

A quantitative analysis of illusion magnitude changes induced by rotation of contextual distractor

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In the present study, the predictions of the computational model of centroid extraction were verified in psychophysical examination of the length illusion induced by stimuli comprising the conventional or asymmetric Müller-Lyer wings as the contextual distractors. In experiments, the illusion magnitude changes evoked by rotation of distractors with different spatial parameters were quantitatively determined. It was demonstrated that the model calculations adequately account for the illusion magnitude variations shown by all the subjects for all modifications of stimuli. A good correspondence between the experimental and theoretical data supports the suggestion that local positional biases caused by the neural processes of automatic centroid extraction can be one of the main reasons of emergence of illusions of the Müller-Lyer type.

Key words: length illusion, distractor rotation, perceptual positional shift, centroid.

INTRODUCTION

According to the hypothesis on the indirect positional coding *via* centroids (Morgan et al. 1990, Morgan and Glennerster 1991), the phenomenon of the Müller-Lyer and related illusions of extent can be explained by spatial assimilation of neural excitations evoked by the neighboring parts of the stimulus. Due to this assimilation, the visual system fails to locate the figure terminators (wings apexes) independently from the adjacent contextual distractors (wings themselves), and the judgments of the distances between the terminators are biased toward the distances between the centers-of-masses (centroids) of the distractors (Fig. 1, upper). Thus, the “centroid” approach assumes that the illusions emerge mostly because of the terminators localization errors, and thereby differs essentially from highly popular the “perspective” explanation (Gregory 1968, Barrow and Tenenbaum 1981, Redding et al. 1997, Gillam 1998, Nanay 2009, Redding and Vinson 2010), which supposes a homogeneous stimuli resizing triggered by implicit 3D interpretation of the flat drawings.

Earlier, the suggestion on the terminators biases was experimentally confirmed in the studies of the Müller-Lyer and Judd figures with markers dividing stimulus shaft into equal-appearing segments (Morgan et al. 1990, Post et al. 1998, Predebon 2001). Substantial positional shifts of the terminators were obtained for the Müller-Lyer figure with the wings placed on only one side of the stimulus (Greene and Nelson 1997, Welch et al. 2004, Predebon 2005), and for a single set of the wings positioned within an imaginary reference rectangle (Bulatov et al. 2013). The “centroid” approach has been successfully applied to the interpretation of the results of experiments with additional non-target dots in the Müller-Lyer (Searleman et al. 2005), Ponzo and horizontal-vertical illusions (Searleman et al. 2009). Authors (Schloss et al. 2014) of newly discovered the configural shape illusion (distorting of the shape of a target object by adjacent objects) claim that the most plausible explanation for the effect can be derived by considering edge localization errors caused by spatial assimilation (i.e., like in the case of illusions of the Müller-Lyer type). Experiments with stimuli composed of three dots forming an imaginary rectangular triangle (Bulatov et al. 2012) have shown that the perpendicularity misjudgments can be explained by local positional biases, and the magnitude of these distortions is com-

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measurable with that of the illusions of length. The results of a number of psychophysical studies of inherently non-illusory stimuli demonstrated that perception of spatial separation of visual objects is strongly affected by neural processes of their centroids localization (Watt and Morgan 1984, 1985, Hirsch and Mjolsness 1992, Morgan et al. 1994, Badcock et al. 1996, Whitaker et al. 1996, Akutsu et al. 1999, Baud-Bovy and Soechting 2001, McGraw et al. 2003, Wright et al. 2011).

Recently, in order to verify whether the center-of-mass alterations can be one of the most important causes of the Müller-Lyer and related illusions, a quantitative model of centroid extraction has been developed (Bulatov et al. 2009, 2010). The model procedure includes a weighted spatial pooling (within a certain attentional window centered with stimulus terminator) of the neural excitations evoked by stimulus elements, and finding the centroid of the pooling (by the two-dimensional convolution of its spatial profile with that of a certain summation unit). It was also assumed that the size of area of centroid extraction (cluster of summation units within relevant attentional window) grows linearly with visual eccentricity. The model was applied in interpretation of results of experiments with figures of the Müller-Lyer type comprising either separate dots or segments of lines or closed two-dimensional shapes (both outlined and uniformly filled), and the calculations agreed pretty well with the experimental data (Bulatov et al. 2009, 2010, 2015). The “centroid” approach looks still more preferable due to biological significance of the mechanism of automatic centroid extraction that enables fast and reliable assessment of location of visual objects independently of their size, shape complexity, and illumination conditions (Morgan et al. 1990).

Previous psychophysical examination (Bulatov et al. 2011) of the Brentano figures with rotating contextual flanks (either the Müller-Lyer wings or arcs of a circle) has yielded cosine-like changes of the illusion magnitude, and thereby largely confirmed the crucial point in the “centroid” explanation concerning the shifts of the stimulus terminators toward the centers-of-masses of adjacent distractors. However, according to the basic equations of the model, almost a pure cosine modulation of the illusion magnitude is valid only for the tilting of a small single-dot (or single-wing) distractor. In the case of rotation of relatively large compound contextual flanks consisting of elements with different orientation, a detailed analysis of the model predictions offers a much more sophisticated pattern of

functional dependencies; therefore, it appears quite reasonable to check experimentally the validity of these theoretical propositions.

For this purpose, in the current study we have performed a psychophysical investigation of the illusory effects induced by stimuli (Fig. 1, lower) of the Brentano type comprising a single set of the symmetric or asymmetric Müller-Lyer wings as a contextual distractor. In experiments with all stimuli modifications, we used the same independent variable, the tilt angle, φ of the distractor’s bisector relative to the horizontal stimulus axis. The use of stimuli comprising a single central distractor actually eliminates the manifestation of positional shifts for the lateral stimulus terminators and, thereby, considerably facilitates subsequent theoretical interpretation of experimental results.

