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Abstract—This paper introduces a multimode transmultiplexer (TMUX) structure capable of generating a large set of user-bandwidths and center frequencies. The structure utilizes fixed integer sampling rate conversion (SRC) blocks, Farrow-based variable interpolation and decimation structures, and variable frequency shifters. A main advantage of this TMUX is that it needs only one filter design beforehand. Specifically, the filters in the fixed integer SRC blocks as well as the subfilters of the Farrow structure are designed only once. Then, all possible combinations of bandwidths and center frequencies are obtained by properly adjusting the variable delay parameter of the Farrow-based filters and the variable parameters of the frequency shifters. The paper includes examples for demonstration. It also shows that, using the rational SRC equivalent of the Farrow-based filters, the TMUX can be described in terms of conventional multirate building blocks which may be useful in further analysis of the overall system.

Index Terms—Multimode communications, transmultiplexers (TMUXs), sampling rate conversion (SRC).

I. INTRODUCTION

A CURRENT focus in the communications area is to develop flexible radio systems which aim to seamlessly support services across several radio standards [1]. A major part of this area is to cost-efficiently implement multimode (multistandard) transceivers. The simplest approach to cope with multimode problems is to use a custom device for each communications mode. However, with the growing number of standards and communications modes, as well as the growing demand for more and more functionality, this approach is becoming increasingly unacceptable both in terms of manufacturing cost and energy consumption. Thus, it is vital to develop new low-cost multimode terminals.

This paper deals with transmultiplexers (TMUXs) which allow various signals (users) to share a single channel and thus constitute fundamental building blocks in communications systems. Popular communications techniques such as code-division multiple access (CDMA), time-division multiple access (TDMA), and frequency-division multiple access (FDMA) constitute special cases of a general TMUX setup [3]. Multimode communications systems require multimode TMUXs that support different bandwidths which may vary with time, as users can request any bandwidth at any time. For example, a communications channel can be shared by three users that simultaneously transmit video, text, and voice. This means that each user occupies a specific portion of the channel and this portion may vary with time. Examples of such

dynamic communications systems are the network topologies proposed by the European Space Agency (ESA) for broadband satellite-based systems which aim to provide a global coverage for dynamic multistandard communications [4].

A digital TMUX is the dual of a multirate filter bank (FB) and is composed of a synthesis FB followed by an analysis FB. The synthesis (analysis) FB is constructed as a parallel connection of a number of branches, each branch being realized by digital bandpass interpolators (decimators). Multimode TMUXs thus require interpolators and decimators with variable parameters. These blocks can be constructed using variable upsamplers (downsamplers) and bandpass filters which have variable center frequencies and bandwidths. However, when the number of modes increases, the degree of variability grows which implies that the implementation complexity of such an approach may become intolerably high. Particularly, it may be necessary to use very high interpolation and decimation factors to obtain the desired bandwidth and center frequency, which may not be practically feasible.

To get around this problem, we introduce an alternative structure for multimode TMUXs. This structure utilizes fixed integer sampling rate conversion (SRC) blocks, Farrow-based variable interpolation and decimation structures, and variable frequency shifters. This TMUX is capable of generating a large set of user-bandwidths and center frequencies with relatively simple building blocks. A main advantage of the structure is that it suffices to design the filters involved only once beforehand. All possible combinations of bandwidths and center frequencies are then obtained by properly adjusting the variable delay parameter of the Farrow-based filters and the variable parameters of the frequency shifters.

Following this introduction, Section II introduces the proposed multimode TMUX structure whereas Section III deals with its design. After a discussion on implementation complexity in Section IV, two applications of the proposed TMUX are covered in Section V. Section VI shows how the TMUX can be described in terms of conventional multirate building blocks. Finally, Section VII concludes the paper.

II. PROPOSED MULTIMODE TMUX STRUCTURE

In this section, we will introduce a multimode TMUX which can generate arbitrary bandwidths and center frequencies.[1] The multimode scenario considered here has been outlined in [5]. In line with that scenario, we define a granularity band[2] as the minimum bandwidth a user can occupy and assume that users are separated by guard bands which means that the TMUX is slightly redundant. In the proposed multimode TMUX, a small redundancy (oversampling) is needed to be able to generate all

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[1]The bandwidths and center frequencies are in practice limited to rational numbers due to finite precision (wordlength). However, at the expense of additional implementation cost, any desired precision can be achieved.

