{Lb} = \frac{U_{bc}}{Z_R} e^{-j\theta} = BD \frac{U_{ac}}{Z_L} e^{-j\theta} \quad (3.15b)$$

$$I_A = \frac{1}{K_T} I_{ac} = \frac{1}{K_T} I e^{j(\frac{\pi}{3}-\theta)} \quad (3.16a)$$

$$I_B = \frac{1}{K_T} I_{bc} = \frac{1}{K_T} BD I e^{-j\theta} \quad (3.16b)$$

$$I_C = -(I_A + I_B) \quad (3.16c)$$

Similarly, the three phase apparent powers the utility grid ( $S_A$ ,  $S_B$  and  $S_C$ ), the maximum capacity utilization ( $S_R$ ), and the transformer winding capacity ( $S_1$  and  $S_2$ ), the transformer capacity ( $S_T$ ) and the line capacity ( $S_L$ ) can be computed respectively as follows:

$$S_A = V_A I_A^* \quad (3.17a)$$

$$V_A = K_T \frac{U_{ac}}{\sqrt{3}} e^{j\frac{\pi}{6}} = K_T \frac{U_{bc} e^{j\frac{\pi}{3}}}{\sqrt{3}} e^{j\frac{\pi}{6}} = K_T \frac{U_{bc}}{\sqrt{3}} e^{j\frac{\pi}{2}}$$

$$I_A^* = \frac{1}{K_T} I e^{-j(\frac{\pi}{3}-\theta)} = \frac{1}{K_T} I e^{j(\theta-\frac{\pi}{3})}$$

$$S_A = \frac{E}{\sqrt{3}} I e^{j(\theta+\frac{\pi}{6})} \quad (3.17b)$$

$$S_B = V_B I_B^* \quad (3.18a)$$

$$V_B = K_T \frac{U_{bc}}{\sqrt{3}} e^{-j\frac{\pi}{6}}$$

$$I_B^* = \frac{1}{K_T} I^* = \frac{1}{K_T} BD I e^{j\theta}$$

$$S_B = \frac{E}{\sqrt{3}} BD I e^{j(\theta-\frac{\pi}{6})} \quad (3.18b)$$

$$S_C = V_C I_C^* \quad (3.19a)$$

$$\begin{aligned}
V_C &= -K_T \frac{U_{ac}}{\sqrt{3}} e^{-j\frac{\pi}{6}} = -K_T \frac{U_{bc}}{\sqrt{3}} e^{j\frac{\pi}{6}} \\
I^*_C &= -(I^*_A + I^*_B) = -\left(\frac{1}{K_T} I e^{j(\theta-\frac{\pi}{3})} + \frac{1}{K_T} BDI e^{j\theta}\right) \\
S_C &= \frac{1}{K_T} \left\{ -K_T \frac{E}{\sqrt{3}} e^{j\frac{\pi}{6}} \right\} * \left\{ -\left( I e^{j(\theta-\frac{\pi}{3})} + BDI e^{j\theta} \right) \right\} \\
S_C &= \frac{E}{\sqrt{3}} I \left\{ 1 e^{j(\theta-\frac{\pi}{6})} + BD e^{j(\theta+\frac{\pi}{6})} \right\} \tag{3.19b}
\end{aligned}$$

$$S_R = |S_A + S_B + S_C| = (1 + BD)EI \tag{3.20}$$

The above equations show that the primary sides,  $S_A$  and  $S_B$ , of the Vv-transformer supply the two feeding sections individually at different power factors, one at  $\theta+30^\circ$  and the second at  $\theta-30^\circ$  while the third phase,  $S_C$ , tries to balance the unbalancing degree effect. The maximum transformer capacity is the algebraic summation of the two single phase transformer winding capacity under balanced load conditions at both secondary sides of the power substation.

$$S_1 = S_2 = EI e^{j\theta} \tag{3.21}$$

$$S_T = |S_1| + |S_2| = 2EI \tag{3.22}$$

The line capacity is calculated based on  $I_C$  which is the maximum current at the utility grid.

$$S_L = |3V_C I^*_C| = 3EI \tag{3.23}$$

### 3.4 Utilization Factors

#### a) Maximum Capacity Utilization

This parameter presents the maximum capacity utilization of the utility grid including the traction transformer and transmission line. It is influenced by load harmonics as well as the unbalance loading and can be calculated by the following equation:

$$S_R = S_A + S_B + S_C \tag{3.24}$$

Where:  $- S_A, S_B$  and  $S_C$  are the phase apparent powers.

#### b) Transformer Capacity

The transformer capacity is calculated considering maximum load balance condition and computed as follows:

$$S_T = \sum_{j=1}^{\infty} S_j \tag{3.25}$$

Where:  $- S_j$  is the transformer winding capacity.

### c) Line Capacity

Assuming maximum load balance demand calculated between three phase currents (A, B, C) to be  $I_{MAX}$ , it is calculated as:

$$S_L = 3VI_{MAX}^* \quad (3.26)$$

According to the above definitions, the Transformer Utilization Factor (TUF) and the Line Utilization Factor (LUF), respectively, can be defined as:

$$TUF = \frac{S_R}{S_T} \quad (3.27)$$

$$LUF = \frac{S_R}{S_L} \quad (3.28)$$

## 3.5 Traction Power Distribution and Feeding Mode

Power supply mode is the connection mode between power grid and the traction power supply system network. It depends on voltage level of the traction load and power grid distribution. In AC traction power supply system power supplied to traction motors can apply several feeding modes, such as direct power supply mode (TR), direct power supply with return wire mode (DR); boost transformer power supply (BT), auto transformer power supply mode(AT), and so on. Power supply mode selection is a design issue determined by many factors that need to be considered for effective power distribution to meet the transportation plan required.

Some of the design requirements that have to be compared in the engineering selection process are traction network complexity, traction network equivalent impedance voltage drop and electrical losses, over loading, feeder voltage, feeding section length; i.e. number of BT and AT, rail potential and communication interference, corridor capacity required; i.e. number of trains, operation safety and train speed, power reliability, etc. Figure 3.6 below outlines the four basic power supply modes listed above.

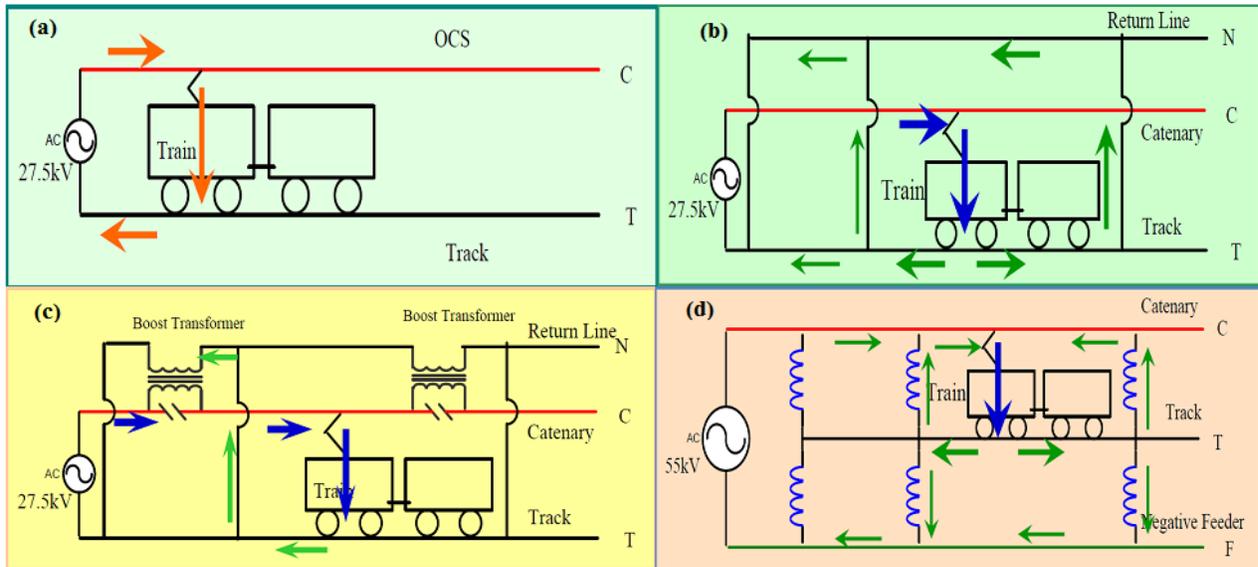


Figure 3.6 Traction Power Supply Modes: (a) Direct Power Supply (b) Direct Power with Return Wire (c) Boost Transformer and (d) Power Supply with Auto Transformer

Compared to each other the above power supply modes give different results determined by the arrangement of traction network components and the available utility grid side power. Hence, selection of either of the supply modes is the merit of the owner of transport service.

### 3.6 Traction Power Flow modeling

Traction power flow analysis requires accurate modeling of all the components in the electrified railway system. The general power system study diagram and the corresponding equivalent circuit shown in Figure 3.1 shows the main power supply subsystem components to be modeled include: the grid side transmission line components, the traction transformer, traction power feeding, the traction power return system and the load. The grid side transmission line and traction transformer component parameters are static and easily obtained directly either by analytical calculations fulfilling design requirements or from name plate readings for the models while feeder line, return wire and traction load components vary with the instantaneous location of the load, train, which needs dynamic modeling for each mode of operation.

Although the properties of the AC railway power feeding systems are intrinsically nonlinear, there are some simplifications of the power network modeling. In this simplified modeling approach, impedances of the overhead catenary wire, rails, return conductor and other equipment such as Boost transformers (BTs) are all added in series so that a single-phase AC equivalent

circuit consisting of an average lumped conductor parameters is formed as shown in Figure 3.7 [21]. This approach is adequate to calculate voltages across trains and phase-to-ground, substation and section post voltages [21]. However, rail potential, rail-to-earth leakage current, Electromagnetic Interference (EMI) effects, etc. cannot be determined using this simplified model [21] and are not included in the model of this research.

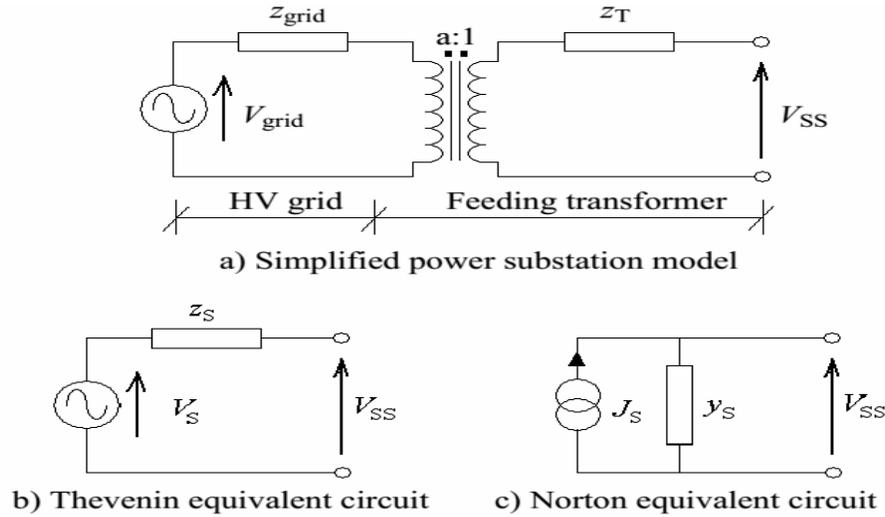


Figure 3.7 Power Substation Model

$$V_s = \frac{V_{grid}}{a}; Z_s = Z_T + \frac{Z_{grid}}{a^2}, y_s = \frac{1}{Z_s} \quad (3.29)$$

The other key power system component is the electric train which requires a simple model to reduce problem complexity and overall execution time [21]. AC locomotives are classified into two main categories: diode or thyristor phase-angle controlled locomotives (PAC locomotives) and a modern PWM (Pulse Width Modulated) locomotive [21]. The Power factor during service for the former locomotive type is typically between 0.5 and 0.85 which leads to a significant amount of reactive power being drawn from the power supply while in the later type a PWM converter replaces the phase-angle controlled converter and can operate at high power factors, such as 0.95 -1.00 [21]. Notably, with a suitable PWM converter design, it can also operate at leading power factor [21]. The general locomotive block diagram and the corresponding different equivalent circuit models used for analytical calculation and simulation purposes are presented in Figure 3.8 below [21].

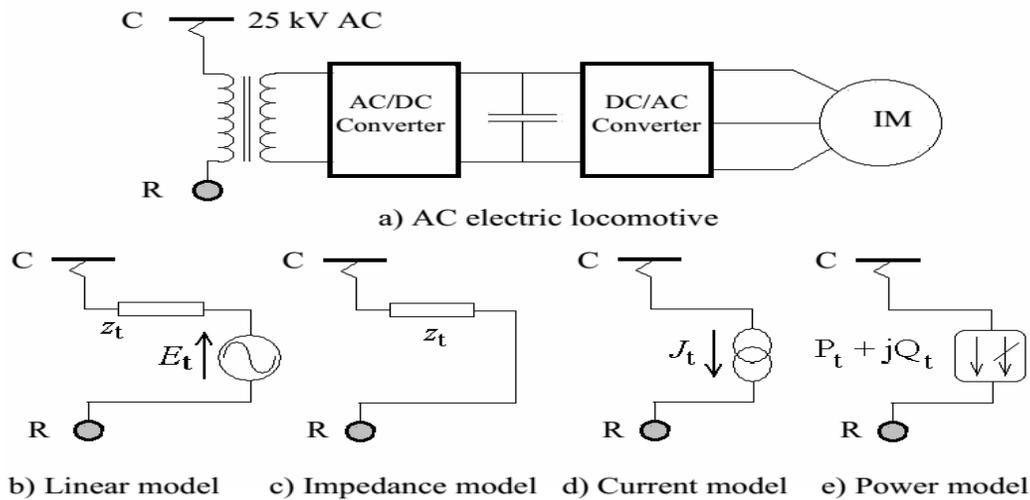


Figure 3.8 Four locomotive Models for Power Flow Calculation

In practice, only four electrical quantities (Pantograph voltage-magnitude, input current-magnitude, power and power factor) can be either measured on-board a train or obtained from analytically calculation or the design requirements and any model that uses a well-chosen sub-set of these quantities is satisfactory [21]. The four basic load models are discussed as follow [21]:

a) Linear Model

Although the voltage source ( $E_t$ ) and the series impedance ( $Z_t$ ) representing this model can be determined as a function of speed, pantograph voltage and tractive effort over the whole range of operation, direct calculation cannot be done because of the phase angle of the internal voltage source, which must be referenced to the substation. Thus, a look-up table is used to define the model parameters at any instance according to train speed, voltage and tractive effort. The linear model is shown in Figure 3.8 (b).

b) Impedance or Admittance Model

The effective model is the one that uses only the four measured quantities. The impedance magnitude is calculated by the ratio between the voltage and current magnitudes whereas the phase angle is simply determined from the power factor interpretation. The impedance model is given in Figure 3.8 (c).

c) Constant Current Model

Although the current model is not appropriate due to the unknown phase angle, a numerical database stored for this model with respect to train speed, pantograph voltage and tractive effort

like the linear model is another effective way. The lookup table is required to obtain its parameters at any simulated time step. Figure 3.8 (d) presents the current model.

#### d) Constant Power Model

This model is widely used in three-phase power flow problems. Powers and power factor are the two quantities that can be measured by the on-board traction controller. Furthermore, applying tractive effort versus speed and train running information with the Newton's Second Law of motion, the model parameters, power and power factor can alternatively be calculated. Figure 3.8 (e) shows the power model. In this research constant power model is used to model all the traction power supply system components.

### 3.7 Traction Network Model

The modeling of traction network includes obtaining traction network conductor parameters, calculation of the per unit length impedance, determining the traction network equivalent circuit, analysis of the traction network under different traction power supply modes and traction network voltage loss calculations [27]. Conductor parameters such as cross sectional area, internal resistance at required temperature, etc. are obtained from standard requirement tables while per unit length impedances are calculated by considering the conductors parameters in combination with the network forming conductors arrangement [27].