The main goal of the study was further development and verification of relevance of the model of automatic centroid extraction through examination whether the

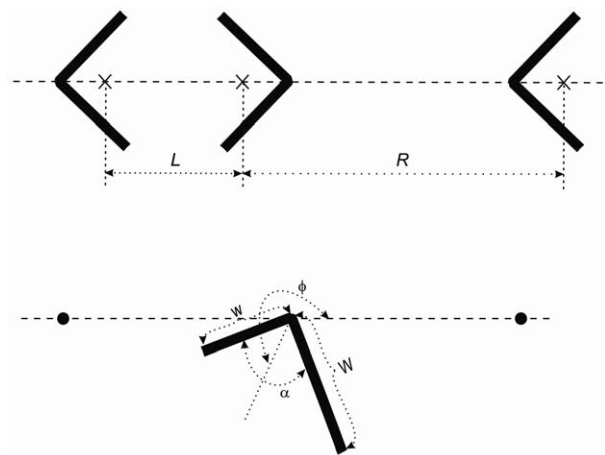


Fig. 1. Various versions of figures of the Müller-Lyer (Brentano) type. The upper part represents the Brentano figure with horizontally oriented contextual flanks. Crosses indicate flanks’ centroids. For illustrative purpose, the centroids’ biases are shown to be exaggerated. The perceived lengths of the left and right stimulus intervals are designated as L and R , correspondingly. The lower part represents the stimulus comprising two lateral dots and a single set of the asymmetric Müller-Lyer wings: w and W refer to the length of the short and long wing, respectively; α is the internal angle of the wings; φ refers to the tilt angle of the wings’ bisector. In experiments, white shapes (luminance 75 cd/m^2) were presented against a dark round-shaped background (5° in diameter and 0.4 cd/m^2 in luminance); dashed and dotted lines, the dimensions were not part of the actual display.

theoretical calculations provide an adequate description of the data obtained in psychophysical experiments. Therefore, a comprehensive analysis of other well-known explanations of the illusions was beyond of the scope of the present paper.

The model predictions

According to the model, which was discussed earlier in more detail (Bulatov et al. 2009, 2010), the hypothetical visual mechanism for extracting object’s centroid position on the arbitrary x -axis can be assembled from evenly distributed (within a Gaussian attentional window centered with the object) summation units each of which possess Laplacian-of-Gaussian, LoG weighting profile along the x -axis, and constant one along the orthogonal y -axis. Analytically, the centroid position can be derived from the convolution of the weighting profile of the summation unit with the function of mass distribution (here mass is considered as the amplitude of the object-evoked neural excitation that is proportional to the object luminance), and corresponds to that of the summation unit with the maximum response. In the case of the elementary stimulus made up of two separate dots (i.e., the terminator and distractor; the terminator-dot is located in the center of the attentional window), the centroid bias, τ of the terminator along the x -axis can be estimated using the following equation:

$$\iint_{-\infty}^{\infty} F'(\tau - x, y)_{\tau} e^{-B(x^2+y^2)} \{M\delta(x, y) + m\delta(x - d\cos(\theta), y - d\sin(\theta))\} dx dy = 0 \tag{1}$$

where $F'(\tau-x, y)_{\tau}$ represents the first derivative (with respect to τ) of the LoG weighting profile of the receptive field of the summation unit; d , the distance between the terminator and distractor; θ , the angle between the chosen reference x -axis and the imaginary line connecting the terminator and distractor; $B=0.5\sigma^{-2}$, where σ is the standard deviation of the Gaussian function of the attentional pooling window and that of the LoG of the summation unit; M and m , are point masses of the stimulus terminator and distractor, respectively; $\delta(x, y)$, the Dirac delta function. In a similar way, the centroid bias along the other axis of the pattern can be obtained by using the system of the summation units oriented correspondingly.

Since the Equation 1 can be solved only numerically, it is assumed (Bulatov et al. 2009, 2010, 2015)

that rather good estimates of the centroid bias can be obtained using the following approximate formula:

$$\begin{aligned} \tau(d, \theta) &= \frac{p(d, \theta)}{m(d)} \cong \\ &\cong \frac{d\cos(\theta)m e^{-Bd^2}}{M+m e^{-Bd^2}} e^{-B\mu(d\cos(\theta))^2 \left(\frac{M-m e^{-Bd^2}}{M+m e^{-Bd^2}}\right)} \approx \\ &\approx \frac{d\cos(\theta)\mu m e^{-Bd^2(1+\mu\cos(\theta)^2)}}{M+m e^{-Bd^2}} \end{aligned} \tag{2}$$

where $p(d, \theta)$ and $m(d)$ represent the first moment of the “pooled” mass of the terminator and distractor and this mass itself, respectively; $\mu \approx 1.5$ is the empirical coefficient.

Assuming a linear superposition of neural responses concerning the intensity of stimulation, the proposed principle of centroid bias calculations can be extended to some other stimuli with different spatial structure. Considering a single set of the asymmetric Müller-Lyer wings (Fig. 1, lower) and taking into account the spread of attentional window (parameter $B=0.5\sigma^{-2}$), the “pooled” mass of the wings (made up of two thin line segments, i.e., when the width of the line, $\lambda \ll \sigma$) can be described as follows:

$$\begin{aligned} m(w, W) &\cong \int_0^w \lambda e^{-Bx^2} dx + \int_0^W \lambda e^{-Bx^2} dx = \\ &= \frac{\lambda}{2} \sqrt{\frac{\pi}{B}} \{ \operatorname{erf}(w\sqrt{B}) + \operatorname{erf}(W\sqrt{B}) \} \end{aligned} \tag{3}$$

where w and W refer to the length of the short and long wing, respectively; $\operatorname{erf}(x)$ is the error function encountered in integrating the normal distribution; the absolute value of the wings luminance is non-essential therefore, with no loss of generality, is arbitrarily set to 1. Considering the numerator in Formula 2, the first moment of mass for the wings can be estimated by using the formula:

$$\begin{aligned} p(w, W, \alpha, \varphi) &\cong \lambda\mu \left(\int_0^w x \cos(\varphi + 0.5\alpha) e^{-Bx^2(1+\mu\cos(\theta+0.5\alpha)^2)} dx + \int_0^W x \cos(\varphi - 0.5\alpha) e^{-Bx^2(1+\mu\cos(\theta-0.5\alpha)^2)} dx \right) = \\ &\frac{\lambda\mu}{2B} \left(\cos(\varphi + 0.5\alpha) \frac{1-e^{-BW^2(1+\mu\cos(\theta+0.5\alpha)^2)}}{1+\mu\cos(\theta+0.5\alpha)^2} + \cos(\varphi - 0.5\alpha) \frac{1-e^{-BW^2(1+\mu\cos(\theta-0.5\alpha)^2)}}{1+\mu\cos(\theta-0.5\alpha)^2} \right) \end{aligned} \tag{4}$$

where α and φ refer to the internal and tilt angle of the distractor, respectively.