[2]In multistandard communications, a granularity band can be chosen such that all bandwidths are rational multiples of the granularity band.
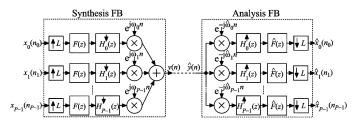
Fig. 1. Proposed multimode TMUX consisting of fixed integer SRC, variable rational SRC, and variable frequency shifters. (Note: $H_p^{\downarrow}$ and $H_p^{\uparrow}$ represent Farrow-based filter for decimation and interpolation.)

possible modes without channel interference and using only one set of fixed filters. Without such an assumption, one would need to redesign the filters for each mode which is cumbersome. In addition, it is well known that redundancy is needed anyhow in communications systems to ensure a high-performance transmission [6]. We also assume that any user p can occupy $R_p(t)$ granularity bands[3] where $1 \leq R_p(t) \leq Q$ with Q being the number of granularity bands in the whole frequency range.

As shown in Fig. 1, the TMUX generates a granularity band through upsampling by L followed by a low-pass filter. As users can have bandwidths that are rational multiples of the granularity band, the Farrow-based filter[4] performs decimation by rational values R_p. To place the users in appropriate positions in the frequency spectrum, variable frequency shifters are utilized. Finally, all users are summed for transmission in the channel. In the analysis FB, the received signal is first frequency shifted such that the desired signal can be processed in the baseband. Then, a Farrow-based interpolator by ratio R_p followed by decimation by L is used to obtain the desired signal. It is also noted that, like e.g., orthogonal frequency-division multiplexing (OFDM)-based TMUXs, the output of the TMUX is complex. Fig. 2 illustrates the principle of the structure by plotting the frequency spectrum at the output of the low-pass filter, Farrow-based filters, and the frequency shifters with a Gaussian input.

A. Channel Sampling Rates

As the proposed TMUX is aimed for a multimode communications system, the users $X_0, X_1, \ldots, X_{P-1}$ can generally have different sampling (data) rates. This means that in one time frame, the time during which the signal is transmitted, the number of samples (and hence, the time index n_p) processed in each branch of the TMUX in Fig. 1 can be different from other branches. Mathematically, the sampling periods of the TMUX inputs, i.e., $T_0, T_1, \ldots, T_{P-1}$, must satisfy

$$T_0 R_0 = T_1 R_1 = \ldots = T_{P-1} R_{P-1} = L T_y \qquad (1)$$

where T_y is the sampling period of $y(n)$.

B. SRC

As shown in Fig. 1, integer interpolation and decimation by L requires low-pass filters $F(z)$ and $\hat{F}(z)$, respectively.[5] The

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[3]The value of $R_p(t)$ is constant during the time frame in which user signal p is transmitted. In later discussions, the time index t will be omitted.

[4]In brief, the Farrow structure is composed of a set of fixed linear-phase filters and adjustable multipliers and performs arbitrary rational SRC [7]–[10].

[5]In this paper, we have used finite-length impulse response (FIR) filters, but infinite-length impulse response (IIR) filters can be used as well.
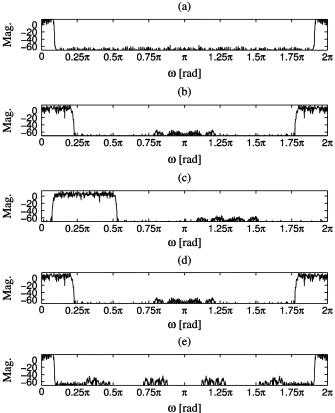

Fig. 2. (a) Output of integer interpolator. (b) Output of Farrow-based decimator. (c) and (d) Outputs of frequency shifters. (e) Output of Farrow-based interpolator. $L = 12, R_p = 2.5, \omega_p = 0.3\pi, \hat{\omega}_p = 0.3\pi$.