Selection of the conductor type depends on the electrical characteristics, mechanical strength and working temperature. For instance, copper conductor, the most popular material used to produce the contact wires has an outstanding electrical conductivity and holds at the same time very good mechanical properties, which, however, are reduced at elevated temperatures of 80°C [27]. Due to this fact, the copper contact wires cannot be too strongly loaded, which in turn, influence the maximum speed boundary for trains to be around 160 km/h [27]. The resolution to this trouble is to add to copper the small quantities of alloy additives, which by no means or to some extent lower the conductivity of conductor, while rising its mechanical properties and increasing the working temperature [27]. Therefore, CuAg0, 10 type conductors have a larger current transporting capacity, which translates directly to increase train speed even up to 250 km/h. Table 3.2a below give typical conductor parameters [27]. The parameters of the overhead wire are based on the resistance (power loss) and inductance, which can be written as  $Z$  [27]. In fact both self-impedance and mutual impedances are influenced by the ground return current

[27]. The method which is applied for calculation of parameters is the well-known Carson's method, according to which, the formulas of the wire's self and mutual impedance, respectively, in AC frequency are [27]:

$$Z_{ii} = r_i + 0.049 + j0.14461g \frac{D_g}{R_{ei}} \left( \frac{\Omega}{km} \right) \quad (3.30)$$

$$Z_{ij} = 0.05 + j0.14461g \frac{D_g}{d_{ij}} \left( \frac{\Omega}{km} \right) \quad (3.31)$$

where:  $D_g = \frac{0.2085}{\sqrt{f\delta 10^{-9}}} cm$

= 930mm (in the absence of information) Distance b/n conductor & ground

$f$ : current frequency,  $\delta$  the ground conductivity [ $(\Omega cm)^{-1}$ ];

$r_i$ : wire's internal resistance and  $R_{ei}$  wire's equivalent radius (mm)

$d_{ij}$ : the distance between wire  $i$  and  $j$

Table 3.2 Electrical parameters for Contact Wire and Catenary [27]

(a) parameter of contact wire and catenary

Models	AC resistance ( $\Omega/km$ ) $r_i$	Equivalent radius (mm) $R_{ei}$
Copper contact line TCG-100	0.179	4.6
Steel, aluminum contact line GLCA-100/215	0.184	8.56
Bare copper stranded wire TJ-95	0.20	4.74
Copper and aluminum wire LGJ-100	0.255	7.22
Steel strand GJ-100	1.45	6.18

(b) Equivalent radius

Steel rail number	G(kg/m)	$R_{g0}$ (mm)	$R_{eg}$ (mm)
P <sub>43</sub>	44.65	140	11.2
P <sub>50</sub>	50.51	152	12.2

The impedance of the overhead catenary feeding system can be modeled by the equivalent component parameters as described in Figure 3.8. In linear model by using the instantaneous voltage drop; impedance model by characteristic impedance obtained from type of conductor used for the OCS and the arrangement of the conductor; constant current model; and constant power model by the power losses obtained analytical from the instantaneous location of the load. In any case the model has to be simple as much as possible similar to other component models. The general system model for single train in single track section using constant power model is shown as in Figure 3.9 below. Notice, in the model, as outlined above power losses are used to represent line and traction substation effects.

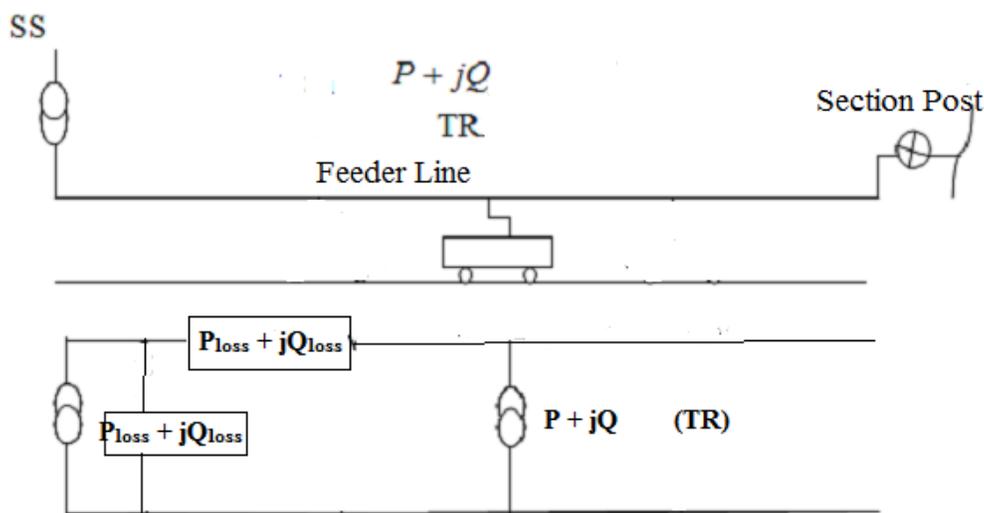


Figure 3.9 The overall Traction System Model in the Load Side for Power Flow Analysis

### 3.8 Power Quality Issues in Traction Power Supply System

An electrified traction power substation feeds non-linear and time varying single-phase traction load which injects large negative sequence component, undesirable harmonic current and unacceptable total demand distortion (TDD) to the utility grid [28]. Even though sequential phase rotation methods are used to reduce unbalances on the utility, the effects of voltage and current unbalances are significant when traction loads are running in a particular section of the line increasing the instantaneous current drawn in the section [28]. These problems, which result due to load non-linearities, cause additional system losses, rotating machine overheating, malfunctioning of protection relays and measuring instruments and further reduce the capacity of traction transformers and feeder lines [28].

Furthermore, in contrast to the common fixed single phase or three phase electrical energy consumers, traction loads are single phase electrical energy consumers, subjected to rapid fluctuations mainly due to route conditions- running speed, grade and curves- number of trains in the feeding section; change in the train's load, etc. Hence, train's power requirement changes with train's curvilinear path and operation modes-start, free-run, coast and stop- making the current drawn random variable. Owing to these load characteristics and many more active and passive electronic components used in the locomotive power conditioning, traction loads inject negative phase sequence currents, harmonics, phase unbalance effects, etc. to the utility side power supply system. Hence, in addition to near and far future load requirements, the utility side power system study need to consider all these effects outlined here above and a lot not stated here too.

Generally, the single-phase load characteristics of the train, connection scheme, curvilinear path of the train, train loading, power supply configuration, etc. try to worsen power quality seen by the utility grid [23]. The main power quality measures in electrified railway systems are voltage unbalance and harmonic distortions [23] [28] [29].

### 3.8.1 Voltage Unbalance

A three-phase power system is called balanced or symmetrical if the three-phase voltages and currents have the same amplitude and are phase shifted by  $120^\circ$  with respect to each other; however, if either or both of these conditions are not met, the system is called unbalanced or asymmetrical [28]. The system operator tries to provide a balanced system voltage at the point of common coupling (PCC) between the distribution grid and the customer's internal network. These voltage unbalances lead to power quality problems that affect the proper operation of the equipment connected at PCC or in other parts of the power system. Under normal conditions, these voltages are determined by the [28]:

- Terminal voltages of the generators
- Impedance of electricity system
- Currents drawn by the loads throughout the transmission and distribution grid.

Even though unbalances are observed to occur at all points before and after common coupling point, in most practical cases, the asymmetry of the loads, after common coupling point, is the main cause of unbalance [28]. In practical planning analysis, wherever possible, applying the method of phase rotation, efforts are made to distribute the single phase loads uniformly over the

three phases to ensure expected values of the loads in each phase will be approximately equal [28]. Even such efforts are made to keep the average loads in the three phases the same, the instantaneous power demands in the three phases differ from each other, leading to unbalanced voltages at the point of common coupling [28].

Asymmetric loads such as traction loads are the main sources of voltage unbalances at PCC. Besides being single phase loads they are time variant following the geographical feature of its path. Moreover, they are connected to the utility grid three-phase system phase-to-phase for single phase traction transformers and to the three-phases in three phase traction transformers but still the load effect is phase-to-phase due to transformer arrangements. The unequal phase loading of the three phases causes the utility system currents to be unbalanced. The flow of different currents in each phase results in unequal voltage drop in the three-phase utility network and this causes the utility voltages to be unbalanced. When loads on the zones of the traction substation are the same and equal i.e. when left arm and right arm load currents are same, the unbalance impact injected to the utility grid will be minimized [29].

The degree of unbalance of a three-phase voltage is often measured based on the ratio of its negative-to-positive-sequence components termed as the unbalance factor (UBF) [28]. The unbalance ratio using voltage terms,  $U^{(2)}$  and  $U^{(1)}$  negative and positive phase sequence voltages respectively, is expressed as:

$$UBF = \frac{U^{(2)}}{U^{(1)}} \quad (3.32)$$

Another approach to predict the level of voltage unbalance, especially for traction loads connected between two of the three phases at PCC through single phase, three phase Vv and Scott/Le Blanc traction transformer respectively, is [29]:

$$UBF[\%] = \frac{S_L}{S_k} * 100 \quad (3.33)$$

$$UBF[\%] = \left| \frac{S_{L2} + a^2 S_{L1}}{S_k} \right| * 100 \quad (3.34)$$

$$UBF[\%] = \left| \frac{S_{L2} - S_{L1}}{S_k} \right| * 100 \quad (3.35)$$

In some cases, especially for power converters, the information regarding phase shift between negative and positive sequence voltages is very important and in these conditions the determination of complex voltage UBF can ensure useful data for analyzing the unbalance perturbation injected in the supply system [29]. The complex voltage unbalance factor is given by [29]:

$$\overline{UBF} = UBF \cdot e^{j\psi_u} \quad (3.36)$$

The exact method for determining the voltage unbalance factor, after measuring the RMS values of the line voltages, is [29]

$$UBF[\%] = \sqrt{\frac{1 - \sqrt{3 - 6\beta_u}}{1 + \sqrt{3 - 6\beta_u}}} * 100 \quad (3.37)$$

With:

$$\beta_u = \frac{(V_{AB}^4 + V_{BC}^4 + V_{CA}^4)}{(V_{AB}^2 + V_{BC}^2 + V_{CA}^2)^2} \quad (3.38)$$

The phase shift is given as:

$$\psi_u = \tan^{-1} \left( \frac{\sqrt{3((V_{AB}^2 - V_{CA}^2))}}{(V_{AB}^2 + V_{BC}^2 + V_{CA}^2)} \right) \quad (3.39)$$

Where:  $V_{AB}$ ,  $V_{BC}$  and  $V_{CA}$  are line-to-line RMS voltages.

The exact expressions, Eq. (3.37), Eq. (3.38) and Eq. (3.39) for computing the voltage UBF are difficult to use in practice [29]. An acceptable compromise brings to an approximate analytical expression of the voltage UBF derived to the series development of Eq. (3.37) [29]. The value of the UBF depends on the voltage level to which the load is connected [29]. European countries assume UBF% less than or equal to 2% for LV and MV; less than or equal to 1% for HV; particularly United Kingdom adopted UBF% less than or equal to 2% for HV and France, for the case of High Speed Trains (HST) too adopted UBF% less than or equal to 1% for HV for period higher 15min and UBF% less than or equal to 1.5% for HV for period less than 15min [29].

Unbalances could be mitigated using different approaches with different degrees of technical complexity. The first and most basic solution is to rearrange or redistribute the loads in such a way that the system becomes more balanced. For certain applications, there is a possibility of reducing unbalance by changing the operating parameters [28]. In order to reduce the influence of negative sequence currents, causing negative sequence voltage drops, on the supply voltage, low internal system impedance is required. This may be achieved by connecting the unbalanced

loads at points with higher short circuit level, or by other system measures to reduce the internal impedance. Another type of mitigation technique is the use of special transformers, such as Scott- and Vv- connected transformers.

### 3.8.2 Harmonics

Harmonics in power distribution system are current or voltage wave forms that are integer multiples of fundamental frequency. The non-linear load connected to the power system distribution will generate harmonics current and voltage. The harmonics current injected on power distribution system caused by nonlinear load, and they can damage equipment overtime by sustained overheating or cause sudden failures due to resonant conditions further reducing system performance. Different terms are used to quantify the distortion some according to the analysis method intended such as harmonic distortion (HD), total harmonics distortion (THD) and total demand distortion. Total Demand Distortion is harmonic current distortion in % of maximum demand load current (15 or 30 min demand) [30]. TDD is the root-sum square, or square root of the sum of individual harmonic components squared which is very much like THD [30]. The terms THD and TDD are calculated, respectively, in terms of current as [30]:

$$THD = \frac{\sqrt{I_2^2 + I_3^2 + I_4^2 + \dots}}{I_1} \quad (3.40)$$

$$TDD = \frac{\sqrt{I_2^2 + I_3^2 + I_4^2 + \dots}}{I_L} \quad (3.41)$$

Another distortion quantifier term is individual distortion, HD, which is the distortion of each harmonic current or voltage wave form with respect to the fundamental component and calculated as using current terms [31]:

$$HD_h = \frac{I_h}{I_1} 100\% \quad (3.42)$$

Where;  $I_h$  in Equations (3.40-3.41) refers the  $h^{\text{th}}$  harmonic components current values obtained by Fourier series expression of the load current. Notice that, in traction load analysis only odd harmonic current and voltage wave forms exist and produce the distortion effect to the fundamental system component, see Appendix C.

### 3.9 Railway Electrification System Design Considerations

A railway electrification system designed for a particular transportation service planned by the required transportation capacity has to provide a reliable, cost effective, stable, efficient, and economical operational features required from the system throughout its life time. Moreover, rail electrification design should consider the rated current and nominal voltage used for train traction. To increase the operational performance of electrified trains, sufficient current-carrying capacity should be provided through overhead contact lines to train engines. Contact lines should be installed to provide short-circuit current capacity. Overhead contact lines should be designed to be compatible with rail track design, gauge clearance, and geometric alignments (horizontal and vertical alignments). Maximum train length and train operating speed affect the length of platforms, spans of overhead contact lines, locations of signals, etc. [15].

For any stable system designed satisfying all the requirements outlined above it is natural and practicable that the demand continually grows and nearly reaches the limit of traction power supplies that were previously designed [21]. Thus the existing electrification systems need to be upgraded to handle the new added power requirement. Nevertheless, to upgrade the railway system is obviously critical [21]. Technically, many related approaches have to be taken into account [21]. The replacement of feeding cables or substation transformers is one of many possible solutions to increase power-feeding capacity but this inevitably interrupts the services [21] and cost wise it is not a profitable solution. The other approach that is widely applied to typical railway systems is the use of additional reactive power sources, e.g. SVC (Static var Compensator) [21]. Anyway the selected upgrading solution shouldn't be costly, interruptive, should be compatible with the existing reference system, etc.

### 3.10 Ethiopian Railway Traction Power Supply System Plan

Ethiopia/Sebeta ~ Djibouti/Nagad New Standard-gauge Railway starts from Sebeta Station in the south, runs towards northeast, passes through Addis Ababa and Adama, reaches Mieso, then crosses the border of Ethiopia and Djibouti at Dawalle, and gets to Nagad of Djibouti, which is an important part of the long-term railway network planning of Ethiopia with the following main lines technical scopes [8] [9] [31]:

- Sebeta~Mieso, length of the line is about 327km;
- Mieso~Dewalle (located in Ethiopia and on Ethiopia-Djibouti border), about 343km;
- Dewalle~Nagad (front-port station of Djibouti), about 82km long.

These three give a total distance of about 752km.

Table 3.3 Ethio-Djibouti Railway General System Technical Specifications [31]

Description	Sebeta ~ Adama (included)	Adama ~ Nagad
Gauge	1435mm	1435mm
Number of main line	Double line	Single Line
Target speed	Passenger: 120km/h Freight: 80km/h	Passenger: 120km/h Freight: 80km/h
Minimum curve radius	800m	800m
Maximum gradient	Ruling gradient 9‰, pusher grade 18.5‰	Ruling gradient 9‰, pusher grade 18.5‰
Traction type	Electric	Electric
Locomotive type	HXD series	HXD series
Traction mass	Preliminary stage: 3500t Long-term: 4000t	Preliminary stage: 3500t Long-term: 4000t
Length of arrival & departure track	850m (Dual loco. 880m)	850m (Dual loco. 880m)
Distance between lines	4.0m	-
Block type	Semi-automatic block	Semi-automatic block

The Ethiopia's national rail network, Ethio-Djibouti corridor in particular, traction network feeding system is proposed to adopt, 1x25kV 50Hz world standard, direct feeding system with return wire power supply mode [8] [9] [23], see Figure (3.6b). The single phase 50Hz power supply for railway traction at 25kV is obtained, from 132kV three phase grid systems supplied by Ethiopian Electric Power Corporation (EEPCo) through a step down transformers, single phase traction transformer at Sebeta and Nagad (end substations) and three phase Vv-connected traction transformers at the rest substations [8] [9] [23]. Table 3.4 below gives the main technical descriptions of the corridor power supply system specifications considered.

Table 3.4 Ethio-Djibouti Railway Power Supply System Technical Description [8] [9] [31]

Description	Sebeta~Nagad
<b>Traction Power Supply System</b>	
Feeding System-the standard	25kV, 50Hz Direct power feeding with return wire mode
Nominal Voltage	25kV
Short-term	29kV (maximum)
Operating voltage	20kV (minimum)
Abnormal operating voltage	19kV
Insulator	Porcelain
Number and location of substations (with average distance of 35.81km)	<b>21+1</b> ; at :Sebeta, Indode, Bishoftu, Mojo, Merebe, Senga Beret, Metehara, Awash, Sirba-Kunkur, Mieso, AK313 (long term), Afdem, Gota, Hurso, Lonnis, Mile, Adigala, Lasarat, Suburban 8, Alisabieh, Holhol, and Nagad
<b>Overhead Contact System (OCS)</b>	
Operation Mode	<b>Unilateral</b> for single track
	<b>Bilateral</b> for double track
Suspension type	Full compensation simplified catenary
Tensioning mechanism	Auto-tensioned catenary
Contact wire height	6000mm (at contact point)
Minimum clearance	5750mm
System height	1400mm
Shortest dropper length	500mm
Vehicle loading height	5300mm (no longer than)
Span (at straight line)	60m
Maximum span	65m (not longer than)
Overlapping section length	Not greater than 2x800m for bilateral power supply operation
	50% of bilateral for unilateral power supply operation
Return line	Insulated installation
<b>Traction Substation</b>	
Primary side voltage	<b>Two</b> separate and reliable <b>132kV</b> from utility side
	Adopts phase sequence rotation mode
	System short circuit capacity <b>400MVA</b>
Utility grid side substations	<b>1x400kV, 2x230kV and 6x132kV</b>
Traction Transformer	Single-phase (at Sebeta and Nagad Substations)
	Three-phase, <b>Vv</b> -connection (at the rest stations)
	Fixed reservation system (as hot spare)
	Oil-immersed self-cooled and reserved wind-cooled condition
	Outdoor layout
	<b>27.5kV</b> secondary side voltage
	<b>20MVA</b> with overload multiplier <b>2.0</b>
132kV traction transformer impedance voltage percentage 8.4%	
Power compensation	Compensator is installed at substations

## Chapter Four

### Ethiopian Railway Train Dynamics Overview and Power Analysis

#### 4.1 Single Track Operational Features and Assumptions

Train Operation over single track is much more difficult than over double track as it is more inherently dependent on trains meeting at fixed position at defined times [13]. Bidirectional traffic operation on a single track does not require the single track section (STS) to be divided in to several block sections to increase capacity [13]. This is because when a train occupies the STS, the train from the opposite direction is not allowed to enter the STS before the first train has exited the STS and released it. Conversely, it does not allow for two succeeding trains. In this report, for the calculations done on single track, it is assumed that alternate trains will run in the opposite direction and that all trains enter and exit the STS at full speed. Hence, the headway distance, the safe distance between two following trains, is assumed to be the block distance while the headway time is determined by the average speed of the train in the line section. Therefore, **the utilized capacity of the single track line is also speed dependent**. To **increase capacity over single track** it is necessary to operate trains near their design speed, which results in an increase in train frequency or increase train length or add more carts in order to accommodate more load or increase proportion of multiple track section length, if any, etc. In this research the first approach is proposed, operate trains near their design speed at all possible time.