Then, the centroid bias of the stimulus terminator (i.e., the perceptual displacement of the vertex of the wings) can be evaluated by dividing the first moment of mass with the mass itself:

$$\tau(w, W, \alpha, \varphi) = \frac{p(w, W, \alpha, \varphi)}{m(w, W)} \quad (5)$$

As can be seen from Formulas 3 and 4, the dependence of the centroid bias on the tilt angle, φ can be considered as a superposition of two cosine functions (symmetrically shifted by $\pm 0.5\alpha$ relative to φ), whose amplitudes, in turn, depend nonlinearly on the tilt angle. In addition, both the masses and the first moments of masses are proportional to λ ; therefore, λ is eliminated in formula (5), and the value of centroid bias should not depend on the line width.

The calculated alterations of the centroid bias as a function of the tilt angle of a single set of the Müller–Lyer wings (symmetric and asymmetric) are shown in Figure 2. It is noteworthy that for the relatively short (compared with the size of the attentional window, $4 \times \sigma$, i.e., the spread of Gaussian with two standard deviations on either side of the mean) symmetric Müller–Lyer wings, the model predicts the changes of the bias, which are quite similar to the cosine functions with appropriate amplitudes (Fig. 2, upper). For the longer wings, the similarity dramatically diminishes because of the appearance of plateaus on the curves in the regions near 0° (180°) and 90° (270°) for the acute- and obtuse-angle wings, respectively (Fig. 2, middle). In the case of the asymmetric Müller–Lyer wings, the model calculations provide the shapes of the curves (Fig. 2, lower) that are even more sophisticated in comparison with a simple cosine modulation of the centroid bias of the stimulus terminator.

It should be noted that in order to measure the effects of illusion (i.e., to perform length-matching or length-bisection task), stimuli typically used in experiments comprise three terminators (the Brentano arrangement), which are located at different visual eccentricities (since all terminators cannot be foveated simultaneously). In accordance with the “centroid” explanation of the illusions of extent, the visual system identifies the distractor-evoked bias of the centroid of the stimulus terminator with its positional shift, and that leads to the misestimat-

ing of relevant spatial intervals. In turn, the value of the centroid bias depends (Formula 5) on the size of the distractor-related area of centroid extraction, which (under the model assumption) grows linearly with distance from the fovea; therefore, the illusion magnitude may vary depending on the actual direction of the observer’s gaze. Nonetheless, although under real experimental conditions the observers can move their gaze more or less freely, a certain correspondence between the model calculations and measured changes in illusion magnitude can be expected.

METHODS

Apparatus

The experiments were carried out in a dark room (the surrounding illumination < 0.2 cd/m²). A Sony SDM-HS95P 19-inch LCD monitor (spatial resolution 1280×1024 pixels, frame refresh rate 60 Hz) was used for the stimuli presentations. A Cambridge Research Systems OptiCAL photometer was applied to the monitor luminance range calibration and gamma correction. A chin and forehead rest was used to maintain a constant viewing distance of 300 cm (at this distance each pixel subtended about 0.33 min of arc); an artificial pupil (an aperture with a 3 mm diameter of a diaphragm placed in front of the eye) was applied to reduce optical aberrations.

Stimuli were presented in the center of a round-shaped background of 5° in diameter and 0.4 cd/m² in luminance (the monitor screen was covered with a black mask with a circular aperture to prevent observers from being able to use the edges of the monitor as a vertical/horizontal reference). For all the stimuli drawings, the Microsoft GDI+ antialiasing technique was applied to avoid jagged-edge effect.

Stimuli

The stimuli (luminance 75 cd/m²) used in the experiments consisted of two lateral dots and a single central set of the Müller-Lyer wings (either symmetric or asymmetric), which were arranged horizontally according to the Brentano pattern (Fig. 1, lower). The dots and the apex of the wings (which is physically inseparable from the distractor, i.e., from the wings them-

selves) were considered as terminators specifying the ends of the left and right stimulus intervals. The shaft line was absent, the thickness of the wing-lines and diameter of the dots was 1 min of arc; the length of the stimuli (the distance between the lateral terminators) was 200 min of arc.

In all experiments, the tilt angle, φ (the independent variable) of the bisector of the central contextual flank was altered in a random fashion from 0° to 360° with respect to the horizontal axis. In the first two series of

experiments, the length of the symmetric Müller-Lyer wings was fixed at 8 min of arc (in the first series, the internal angle of the wings, α was 30° , and in the second one, 120°). In the third ($\alpha=30^\circ$) and fourth ($\alpha=120^\circ$) series, the length of the symmetric wings was changed to 30 min of arc. In the fifth ($\alpha=30^\circ$) and sixth ($\alpha=120^\circ$) series, the asymmetric Müller-Lyer wings were used, and the length of the longer wing was set to 30 min of arc, whereas the shorter one was equal to 8 min of arc.