stopband edges of these filters are defined as $(\pi(1 + \rho))/(L)$ where the parameter $0 < \rho < 1$ denotes the roll-off. This also sets the value of the granularity band $(2\pi(1 + \rho))/(L)$. Further, SRC by the rational value R_p is performed by the Farrow-based filters resulting in the set of user bandwidths[6] $B_p = (2\pi R_p(1 + \rho))/(L), p = 0, 1, \ldots, P - 1$. The Farrow-based filter utilizes linear-phase FIR subfilters $G_k(z)$ and its transfer function is given by[7]

$$H(z, \mu) = \sum_{k=0}^{K} \mu^k G_k(z), \qquad |\mu| \leq 0.5. \qquad (2)$$

The subfilters are either symmetric (for k even) or antisymmetric (for k odd) and μ is a fractional delay value [7]. Here, each of systems $H_p^{\downarrow}(z)$ and $H_p^{\uparrow}(z)$ employs a filter with a transfer function given by (2) and performs decimation and interpolation, respectively. The fractional delay value[8] for decimation and interpolation is given by [10]

$$[n_{\text{in}} + \mu(n_{\text{in}})]T_{\text{in}} = n_{\text{out}}T_{\text{out}} \Leftrightarrow \mu(n_{\text{in}}) = n_{\text{out}}\frac{T_{\text{out}}}{T_{\text{in}}} - n_{\text{in}}$$

$$(3)$$

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[6]The narrow-band output of $F(z)$, as in Fig. 2(a), can be expanded by any ratio R_p through $H_p^{\downarrow}(z)$, as in Fig. 2(b). Thus, for fixed values of L and ρ, it is the values of R_p that determine the set of P user bandwidths.

[7]If IIR variable fractional-delay filters are used, care must be taken to avoid transients as the delay parameter may change for every time index.

[8]The fractional delay value defines the time difference between each input sample of the SRC block and the corresponding output sample.

where $n_{\text{out}}(n_{\text{in}})$ is the output (input) sample index and $T_{\text{out}} = (T_{\text{in}})/(R_p)$. In other words, for $H_p^\uparrow(z)$, we have $T_{\text{out}} < T_{\text{in}}$ whereas for $H_p^\downarrow(z)$, the relation $T_{\text{out}} > T_{\text{in}}$ holds.

III. FILTER DESIGN

The filters $F(z)$ and $\hat{F}(z)$, respectively, suppress the channel cross talk and make the overall transfer functions between $x_p(n_p)$ to $\hat{x}_p(n_p)$ approximate unity. As the TMUX is slightly redundant, the level of cross talk and the aliasing, resulting from the rational SRC, is determined by the stopband attenuation of these filters and can thus easily be suppressed to any desired level. Further, ignoring the rational SRCs, it is well known that the transfer function from $x_p(n_p)$ to $\hat{x}_p(n_p)$ is the zeroth polyphase component of the cascaded filter $F(z)\hat{F}(z)$ [6]. To make this polyphase component unity,[9] $F(z)\hat{F}(z)$ must be an Lth-band filter. The filters $F(z)$ and $\hat{F}(z)$ should thus be designed so that: 1) the zeroth polyphase component of $F(z)\hat{F}(z)$ approximates unity and 2) the stopband attenuation of $F(z)$ and $\hat{F}(z)$ is high enough. This also holds when the rational SRCs are present provided that the Farrow-based filter in (2) approximates a fractional-delay filter with delay μ throughout its respective frequency band. For the Farrow-based filter in the synthesis FB, only the granularity band needs to be covered whereas in the analysis FB, the whole band except for a small band near π must be covered. The reason is that the output of each integer interpolator (in the synthesis FB) is bandlimited to the granularity band. However, in the the analysis FB, the sum of user signals is processed by the Farrow-based filter and therefore, this sum determines the frequency band. Consequently, the complexity of the Farrow-based filter in the synthesis FB will be less than that of the analysis FB.