Figure 4.1 Single Track Section and Passing Loops [13]

Another main operational features of single track rail systems is passing loops, also known as crossing loops, meetings points and sidings, -are short sections of an additional track adjacent to the main line. The passing loops are connected at both ends to the main line with turnouts where two trains travelling in opposite directions can pass each other, or trains travelling in the same direction overtake each other. As shown in the above figure passing loops are placed at the end of single track section. For the calculations presented in this report, in the Ethio-Djibouti corridor in general, passing loops are used either as train loading-unloading stations or simple train meeting and passing or over taking stations.

## 4.2 Track Line Sections under Consideration

The line section selected for analytical operation of train dynamics and the corresponding power consumption, the Awash~Sirba Kunkur~Sebaka line subsection is feed by two three phase Vv-connected traction transformer substations located at Awash, 11.4km of the line section, and at Sirba Kunkur, 22.38km (Awashit~Sirba Kunkur) and 11km (Sirba Kunkur~Sebaka) of the line section. Besides supplying the line section, the stations are designed for intermediate station, Awash, and passing station, Sirba Kunkur. Neutral sections or section posts are installed near Awashit and Sebaka to isolate the two feeding sections. The above described line section is selected for analysis purpose because of the geographical features observed on the preliminary design which is the most continuous gradient, down link (Addis Ababa to Mieso direction), and is part of the Ethiopian National Railway Plan on which Addis Ababa/Sebeta~Nagad and Awash~Woldiya/Hara Gebeya~Mekelle corridors share making it to be the most congested section in the future mid-to-long term plans since it is the immediate path to the port. Moreover, this thesis report principally focuses on the traction power consumption analysis for freight trains to investigate the operational speed in the line section and the corridor in general, to make the section occupancy time as short as possible increasing the frequency of services.

Traction power analysis requires accurate or reasonably approximated train dynamics models model analysis methods. Newton's Second law of Motion is used as train dynamics model and analytical method is used to evaluate the model in this report, within accepted level of dynamic assumptions. Moreover, the net tractive effort (drawbar) used to accelerate the train is taken to be the algebraic difference between tractive effort given by Eq. (2.7)-speed independent or Eq. (2.11)-speed dependent tractive effort and total resistances Eq. (A.7), see Appendix A. This relationship is applicable in the first two regions of induction motor operation; before balancing speed is reached.

Having these, now let us evaluate the general case for the three operation modes; namely: powering mode, coasting mode and breaking mode. Powering mode has three conditions when tractive effort is constant; is a function of speed; and when it balances the total drag force. The first two conditions forming accelerating mode and the last being constant speed condition. In coasting mode except the drag force power supply is disconnected. Similarly, during breaking only drag and break forces act on the train. Hence, these conditions determine the form of the

general net force acting on the train given by the following set of Eq. (4.1a) through Eq. (4.3). Notice in the models the maximum gradient and curve resistances at 800m are considered.

$$F_{net,1} = \frac{1000}{3.6} M_e \alpha_1 = 277.8 M_e \alpha_1 \quad (4.1a)$$

$$F_{net,2}(v) = \frac{3.6 * P}{v} - (785.6594 + 0.2825v + 0.02818v^2) kN \quad (4.1b)$$

$$F_{net,3}(v) = 0 = \frac{3.6 * P}{v_{max}} - (785.6594 + 0.2825v + 0.02818v^2) kN \quad (4.1c)$$

$$F_{net,4}(v) = -(F_{grade+curve} + 24.7212 + 0.2825v + 0.02818v^2) kN \quad (4.2)$$

$$F_{net,5}(v) = -B_e - (F_{grade+curve} + 24.7212 + 0.2825v + 0.02818v^2) kN \quad (4.3)$$

### 4.3 Tractive Effort versus Speed Curve Estimation

The tractive efforts versus train speed curve obtained by using Matlab plot of Eq. (2.7), Eq. (2.11) and Eq. (A.7), see Appendix A, is presented below for the two operating regions of traction motors. Eq. ((4.1a) through Eq. (4.3)) are evaluated using this graph to calculate running time and distances.

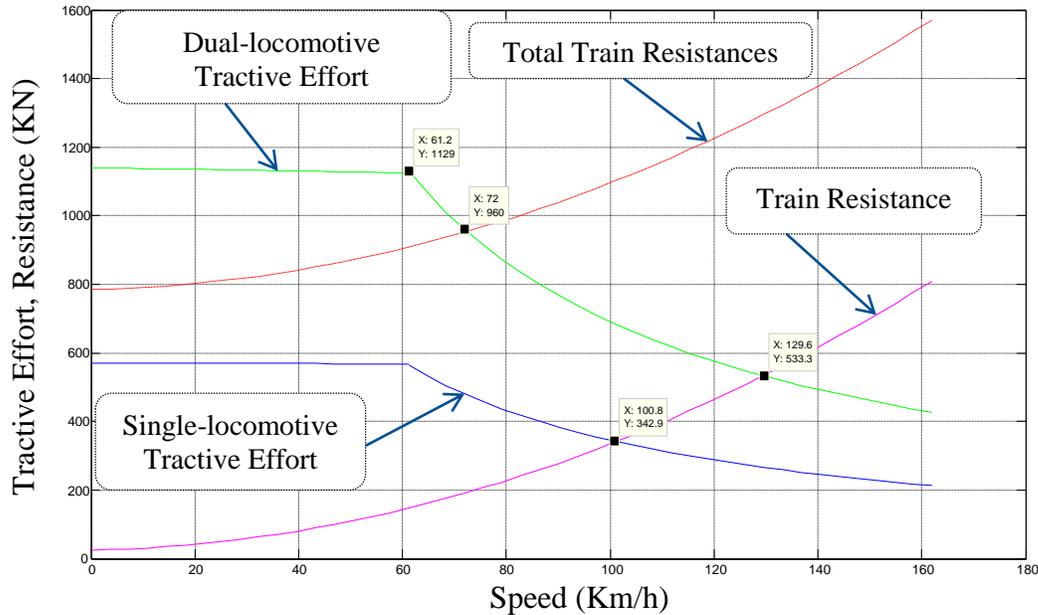


Figure 4.2 Tractive Effort Total Train Resistance versus Train Speed

#### 4.4 Running Time and Distance in the Driving Modes

The two physical quantities that could be calculated from train dynamics; namely: running time and running distance are obtained analytically using Eq. (2.15) and Eq. (2.16). Henceforth, these two quantities in the line subsection are calculated and the results are used for further energy calculation in the respective subsections.

#### Awash~Sirba Kunkur Line Subsection

##### 4.4.1 Accelerating to the Maximum Speed

During acceleration the train follows two conditions as described in Eq. (4.1a) and Eq. (4.1b) when the power drawn from the traction network increases gradually. The train driver has to control the tractive force so that the target accelerations of 0.2 to 0.4 kmphps for goods and 0.3 to 0.5 kmphps for parcel transportation may not be exceeded. The gradient near Awash Substation is observed to be 6.1 in 1000 and used to calculate grade resistance, see Eq. (A.3).

Constant tractive effort condition, Eq. (4.1a):

$$F_{net,1} = TE_{max} - R_e(0) = (570 - 264.0852)kN = 305.9148$$

To maintain an acceleration of nearly 0.3kmphps, the second locomotive needs to add about 50kN of force at the start or the two locomotives share a net force of 354kN and resistance force of 264kN. The force control is under the supervision of the driver.

$$\begin{aligned} \alpha_1[kmphps] &= \frac{F_{net}}{277.8M_e} = \frac{355}{277.8 * 4.4} = 0.3 \\ t_{acc,1}[s] &= \frac{M_e}{3.6} \int_0^{61.2} \frac{dv}{F_{net}} = \frac{277.8 * 4.4}{355} \int_0^{61.2} dv \\ &= 3.4496 * 61.2 = 211s = 3.5min \\ s_{acc,1}[m] &= \frac{M_e}{(3.6)^2} \int \frac{v}{F_{net}} dv = \frac{3.4496}{3.6} \int_0^{61.2} v dv \\ &= \frac{0.9582}{2} v^2 = 0.4791 * (61.2)^2 = 1794m \end{aligned}$$

In the constant power region, tractive effort is a function of speed, Eq. (4.1b), the traction motor is already supplied with its rated power of 9.6MW. Therefore, the power to weight ratio in the integral is 489.2966.

$$\begin{aligned}
t_{notch-up}[s] &= \frac{1100W_t}{3.6 * 9.81W_t} \int_{61.2}^{v_{max}} \frac{v dv}{\left(\frac{3.6 * P}{W_t} - (20.0219 + 0.0072v + 0.0007181v^2)v\right)} \\
&= 31.1474 \int_{61.2}^{72.4} \frac{1}{3.6} \left( \frac{v}{\frac{P}{W_t} - 5.5616v - 0.002v^2 - 0.0001995v^3} \right) dv
\end{aligned}$$

Since the integrand in the above integral equation is continuous over the integration range its limit exists. Hence, from the definition of definite integral; using integration as the sum of limit:

$$\int_a^b f(v)dv = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(v_i)\Delta v$$

where  $\Delta v = \frac{b-a}{n}$  and  $v_i = a + i * \Delta v$

$$f(v) = \frac{1}{3.6} \left( \frac{v}{489.2966 - 5.5616v - 0.002v^2 - 0.0001995v^3} \right)$$

for  $n = 140$ ;  $\Delta v = \frac{72 - 61.2}{140} = 0.1 \text{ kmph}$  and  $v_i = 61.2 + i * \Delta v$

$$t_{notch-up}[s] = 31.1474 \sum_{i=1}^{140} f(v_i) * \Delta v = 323.8142 \text{ sec} = 5.3969 \text{ min}$$

Similarly, the distance covered will be

$$\begin{aligned}
s_{notch-up}[m] &= 8.6521 \int_{61.2}^{72.4} \frac{1}{3.6} \left( \frac{v^2}{489.2966 - 5.5616v - 0.002v^2 - 0.0001995v^3} \right) dv \\
&= 6318 \text{ m}
\end{aligned}$$

Train acceleration during notch-up period will be

$$\alpha_2[\text{kmphps}] = 3.6 * \left\{ \frac{v_f^2 - v_i^2}{2s} \right\} = 3.6 * \left\{ \frac{\left(\frac{72.4}{3.6}\right)^2 - \left(\frac{61.2}{3.6}\right)^2}{2 * 6318} \right\} = 0.03289$$

#### 4.4.2 Constant Speed

At the end of two acceleration regimes, the train runs at the maximum speed with zero acceleration until coasting or braking is applied by the driver.

$$3.6 * \left( \frac{P}{W_t} - 5.5616v_{max} - 0.002v_{max}^2 - 0.0001995v_{max}^3 \right) W_t = 0$$

$$0.0001995v_{max}^3 + 0.002v_{max}^2 + 5.5616v_{max} - 489.2966 = 0$$

Solving this polynomial equation gives a maximum speed of 72.4kmph over the maximum gradient and minimum curve radius. As a result trains will not run at their design speed of 80kmph over the maximum drag forces which marks the real gap between design and operation speed provide that the traction power supply system is feeding the train at its rated power. However, train speed on the level ground part of the route is obvious to be more than this maximum value. This is seen on tractive effort versus speed curve, Figure 4.2.

During constant speed operation, the train is expected to run the total distance between stations or passing loops minus the sum of the two accelerations, coasting and breaking distances. However, since the line section is feed by two substations it is necessary to break the whole distance in to subsections feed by each transformers. Hence, considering down-link moving train, a train moving from Sebeta to Mieso, the feeding substation is Awash Substation with a total distance of 11.4km. Therefore the distance difference is expressed as:

$$S_{cont,spd} = S_{total} - (S_{acc,1} + S_{notch-up}) = 11400 - (1794 + 6318) = 3287m$$

The time it will take the train to cover this distance will be

$$t_{const,spd} = \frac{S_{cont,spd}}{v_{max}} = 3.6 \frac{3287}{72.4} sec = 163.4420sec = 2.7240min$$

After 11.4km onwards the train is feed by Sirba Kunkur Substation for a distance of feeding section minus coasting and breaking. Thus, the running distance and time are respectively given:

$$S_{cont,spd} = 19780m \text{ and } t = \frac{S_{cont,spd}}{v_{max}} = 3.6 \frac{19780}{72.4} sec = 983.5519sec = 16.3925min$$

#### 4.4.3 Coasting

As it is shown in Section 4.4.4 at planned stoppages the train shouldn't be braked at 72kmph to stop it in the stations' boundary since the maximum length of sidings, main multiple track section, in the station is designed to be 1100m. But the reaming sidings are 800m for single locomotives and 850m for double locomotives. Therefore, added safety distance in order to avoid side collision at switches, it is better to break the train at or near 62kmph. To fulfill this condition, the train has to coast from 72.4kmph to 62kmph and the coasting time and distance is calculated as follow:

$$t_{coast}[s] = \frac{M_e}{3.6} \int_{72.4}^{62} \frac{dv}{F_{net,4}} = \frac{M_e}{3.6} \int_{72.4}^{62} \frac{-dv}{(103.7212 + 0.2825v + 0.02818v^2)kN}$$

$$\text{but; } \int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right)$$

$$t[s] = -1222.32 * 0.5869(0.83674 - 0.90773) = 50.92$$

$$s_{coast}[m] = \frac{M_e}{(3.6)^2} \int_{72.4}^{62} \frac{v}{F_{net,4}} dv = \frac{M_e}{(3.6)^2} \int_{72.4}^{62} \frac{-v dv}{(103.2012 + 0.2825v + 0.02818v^2)kN}$$

$$\text{but; } \int \frac{x}{ax^2 + bx + c} dx = \frac{\ln(ax^2 + bx + c)}{2a} - \frac{b}{a\sqrt{4ac - b^2}} \tan^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right)$$

$$s_{coast}[m] = -339.5062(96.4543 - 99.4568 - 2.94192(0.83674 - 0.90773)) = 948.4689$$

Therefore, to stop the train at the targeted point of 1151.2101 after break is applied the train has to be coasted from maximum speed to 62kmph which will last 50.92sec and the overall distance, coasting distance added to breaking distance, gives 2099.679m. As a result, deceleration during coasting is given as:

$$\beta_1[\text{kmphps}] = 3.6 * \left\{ \frac{v_f^2 - v_i^2}{2s_{coast}} \right\} = 3.6 * \left\{ \frac{\left(\frac{62}{3.6}\right)^2 - \left(\frac{72.4}{3.6}\right)^2}{2 * 948.4689} \right\} = -0.2047$$

#### 4.4.4 Braking

During breaking power is disconnected and break is applied, hence resistance and brake act together to halt the train from the speed break is applied to standstill.

$$\begin{aligned} s_{br}[m] &= \frac{M_e}{(3.6)^2} \int_{v_{br}}^0 \frac{v}{-B_e - (103.2012 + 0.2825v + 0.02818v^2)kN} dv \\ &= \frac{4400}{(3.6)^2} \int_{v_{br}}^0 \frac{-v}{B_e + F_{grad+curv} + 24.7212 + 0.2825v + 0.02818v^2} dv \end{aligned}$$

$$\text{but; } \int \frac{x}{ax^2 + bx + c} dx = \frac{\ln(ax^2 + bx + c)}{2a} - \frac{b}{a\sqrt{4ac - b^2}} \tan^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right)$$

$$= -339.5062 \left\{ \frac{\ln c}{2a} - \frac{\ln(c + bv_{br} + av_{br}^2)}{2a} - \frac{b}{a\sqrt{4ac - b^2}} \left( \tan^{-1} \left( \frac{b}{\sqrt{4ac - b^2}} \right) - \tan^{-1} \left( \frac{2av_{br} + b}{\sqrt{4ac - b^2}} \right) \right) \right\}$$

Similarly, breaking time is evaluated as;

$$t_{br}[s] = \frac{M_e}{3.6} \int_{v_{br}}^0 \frac{dv}{F_{net,5}} = 277.8 * 4.4 \int_{v_{br}}^0 \frac{-dv}{B_e + F_{grd+curv} + 24.7212 + 0.2825v + 0.02818v^2} dv$$

$$\text{but; } \int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right)$$

$$t_{br}[s] = -1222.32 \frac{2}{\sqrt{4ac - b^2}} \left\{ \tan^{-1} \left( \frac{b}{\sqrt{4ac - b^2}} \right) - \tan^{-1} \left( \frac{2av_{br} + b}{\sqrt{4ac - b^2}} \right) \right\}$$

$$F_{net,5}(v) = B_e + F_{grd+curv} + 24.7212 + 0.2825v + 0.02818v^2 \quad (4.4)$$

Break force and tractive effort are under the control of the driver. Hence, for each driving mode break force varies from zero to the maximum value of 800kN for dual locomotive and 400kN for single locomotive. In the next calculation due to the need for the control of deceleration between 0.2kmphps to 0.5kmphps, break force 400kN is applied for which case deceleration is calculated to be 0.4638kmphps. The drag forces, grade and curvature are incidental forces and they may or may not exist in the route section the break is to be applied. Near the station grade is 2 in 1000m.