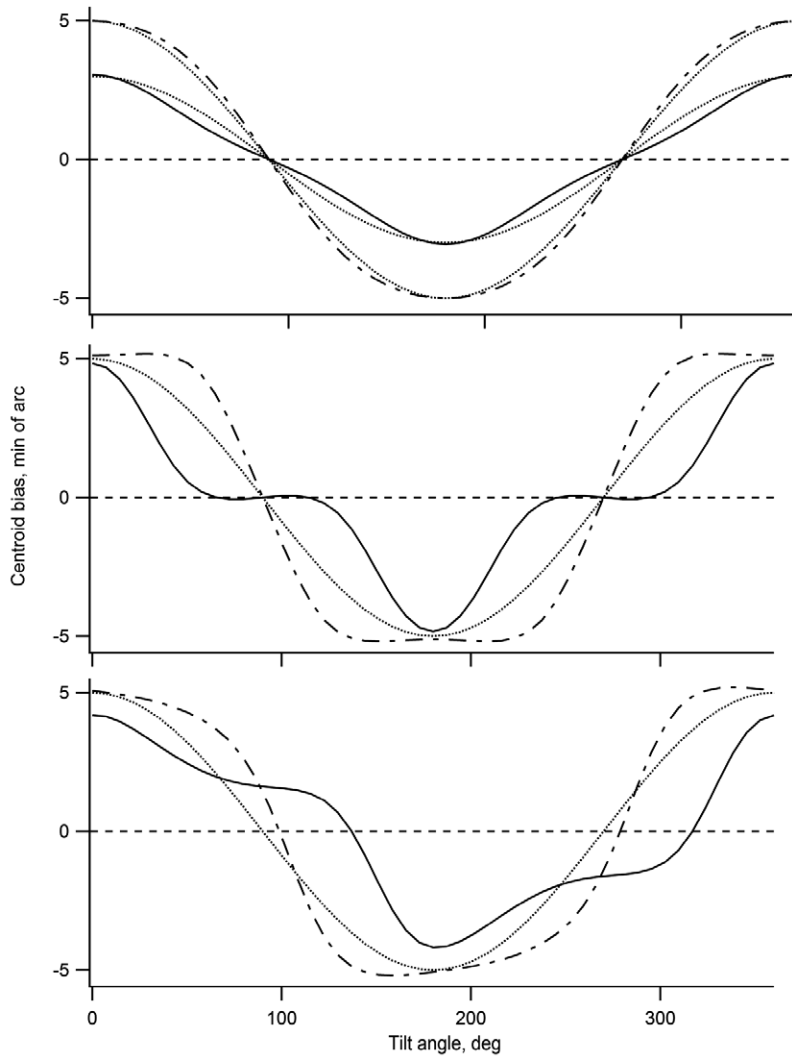


Fig. 2. Diagrams illustrating the model predictions for the centroid bias alterations. In calculations of the magnitude of centroid bias as a function of the tilt angle, φ of contextual distractor, formula 5 was used (in all cases, the standard deviation, σ of the Gaussian function of the attentional pooling window was equal to 10 min of arc). The upper graph, tilting of the short symmetric Müller-Lyer wings (length 8 min of arc): the internal angle, α equal to 30° (dash-dot curve) or to 120° (solid curve). The middle graph, tilting of the long symmetric Müller-Lyer wings (length 30 min of arc): the internal angle, α equal to 30° (dash-dot curve) or to 120° (solid curve). The lower graph, tilting of the asymmetric Müller-Lyer wings (lengths 8 and 30 min of arc): the internal angle, α equal to 30° (dash-dot curve) or to 120° (solid curve). In all graphs, dotted curves represent the cosine functions with appropriate amplitudes.

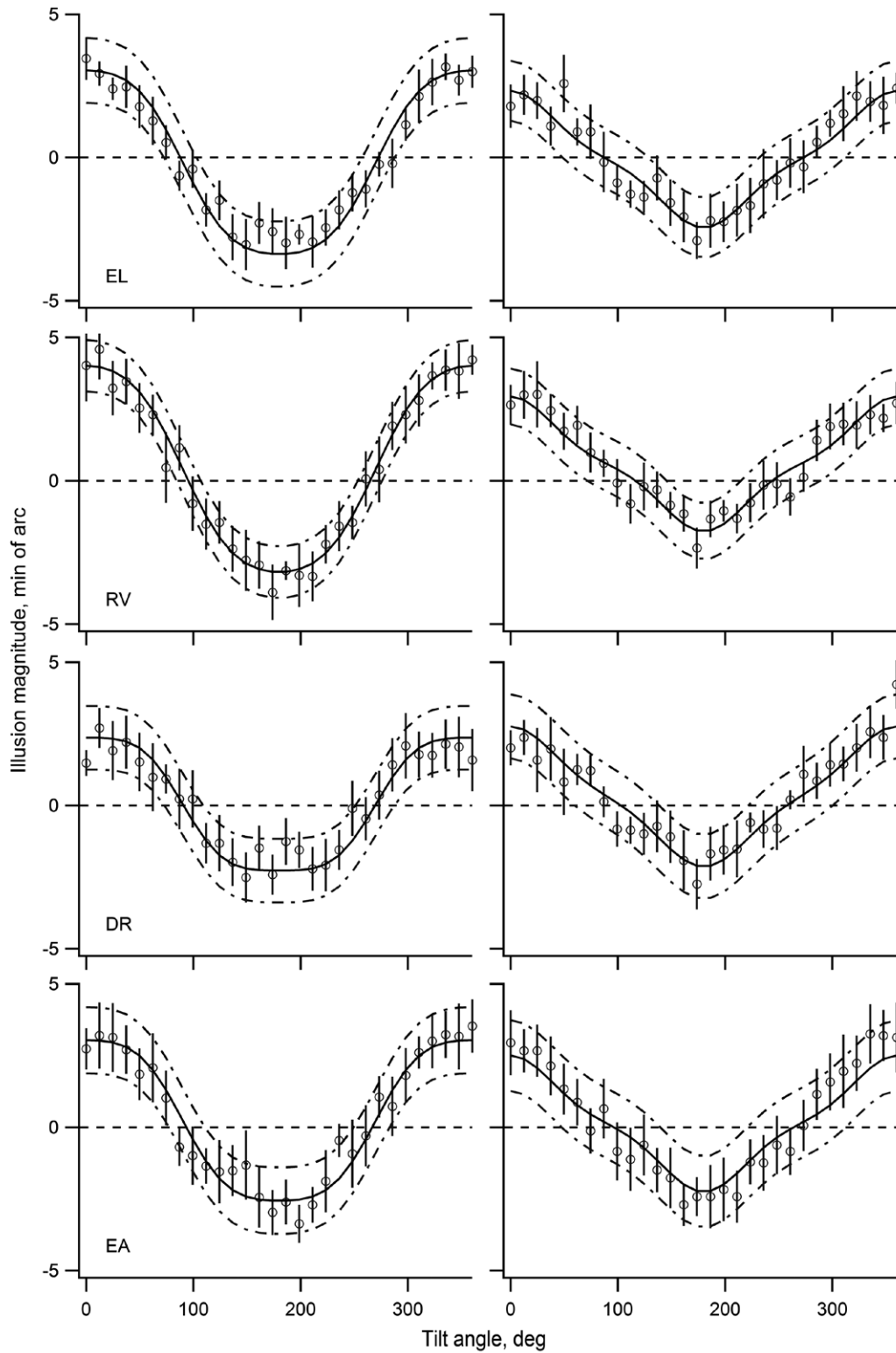


Fig. 3. The illusion magnitude as a function of the tilt angle of the short symmetric Müller-Lyer wings. In columns: left, the results (circles) for tilting of the acute- ($\alpha=30^\circ$) and, right, obtuse-angle ($\alpha=120^\circ$) wings. Solid curves, the least squares fitting of Eq. 6 to the experimental data; dash-dot curves, confidence intervals of the fitting. The length of the wings 8 min of arc. Error bars, \pm one standard error of the mean (SEM). Subjects: EL, RV, DR, and EA.