The discussion above reveals that the proposed TMUX can be designed by determining $F(z)$ and $\hat{F}(z)$ such that
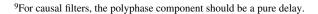
$$|[F(e^{j\omega})\hat{F}(e^{j\omega})]_{\text{zeroth}} - 1| \le \delta_1, \qquad \text{for } \omega \in [0, \pi]$$
$$|F(e^{j\omega})| \le \delta_2, |\hat{F}(e^{j\omega})| \le \delta_3, \qquad \text{for } \omega \in [0, \omega_1] \quad (4)$$

where $\omega_1 = (\pi(1+\rho))/(L)$, and $[F(e^{j\omega})\hat{F}(e^{j\omega})]_{\text{zeroth}}$ denotes the zeroth polyphase component of $F(z)\hat{F}(z)$. In addition, the Farrow-based filter in the synthesis/analysis FB, i.e., $H(z, \mu)$ should be designed such that

$$|H(e^{j\omega}, \mu) - e^{-j\omega\mu}| \le \delta_4, \qquad \text{for } \omega \in [0, \omega_2] \quad (5)$$

for all $\mu \in [-0.5, 0.5]$. Additionally, $\omega_2 = \omega_1$ for the synthesis FB whereas in the analysis FB, it is the width of the spectrum of $y(n)$ in Fig. 1 that determines ω_2. For example, at a typical spectrum utilization percentage of 90%, we have $\omega_2 = 0.9\pi$.

It is well known that all δ_i, $i = 1, 2, 3, 4$, in (4) and (5) can be reduced to any desired levels by simply increasing the filter order. It must be noted that, to design the TMUX, there is a need to solve : 1) one filter design problem to get the filter pair $F(z)$ and $\hat{F}(z)$ as in (4) and 2) two filter design problems to get the subfilters of the Farrow structures in the synthesis and analysis FBs as in (5). Having solved these problems once, it is only the values of the fractional delays and the parameters of the variable frequency shifters that change for every new configuration

[9]For causal filters, the polyphase component should be a pure delay.
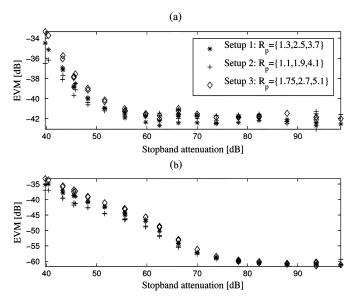

Fig. 3. EVM at different stopband attenuations of $F(z)\hat{F}(z)$ with fixed errors in $[F(e^{j\omega})\hat{F}(e^{j\omega})]_{\text{zeroth}}$ and $H(e^{j\omega}, \mu)$. (a) $\delta_1 = \delta_4 = 0.01$. (b) $\delta_1 = \delta_4 = 0.001$.

of standards. The filter pair $F(z)$, $\hat{F}(z)$ can for example be designed as outlined in [11] whereas the Farrow-based filters may be designed as described in [7]. In other words, the proposed multimode TMUX can be designed to approximate perfect recovery as close as desired for all possible modes by separately solving three conventional filter design problems.

A. Example

As discussed in the previous section, the proposed multimode TMUX can approximate perfect recovery as close as desired by proper design of the filters in the fixed and integer SRC blocks. To illustrate this fact, a series of filters with fixed $\delta_1 = \delta_4 = \{0.01, 0.001\}$, $\omega_1 = 0.0875\pi$, $\omega_2 = 0.9\pi$, and $L = 12$ are assumed. As the stopband attenuation of $F(z)$ and $\hat{F}(z)$ suppresses channel cross talk, they have been designed with different values of $\delta_2 = \delta_3$. Thus, there are similar constraints on $[F(e^{j\omega})\hat{F}(e^{j\omega})]_{\text{zeroth}}$ and $H(e^{j\omega}, \mu)$ with the stopband attenuation of $F(z)$ and $\hat{F}(z)$ being the only parameter that changes. In addition, the error vector magnitude (EVM), a metric of transmitter signal quality in modern communications, is used [2]. EVM gives a statistical estimate of the error vector normalized by the magnitude of the ideal signal and is defined as

$$\text{EVM}_{\text{rms}} = \sqrt{\frac{\sum_{k=0}^{N_s-1} |s(k) - s_{\text{ref}}(k)|^2}{\sum_{k=0}^{N_s-1} |s_{\text{ref}}(k)|^2}}$$

(6)

where $s(k)$ and $s_{\text{ref}}(k)$ are the length-N_s measured and ideal complex sequences, respectively. Fig. 3 shows the average EVM values for three multimode setups in a 16-QAM signal. It illustrates the fact that the EVM resulting from the TMUX can be made as small as desired for all possible modes by decreasing δ_i, $i = 1, 2, 3, 4$.