In train operation there are two types of train stoppages; namely: planned stoppages which are operated at stations and passing loops and incidental stoppages which are under the control of emergencies and signal aspects due to the state of the track ahead of the train. At planned stoppages, there are not grade and curve resistances. Hence, the constant in the integrand, c, will be reduced to only train resistance and break force. Incidental stoppages could happen at any point in the route and full net force of breaking is necessary. In this thesis report, calculations for both cases of stoppages are done as follows. Rearranging Eq. (4.4) as in Eq. (4.5), are presented in the Table 4.1 and in the Table 4.2 respectively.

$$F_{net,5}(v) = 503.2012 + 0.2825v + 0.02818v^2 \quad (4.5)$$

Table 4.1 Breaking Distance and Time near Sirba Kunkur Passing Station

coefficients	a=0.02818; b=0.2825 and c=503.2012kN				
Break speeds (kmph)	V <sub>max</sub> =72	67	62	57	52
$\frac{\ln c}{2a}$	110.3795				
$\frac{\ln F_{net,4}(v)}{2a}$	115.4493	114.8840	114.3399	113.8195	113.3256
$\frac{b}{a\sqrt{4ac - b^2}}$	1.3320				
$\frac{2}{\sqrt{4ac - b^2}}$	0.2657				
$\tan^{-1}\left(\frac{b}{\sqrt{4ac - b^2}}\right)$	0.0375				
$\tan^{-1}\left(\frac{2av_{br} + b}{\sqrt{4ac - b^2}}\right)$	0.5231	0.4946	0.4651	0.4348	0.4035
S <sub>br</sub> [m]	1501.6294	1322.5949	1151.2101	988.2334	834.7059
t[s]	157.7085	148.4525	138.8718	129.0313	118.8660

Table 4.2 Breaking Distance and Time at any point in the Route Section

coefficients	a=0.02818; b=0.2825 and c=24.7212+F <sub>br</sub> +grade & curve forces(kN)				
Break speeds (kmph)	V <sub>max</sub> =72	67	62	57	52
$\frac{\ln c}{2a}$	130.7444				
$\frac{\ln F_{net,5}(v)}{2a}$	132.5153	132.3014	132.0994	131.9100	131.7334
$\frac{b}{a\sqrt{4ac - b^2}}$	0.75				
$\frac{2}{\sqrt{4ac - b^2}}$	0.1496				
$\tan^{-1}\left(\frac{b}{\sqrt{4ac - b^2}}\right)$	0.0211				
$\tan^{-1}\left(\frac{2av_{br} + b}{\sqrt{4ac - b^2}}\right)$	0.3140	0.2948	0.2754	0.2558	0.2359
S <sub>br</sub> [m]	526.6505	458.9190	395.2786	335.9658	281.0772
t <sub>br</sub> [s]	53.5594	50.0485	46.5011	42.9170	39.2781

Deceleration during breaking is given as:

$$\beta_2[\text{kmphps}] = 3.6 * \left\{ \frac{v_f^2 - v_i^2}{2s_{br}} \right\} = 3.6 * \left\{ \frac{(0)^2 - \left(\frac{62}{3.6}\right)^2}{2 * 1151.2101} \right\} = -0.4638$$

### Sirba Kunkur~Sebaka~Asabot~Mieso Line Subsection

In this line subsection train dynamics analysis accelerating modes of operation is assumed to be the same as the former line subsection; in the Awash~Sirba Kunkur Line Subsection, hence, accelerations; accelerating time and running distances remain same. Following the same procedures as in the Awash~Sirba Kunkur Line Subsection for constant speed, coasting and breaking regimes running time and running distances are obtained by rearranging Eq. (4.4) as in Eq. (4.6) and evaluating it for  $B_e$  of 200kN as shown below. Notice, near Mieso station, Down Link direction, gradients are negative. Hence, support tractive effort. Evaluating the integrals for running time and distance results are presented in Table (4.3); in the calculation break force is taken to be 200kN.

$$F_{net,5}(v) = -200 - (495.6012 + 0.2825v + 0.02818v^2)kN \quad (4.6)$$

Table 4.3 Breaking Distance and Time near Mieso Station

coefficients	a=0.02818; b=0.2825 and c=695.6012kN		
Break speeds (kmph)	$V_{max}=72$	67	62
$\frac{\ln c}{2a}$	116.1245		
$\frac{\ln F_{net,5}(v)}{2a}$	119.9663	119.493	119.0749
$\frac{b}{a\sqrt{4ac - b^2}}$	1.1327		
$\frac{2}{\sqrt{4ac - b^2}}$	0.2260		
$\tan^{-1}\left(\frac{b}{\sqrt{4ac - b^2}}\right)$	0.0319		
$\tan^{-1}\left(\frac{2av_{br} + b}{\sqrt{4ac - b^2}}\right)$	0.4580	0.4300	0.4033
S[m]	1140.4545	990.5338	858.8540
t[sec]	117.7077	109.9729	102.5971

Breaking deceleration will be

$$\beta_2[\text{kmphs}] = 3.6 * \left\{ \frac{v_f^2 - v_i^2}{2s_{br}} \right\} = 3.6 * \left\{ \frac{(0)^2 - \left(\frac{72}{3.6}\right)^2}{2 * 1140.4545} \right\} = -0.6313$$

Note: the above deceleration result indicates that near Asabot Station it is possible to break trains from the maximum speed, hence, there is not any coasting operation.

Table 4.4 the overall Line Section Acceleration Deceleration Running Time and Distance

Section	Awash to Sirba Kunkur					Sirba Kunkur~Sebaka			Sebaka Asabot	Asabot~Mieso	
$\alpha_1$ [km/h/s]	0.3	-	-	-	-	0.3	-	-	-	-	-
$\alpha_2$ [km/h/s]	-	0.033	-	-	-	-	0.033	-	-	-	-
$\beta_1$ [km/h/s]	-	-	-	0.2	-	-	-	-	-	-	-
$\beta_2$ [km/h/s]	-	-	-	-	0.46	-	-	-	-	-	0.63
Time (sec)	211	323.8	1147	50.9	139	211.1	323.8	143.3	977.27	577	118
Distance (m)	1794	6318	23067	949	1151	1794	6318	2882	19654	11605	1141

#### 4.5 Speed-Time Curve Estimation

Using the above analytical results train speed profile over the route section is plotted as shown below. The graph shows that trains cover the distance between Awash; Intermediate Station and traction Substation, and Sirba Kunkur; Passing Station and Traction Substation, of 33.28km, nearly within 31.20min. Similarly, to cover the distance between Sirba Kunkur and Mieso, 43.96km, it takes it 39.17min. Hence, the total time taken to cover 77.24km in the down link direction is 70.37min.

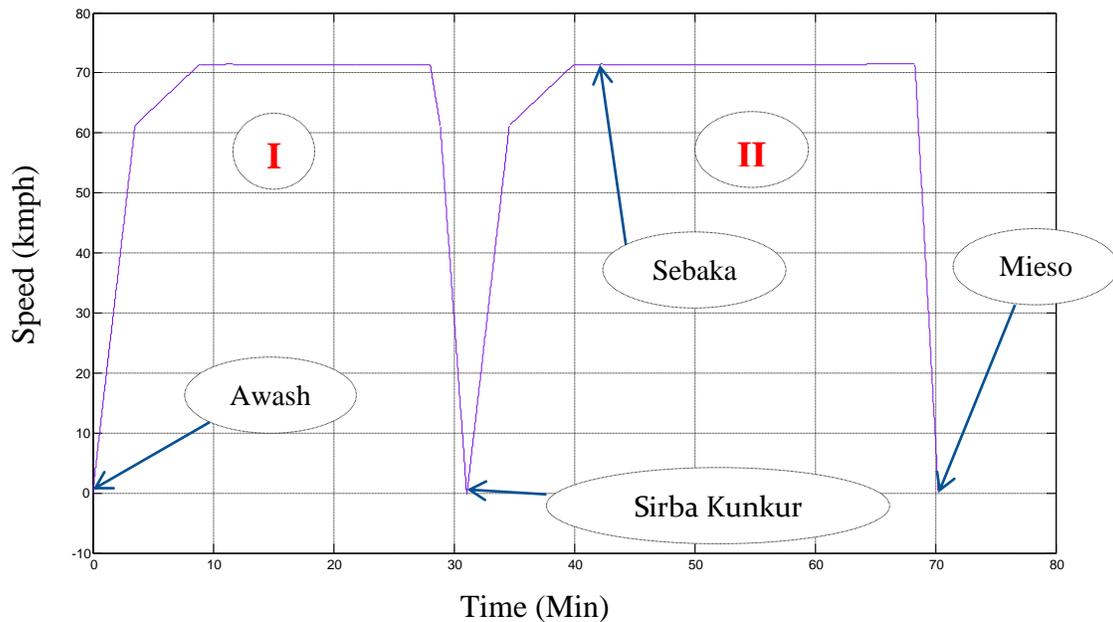


Figure 4.3 Speed versus Time Curve of the Line Section

The two subsections distance covered by the train = **I** + **II** = 33.28km+43.96km = 77.24km, the average and schedule speed of the train are calculated as:

- ✓ Actual run time = 31.2min+39.17min = 70.37min
- ✓ Added time = Train Stop+Departure+Crossing+Arrival at different time = 10min

$$v_{av} = \frac{S}{t_{actu}} = \frac{77.24km}{70.37min} * 60 \frac{min}{h} = 65.86kmph$$

$$v_{sch} = \frac{S}{t_{actu} + t_{add}} = \frac{77.24km}{10min + 70.37min} * 60 \frac{min}{h} = 57.66kmph$$

Schedule speed is used to estimate the number of trains that could be operated over the section. Hence, each train occupies the line subsection for 80.37min per day or 1.3395h per day. Dividing the 24h of the day for each train the total pair of the train that could be dispatched in the line subsection is calculated as:

$$Train\ pair = \frac{24h}{1.3395h} = 17.92 \cong 18$$

In the calculation train pair is considered because two trains run in the total line subsection at a time. The technical document prepared by the construction firm and approved by the corporation has proposed 19 train pairs to be operated in the short term plan.

#### 4.6 Traction Power Consumption Analysis

On the course of its movement the energy required by the train, the traction system in general, varies randomly making the current drawn from the supply main random too. Furthermore, trains are connected to the traction power supply network only during acceleration and constant speed operation regimes. In these regimes train power is the product of tractive effort and train speed. Where tractive effort is the sum of all the forces required to accelerate the train; to overcome gradient, train and curve resistances. Thus, train power consumption analysis in these two regimes is done as follows:

$$\begin{aligned} TE &= F_{acc} \pm F_g + F_{TR} + F_C & (4.7) \\ &= 277.8M_e\alpha \pm MgD + (0.63 + 0.0072v + 0.0007181v^2)W_t \\ &= (277.8\frac{\alpha}{g} \pm D + 0.63 + 0.0072v + 0.0007181v^2)W_t \\ &= (31.1498\alpha \pm D + 0.63 + 0.0072v + 0.0007181v^2)W_t \end{aligned}$$

for  $0 \leq v \leq 61.2\text{kmph}$ ;  $\alpha = 0.3\text{kmphps}$  and  $D = 6.1$

$$P * \eta_t = 10.9(16.0749v + 0.0072v^2 + 0.0007181v^3)\text{kW} \quad (4.8a)$$

for  $61.2 < v \leq 72.4\text{kmph}$ ;  $\alpha = 0.033\text{kmphps}$  and different  $D$  values

$$P * \eta_t = 10.9((1.6579 \pm D)v + 0.0072v^2 + 0.0007181v^3)\text{kW} \quad (4.8b)$$

for  $v = 72.4\text{kmph}$ ;  $\alpha = 0$ ; constant speed and different  $D$  values

$$P * \eta_t = (\pm 0.7892 * D + 3.8790)\text{MW} \quad (4.8c)$$

The plot of Eq. (4.8a), Eq. (4.8b) and Eq. (4.8c) for an efficiency of 95% is given as:

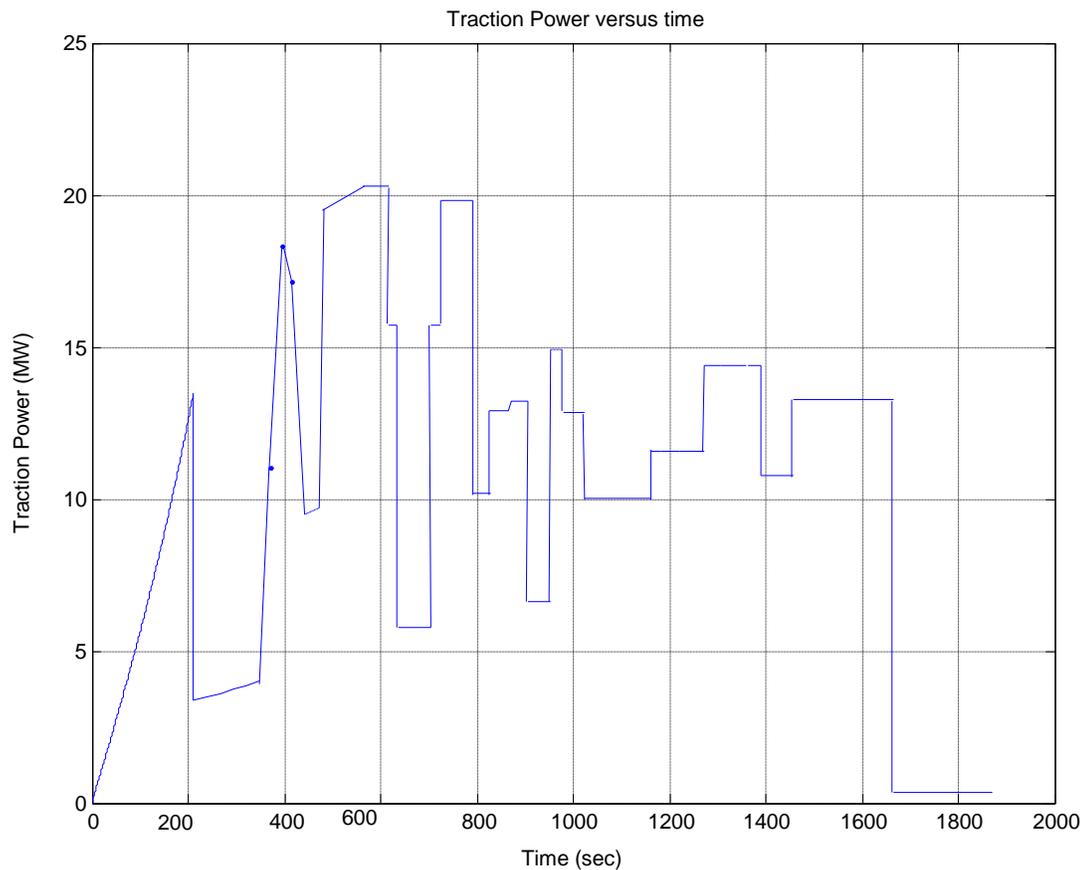


Figure 4.4 Traction Power versus Time

This graph shows the maximum power required at the end of acceleration is 20.2937MW when divided by power factor gives 21.3618MVA. This value indicates the traction transformer is not overloaded as it is expected to be about twice its rating.

From Figure 4.4, train instantaneous power with respect to time (t) graph, the speed dependent power equation in each subsection of train movement is easily obtained using slope point method of equation generation. Using this result train energy consumption is obtained by integrating the power equation in time.

Points (0, 0) → (211, 13.4888), t in sec and P in MW

$$P = 0.06393t$$

$$E = \int_0^{211} P dt = 0.03196t^2$$

$$= 0.03196(211^2 - 0^2)$$

$$= 1423.0767MJ$$

$$= \frac{1423.0767}{3.6} kWh$$

$$= 395.2991kWh$$

Flowing similar procedure it is possible to calculate energy consumed during each regime of motion. Hence, total energy consumed by the train along its path is calculated to be

$$E_{Total} = 4169.453kWh$$

The energy required per year is calculated as

$$E_{Year} = 18 \frac{trains}{day} * \frac{365days}{year} * \frac{4169.453kWh}{train} = 2739.3306 * 10^4 \frac{kWh}{yaer}$$

If we add about 15% of this for auxiliary energy consumers, hence, total energy required from Sirba Kunkur sub station will be  $3150.2302 * 10^4 kWh$  per year. Technical document prepared by the construction firm indicates that the short term energy required from this substation is  $3095 * 10^4 kWh$  per year [37].