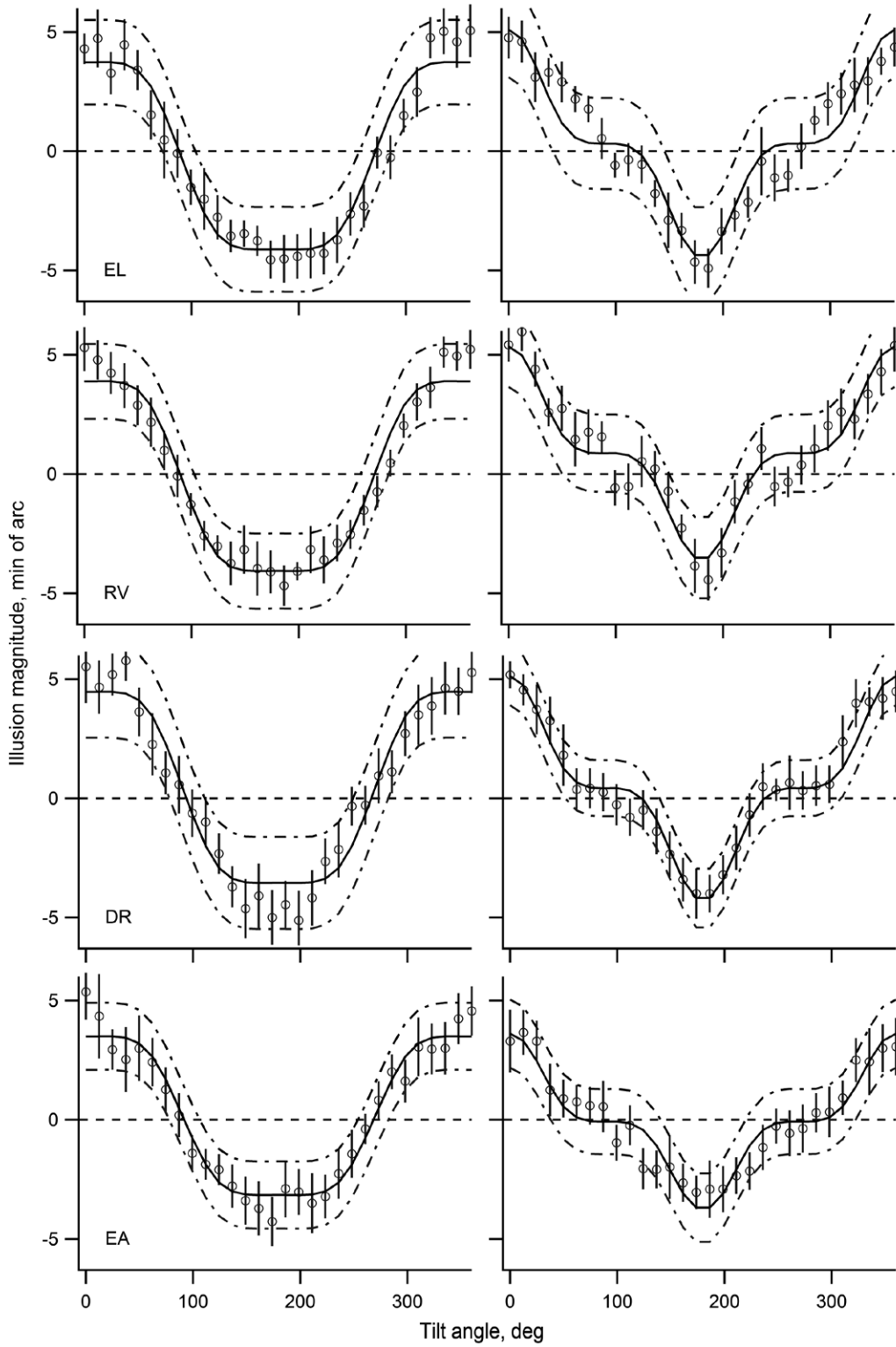


Fig. 4. The illusion magnitude as a function of the tilt angle of the long symmetric Müller–Lyer wings. In columns: left, the results (circles) for tilting of the acute- ($\alpha=30^\circ$) and, right, obtuse-angle ($\alpha=120^\circ$) wings. Solid curves, the least squares fitting of Eq. 6 to the experimental data; dash-dot curves, confidence intervals of the fitting. The length of the wings 30 min of arc. Error bars, \pm one standard error of the mean (SEM). Subjects: EL, RV, DR, and EA.

Procedure

To establish functional dependence of the illusion magnitude on the tilt angle of the distractor, the bisection procedure was used. The subjects were asked to manipulate the keyboard buttons “←” and “→” to move the Müller-Lyer wings into a position that makes both stimulus intervals perceptually equal in length; the deviation of the position of the central terminator from the physical midpoint between the lateral terminators was considered as the value of the illusion magnitude. A single button push varied the position of the terminator by one pixel corresponding approximately to 0.33 min of arc. The initial length differences between the left and right stimulus intervals were randomized and distributed evenly within a range of ± 10 min of arc.

The subjects were encouraged to maintain their gaze on the central stimulus terminator, however, observation time was not limited, and subjects' eye movements were not registered. An experimental run comprised 60 stimulus presentations, i.e., 30 different values of the independent variable were taken (in a random order) twice. Each observer carried out at least five experimental runs on different days. Ten trials went into each data point analysis, and in the data graphs, the error bars depict \pm one standard error of the mean (SEM).

Subjects

Four subjects: EL, RV, DR, and EA participated in the experiments. All participants reported normal or corrected-to-normal vision. Subjects RV and EA were naïve with respect to the goal of the study. Viewing was monocular, and the right eye was always tested irrespective of whether it was the leading eye or not. All subjects gave their informed consent before taking part in the experiments performed in accordance with the ethical standards of the Declaration of Helsinki.

RESULTS

Experimental data

The aim of the first two series of experiments was to determine quantitatively the magnitude of the illusion of extent as function of tilting of the small (wings length 8 min of arc) symmetric contextual

distractor. According to the model predictions (Fig. 2, upper), for both values of the internal angle of the Müller-Lyer wings ($\alpha=30^\circ$ in the first series of experiments, and $\alpha=120^\circ$ in the second one) the shape of functional dependencies similar to a cosine was expected.

For all subjects, the experimental results showed rather symmetrical curves (with respect to 180°) with parts comprising positive and negative values (Fig. 3).

