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Modelling of microstructural effects on magnetic hysteresis properties

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Abstract

In this paper, the relationship between microstructural properties of steels and the material parameters in the Preisach model and in the Jiles–Atherton (JA) model is discussed, in the instance where both models describe quasi-static hysteretic magnetic behaviour. It is shown how the material parameters in both hysteresis models should be modified to reflect their dependence on dislocation density and grain size. The dependence of the Preisach material parameters on these microstructural features is identified starting from hysteresis loops calculated by the microstructurally dependent modified JA model. For the Preisach model, a Lorentzian distribution function is used for the distribution function. This makes it possible to compare predictions here to results of an earlier paper in which the Lorentzian distribution was used for Preisach fits to experimental data for steels of different grain sizes. Also, in a different earlier paper, it was shown how the Lorentzian distribution can be formulated so that it connects with salient features of the JA model. The procedure in this paper enables one to examine and predict microstructural variations of Preisach parameters in steels not only for the case of grain size variation but also for the case of variation in dislocation density.

1. Introduction

In steels, properties like magnetization curves, coercive field, permeability and loss are principally determined by the microstructure. The microstructure affects magnetization processes, like domain wall pinning and domain wall motion, which are responsible for hysteretic behaviour. Two important features of the microstructure are: average diameter of the grains ϕ and dislocation densities ζ_d . Grain boundaries present an obstacle to domain wall motion and act as pinning centres. Consequently, when the grain size decreases, the pinning of domain walls due to grain boundaries increases. Moreover, when the dislocation density increases, dislocations begin to get entangled, forming strong pinning centres for domain walls, and so impeding wall motion.

This paper discusses the relation between the microstructural features ϕ and ζ_d and the material parameters in the Jiles–Atherton (JA) model [1, 2] and the Preisach model [3–5]. The material-dependent parameters of the JA model were modified to study variation of magnetic properties with grain size and dislocation density [6]. In [7], the effect of the average grain size ϕ and crystallographic texture on the distribution function

(PDF) in the Preisach model was identified by a large number of measurements. Here, we interrelate the results of [6, 7]. For this, the magnetization loops from the microstructurally dependent JA model are used to identify the microstructure dependence of the material parameters of the Preisach model via the fitting of Preisach loops to the JA loops. Indeed, the JA loops show distinct changes due to changing microstructural features and those same tendencies must, therefore, appear in the Preisach magnetization loops, generated from a systematic sampling of the internal loops from the JA model. Finally, microstructure dependence must also appear in the Preisach material parameters used in generating the fitted Preisach loops. Various connections between the Preisach model and the JA model are discussed in the literature [8–10] and we shall return to such papers in our discussion later.

2. Hysteresis models

2.1. JA model

In the JA model [1, 2], the total magnetization M is the sum of a reversible (M_{rev}) and an irreversible (M_{irr}) component.

These components are given by

$$M_{\text{rev}} = c_j(M_{\text{an}} - M_{\text{irr}}), \quad (1)$$

$$M_{\text{irr}} = M_{\text{an}} - \frac{k_j \delta}{\mu_0} \frac{dM_{\text{irr}}}{dH_e}. \quad (2)$$

Here, M_{an} is the anhysteretic magnetization, given as

$$M_{\text{an}}(H_e) = M_{\text{sj}} L\left(\frac{H_e}{a_j}\right), \quad (3)$$

where the function $L(x) = \coth x - 1/x$ is the so-called Langevin function, and where H_e is the effective magnetic field inside the material, i.e.

$$H_e = H + \alpha_j M_{\text{an}}. \quad (4)$$

M_{sj} , c_j , a_j , k_j and α_j are all parameters of the material. The number δ , taking the value $+1$ or -1 , depending on whether H is increasing or decreasing, corresponds mathematically with the hysteresis. Equation (2) is actually a differential equation. A more transparent form of equation (2) is given by [1, 2]

$$\frac{dM_{\text{irr}}}{dH} = \frac{M_{\text{an}} - M_{\text{irr}}}{(k_j \delta / \mu_0) + \alpha_j (M_{\text{an}} - M_{\text{irr}})}. \quad (5)$$

We expect that microstructure will affect all the parameters subscripted by j , although some more than others. In particular, grain size and dislocation density have important effects on the domain wall pinning parameter k_j and the effective field scaling parameter a_j , which is a function of the domain density.