## 4.7 Average Power Calculations

Train average power in each subsection of its movement is calculated in this section and this power is used to calculate train currents, catenary voltages and voltage drops.

### Awash~Awashit Subsection (11.4km)

#### 4.7.1 Accelerating to the Maximum Speed

During acceleration to the maximum speed the train passes two acceleration characteristics. One is determined by the net maximum tractive effort and the second slow advance determined by residual force during constant power motion. Power demand and the corresponding energy consumption during each acceleration scheme are analyzed as follows:

#### During first acceleration scheme:

$$Work = F_{net,1} S_{acc,1} = 355kN * 1794m = 635.858MJ$$

$$E_{acc,1} = \frac{635.858}{3.6} kWh = 176.6272kWh$$

$$P_{acc,1} = \frac{635.858 * 10^6 J}{211s} = 3.0119MW$$

For a train heading from Awash to Sirba Kunkur Substation the gradient for each line subsection:

Gradient	1	6.1	-16.5	-14.5	-18.1	-15	-3	9	18.1	16.3	6.3	18.3	18.5	13	1
Distance	1050	500	150	850	710	590	800	500	400	500	750	1650	1050	400	700

The route profile prepared by the construction firm shows that the line subsection starting from Awash Station is low gradient of 1.0 and 6.1 for 1050m and 500m respectively. Hence, the energy required to overcome the two gradients is

$$E_{grade} = W_t \sum_{i=1}^2 D_i S_i = 4 * 9.81(1.0 * 1050 + 6.1 * 500)kJ = 160.884MJ$$

$$E_{grade} \text{ in } kWh = \frac{160.884}{3.6} kWh = 44.69kWh$$

$$t_{D1} = \frac{1050}{1794.4819} * 211.1155 = 123.5294 \text{ sec}; t_{D2} = \frac{500}{1794.4819} * 211.1155 = 58.8235 \text{ sec}$$

$$P_{D1} = \frac{4 * 9.81 * 1050kJ}{123.5294sec} = 0.3335MW \text{ and } P_{D2} = \frac{4 * 9.81 * 6.1 * 500kJ}{58.8235sec} = 2.0346MW$$

In the line subsection the minimum curve radius is 1600m; hence it is possible to ignore its effect. Thus power consumed to overcome train resistance will be

$$\begin{aligned}
P_{tr} &= \frac{1}{3.6} \int_0^{61.2} R_t(v) dv = \frac{W_t}{3.6} \int_0^{61.2} (0.63 + 0.0072v + 0.0007181v^2) dv \\
&= \frac{W_t}{3.6} \left\{ 0.63v + \frac{0.0072}{2} v^2 + \frac{0.0007181}{3} v^3 \right\} \\
&= 1.1653 MW \\
E_{tr} &= P_{tr} * t_{tr} = 1165.3 * \frac{211.1155}{3600} kWh = 68.3369 kWh
\end{aligned}$$

Therefore, total power and energy required at the end of first acceleration will be

$$\begin{aligned}
P_{total} &= P_{acc,1} + P_{D1} + P_{D2} + P_{tr} = 6.5453 MW \\
E_{total} &= E_{acc,1} + E_{grade} + E_{tr} = 289.6541 kWh
\end{aligned}$$

### During second acceleration scheme:

The net consumed power during acceleration to the maximum speed is calculated as

$$\begin{aligned}
P_{notch-up} &= \frac{1}{3.6} \int_{61.2}^{72.4} (TE - R_e) dv \\
&= \frac{1}{3.6} \int_{61.2}^{72.4} \left( \frac{3.6 * P_{rated}}{v} - (785.6594 + 0.2825v + 0.02818v^2) kN \right) dv \\
&= P_{rated} \ln v - \frac{1}{3.6} \left( 785.6594v + \frac{1}{2} 0.2825v^2 + \frac{1}{3} 0.02818v^3 \right) kW \\
&= P_{rated} \ln \frac{72.4}{61.2} \\
&\quad - \frac{1}{3.6} \left( 785.6594 * 10.8 + \frac{1}{2} 0.2825(72.4^2 - 61.2^2) \right) \\
&\quad + \frac{1}{3} 0.02818(72.4^3 - 61.2^3) kW \\
P_{notch-up} &= 331.1377 kW = 0.3311 MW \\
E_{notch-up} \text{ in kWh} &= Pt = \frac{331.1377 * 323.8142}{3600} kWh = 29.7853 kWh
\end{aligned}$$

During second acceleration scheme gradients occur five times with 9, 18.1, 16.3, 6.3 and 18.3 for a distance of 500m, 400, 500m, 750m and 500m respectively. Hence, required energy will be:

$$E_{grade} = W_t \sum_{i=1}^5 D_i S_i = 1324.9386 MJ$$

$$E_{grade} \text{ in } kWh = \frac{1324.9386}{3.6} kWh = 368.0385 kWh$$

The time the train will stay in the gradient regions will be determined from total time as

$$t_i = \frac{\sum_{i=1}^5 S_i}{S_{total}} * t_{notch-up} = \frac{2650}{6318} * 323.8142 sec = 135.8195 sec$$

$$P_{grade} = \frac{1324.9386 MJ}{135.8195 sec} = 9.7551 MW$$

By similar approach followed in the previous acceleration, the energy required to overcome resistances at curves is ignored due to  $R_{min}$  is 1600m. Thus, power consumed to overcome train resistance:

$$\begin{aligned} P_{tr} &= \frac{1}{3.6} \int_{61.2}^{72.4} R_t(v) dv \\ &= \frac{W_t}{3.6} \int_{61.2}^{72.4} (0.63 + 0.0072v + 0.0007181v^2) dv \\ &= \frac{W_t}{3.6} \left\{ 0.63v + \frac{0.0072}{2} v^2 + \frac{0.0007181}{3} v^3 \right\} \\ &= 0.5277 MW \end{aligned}$$

$$E_{tr} = P_{tr} t_{tr} = 527.7 * \frac{323.8142}{3600} kWh = 47.4682 kWh$$

Total power and energy consumed at the end of second acceleration are:

$$P_{total} = P_{notch-up} + P_{grade} + P_{tr} = 10.6139 MW$$

$$E_{total} = E_{notch-up} + E_{grade} + E_{tr} = 445.292 kWh$$

#### 4.7.2 Constant Speed

During constant speed motion energy required from the traction system is energy to overcome train and grade resistances to maintain motion at 72.4kmph. As a result, the resistance power is:

$$P_{tr} = \frac{W_t}{3.6} R(v) * v = \frac{W_t}{3.6} (0.63 + 0.0072v + 0.0007181v^2)v = 3.8790 MW$$

Energy consumed will be obtained after calculating tractive effort as follow:

$$TE = \frac{P_{tr}}{v} = \frac{3.6 * 3.8790 MW}{72.4} = 192.0825 kN$$

$$E_{tr} = TE * s = 192.0825 * 3287kJ = 631.3752MJ$$

$$E_{tr} \text{ in } kWh = \frac{631.3752}{3.6} kWh = 175.3820kWh$$

There are four gradients in the line subsection, 18.3, 18.5, 13.0 and 1.0 for a distance of 1138m, 1050m, 400m and 700m respectively and the energy required to overcome grade resistance is

$$E_{grade} = W_t \sum_{i=1}^4 D_i S_i = 1810.9417MJ$$

$$E_{grade} \text{ in } kWh = \frac{1810.9417}{3.6} = 503.0394kWh$$

Time elapsed to cover this motion will be  $t_{grade} = \frac{s_{grade}}{v_{max}} = 163.4419sec = 2.7249min$  and

hence the corresponding consumed power will be

$$P_{grade} = \frac{1810.9417MJ}{163.4420sec} = 11.0767MW$$

Total power and energy consumed in this line subsection will be:

$$P_{total} = P_{grad} + P_{tr} = 14.9557MW$$

$$E_{total} = E_{grad} + E_{tr} = 679.115kWh$$

### **Awashit~Sirba Kunkur Subsection (19.78km of 21.88km; 2.1km coasting and breaking)**

Similarly, since the line subsection, Awashit~Sirba Kunkur is feed by the Sirba Kunkur Substation, the energy required from traction substation is only to overcome train and gradient resistances. Hence, power consumed by train resistance remains same as the previous subsection, since it is moving with same speed, while gradient resistance power requirement is calculated for each grade in the section as follows: note there are 25 positive gradient line subsections and in the calculation no speed loss is assumed at section post or neutral section.

Gradient	1	13	18.5	18.3	6.3	10	-2	10	2	12	9.5	6.5
Distance	700	400	700	650	750	500	500	500	1000	500	900	700
Gradient	3.5	6	8	-4	8	11.5	12.5	14.5	8.5	10	-2	-5
Distance	700	1400	400	800	500	500	700	600	500	600	500	400
Gradient	7	18.3	15	3	12	2	-2	1	-			
Distance	400	1200	400	900	400	800	500	2100				

$$E_{g,cons.speed} = W_t \sum_{i=1}^{25} D_i S_i = 6143.3892MJ$$

$$E_{g,cons.speed} \text{ in } kWh = \frac{6143.3892}{3.6} = 1706.497kWh$$

$$\text{Time elapsed } t = \frac{14500m}{19780m} * 983.5519sec = 721.0062sec$$

$$P_{g,cons.speed} = \frac{E_{g,cons.speed}}{t} = \frac{6143.3892MJ}{721.0062sec} = 8.5206MW$$

$$P_{tr} = \frac{W_t}{3.6} R(v) * v = \frac{W_t}{3.6} (0.63 + 0.0072v + 0.0007181v^2)v = 3.8790MW$$

$$E_{tr,cons.speed} \text{ in } kWh = \frac{3879.0}{3600} * 983.5519kWh = 1059.7772kWh$$

Total power and energy consumed in this line subsection will be:

$$P_{total} = P_{g,cons.speed} + P_{tr} = 12.3996MW$$

$$E_{total} = E_{g,cons.speed} + E_{tr} = 2766.2742kWh$$

## Sirba Kunkur~Sebaka Line Subsection

This subsection is feed by Sirba Kunkur Substation (11.0km). Traction power consumption in the line section is calculated assuming same accelerations, 0.3kmphps and 0.03289kmphps during acceleration and notching-up and results are presented in the Table 4.6.

Gradient	1	5.5	10	18.3	16	4	8	9.5	12.5	14	9	2.5
Distance	1000	500	700	1500	450	1250	1400	800	600	400	400	250
Gradient	14	2.5	13	1								
Distance	100	497	403	1450								

## 4.8 Electric Traction Load Model

### 4.8.1 Locomotive Model

Due to the dynamic load nature of the train, traction model for each line subsection varies according to the requirement of the train movement in the subsection. Hence, the traction load model assuming 0.95 power factor and using constant power model [21], is done as follows:

$$S_{traction} = \frac{P}{pf} \text{ and } Q = \sqrt{S^2 - P^2} = P \sqrt{\left(\frac{1}{pf^2} - 1\right)} = 0.3286 * P \quad (4.9)$$

$$I_{traction} = \frac{S_{traction}}{U_{Li}} \angle -\cos 0.95^{-1} \quad (4.10)$$

In Eq. (4.10) above the load voltage,  $U_{Li}$  is not known to calculate traction current. Hence, voltage drop approach is used as shown below to find  $U_{Li}$  and  $I_{traction}$  traction in turn.

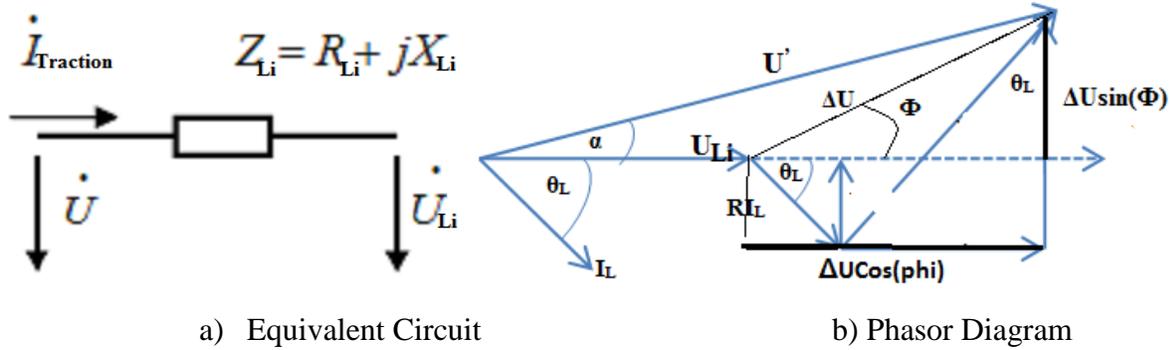


Figure 4.5 Voltages and Load Current Phasor Diagram at Transformer Secondary

$$|i| = \frac{|\dot{U} - \dot{U}'|}{|Z_{Li}|} = \frac{|\Delta U|}{Z_{Li}} = \frac{\dot{U}'}{|Z_L|} = \frac{S}{\dot{U}'} \quad (4.11)$$

$\Phi = \cos^{-1} 0.85 - \cos^{-1} 0.95 = 13.6^\circ$ ; where 0.85 is impedance power factor

$$\alpha = \cos^{-1} \left( \frac{U_{Li} + |\Delta U| \cos 13.6}{27.5} \right) \quad (4.12)$$

$$|\Delta U| = \frac{S_{Li} Z_{Li}}{U_{Li}} \quad (4.13)$$

$$27.5^2 = \left( U_{Li} + \frac{S_{Li} Z_{Li}}{U_{Li}} \cos 13.6 \right)^2 + \left( \frac{S_{Li} Z_{Li}}{U_{Li}} \sin 13.6 \right)^2$$

$$U_{Li}^4 + (2S_{Li} Z_{Li} \cos 13.6 - 27.5^2) U_{Li}^2 + (S_{Li} Z_{Li})^2 = 0 \quad (4.14)$$

let  $U_{Li}^2 = x$

Evaluating Eq. (4.14) for each section complex power and the corresponding equivalent impedance, see Eq. (4.15) below for equivalent impedances. Equation (4.18) is reduced to

$$x^2 + (2S_{Li} Z_{Li} \cos 13.6 - 27.5^2)x + (S_{Li} Z_{Li})^2 = 0 \quad (4.15)$$

Using the quadratic equation solution method:

$$x = \frac{-(2S_{Li} Z_{Li} \cos 13.6 - 27.5^2) \pm \sqrt{(2S_{Li} Z_{Li} \cos 13.6 - 27.5^2)^2 - 4(S_{Li} Z_{Li})^2}}{2}; MV$$

$$U_{Li} = \sqrt{x} = \sqrt{\frac{-(2S_{Li} Z_{Li} \cos 13.6 - 27.5^2) \pm \sqrt{(2S_{Li} Z_{Li} \cos 13.6 - 27.5^2)^2 - 4(S_{Li} Z_{Li})^2}}{2}}; kV$$

Taking only the plus sign only catenary voltage and load or traction currents are calculated as shown in Table 4.5 below.