As can be seen from the graphs, for both the acute and obtuse internal angle of the Müller-Lyer wings the illusions' extreme values (absolute values about 3–4 min of arc, the data slightly differs for different subjects) were established with the horizontal orientation (0° and 180°) of the distractor bisector. The illusion magnitude diminished when the distractor declined from the horizon and decreased to zero when the tilt angle approached 90° or 270° (vertical orientations of the bisector).

As well as in previous two series of experiments, in the third and fourth series either the acute- or obtuse-angle ($\alpha=30^\circ$ in the third, and $\alpha=120^\circ$ in the fourth series) symmetric Müller-Lyer wings were used; however, the length of the wings was enlarged to 30 min of arc. According to the model predictions (Fig. 2, middle), it was expected that for these longer wings the similarity of the experimental curves to a cosine function should be considerably disrupted because of the emergence of plateaus on the curves in the regions near horizontal (0° and 180°) and vertical (90° and 270°) orientations for the acute- and obtuse-angle distractors, respectively.

As can be seen in Figure 4, for both the acute- and obtuse-angle wings the illusion magnitude diminishes to zero for the tilt angle approaching 90° or 270° (i.e., for vertically oriented distractor). However, even a slight deviation from the vertical induces significant change of the illusion caused by the acute-angle wings (Fig. 4, left column), whereas for stimuli comprising the obtuse-angle wings (Fig. 4, right column) the slope of experimental curves remains close to zero for distractor inclinations within a range of about $\pm 20^\circ$. On the contrary, the illusion magnitude varies relatively little for the near-horizontal (within a range of approximately $\pm 20^\circ$) orientation of the acute-angle distractors.

In the fifth and sixth series of experiments, the illusion magnitude changes caused by the rotation of the asymmetric Müller-Lyer wings were examined. The

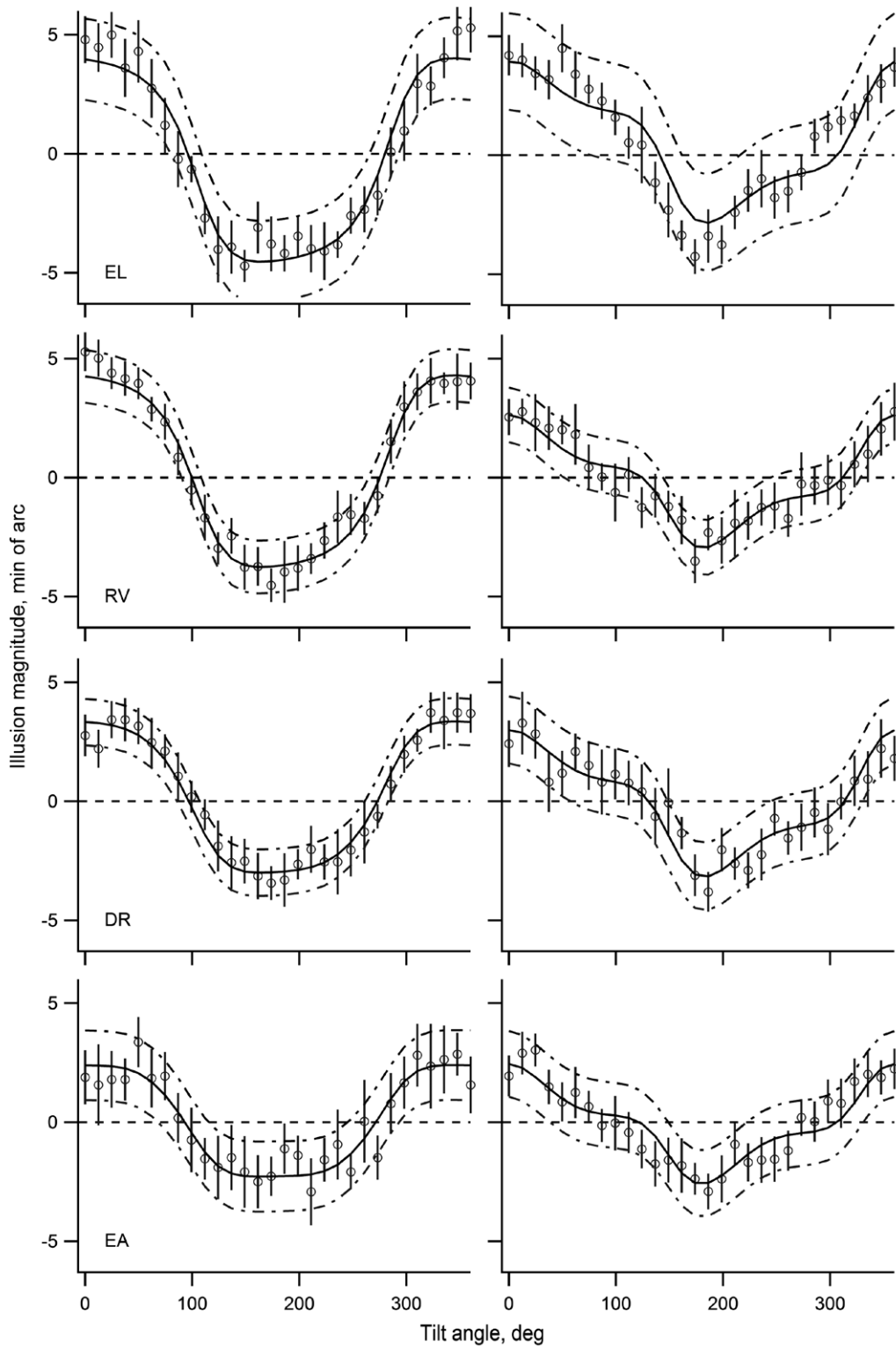


Fig. 5. The illusion magnitude as a function of the tilt angle of the asymmetric Müller–Lyer wings. In columns: left, the results (circles) for tilting of the acute- ($\alpha=30^\circ$) and, right, obtuse-angle ($\alpha=120^\circ$) wings. Solid curves, the least squares fitting of Eq. 6 to the experimental data; dash-dot curves, confidence intervals of the fitting. The length of the short and long wing 8 and 30 min of arc, respectively. Error bars, \pm one standard error of the mean (SEM). Subjects: EL, RV, DR, and EA.

length of the long wing was fixed at 30 min of arc, and the length of the short one, at 8 min of arc; as well as in previous series, either the acute- or obtuse-angle wings were used ($\alpha=30^\circ$ in the fifth, and $\alpha=120^\circ$ in the sixth series). According to the predictions (Fig. 2, lower), at least in the case of the obtuse-angle wings considerably asymmetric (with respect to 180°) patterns of the dependence of the illusion magnitude on the distractor' tilt angle were expected.