2.2. Preisach model

The Preisach model, initially introduced by Preisach [3], is another accurate method of describing the scalar hysteresis effects in magnetic materials. According to Preisach's approach, the hysteresis model gives as response the magnetization M as a function of the applied magnetic field H and its history H_{last} . It rests upon the idea of a material structure containing an infinite set of magnetic dipoles. Each dipole has a rectangular nonsymmetric hysteresis loop defined by two characteristic parameters, which are denoted by α_p and β_p ($\beta_p \leq \alpha_p$). In the Preisach model, the state ϕ_p of the dipole at time point t only may take the value $+1$ or -1 . Explicitly,

$$\phi_p(t) = \begin{cases} +1, & \text{in case } H(t) > \alpha_p \text{ or} \\ & (\beta_p < H(t) < \alpha_p \text{ and } H_{\text{last}} > \alpha_p), \\ -1, & \text{in case } H(t) < \beta_p \text{ or} \\ & (\beta_p < H(t) < \alpha_p \text{ and } H_{\text{last}} < \beta_p). \end{cases} \quad (6)$$

Here H_{last} is the last extreme value of the magnetic field kept in memory outside the interval (β_p, α_p) and which physically is remembered in the domain structure of the material. In the moving Preisach model [11], the applied magnetic field H in (6) is replaced by the effective field $H_e(t) = H(t) + cM(t)$. Here, the effective field $H_e(t)$ is assumed to be equal to the effective field in the JA model; thus $c = \alpha_j$.

The density of the dipoles is represented by the Preisach distribution function $P(\alpha_p, \beta_p)$, (PDF), characterizing the material. The resulting magnetization M of the entire material is obtained from the accumulated magnetization of all the

dipoles. In the moving Preisach model, the magnetization is then given by

$$M(H_e(t), H_{e,\text{past}}(t)) = \frac{1}{2} \int_{-\infty}^{+\infty} d\alpha_p \int_{-\infty}^{\alpha_p} d\beta_p P(\alpha_p, \beta_p) \times \phi_p(\alpha_p, \beta_p, H_e(t), H_{e,\text{past}}(t)). \quad (7)$$

In order to quantify how the PDF changes due to a variation of grain size or of dislocation density, a Lorentzian PDF is considered [12]:

$$P(\alpha_p, \beta_p) = \frac{k_{1p}}{(1 + ((\alpha_p - a_p)/b_p)^2)(1 + ((\beta_p + a_p)/b_p)^2)} + \delta_{\alpha_p \beta_p} \left(\frac{k_{2p}}{1 + (\alpha_p/c_p)^2} \right), \quad \alpha_p \geq \beta_p, \quad (8)$$

with $\delta_{\alpha_p \beta_p}$ denoting the Kronecker delta. It was proved in [12] that the Lorentzian distribution formulated in this way is a suitable distribution function to describe the experimentally obtained magnetization loops of steels. In particular, the physical information contained in the longer tail of the Lorentzian distribution enables a better fit to experimentally obtained hysteresis data than would be enabled for other types of distributions, e.g. the Gaussian distribution. Indeed, such a Lorentzian distribution was used, for example in fitting Preisach loops to experimental hysteresis curves for steels of different grain sizes [7]. Finally, in [8], it is shown how the above-formulated Lorentzian distribution can be related to important features of the JA model, and for this reason, is an appropriate choice of distribution function to be used in connection with relating the Preisach model to the JA model, despite the fact that another distribution function has also been used [9, 10] in connecting with other features. More than one distribution function might be expected to be available for relating the two models because the fit between the JA model and the Preisach model cannot be exact when considering the major cycle as well as symmetric internal loops as we did in the paper.

3. Influence of microstructure on material parameters

In order to see how the material parameters of the JA model (i.e. M_{sj} , a_j , α_j , k_j and c_j) and those of the Preisach model (i.e. a_p , b_p , c_p , k_{1p} and k_{2p}) are related and how they are influenced by microstructural features (like average grain size and dislocation density), a set of numerical experiments was performed.