IV. IMPLEMENTATION AND DESIGN COMPLEXITY ISSUES

In the previous section, it was shown and demonstrated that the proposed TMUX can be designed to have as small errors as

desired for all possible modes through three separate filter designs. This is attractive compared to solutions that require either one set of filters for each mode or on-line filter design whenever a new mode is desired. However, there is still room for complexity reductions by modifying the new structure. Details are beyond the scope of this paper which aims to outline the main course to follow when implementing multimode TMUXs, but we will in this section point out some possible ways to reduce the complexity and issues for future research.

A motivation to using integer interpolation in the synthesis FB, to generate signals with the spectral width of a granularity band, is that regular integer-interpolation structures are more efficient than Farrow-based structures when it comes to implementing an interpolator with a relatively large conversion ratio L [8]. This is true if multistage interpolation structures [11] are utilized which should be done for larger values of L. If the bandwidth of the users often matches the granularity band, this option (and the dual in the analysis FB) appears the most natural choice. On the other hand, if the users often occupy wider bandwidths than the granularity band, then it may be worth to use a smaller L in the integer-conversion stages. The Farrow-based filter in the synthesis/analysis FBs can then both work either as interpolator or decimator. In this way, one can find the best tradeoff between the complexity of the integer-conversion part and the rational-conversion part in order to reduce the overall complexity. Some results are available for interpolators and decimators [8] but the problem is more complex here as we deal with TMUXs. It is thus of interest to extend the results of [8] to multimode TMUXs. It is noted that, as the overall optimum will depend on how often the users take on narrow or wide bandwidths, it is not a trivial task to derive it mathematically.

Another issue is the filter design. In the previous section, we outlined the separate filter design which is attractive as known techniques can be adopted. Although this gives us a good sub-optimum overall solution, it is slightly overdesigned and has a somewhat higher complexity than necessary. To reduce the complexity, one can design all filters simultaneously which can, in principle, be done using standard nonlinear optimization techniques. This has successfully been used for fixed FBs and TMUXs [12], but the problem is much more complex here as we deal with multimode TMUXs. This implies that, in the optimization, the pre-specified requirements must be satisfied for all possible modes. Consequently, the number of constraints grows with the number of modes. Simultaneous optimization may therefore be practically feasible only for problems that have a few modes. However, it is interesting to investigate how many modes one can handle using simultaneous optimization.

V. TMUX APPLICATION

In this section, two applications of the proposed TMUX are considered. Having designed the TMUX for a specific EVM[10] (e.g., -100dB achieved with $\delta_i = 10^{-5}, i = 1, 2, 3, 4$), the setup in Fig. 4(a) can be used for functionality/performance test of the flexible frequency-band reallocation (FFBR) network defined in [5]. According to [2], and as shown in Fig. 4(b), the values for the granularity and guard band are chosen as $(2\pi)/(Q)$ and $(2\pi\epsilon)/(Q)$ where $0 < \epsilon < 1$. To verify the

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[10]If the TMUX is designed to have a very small EVM, it can detect larger errors that arise due to the system under test.
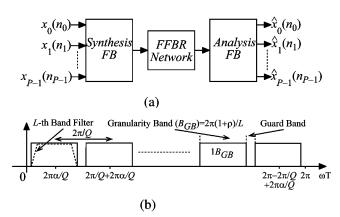

(a)

(b)

Fig. 4. Test setup and the formulation of guard/granularity band. (a) Setup for functionality/performance test. (b) Guard band, granularity band, and the Lth band filter. As $(2\pi)/(Q) - (2\pi\epsilon)/(Q) = (2\pi(1+\rho))/(L)$, we have $\rho = (L(1-\epsilon))/(Q) - 1$.
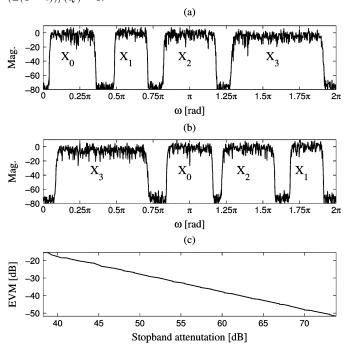

Fig. 5. (a) and (b) Functionality test of a FFBR network using the proposed TMUX. (c) Performance test of a FFBR network using the proposed TMUX. $L = 12, Q = 10, \epsilon = 0.125$.