Table 4.5 Traction Load Dynamic Models in Awash~Sirba Kunkur Line Subsection

Subsection	Awash~Awashit			Awashit~Sirba Kunkur
	Acceleration 1	Acceleration 2	Constant Speed	Constant Speed
Distance (km)	1.794	6.318	3.288	21.88
P in (MW)	6.5453	10.6139	14.9557	12.3996
Q in (MVarac)	2.1509	3.4881	4.9144	4.0756
$S_{\text{Traction}}$ (MVA)	6.8897	11.1725	15.7428	13.0522
$I_{\text{Traction}}$ (A)	252	431	653	604
$U_{Li}$ (kV)	27.2993	25.9499	24.1062	21.6074

Table 4.6 Traction Load Dynamic Models in Sirba Kunkur~Sebaka Line Subsection

Subsection	Sirba Kunkur~Sebaka		
	Acceleration 1	Acceleration 2	Constant Speed
P in (MW)	6.1753	9.529	9.1212
Q in (MVarac)	2.0292	3.1312	2.9972
$S_{\text{Traction}}$ (MVA)	6.5003	10.0305	9.6013
$I_{\text{Traction}}$ (A)	238	384	374
$U_{Li}$ (kV)	27.3108	26.1176	25.6749

#### 4.8.2 Traction Network Model

Traction network of the Ethio-Djibouti corridor employs suspension type, full compensation simple catenary, overhead contact system arrangement, bilateral and unilateral operation modes for double and single line sections respectively. The double line extends from Sebeta to Adama (included) while the single line goes through Adama (excluded) to Nagad (included). The line section under investigation, Awash~Sirba Kunkur~Sebaka, is found in the single line section of the corridor. In the report of [27], the equivalent impedance, the power factor is 0.85 [8] [9], is calculated to be  $0.141072+j0.433479 \Omega/km$ ; for single line and  $0.130081+j0.392381 \Omega/km$  for double line using the following technical parameters of the electrification system: the contact line using CTAH-120,  $r_j = 0.2 \Omega/km$ ,  $R_{ej} = 4.74mm$ , the carrier cable is; JTMH-95,  $r_c = 0.211 \Omega/km$ ,  $R_c = 4.4mm$ , negative feeder is; TGJ-185,  $r = 0.163 \Omega/km$ ,  $R = 9.03mm$ , contact line from the orbital plane of average height  $H = 5800mm$ ; the track is  $50kg/m$ ,  $r_g = 0.18 \Omega/km$ ,  $X_{Ng} = 0.18 \Omega/km$ ,  $R = 96.5mm$ ,  $d_g = 1435mm$ , the ground conductivity is  $\delta = 10^{-4} \Omega/km$ ; track center distance is 5m. Hence, equivalent impedances per unit length in phasor form for single and double track respectively obtained to be [27]:

$$Z_{eq, single} = \frac{0.455857e^{j71.97^\circ} \Omega}{km} \quad (4.16)$$

$$Z_{eq,double} = \frac{0.413381e^{j71.66^\circ} \Omega}{km} \quad (4.17)$$

In constant power modeling traction network is modeled using losses over the line section. Therefore, since the current varies per line subsection according to the requirement of train movement, power loss is calculated as:

$$S_{MVA,loss} = \Delta U_{Li} I_{Li}^* = 0.455857 \left| \frac{I_{Li}}{1000} \right|^2 \angle 31.79^\circ \frac{MVA}{km} \quad (4.18)$$

Table 4.7 Main Line Overhead Contact System Model for Awash~S/Kunkur Line Subsection

Subsection	Awash~Awashit			Awashit~Sirba Kunkur
	Acceleration 1	Acceleration 2	Constant Speed	Constant Speed
Operation mode	Acceleration 1	Acceleration 2	Constant Speed	Constant Speed
Distance (km)	1.794	6.318	3.288	21.88
Time (hr.)	0.0586	0.0899	0.0454	0.2003
$S_{loss}$ (MVA)	0.0519	0.6869	2.2159	3.6387
P (MW)	0.0441	0.5839	1.8836	3.0929
Q in (MVArac)	0.0273	0.3618	1.1672	1.9168
$\Delta U$ (kV) $\angle \Phi$	0.2061 $\angle 13.6^\circ$	1.5938 $\angle 13.6^\circ$	3.3935 $\angle 13.6^\circ$	6.0244 $\angle 13.6^\circ$

Table 4.8 Main Line Overhead Contact System Model for S/Kunkur~Sebaka Line Subsection

Subsection	Sirba Kunkur~Sebaka		
	Acceleration 1	Acceleration 2	Constant Speed
Operation mode	Acceleration 1	Acceleration 2	Constant Speed
Distance (km)	1.794	6.318	2.882
Time (hr.)	0.0586	0.0899	0.0398
$S_{loss}$ (MVA)	0.0463	0.5453	0.7010
P (MW)	0.0394	0.4635	0.5959
Q in (MVArac)	0.0244	0.2873	0.3693
$\Delta U$ (kV) $\angle \Phi$	0.1946 $\angle 13.6^\circ$	1.4200 $\angle 13.6^\circ$	1.8744 $\angle 13.6^\circ$

During each movement regime the load seen by the traction transformer is given in table below:

$$S_{total,MVA} = S_{traction,MVA} + S_{loss,MVA} \quad (4.19)$$

$$= P_{traction,MW} + P_{loss,MW} + j(Q_{traction,MVArac} + Q_{loss,MVArac})$$

$$\theta_{Li} = \theta_L + \alpha \quad (4.20)$$

Table 4.9 Main Line Overhead Contact System plus Locomotive Model (Awash~S/Kunkur)

Subsection	Awash~Awashit			Awashit~Sirba Kunkur
	Acceleration 1	Acceleration 2	Constant Speed	Constant Speed
Operation mode				
$P_{Total}$ (MW)	6.5894	11.2078	16.8393	15.4925
$Q_{Total}$ (MVarac)	2.1782	3.8499	6.0816	5.9924
$S_{Total}$ (MVA)	6.9400	11.8506	17.9039	16.6110
Power Factor $\cos^{-1}(\theta_{Li})$	0.9484	0.9473	0.9207	0.9325
$\bar{U}$ (kV) $\angle\alpha$	$27.5\angle 0.3^{\circ}$	$27.5\angle 0.49^{\circ}$	$27.5\angle 4.78^{\circ}$	$27.5\angle 2.98^{\circ}$

Table 4.10 Main Line Overhead Contact System plus Locomotive Model (S/Kunkur~Sebaka)

Subsection	Sirba Kunkur~Sebaka		
	Acceleration 1	Acceleration 2	Constant Speed
Operation mode			
$P_{Total}$ (MW)	6.2147	9.9925	9.7171
$Q_{Total}$ (MVarac)	2.0536	3.4185	3.3665
$S_{Total}$ (MVA)	6.5452	10.5611	10.2837
Power Factor $\cos^{-1}(\theta_{Li})$	0.9493	0.9459	0.9451
$\bar{U}$ (kV) $\angle\alpha$	$27.5\angle 0.12^{\circ}$	$27.5\angle 0.73^{\circ}$	$27.5\angle 0.88^{\circ}$

### 4.8.3 Traction Substation and Utility Grid Model at Load Side

Traction transformers installed at Awash and Sirba Kunkur Substations are Vv-transformers with rating of 20MVA. Similarly primary side short circuit capacity is taken to be 400MVA. Assuming secondary side no-load (open circuit voltage) to be 27.5kV, the transformer and system impedance parameters observed at secondary winding side are:

$$Z_T = jX_T = \frac{U_k \% U_{2N}^2}{100 S_T} = \frac{10.5 * (27.5kV)^2}{100 * 20MVA} = j3.9703\Omega$$

$$Z_s = jX_s = \frac{U_{2N}^2}{S_k} = \frac{(27.5kV)^2}{400MVA} = j1.891\Omega$$

$$Q_{trafo} = j \frac{\Delta U^2}{Z_T}$$

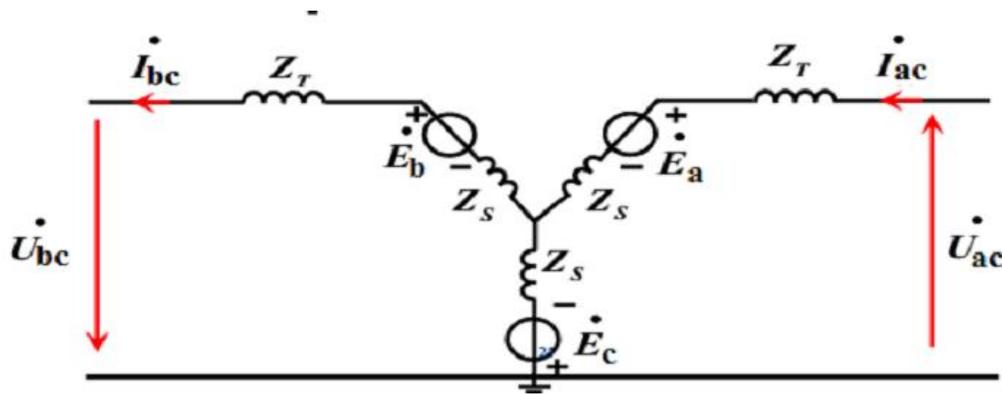
Table 4.11 Voltage Drops and Power Losses at S/Kunkur Station Transformer, Left Arm

Subsection	Awash~Awashit			Awashit~Sirba Kunkur
	Acceleration 1	Acceleration 2	Constant Speed	Constant Speed
$\Delta U_{Trafo}$ (kV) $\angle 90 - \theta^{\circ}$	$1.001\angle 71.51^{\circ}$	$1.712\angle 71.32^{\circ}$	$2.5926\angle 67.03^{\circ}$	$2.3981\angle 68.83^{\circ}$
$Q_{loss}$ (MVarac)	0.2523	0.7375	1.630	1.4484

Table 4.12 Voltage Drops and Power Losses at S/Kunkur Station Transformer, Right Arm

Subsection	Sirba Kunkur~Sebaka		
Operation mode	Acceleration 1	Acceleration 2	Constant Speed
$\Delta U_{\text{Trafo}}$ (kV) $\angle 90 - \theta^\circ$	$0.9449 \angle 71.68^\circ$	$1.5246 \angle 71.07^\circ$	$1.4849 \angle 70.93^\circ$
$Q_{\text{loss}}$ (MVA <sub>rac</sub> )	0.2684	0.6384	0.5854

To calculate transformer secondary voltages and the corresponding utility grid side voltage and currents, Phase and line values, three phase Vv-connected traction substation model of Figure 3.5 is used and results are outlined as shown below.



Writing the port equation for Figure 3.5, the Equivalent Circuit,

$$U_{ac} = E_{ac} - (2Z_s + Z_T)I_{ac} + Z_s I_{bc} \quad (4.21)$$

$$U_{bc} = E_{bc} - (2Z_s + Z_T)I_{bc} + Z_s I_{ac} \quad (4.22)$$

where:  $|U_{ac}| = |U_{bc}| = |\mathbf{U}'|$  (kV) = 27.5kV; the no load voltage

$|I_{bc}| = |I_{Li}|$  at an angle of the load at the transformer secondary

Rearranging Eq. (4.21) and Eq. (4.22) to evaluate for  $E_{ac}$  and  $E_{bc}$  respectively:

$$E_{ac} = U_{ac} + (2Z_s + Z_T)I_{ac} - Z_s I_{bc} \quad (4.23)$$

$$E_{bc} = U_{bc} + (2Z_s + Z_T)I_{bc} - Z_s I_{ac} \quad (4.24)$$

Eq. (4.23) and Eq. (4.24) are evaluated assuming only 384A for  $I_{ac}$  and 604A for  $I_{bc}$  and hence load angles will be  $18.93^\circ$  and  $21.17^\circ$  respectively.

$$E_{ac} = 27.5 + 7.7523|I_{ac}|(\cos(90 + \theta_{ac}) + j\sin(90 + \theta_{ac})) \\ - 1.891|I_{bc}|(\cos(90 + \theta_{bc}) + j\sin(90 + \theta_{bc}))$$

$$= 27.5 + 7.7523|I_{ac}| \cos(108.93) - 1.891|I_{bc}| \cos 111.17 + j(7.7523|I_{ac}| \sin(108.93) - 1.891|I_{bc}| \sin(111.17))$$

$$E_{ac} = 26.9467 + j3.8810 \text{ kV} = 27.2248 \angle 8.20^\circ \text{ kV} \quad (4.25)$$

Similarly,

$$\begin{aligned} E_{bc} &= 27.5 + 7.7523|I_{bc}|(\cos(90 + \theta_{bc}) + j\sin(90 + \theta_{bc})) - 1.891|I_{ac}|(\cos(90 + \theta_{ac}) + j\sin(90 + \theta_{ac})) \\ &= 27.5 + 7.7523|I_{bc}|\{\cos(111.17) + j\sin(111.17)\} - 1.891|I_{ac}|\{\cos 108.93 + j\sin 108.93\} \\ &= 27.5 + 7.7523|I_{bc}| \cos(111.17) - 1.891|I_{ac}| \cos(108.93) + j\{7.7523|I_{bc}| \sin(111.17) - 1.891|I_{ac}| \sin(108.93)\} \\ &= 26.0446 + j3.6795 \text{ kV} = 26.3032 \angle 8.04^\circ \text{ kV} \end{aligned} \quad (4.24)$$

$U_{ac}$ ,  $U_{bc}$ ,  $I_{ac}$  and  $I_{bc}$  are shown below by phasor diagram using  $U_{bc}$  as reference voltage

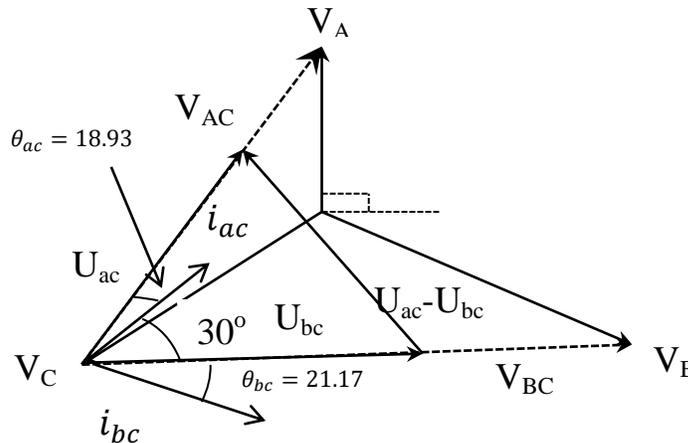


Figure 4.6 Source Voltages and Load Current Phasor Diagram at Transformer Secondary

Using Eq. (3.4) balancing degree is calculated as

$$\begin{cases} |I_{ac}| \leq |I_{bc}| ; 384 < 604 \\ BD_1 = \frac{|384|}{|604|} = 0.6358, 0 \leq BD_1 \leq 1 \end{cases} \quad (4.26)$$

Using Eq. (3.16a), Eq. (3.16b), and Eq. (3.16c) primary side phase currents are calculated as

$$I_A = \frac{1}{K_T} BD_1 I_{bc} e^{j(\frac{\pi}{3} - \theta_{ac})} = \frac{384 e^{j41.07}}{4.8} = 80 \angle 41.07^\circ A \quad (4.27)$$

$$I_B = \frac{1}{K_T} I_{bc} e^{-j\theta_{bc}} = \frac{604 e^{-j21.17}}{4.8} = 126 \angle -21.17^\circ A \quad (4.28)$$

$$\begin{aligned} I_C &= -(I_A + I_B) = -(80 \angle 41.07^\circ + 126 \angle -21.17^\circ) A \\ &= -(177.8092 + j7.0553) A = -178 \angle 2.27^\circ A = 178 \angle 182.27^\circ A \end{aligned} \quad (4.29)$$

Similarly, primary side voltages are

$$\begin{aligned} V_A &= K_T \frac{U_{ac}}{\sqrt{3}} e^{j\frac{\pi}{6}} = K_T \frac{U_{bc} e^{j\frac{\pi}{3}}}{\sqrt{3}} e^{j\frac{\pi}{6}} = K_T \frac{U_{bc}}{\sqrt{3}} e^{j\frac{\pi}{2}} \\ &= 4.8 \frac{27.5 e^{j92.98}}{\sqrt{3}} kV = 76.2102 e^{j92.98} kV \end{aligned} \quad (4.30)$$

$$V_B = K_T \frac{U_{bc}}{\sqrt{3}} e^{-j\frac{\pi}{6}} = 4.8 \frac{27.5 e^{-j27.02}}{\sqrt{3}} kV = 76.2102 e^{-j32.98} kV \quad (4.31)$$

$$\begin{aligned} V_C &= -U_{bc} e^{j\frac{\pi}{6}} = -U_{ac} e^{-j\frac{\pi}{6}} \\ &= -4.8 \frac{27.5 e^{j32.98}}{\sqrt{3}} kV = 76.2102 e^{j212.98} kV \end{aligned} \quad (4.32)$$

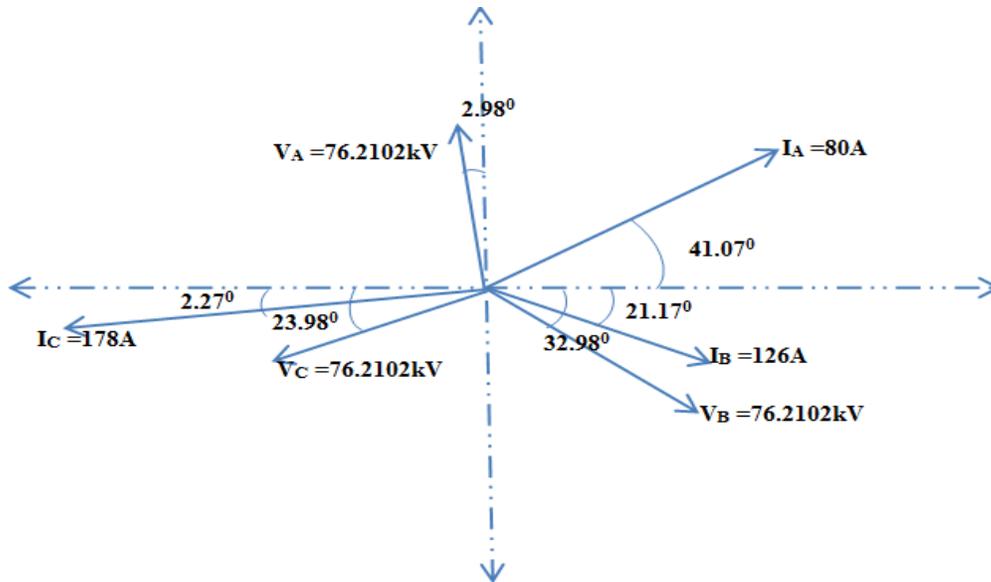


Figure 4.7 Utility Grid Side Fundamental Voltage and Current Phasor Diagram

#### 4.9 System Evaluation for Disturbances

Using symmetrical components

$$\begin{bmatrix} \dot{V}_0 \\ \dot{V}_+ \\ \dot{V}_- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \dot{V}_A \\ \dot{V}_B \\ \dot{V}_C \end{bmatrix} \quad (4.33)$$

$$\text{where } a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = 1e^{j120} \text{ and } a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = 1e^{j240}$$

$$\begin{aligned} V_0 &= \frac{1}{3} (76.2102e^{j92.98} + 76.2102e^{-j32.98} + 76.2102e^{j212.98})kV \\ &= \frac{76.2102}{3} (-0.05199 - j0.09004)kV = 25.4034e^{j59.99}kV \end{aligned}$$

$$\begin{aligned} V_+ &= \frac{1}{3} (76.2102e^{j92.98} + 76.2102e^{j87.02} + 76.2102e^{j452.98})kV \\ &= \frac{76.2102}{3} (-0.05199 + j2.9959)kV = 76.1175e^{j90.99}kV \end{aligned}$$

$$\begin{aligned} V_- &= \frac{1}{3} (76.2102e^{j92.98} + 76.2102e^{j207.02} + 76.2102e^{j332.98})kV \\ &= \frac{76.2102}{3} (-0.05199 + j0.09004)kV = 2.6413e^{j120}kV \end{aligned}$$

the unbalance coefficient is calculated as, using Eq. (3.32)

$$\left| \frac{V_-}{V_+} \right| = \frac{2.6413}{76.1175} 100\% = 3.47\% \quad (4.34)$$

The above results and Figure 4.7 show neither phase angles nor magnitudes of both current and voltage are balanced; hence, the utility grid system suffers from unbalanced disturbances. To eliminate these effects is inevitable.