In [6], it was observed that a modification of the grain size or of the dislocation density corresponds to a variation of the JA material parameters k_j and a_j according to

$$k_j = k_{j0} \left(G_1 + \frac{G_2}{\phi} \right) \sqrt{\zeta_d}, \quad (9a)$$

$$a_j = a_{j0} \left(G_1 + \frac{G_2}{\phi} \right) \sqrt{\zeta_d}. \quad (9b)$$

The remaining parameters in the JA model are independent of ϕ and ζ_d . The reason for the formulation of k_j as in equation (9a) is that parameter k_j is a measure of the width of the hysteresis and is proportional therefore to the coercivity H_c [2], and the consensus of prior experimental work [6, 13–16] is that the coercivity predominantly has the dependence on grain size and

dislocation density exhibited in equation (9a). The reason that parameter a_j has the dependence in equation (9b) is that the parameter a_j is known to be proportional to the domain density in the demagnetized state [2] and the domain density in the demagnetized state should be proportional to the pinning site density, which, in turn, is proportional to k_j . Note that experimental fitting is what drives the dependences used for k_j and a_j . Thus, hysteresis curves computed from the JA model should reflect these experimental dependences.

For a set of values of the grain size ϕ and of the dislocation density ζ_d , the magnetization major cycle as well as symmetric internal magnetization loops are calculated using the microstructurally dependent modified JA model. Next, the five material parameters in the Preisach model, defined in (8), i.e. a_p , b_p , c_p , k_{1p} and k_{2p} , are fitted in such a way that a good agreement between the Preisach loops and the original JA loops was obtained, for various values of ϕ and ζ_d .

At the start of the fitting procedure—for a fixed value of ϕ and ζ_d —a five-dimensional mathematical subspace with coordinate axes a_p , b_p , c_p , k_{1p} and k_{2p} is defined by choosing in a proper way an upper and lower limit for each material parameter in the Preisach model. In order to obtain numerically the set of value for a_p , b_p , c_p , k_{1p} and k_{2p} corresponding to the best fit, we discretize the whole five-dimensional subspace by defining a regular grid of discrete points. The set of values for the five Preisach parameters which give the best agreement between the Preisach magnetization loops and the JA loops is obtained using a least-squares method. This least-squares method minimizes $\sum_i (M_{\text{Preis}}(H_i) - M_{\text{Jiles}}(H_i))^2$ and takes into account all discrete points of the regular grid defined in the whole five-dimensional subspace. $M_{\text{Preis}}(H_i)$ is obtained from (7) and (8) while $M_{\text{Jiles}}(H_i)$ is calculated using (1)–(5). H_i , $i = 1, 2, 3, \dots, N_k$, are the discrete values for the magnetic fields considered in the major cycle and the symmetric internal magnetization loops during the fitting process. In doing the fitting, we tested several discretization meshlengths in order to assure that large changes in the fitted Preisach parameters did not occur in going from one meshlength to another.

Figure 1 depicts the correspondence between the JA magnetization loops and the Preisach magnetization loops for different excitation levels for ϕ and ζ_d equal to $8 \mu\text{m}$ and $5 \times 10^{10} \text{m}^{-2}$, respectively (major cycle and symmetric internal magnetization loops). The points shown are for the original JA loops, and the curves shown are for the Preisach loops. A good correspondence between the two models is obtained also with respect to coercive field H_c , differential permeability μ_{H_c} at coercive field H_c , remanent induction B_r , hysteresis loss W_h , and other specific material properties that are usually considered when studying the influence of microstructure. By incorporating variation of grain size and dislocation density into the systematic behaviour of the JA loops, we are able to develop information about the effect of microstructure on the systematic behaviour of the Preisach loops, and thus we can obtain information about how the Preisach parameters are affected by microstructural change.

Two cases were considered: first, variation of the grain size with constant dislocation density and, second, variation of dislocation density with constant average grain size. In this way, the influence of both microstructural properties may be separated. In both cases, the JA parameters were chosen

as follows: $M_{sj} = 1.585 \times 10^6 \text{ A m}^{-1}$, $\alpha_j = 0.844 \times 10^{-5}$, $c_j = 0.25$, $G_1 = 0.2236 \times 10^{-5} \text{ m}$, $G_2 = 4.472 \times 10^{-11} \text{ m}^2$, $k_{j0} = 1200 \text{ A m}^{-1}$, $a_{j0} = 1100 \text{ A m}^{-1}$. The JA parameters k_{j0} and a_{j0} were chosen so as to let k_j and a_j vary over a range which realistically corresponds to fits to known steels [2, 17]. The values for G_1 and G_2 are the same values as used in [6]. The parameters M_{sj} , α_j and c_j were chosen similarly to what was used in earlier papers [2, 17].