functionality of the FFBR network, four different user signals $\{X_0, X_1, X_2, X_3\}$ with $R_p = \{1.75, 1.25, 2, 3.5\}$ resulting in subcarrier frequencies $\omega_p = \{0.2\pi, 0.6\pi, \pi, 1.6\pi\}$ are assumed. The frequency spectrum of the input and the multiplexed output of the FFBR network with $Q = 10$ granularity bands are, respectively, shown in Fig. 5(a) and (b). The scenario of FBR [2] shown in Fig. 5(b), results in $\hat{\omega}_p = \{\pi, 1.8\pi, 1.4\pi, 0.4\pi\}$. To illustrate the noise behavior of the FFBR network, Fig. 5(c) shows the values of EVM for different prototype filter stopband attenuations assuming a 16-QAM signal. As can be seen, the stopband attenuation of the FFBR network's prototype filter is the main source of aliasing suppression [5].

VI. ANALYSIS OF THE NONUNIFORM TMUX USING MULTIRATE BUILDING BLOCKS

This section shows that the proposed TMUX can alternatively be described in terms of conventional multirate building blocks which may be useful in further analysis of the overall
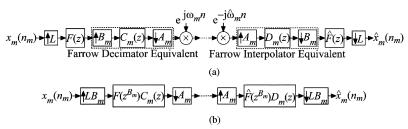
Fig. 6. Equivalent paths between $x_m(n_m)$ and $\hat{x}_m(n_m)$. Considering only one branch of the TMUX, the frequency shifters can be removed. (a) Equivalent path between $x_m(n_m)$ and $\hat{x}_m(n_m)$. (b) Simplified equivalent path between $x_m(n_m)$ and $\hat{x}_m(n_m)$.

system. This is done by utilizing the rational SRC equivalent of the Farrow-based filter [9]. In each branch of the TMUX, the Farrow-based filter for interpolation by $R_p = (A_p)/(B_p)$ can be modeled as the cascade of upsampling by A_p, FIR filter $D_p(z)$, and downsampling by B_p. Similarly, a cascade of upsampling by B_p, FIR filter $C_p(z)$, and downsampling by A_p can be used to model decimation by $R_p = (A_p)/(B_p)$. Consequently, each branch of the TMUX between $X_m(z)$ and $\hat{X}_m(z)$ can be drawn as shown in Fig. 6 and is similar (with some differences as outlined in [2]) to the TMUX discussed in [13].

Hence, using the analysis in [13], the blocked transfer function of the TMUX in Fig. 1 can be written as $\underline{T}(z) = \underline{\Phi}(z)\underline{\Psi}(z)$ where

$$\underline{\Phi}(z) = [\phi_0(z)\,\phi_1(z)\,\cdots\,\phi_{P-1}(z)]^T$$
$$\underline{\Psi}(z) = [\psi_0(z)\,\psi_1(z)\,\cdots\,\psi_{P-1}(z)]. \qquad (7)$$

For the proposed TMUX, we have

$$\phi_p(z) = D_p(z)\hat{F}(z^{B_p}), \psi_p(z) = F(z^{B_p})C_p(z). \qquad (8)$$

Assuming the desired user $X_d(z)$, the TMUX output can be written as $\hat{X}_d(z) = V_{dd}(z)X_d(z) + \sum_{i=0,i \neq d}^{P-1} V_{id}(z)X_i(z)$ where $V_{dd}(z)$ and $V_{id}(z)$ represent inter-symbol interference (ISI) and inter-carrier interference (ICI), respectively. In general, it is desired to have $|V_{dd}(z) - z^{-\eta_d}| \leq \delta_{d1}$ and $|V_{id}(z)| \leq \delta_{d2}$ with δ_{d1} and δ_{d2} being the allowed ISI and ICI.

Although the same analysis methods as for existing TMUXs can be used here, the implementation is different. In other words, the conventional rational-conversion building blocks (upsamplers, downsamplers, and frequency selective filters) are only used for the analysis whereas the TMUX implements these blocks implicitly using integer-conversion blocks and Farrow-based rational-conversion blocks.

VII. CONCLUSION

A nonuniform TMUX capable of generating arbitrary bandwidths was introduced and analyzed. As illustrated by the ex-

amples, the TMUX can approximate perfect recovery as close as desired for any configuration of standards.

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