To analyze the effect of harmonics due to the rectifier on the utility grid, let's express each primary side phase current by the Fourier series expression in Eq. (B.9), see Appendix B.

$$I(t) = I_o + \sqrt{2} \sum_{h=1 \text{ \& } odd}^{\infty} I_h \sin(h\omega t + \beta_h)$$

$$\text{where: } I_o = \frac{I_{max}}{2\sqrt{2}}; I_h = \frac{-4I_{max}}{\pi h(h^2 - 4)\sqrt{2}} \text{ and } \beta_h \text{ is load angle}$$

**On Phase A**

$$\begin{aligned}
I_A(t) &= \frac{I}{2} + \sqrt{2} \left( \frac{4I}{3\pi} \sin(\omega t + 41.07) - \frac{4I}{15\pi} \sin(3\omega t + 123.21) \right. \\
&\quad \left. - \frac{4I}{105\pi} \sin(5\omega t + 205.35) \right) \dots \\
&= \frac{4\sqrt{2}I}{\pi} \left\{ \frac{\pi}{8\sqrt{2}} + \frac{1}{3} \sin(\omega t + 41.07) - \frac{1}{15} \sin(3\omega t + 123.21) \right. \\
&\quad \left. - \frac{1}{105} \sin(5\omega t + 205.35) - \frac{1}{315} \sin(7\omega t + 289.49) \dots \right\} \\
&= 40 + 48.0169 \sin(\omega t + 41.07) - 9.6034 \sin(3\omega t + 123.21) \\
&\quad - 1.3719 \sin(5\omega t + 205.35) - 0.4573 \sin(7\omega t + 289.49) \dots
\end{aligned}$$

For  $\omega t = 2\pi 50t = 2\pi$ , using the property  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

For all  $h$ ;  $\sin h2\pi = 0$  and  $\cos h2\pi = 1$

$$\begin{aligned}
I_A(t) &= 40 + 48.0169 \sin(41.07) - 9.6034 \sin(123.21) - 1.3719 \sin(205.35) \\
&\quad - 0.4573 \sin(289.49) \dots
\end{aligned}$$

**For Phase B**

$$\begin{aligned}
I_B(t) &= \frac{I}{2} + \sqrt{2} \left( \frac{4I}{3\pi} \sin(\omega t - 21.17) - \frac{4I}{15\pi} \sin(3\omega t - 63.51) - \frac{4I}{105\pi} \sin(5\omega t - 105.85) \right) \dots \\
&= 63 + 75.6266 \sin(\omega t - 21.17) - 15.1253 \sin(3\omega t - 63.51) \\
&\quad - 2.1608 \sin(5\omega t - 105.85) - 0.7203 \sin(7\omega t - 148.19) \dots
\end{aligned}$$

$$\begin{aligned}
I_B(t) &= 63 + 75.6266 \sin(-21.17) - 15.1253 \sin(-63.51) - 2.1608 \sin(-105.85) \\
&\quad - 0.7203 \sin(-148.19) \dots
\end{aligned}$$

**For Phase C**

$$\begin{aligned}
I_C(t) &= \frac{I}{2} + \sqrt{2} \left( \frac{4I}{3\pi} \sin(\omega t + 182.27) - \frac{4I}{15\pi} \sin(3\omega t + 546.81) \right. \\
&\quad \left. - \frac{4I}{105\pi} \sin(5\omega t + 911.35) \right) \dots \\
&= \{89 + 106.8375 \sin(182.27) - 21.3675 \sin(546.81) - 3.0525 \sin(911.35) \\
&\quad - 1.0175 \sin(1275.89) \dots \}
\end{aligned}$$

Table 4.13 Uncompensated Individual and Total Harmonic Analysis on Phase A, B and C

Phase	Order	h=1	h=3	h=5	h=7
A	$ I_h $	31.5462	8.0349	0.5874	0.4311
	HD (%)		25.47	1.86	1.37
	THD (%)		25.57		
B	$ I_h $	27.3115	13.5373	2.0786	0.3799
	HD (%)		49.57	7.61	1.39
	THD (%)		50.17		
C	$ I_h $	4.2317	2.5334	0.6007	0.2786
	HD (%)		59.87	14.19	6.58
	THD (%)		61.88		

Fourier expression shows that odd components of the current and voltage signals exist in the harmonic components. Hence, the individual and total harmonic analysis results in Table 4.13 above show the HD for 3<sup>rd</sup> order on phase A; 3<sup>rd</sup> and 5<sup>th</sup> orders on phases A and B; and 3<sup>rd</sup>, 5<sup>th</sup> and 7<sup>th</sup> orders on phases A, B and C violate the standard limits in Table 1 Appendix C. Similarly, THD on all the three phases are violated. Therefore, to limit the effects of harmonics on the utility grid harmonic elimination is necessary.

#### 4.10 Utilization Factors

The three phase apparent powers of the utility grid ( $S_A$ ,  $S_B$  and  $S_C$ ), the maximum capacity utilization ( $S_R$ ), and the transformer winding capacity ( $S_1$  and  $S_2$ ), the transformer capacity ( $S_T$ ) and the line capacity ( $S_L$ ) can be computed respectively, using Eq. (3.17a) through Eq. (3.23), as:

$$S_A = V_A I_A^* = 76.2102 * 80e^{j51.91} kVA = 6.0968e^{j51.91} MVA$$

$$S_B = V_B I_B^* = 76.2102 * 126e^{-j11.81} kVA = 9.6025e^{-j11.81} MVA$$

$$S_C = V_C I_C^* = 76.2102 * 178e^{-j30.71} kVA = 13.5655e^{-j30.71} MVA$$

$$S_R = |S_A + S_B + S_C| = |24.8235 - j4.0947| MVA$$

$$= 25.1589 MVA$$

The line capacity is calculated based on  $I_C$  which is the maximum current at the utility grid.

$$S_L = |3V_C I_C^*| = 3 * 13.5655 MVA = 40.6965 MVA$$

The maximum transformer capacity is the algebraic summation of the two single phase transformer winding capacity.

$$S_T = |S_1| + |S_2| = 2E_{bc}I_{bc} = 2 * 26.3032 * 604 kVA = 31.7743 MVA$$

Now line and transformer utilization factor are obtained, using Eq. (3.27) and Eq. (3.28), as

$$TUF = \frac{S_R}{S_T} = \frac{25.1589}{31.7743} = 0.7918$$

$$LUF = \frac{S_R}{S_L} = \frac{25.1589}{40.6965} = 0.6182$$

#### 4.11 Power Quality Problems Resolution

As discussed in Section 3.8 of this report and many other reports disclose, AC electrified railway systems show power quality problems such as harmonic current pollution, feeder voltage perturbation, and reactive power demand. These power quality problems have a bad effect on the AC railway system itself as well as other electric systems connected together. Here now voltage unbalance and harmonic levels are investigated while reactive power demand is not considered because power factor reduction observed in power analysis is low.

##### a) Voltage Unbalance Mitigation Methods

A number of technical options may be considered in order to reduce existing or potential unbalance in a supply system:

- ✓ connection of disturbing loads to a different supply point
- ✓ rearrangement of phase connections of one or more loads
- ✓ connection of disturbing load at a higher voltage level
- ✓ provision of phase balancing or filtering equipment

##### b) Harmonic Mitigation Techniques

Regarding harmonic effect mitigations, satisfying various harmonic standards and engineering recommendations such as IEC 1000-3-2, IEEE 519 (USA), AS 2279, D.A.CH.CZ, EN 61000-3-

2/EN 61000-3-12, and ER G5/4 (UK) at the PCC, there are three mitigation techniques in which a large number of technical publications have been reviewed [35]. Namely [35]:

- ✓ Passive techniques,
- ✓ Active techniques, and
- ✓ Hybrid harmonic reduction techniques using a combination of active and passive methods.

Each technique has many approaches for the implementation [35].

In the report of [23], to mitigate harmonic problem, using the method of passive technique, a multi-section filter, each filter section tuned to 4% below the filtered harmonic, was designed to attenuate the 5th, 7th, and 11th harmonics. As a result the measured 7.75% level of THD, from MATLAB/SIMULINK system model, was reduced to lower than 0.5%. For the THD levels observed in this research report too, the same approach shall be used to limit its effect and operate trains at 72.4kmph without imposing any hazardous effect on the utility grid. Since the aim of this research was not to design power quality problem mitigation methods for the traction system under investigation it is not covered in this report, yet, same analysis done in [23] may be used for the solution herein.

## Chapter Five

### Results Conclusions Limitations and Recommendations

#### 5.1 Results and Discussion

As outlined throughout the report train design and operational speeds are limited by many technical and non-technical factors. As a design parameter train speed selection is mainly based on costs related to infrastructure, rolling stock and operation; etc. Conversely, train operational speeds over a track designed for a particular line speed are determined by factors such as route curve, down grade, switch/turnout, block signal, track strength, location of stations, location of passing loops (for single track system), pantograph-contact wire interaction, traction power available over the feeder line at the point of contact between pantograph and contact wire, internal and external factors expressed in equation of motion, etc.

Traction power available at the point of contact, in the form of tractive effort at the motor shaft, is used to initiate, accelerate and maintain train motion overcoming all the resistances along the train path. Hence, traction power requirement is instantaneous due to the continuous changes of train total resistances. Moreover, train dynamics analysis proves that to keep a targeted speed level over train route, traction power requirement is either constant or increases according to the geographical feature of the line. The analytical results of this report disclose that due to a continuous up gradient of the line section under investigation to maintain train speed constant current has to be drawn from the supply system during each course of motion except some low down gradient parts. Hence, train current was observed to increase from 252A at start at Awash Substation to 431A to 653A from Awash Substation and 604A from S/Kunkur Substation. Similarly, in the other side of S/Kunkur Substation, 238A to 384A and down to 374A.

However, even though there is an increase in current demand, the maximum requirement is 653A for 2.724min and 604A for 12.018min, and minimum catenary voltage calculation, 21.6074kV, show that the system is under safe condition, except unbalance and harmonics; since the minimum operating voltage and the maximum 20min network current capacity are 20kV and 848A [8] [9] [31], respectively, which indicate the calculated values are safe values. Moreover, traction substation voltage drops are within the standard limit, maximum drop 2.5926kV which is less than 6.0kV former phase or later phase, see Appendix C Table 2.

Finally, system evaluation for power quality problems, under operation of trains with targeted maximum speed of 72.4kmph indicates that the utility grid side suffers from perturbation caused by non-linear and time-variant load nature of traction loads under investigation.

#### a) Voltage Unbalance Evaluation

Unbalance Factor Evaluation at point of common coupling due to a varying traction loads on Sirba Kunkur Substation, using the analytical Eq. (3.32) results 3.47% which is greater than the expected value of not more than 0.02 or 2%, hence, the system is unbalanced. This is also observed from the phase voltage and current magnitudes and phase angles, Eq. (4.27) through Eq. (4.32) and Figure 4.7, which violates all of the conditions for the balanced system.

#### b) Harmonic Analysis

Table 3.13 gives the analytical evaluation results of individual and total harmonics distortion in percentage of fundamental signal on the three-phases at point of common coupling, the utility grid side. The result indicates the system is highly distorted, Phase C being the most disturbed as it is expected since it shares the two load arms with Phase A and Phase B. The standard requirement is, see Appendix C, a total harmonic distortion (THD) of not more than 5% and individual distortions (HD) of maximum 3.5%. But, Table 4.13 shows THD of 25.57%, 50.17% and 61.88% on Phases A, B and C respectively which badly violate the standard limit. As a result harmonic mitigation technique is inevitable and mitigation methods such as one reviewed in literature [23] is proposed as a solution, hence, harmonic distortion is not the problem at the targeted maximum speed of operation, 72.4kmph.

## 5.2 Conclusions

In this thesis report, keeping the general objective, analyze traction power consumption along the Ethio-Djibouti Railway particularly at Awash~Sirba Kunkur~Sebaka Line Subsection to investigate freight train operational speeds and finally present documented findings that will serve as reference for operation planning process studies; analytical power consumption evaluation for targeted maximum speed of 72.4kmph in the Down link direction, Addis Ababa to Nagad direction, is done. The analysis of the train speed profile results with average train speed of 65.86kmph which improves the proposed average train speed 54kmph [9]. Moreover, power supply system evaluation mainly based on power quality determinants is performed and results indicate that the system is severely suffers from voltage unbalance, harmonics and power factor

reduction. Furthermore, utilization factors show the system may not be utilized effectively unless compensation methods are included.

### 5.3 Limitations and Recommendations

Practical train operational speed investigation is related to many railway system component evaluation and analysis. Traction power analysis proposed and analyzed in this research report, to discover the real possible operational speed, is a very little part of the system. Others, such as

- ✓ Track infrastructure;
- ✓ Rolling stock;
- ✓ Signalling and traffic control;
- ✓ Communication system;
- ✓ Operation mode;
- ✓ Time table scheduling and rescheduling;
- ✓ Transportation plan study and capacity evaluation;
- ✓ Maintenance system;
- ✓ Station design and location;
- ✓ Optimum location and design of passing loops (for single track system);
- ✓ Logistic support
- ✓ Social studies, etc. are prominent considerations that are not discussed in part or as a whole in this research. Anyways the study has to cover all the potential sources of technical and non-technical bottlenecks for the operation of trains at or near their design speed over tracks designed for them and propose a solution to alleviate the problem(s) identified by the study.

Finally, the author recommends that the following points to be further studied in the future to benefit the results of this research and compile the study as a concrete work:

- ✓ The above listed electrified railway subsystem studies may be included in detail;
- ✓ Power disturbance created due to inverters may be included;
- ✓ An appropriate and cost effective power quality problem mitigation method(s) and location of installation may be studied;
- ✓ The study may be supported by software simulations which includes all the effects of main causes for operational speed reduction along the corridor;

## References

- [1] “Electric Traction”, J. Upadhyay and S.N. Mahendra, Allied Publishers Ltd, 2011, ISBN 81-7764-005-4, New Delhi.
- [2] “Impact of Innovation Technology on Complex Systems: The Electrified Railway Supply System” by di Daniela Proto, Dipartimento Di Ingegneria Elettrica - Via Claudio 21 – 80125 Napoli Italy, November 2004.
- [3] “Algorithms for Generating Train Speed Profiles”, Journal of the Eastern Asia Society for Transportation Studies, Vol. 6, pp. 356 - 371, 2005, Jyh-Cherng JONG & Sloan CHANG
- [4] “High speed, energy consumption and emissions” Study and Research Group for Railway Energy and emissions, UIC: International Union of Railways, December 2010.
- [5] “Railway operation analysis Evaluation of quality, infrastructure and timetable on single and double-track lines with analytical models and simulation”, Doctoral thesis Division of Traffic and Logistics Department of Transport and Economics, by Olov Lindfeldt, Stockholm 2010.
- [6] “An Analysis of Single Track High Speed Rail Operation” Dissertation submitted in partial fulfillment of the requirements for the award of MSc in Railway Systems Engineering & Integration, College of Engineering, School of Civil Engineering University of Birmingham, by Sam Paul Singh Pawar, May 2011.
- [7] “High-Speed Railways in China: A Look at Construction Costs” China Transport Topics No. 9, by Gerald Ollivier, Jitendra Sondhi and Nanyan Zhou, World Bank Office, Beijing, July, 2014.
- [8] “Final Evaluation Report on the Feasibility Study of Addis Ababa/Sebeta– Djibouti Railway Project”, China International Engineering Consulting Corporation, September 2012.
- [9] “Ethiopia/Sebeta-Djibouti/Nagad Railway Feasibility Study Part XIII- Locomotive Equipment”, Sebeta-Mieso Section, Ethiopian Railway Corporation, Executive edition, China Railway Yuan Engineering Group Co. Ltd. Chengdu, September 2012,
- [10] <http://en.wikipedia.org/wiki/Transport> access date Dec. 10.2014
- [11] “Railway Signaling and Interlocking, International Compendium”, Gregor Theeg and Sergej Vlasenko, Eurail press, ISBN 978-3-7771-0394-5, 1<sup>st</sup> ed. 2009, Germany.
- [12] “Research on Optimum Speed for High Speed Lines” Volume I, by FCH Authors, UIC Directors of the research In cooperation with the UIC Intercity and High Speed Committee, October 2012.