The results from the fifth series of experiments (Fig. 5, left column) show curves, which are quite similar to those from the third series: the data obtained with stimuli comprising the acute-angle wings demonstrate the illusion magnitude remained almost constant (or varied only slightly) within a range of about $\pm 30^\circ$ with respect to the horizontal orientation of the distractor. In the case of the rotation of the obtuse-angle wings (the sixth series of experiments), the data (Fig. 5, right column) obtained for all observers exhibit a much more complicated asymmetric shape of the experimental curves, which, nevertheless, is in a rather good agreement with that predicted by the model (Fig. 2, lower, solid curve).

Data fitting

In order to check the model predictions quantitatively, we have fitted the experimental data presented in Figures 3–5 with the following common function:

$$I(w, W, \alpha, \varphi) = C - F(w, W, \alpha, \varphi, B) \quad (6)$$

where C refers to a constant shift along the ordinate axis; $F(w, W, \alpha, \varphi, B)$ represents function (5) with additional argument $B=0.5\sigma^2$, where σ refers to the standard deviation of the circular Gaussian profile of the attentional pooling window.

To fit the experimental data, the method of least squares with two free parameters (C and B) was used. A good resemblance between the computational and experimental results was obtained (Fig. 3–5, solid curves); the values of coefficient of determination R^2 in all the cases were higher than 0.8 (Table I). To make a more careful examination of the goodness-of-fit, statistical analysis of the data with the Shapiro-Wilk (assessment of normality of residuals) and chi-square tests was performed (Table I). Besides, for each calculated curve, the asymp-

totic variance-covariance matrix of the parameters estimates was calculated by multiplying a matrix of partial derivatives (Jacobian) of the model's function by the residual mean square. These data allowed an additional evaluation of the goodness-of-fit by calculating confidence intervals for predicted values at each point along the range of the independent variable (Figs 3–5, dash-dot curves).

As can be seen from Table I (parameter C), slight systematic shifts of the fitted curves along the ordinate axis are observer-specific and have opposite directions in different subjects. We therefore assume that these shifts can be explained, mainly, by the inherent inaccuracy of the method of adjustment used in the present study (e.g., errors due to the impossibility of having a strict control on the subjects' attention and gaze fixation during stimulus observations, or errors caused by biases in judgment and decision-making).

DISCUSSION

The aim of the present study was to test whether the predictions of a quantitative model of centroid extraction (Bulatov et al. 2009, 2010, 2013, 2015) are in agreement with experimental data gathered from current psychophysical examination of stimuli with rotating contextual distractors (the symmetric or asymmetric Müller-Lyer wings). The collected data demonstrate (Fig. 3–5, solid curves; Table I) that the model calculations fit properly all variations of illusion magnitude shown by all the subjects for all modifications of the distractors. Therefore, it seems quite reasonable to assume that the results obtained are in line with the explanation based on the idea that the perceptual displacement of stimulus terminators due to centroid biases can be one of the main reasons of the illusion's emergence.

However, some substantial simplifications have been applied in our theoretical approach, and that could cause considerable inaccuracy in estimations. It is obvious that equation (1) gives only a very rough description of the procedure of centroid extraction because a variety of the accompanying neural processes has not been taken into account. For instance, the model was not concerned with the two-dimensional spatial frequency filtering, which is an inherent feature in even the lowest levels of the visual system. Furthermore, the influence of any top-down control

(with the exception of the procedure of attentional pooling) from higher-order visual processing was not considered. An additional simplification is related to formula (2): it was demonstrated earlier (Bulatov et al. 2009, 2010) that this formula provides a rather good fitting of the results of numerical root-finding by equation (1), however, the goodness of fit gradually diminishes with increase of the ratio of masses of the stimulus distractor and terminator.

The other possible inaccuracy in a quantitative interpretation of the present experimental data was associated with inevitable uncertainties due to the impossibility to control the observers' gaze direction. According to the model, the centroid bias (i.e., the illusion magnitude) depends on the size of the distractor-related area of centroid extraction, which grows linearly with visual eccentricity. During the stimulus observations, the subjects were not constrained in moving their eyes more or less freely; consequently, the illusion magnitude could vary depending on the actual direction of the observer's gaze. Therefore, for the sake of simplicity, in the model fittings the experimental data points were considered as presenting some averaged values of illusion magnitude, and when trying to assess the width of relevant area of centroid extraction (the parameter B in formula 6), it was assumed that this area is located at a certain averaged distance from the fovea center.

Despite this simplification, the results of the fitting of the experimental data yielded physiologically quite reasonable parameters consistent with relevant literary data. It was shown (Sagi and Julesz 1986, Nakayama and Mackeben 1989, Intriligator and Cavanagh 2001) that the size of the "spotlight of attention" is about 4 min of arc at the fovea center and scales with eccentricity in peripheral vision with a factor of about 0.5. If one assumes that this scale factor is also applicable for sizes of attentional pooling windows, then the averaged areas of centroid extraction (Table I, last row) can be considered as to be located at visual eccentricities of about 62.6 ± 12.2 , 59.2 ± 9.0 , 56.3 ± 16.0 and 46.4 ± 10.8 min of arc for subjects EL, RV, DR and EA, respectively. In turn, such the values correspond approximately to half the length of the left/right stimulus intervals; therefore, a two-phase sequential procedure of stimulus observation can be suggested: in order to compare the lengths of the intervals, the subjects allocated their gaze alternately between the regions surrounding the midpoints of these intervals. Such an interpretation is consistent with pre-

vious findings concerning the patterns of involuntary eye fixations (Coren 1986), and allows to suggest also that in the case of the smallest distractor (the first series of experiments), the subjects tended to shift their gaze toward the central stimulus terminator (visual eccentricities 43.7 ± 5.7 , 51.8 ± 5.7 , 29.3 ± 4.0 and 36.6 ± 4.8 min of arc for subjects EL, RV, DR and EA, respectively), whereas for the largest one (the fourth series) - toward the lateral dots (visual eccentricities 83.8 ± 12.4 , 77.8 ± 10.5 , 82.3 ± 7.4 and 62.7 ± 8.9 min of arc for subjects EL, RV, DR and EA, respectively).