- [13] “High-speed rail freight” Sub-report in Efficient train systems for freight transport by Gerhard Troche, KTH Railway Group Report 0512, Centre for Research and Education in Railway Engineering at the Royal Institute of Technology Stockholm (Railway Group KTH), Stockholm 2005.
- [14] “Project Report on Design Loads for Bridges on High Speed Routes of 250-350 Km/h” presented by Sri Dipak Roy and Sri Sudhirp Arihar, Indian Railway Institute of Civil Engineering, pune 411001.
- [15] “Electrification of the Freight Train Network from the Ports of Los Angeles and Long Beach to the Inland Empire”, California State Polytechnic University, Pomona College of Engineering Pomona, CA 91768, May 2008.
- [16] “Calculation method for powering a tramway network”, Jakob Edstrand, Master of Science Thesis in Electrical Power Engineering, Department of Energy and Environment, Division of Electric Power Engineering, Chalmers University of Technology, Göteborg, Sweden 2012.
- [17] “Railway Reform: Toolkit for Improving Rail Sector Performance” The International Bank for Reconstruction and Development / The World Bank, 1818 H Street NW, Washington DC 20433, June 2011, [www.worldbank.org](http://www.worldbank.org)
- [18] “CEE 3604 ‘Rail Transportation: Addendum Rail Resistance Equations’” Transportation Engineering A.A Trani, Virginia Tech, access date 21,03,2015.
- [19] “Analytical Solutions for Predicting Train Coasting Dynamics”, by Jyh-Cherng Jong Ph. D., P.E., Research Scientist Civil and Information Technology Research Center Sino-tech Engineering Consultants, Inc.171 Nanking E. RD. SEC. 5Taipei 105, Taiwan Fax: +886-2-27692131 Ext. 20998 E-mail: [jcyjong@sinotech.org.tw](mailto:jcjong@sinotech.org.tw), Proceedings of the Eastern Asia Society for Transportation Studies, Vol.4, October, 2003
- [20] Speed Limit on Railway Curves (Uses of Super-Elevation on Railways)
- [21] “Optimizing AC Electric Railway Power Flow with Power Electronic Control”, Thanatchai Kulworawanichpong; a thesis submitted to The University of Birmingham for the degree of Doctor of Philosophy, November 2003.
- [22] “Railway Electrification: 25KV a.c. Design on British Railway”, Director of Mechanical and Electrical Engineering, British Railways, BR.12034/16

- [23] “Power Quality Issues In Railway Traction System” by Demeke Kebede Hailu, Electrical Power Engineering Dept., College of Engineering, Defense University, Bishoftu, Ethiopia  
Phone number: +251 924 40 9603 E-mail: demeke.kebede@yahoo.com
- [24] “Electrification Systems Technology Assessment Appendix 6” Prepared for METROLINX,  
20 Bay Street, Suite 901, Toronto ON M5J 2N8 Dec. 2010.
- [25] “Concept on Traction Metering Principle” Pattanaik P.K, IEEMA Journal, December 2007,  
pp. 99–103
- [26] “Modeling and comparison of traction transformers based on the utilization factor definitions”, International Review on Modeling and Simulation (IREMOS) Part B, ISSN 1974-9821, Vol.4 N.1, February 2011.
- [27] “Modeling and Simulation of AC traction power Supply system”, by Belay Tibebe Mintesnot, Master Degree Thesis, Southwest Jiaotong University, November, 2013.
- [28] “Mitigation of Voltage Unbalance in Traction System” B.R.LakshmiKantha, Dr. Murugesh Mudaliar, Dr. K.Balaraman, Dr. R.Nagaraja, Proceedings of the World Congress on Engineering and Computer Science 2008, WCECS 2008, October 22-24, 2008, San Francisco, USA.
- [29] “Estimation of Voltage Unbalance in Power Systems Supplying High Speed railway” by Giulio Burchi and Cristian Lazariou, Electrical Power Quality and Utilization, journal Vol. XI, No.2, 2005
- [30] “Application of IEEE Standard 519-1992 Harmonic limits”, Thomas M. Blooming P.E., t.blooming@ieee.org, and Daniel J. Carnovale, P.E., DanielJCarnovale@eaton.com
- [31] “New Standard Gauge Railway” Ethiopia/Sebeta-Djibouti/Nagad Railway Preliminary Design (Revised Draft) Part XI Electrification, China Civil Engineering Construction Corporation, November 2013.
- [32] “Active Control of Reactive Power in a Modern Electrical Rail Vehicle” by Eivind Toreid, Master of Science in Electric Power Engineering, Norwegian University of Science and Technology Department of Electric Power Engineering, June 2011.
- [33] “New Standard Gauge Railway Ethiopia/Sebeta-Djibouti/Nagad Railway Feasibility Study Part XII Fascicule I (Sebeta-Mieso Section) Electrification” Executive Edition Ethiopian Railways Corporation, September, 2012.

- [34] Review Article “Harmonic Mitigation Techniques Applied to Power Distribution Networks” by Hussein A. Kazem, Hindawi Publishing Corporation, Advances in Power Electronics, Volume 2013, Article ID591680,10pages <http://dx.doi.org/10.1155/2013/591680>
- [35] “Electric Rail – Problems of The Controlling And Supplying Equipment” by Nicolîță GUȚĂ and Dan MIHOC, U.P.B. Sci. Bull., Series C, Vol. 72, Iss. 4, 2010 ISSN 1454-234x
- [36] “Design a Realistic Performance of a Passenger Train” by Sergey E. Galushin, Shazly A. Mohammed, Nouby M. Ghazaly, International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622, Vol. 2, Issue 1,Jan-Feb 2012, pp.1088-1098, [www.ijera.com](http://www.ijera.com)
- [37] New Standard Gauge Railway Ethiopia/Sebeta-Djibouti/Nagad Railway Feasibility Study Part I, General Specification Executive Edition, Ethiopian Railways Corporation Sept. 2012

## Appendix A

### Train's Total Resistance Force Derivation

For the analytical calculation, in this report, it is assumed that only one train per section is to be operated at a time due to single-track and semi-automatic block system implementation of the project. Moreover the following technical data are used where necessary: maximum gradient (D) =18.5 in 1000 for dual locomotive and Minimum curve radius=800m; k=700; W:weight per axle in ton=25ton; n:number of axle=6; M=4000ton; g=9.81m/s<sup>2</sup>;  $W_t = Mg$

Total train resistance is calculated using train resistance from AREMA [19], curve resistance from Japan [19] and the general grade resistance formulae. A popular way to determine the maximum starting weight for a vehicle is to look at the power-to-weight ratio [17]. In railroad freight, a ratio of 1 [HP/ton] is a common value while in rapid transit systems the ratio may be as high as 5-10 [HP/ton] [17].

The weight ratio for single and dual locomotives of 9600kW is shown below respectively:

$$W_{ratio} = \frac{9600k}{746} \frac{Hp}{4000ton} = 3.2172 \text{ and } W_{ratio} = \frac{19200k}{746} \frac{Hp}{4000ton} = 6.4343$$

Since the weight ratios are greater than the expected value for freight, starting resistance is neglected in the calculations of this report.

#### Train resistance

$$R_u \left[ \frac{lb}{ton} \right] = 0.6 + \frac{20}{W} + 0.01v + K \frac{v^2}{W * n} \quad (A.1)$$

$$\begin{aligned} R_u \left[ \frac{kg}{ton} \right] &= 0.45 * \left( 0.6 + \frac{20}{25} + 0.01v + 0.0935 \frac{v^2}{25*6} \right) \\ &= 0.63 + 0.0045v + 0.0002805v^2 \end{aligned}$$

$$R[N] = (0.63 + 0.0045v + 0.0002805v^2)W_t; v = \text{speed [mph]}$$

$$\text{But, } v \text{ [mph]} = 1.6 * v \text{ [kmph]}$$

$$R[N] = (0.63 + 0.0072v + 0.0007181v^2)W_t; v = \text{speed [kmph]} \quad (A.2)$$

#### Grade resistance

$$R_g [N] = \pm Mg \sin \theta = \pm W_t D; \quad (A.3)$$

**Curve resistance**

$$R_c [N] = r_c \frac{W_t}{g} \quad (A.4)$$

$$r_c \left[ \frac{\text{kN}}{\text{ton}} \right] = 0.01 \frac{\text{k}}{R_c} = \frac{0.01 * 700}{800} = 0.00875$$

$$r_c \left[ \frac{\text{N}}{\text{ton}} \right] = 8.75 \quad (A.5)$$

$$R_c [N] = \frac{8.75}{9.81} W_t = 0.8919 W_t \quad (A.6)$$

Total train resistance is the sum of train resistance in Eq. (4.2), grade resistance in Eq. (4.3) and curve resistance in Eq. (4.6) and given as shown below:

$$R_e(N) = 0.8919 W_t + \pm W_t D + (0.63 + 0.0072v + 0.0007181v^2) W_t \quad (A.7)$$

$$= (0.8919 \pm D + 0.63 + 0.0072v + 0.0007181v^2) W_t \quad (A.7)$$

Taking the maximum gradient of the route 18.5 in 1000 gives the total train resistance over the line as

$$\begin{aligned} R_e(N) &= (20.0219 + 0.0072v + 0.0007181v^2) W_t \\ &= (785.6594 + 0.2825v + 0.02818v^2) kN \end{aligned} \quad (A.8)$$

As it can be seen from this result, a single locomotive with maximum tractive effort of 570kN cannot overcome the grade resistance at the maximum gradient; hence, another locomotive to push the load is required so that they can share the total load.

## Appendix B

### Fourier series Expression of Rectifiers Output

The immediate electrical power conditioner, on-board the train, has single phase voltage transformer and power electronic circuit employing IGBTs. This power conditioner set does mainly rectification with eliminated ripple level, DC-linkage and inversion to three phase voltage to feed the induction motors driving the locomotive, for AC-locomotives, or the rectified current is feed directly, for DC-locomotives. As seen on block diagram of Figure 3.2 power at 25 kV is taken via a pantograph from the overhead contact wire and fed to the step-down transformer in the locomotive. The low AC voltage so obtained is converted into pulsating DC voltage by means of the rectifier; i.e. modulated and rectified [36]. A Four-Quadrant Chopper Rectifier (FQR) implemented with uncontrolled diodes and IGBTs does modulation and rectification [36]. The modulated and rectified pulsating DC voltage wave form is given by [36]:

$$v_r(t) = \frac{V_{max}}{2} (1 - \cos 2\omega t) \quad (B.1)$$

The DC current driven by this DC voltage is inverted and feed to induction motors to act on the train wheel-rail so that rotational motion is converted to translational motion provide that there is efficient wheel-rail interaction or slip.

As described so far traction load could be source of power quality problems, and it is, to the utility grid due to its non-inevitable characteristics. This power quality problem is expressed in system frequency disturbances, flicker, dips, power factor reduction, harmonics, negative phase sequence current and voltage unbalances.

To analyze the effect of each harmonic component on the supply side it is usual to express the output signals, voltage and current, by the well-known Fourier series expansion of the form:

$$f(t) = a_o + \sum_{n=1}^{\infty} (a_n \cos ht + b_n \sin ht) \quad (B.2)$$

Where the coefficients  $a_o$ ,  $a_n$  and  $b_n$  are given by

$$a_o = \frac{1}{2\pi} \int_0^{2\pi} f(t) dt$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos ht dt$$

$$b_h = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin ht \, dt$$

For the pulsating DC uncontrolled FQR whose output expressed in eq. (B.1)

$$\begin{aligned} a_o &= \frac{1}{\pi} \int_0^{\pi} \frac{V_{max}}{2} (1 - \cos 2wt) dwt \\ &= \frac{V_{max}}{2} \end{aligned} \quad (B.3)$$

$$\begin{aligned} a_h &= \frac{1}{\pi} \int_0^{\pi} \frac{V_{max}}{2} (1 - \cos 2wt) \cos hwt \, dwt \\ &= \frac{V_{max}}{2\pi} \left\{ \int_0^{\pi} \cos hwt \, dwt - \int_0^{\pi} \cos 2wt \cos hwt \, dwt \right\} \end{aligned}$$

using the property  $\cos A \cos B = \frac{1}{2} (\cos(A+B) + \cos(A-B))$

$$= \frac{V_{max}}{2\pi} \left\{ \frac{1}{h} \sin hwt - \frac{1}{2} \left( \frac{\sin(2+h)wt}{2+h} + \frac{\sin(2-h)wt}{2-h} \right) \right\}$$

Since  $\sin(wt)$  is odd function, evaluating the above integrand over the period (0 to  $\pi$ ) gives zero.

$$\text{therefore: } - a_h = 0 \quad (B.4)$$

$$\begin{aligned} b_h &= \frac{1}{\pi} \int_0^{\pi} \frac{V_{max}}{2} (1 - \cos 2wt) \sin hwt \, dwt \\ &= \frac{V_{max}}{2\pi} \left\{ \int_0^{\pi} \sin hwt \, dwt - \int_0^{\pi} \cos 2wt \sin hwt \, dwt \right\} \end{aligned}$$

using the property  $\cos A \sin B = \frac{1}{2} (\sin(A+B) - \sin(A-B))$

$$= \frac{V_{max}}{2\pi} \left\{ \frac{-1}{h} \cos hwt - \frac{1}{2} \left( \frac{-1}{2+h} \cos(2+h)wt + \frac{1}{2-h} \cos(2-h)wt \right) \right\}$$

Evaluating this integral result over the period (0 to  $\pi$ ) gives

$$\begin{aligned} b_h &= \frac{V_{max}}{2\pi} \left\{ \frac{-1}{h} (\cos h\pi - 1) - \frac{1}{2} \left( \frac{-\cos(2+h)\pi}{2+h} + \frac{\cos(2-h)\pi}{2-h} + \frac{1}{2+h} - \frac{1}{2-h} \right) \right\} \\ &= \frac{V_{max}}{2\pi} \left\{ \frac{2}{h} - \frac{1}{2} \left( \frac{2}{2+h} - \frac{2}{2-h} \right) \right\} \end{aligned}$$

$$\begin{aligned}
&= \frac{V_{max}}{2\pi} \left\{ \frac{2}{h} - \frac{1}{2} \left( \frac{4 - 2h - 4 - 2h}{(2+h)(2-h)} \right) \right\} = \frac{V_{max}}{\pi} \left\{ \frac{1}{h} - \left( \frac{h}{(h^2 - 4)} \right) \right\} \\
b_h &= \begin{cases} 0 & ; \text{for } h \text{ even} \\ \frac{-4V_{max}}{\pi h(h^2 - 4)} & ; \text{for } h \text{ odd} \end{cases} \quad (B.5)
\end{aligned}$$

Substituting  $a_o$  and  $b_h$  in the Fourier expression we obtain:

$$v(t) = \frac{V_{max}}{2} + \frac{4V_{max}}{\pi} \sum_{h=1 \& \text{ odd}}^{\infty} \frac{-1}{h(h^2 - 4)} \sin h\omega t \quad (B.6)$$

$V_{max}$  is the line to line voltage since the input to traction transformers is line voltage. Hence, the load side voltage with reference to phase voltage is given by

$$v(t) = \frac{V_{max}}{2} + \frac{4V_{max}}{\pi} \sum_{h=1 \& \text{ odd}}^{\infty} \frac{-1}{h(h^2 - 4)} \sin(h\omega t - h\frac{\pi}{6}) \quad (B.7)$$

Expressed using RMS values as follows:

$$V(t) = V_o + \sqrt{2} \sum_{h \neq 0}^{\infty} V_h \sin(h\omega t + \alpha_h) \quad (B.8)$$

$$\text{where: } V_o = \frac{V_{max}}{2\sqrt{2}}; V_h = \frac{-4V_{max}}{\pi h(h^2 - 4)\sqrt{2}} \text{ and } \alpha_h = -h\frac{\pi}{6}$$

Similarly the current wave form expressed in RMS values is given as:

$$I(t) = I_o + \sqrt{2} \sum_{h \neq 0}^{\infty} I_h \sin(h\omega t + \beta_h) \quad (B.9)$$

$$\text{where: } I_o = \frac{I_{max}}{2\sqrt{2}}; I_h = \frac{-4I_{max}}{\pi h(h^2 - 4)\sqrt{2}} \text{ and } \beta_h \text{ is load angle}$$

## Appendix C

## Harmonics and Traction Transformer Voltage Drop Standard Limits

Table 1 . Harmonic Current Limits for Individual End Users from IEEE 519-1992. [34]

HARMONIC CURRENT DISTORTION LIMITS IN % OF $I_L$						
$v \leq 69kV$						
$I_{sc} / I_L$	$h < 11$	$11 \leq h < 17$	$17 \leq h < 23$	$23 \leq h < 35$	$35 \leq h$	TDD
<20	4.0	2.0	1.5	0.6	0.3	5.0
20-50	7.0	3.5	2.5	1.0	0.5	8.0
50-100	10.0	4.5	4.0	1.5	0.7	12.0
100-1000	12.0	5.5	5.0	2.0	1.0	15.0
>1000	15.0	7.0	6.0	2.5	1.4	20.0
$69kV < v \leq 161kV$						
<20*	2.0	1.0	0.75	0.3	0.15	2.5
20-50	3.5	1.75	1.25	0.5	0.25	4.0
50-100	5.0	2.25	2.0	1.25	0.35	6.0
100-1000	6.0	2.75	2.5	1.0	0.5	7.5
>1000	7.5	3.5	3.0	1.25	0.7	10.0
$v > 161kV$						
< 50	2.0	1.0	0.75	0.3	0.15	2.5
$\geq 50$	3.5	1.75	1.25	0.5	0.25	4.0

Table 2. : The maximum voltage losses permissible value [28]

Transformer connection	Single phase		Three phase	
	Vv	All parallel	The former phase	The later phase
Maximum value (kV)	5.6	6.0	6.0	4.5