In our opinion, the theory of "whole-part determination" (Day 2006) can be considered, to some extent, as a possible alternative explanation of the results obtained in the present study. The theory argues that the illusions of the Müller-Lyer type may occur because the size of the whole figure determines the apparent size of its components. According to this explanation, the figure with the inward-pointing flanks, in overall, occupies less space than that with outward-pointing; therefore, if one figure is physically smaller the size of its component (e.g., shaft-interval) also should be perceived as being smaller. Unfortunately, the theory (like the vast majority of psychological explanations of illusions) does not provide any methodological description of specific algorithms for quantitative assessment of illusion effects. However, if one makes some reasonable assumptions concerning the parameters that determine the apparent size of stimuli components, it is possible to obtain the curves resembling the ones shown in Fig. 2 (top and middle, dash-dot curves) for the symmetric acute-angle wings. Nevertheless, we find it difficult for the theory to resolve the issues concerning the actual value of the illusion magnitude and the shape of the curves for the symmetric obtuse-angle or asymmetric wings.

The assimilation (or integrative field) theory (Pressey and Pressey 1992) can be considered as another potential alternative explanation. The theory does not imply any procedure of centroid extraction; however, it operates with "attentive interactive fields", which in some respects are similar to attentional windows used in our modeling (both the fields and the windows are centered with stimulus terminators and have weighting profiles resembling the bell-shaped one). Nevertheless, when trying to account quantitatively for the effects obtained in the present study, the assimilation theory may face considerable challenges and require a number of additional assumptions, because the interactive field is considered as some abstract "field

Table I

The parameters (the significance level, $\alpha=0.05$) of fitting Eq. 6 to experimental data ¹						
Distractor type	Internal angle	Parameters	Subjects			
			EL	RV	DR	EA
Small symmetric	30°	C	-0.164±0.198	0.415±0.156	0.049±0.194	0.24±0.202
		σ	6.455±0.713	7.485±0.713	4.663±0.499	5.573±0.602
		R ²	0.947	0.979	0.91	0.945
		W _s (P _w)	0.985 (0.941)	0.975 (0.679)	0.975 (0.691)	0.976 (0.702)
		χ^2_s (P _z)	7.891 (1)	2.213 (1)	4.964 (1)	5.933 (1)
	120°	C	-0.058±0.186	0.587±0.173	0.315±0.199	0.127±0.219
		σ	8.058±2.085	7.694±1.728	8.784±2.806	7.906±2.521
		R ²	0.915	0.916	0.902	0.915
		W _s (P _w)	0.939 (0.086)	0.963 (0.379)	0.96 (0.313)	0.95 (0.165)
		χ^2_s (P _z)	5.127 (1)	5.907 (1)	6.277 (1)	4.251 (1)
Large symmetric	30°	C	-0.189±0.312	-0.087±0.276	0.466±0.338	0.176±0.248
		σ	8.34±0.803	8.444±0.706	8.513±0.872	7.066±0.637
		R ²	0.947	0.957	0.941	0.953
		W _s (P _w)	0.967 (0.461)	0.97 (0.534)	0.969 (0.518)	0.962 (0.34)
		χ^2_s (P _z)	9.083 (0.998)	11.637 (0.989)	9.031 (0.999)	4.582 (1)
	120°	C	0.323±0.343	0.878±0.292	0.431±0.211	-0.073±0.245
		σ	11.471±1.553	10.733±1.31	11.293±0.928	8.84±1.11
		R ²	0.898	0.917	0.957	0.907
		W _s (P _w)	0.921 (0.028*)	0.941 (0.095)	0.947 (0.142)	0.971 (0.555)
		χ^2_s (P _z)	19.799 (0.757)	10.167 (0.996)	4.064 (1)	6.621 (1)
Asymmetric	30°	C	-0.257±0.301	0.274±0.195	0.186±0.171	0.062±0.257
		σ	9.17±0.934	8.537±0.594	6.543±0.491	4.822±0.682
		R ²	0.952	0.977	0.97	0.881
		W _s (P _w)	0.97 (0.529)	0.937 (0.077)	0.957 (0.252)	0.98 (0.836)
		χ^2_s (P _z)	6.846 (1)	4.901 (1)	4.116 (1)	4.795 (1)
	120°	C	0.554±0.359	-0.158±0.203	-0.082±0.249	-0.071±0.243
		σ	9.404±1.107	7.485±0.71	8.432±0.809	6.599±0.957
		R ²	0.874	0.908	0.878	0.859
		W _s (P _w)	0.968 (0.477)	0.978 (0.772)	0.968 (0.477)	0.97 (0.539)
		χ^2_s (P _z)	16.82 (0.888)	4.21 (1)	5.64 (1)	6.45 (1)
Averaged size of area of centroid extraction, <i>min of arc</i>			35.28±6.08	33.6±4.52	32.16±8.02	27.2±5.4

¹[C (min of arc)] a constant component; [σ (min of arc)] standard deviation of the Gaussian profile of the attentional pooling window; (R²) coefficient of determination; (W) the Shapiro-Wilk test statistic (df=29); (P_w) the P-value for Shapiro-Wilk test; (χ^2) the chi-square goodness-of-fit test statistic (df=25); (P_z) the P-value of chi-square test.

of probability”, which weighting profile strongly depends on the spatial structure of the contextual distractor. On the contrary, in our model, for all modifications of distractors we used the attentional windows of the same type, i.e., circular Gaussians, whose parameters depend only on retinal eccentricity.

Unlike most other approaches, our model is based on a relatively small set of assumptions and is capable of making quantitative predictions that can be immediately and purposefully tested in experiments. Considering a good agreement between the calculations and experimental data demonstrated in our previous investigations (Bulatov et al. 2009, 2010, 2013, 2015) and in the present communication, one can conclude that the model of automatic centroid extraction adequately explains the magnitude of length misjudgments for a wide variety of stimuli modifications. However, we are convinced that the main finding of the present study is that the model is able to predict not only some general trends, but also reveals a certain “fine structure” of the illusion magnitude changes, what is obviously still unattainable for most of modern widely accepted explanations of illusions of extent of the Müller-Lyer type.

CONCLUSIONS

The predictions of the model of automatic centroid extraction were tested in the psychophysical examinations of the Brentano figure comprising a single set of the symmetric or asymmetric Müller-Lyer wings. It was demonstrated that the model calculations properly account for all illusion magnitude variations induced by distractors’ rotation. A good correspondence between the experimental results and the predictions of our computational model strongly supports the suggestion that the effects of centroid extraction are powerful enough to be considered as one of the main causes of the Müller-Lyer and related illusions of extent. We expect that the computational approach proposed in our modeling can be useful in examining the wide range of stimuli modifications.

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