First, the grain size was varied from 8 to $40 \mu\text{m}$ while the dislocation density remained $\zeta_d = 5 \times 10^{10} \text{m}^{-2}$. The variation of the magnetization loops due to the variation of grain size, as well as the correspondence between the JA loops and the Preisach loops, are illustrated in figure 2. In order

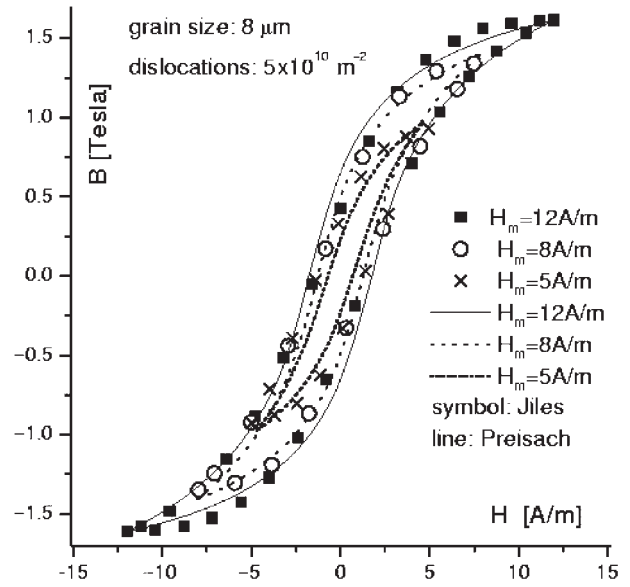


Figure 1. Correspondence between the JA magnetization loops and the Preisach magnetization loops for different field excitation levels when fitting the Preisach material parameters starting from the magnetization loops calculated by the microstructurally dependent modified JA model.

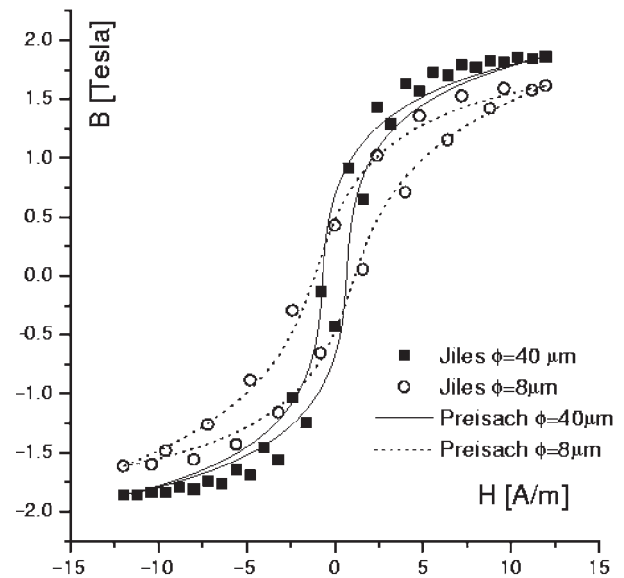


Figure 2. Variation of the magnetization loops with the grain size ϕ .

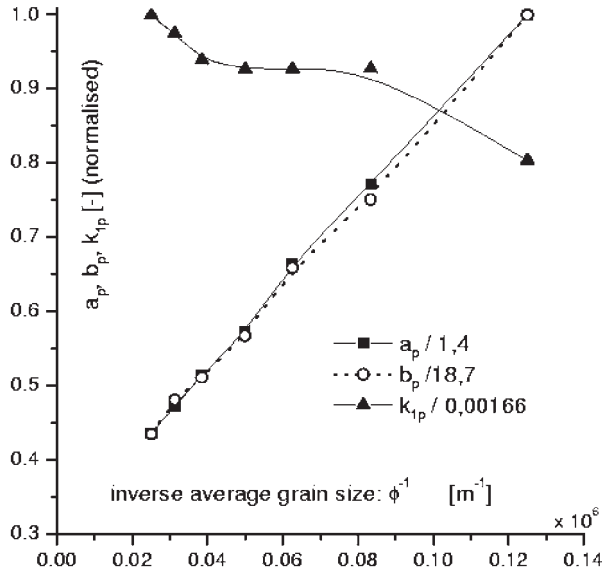


Figure 3. Normalized a_p , b_p and k_{1p} as functions of the inverse grain size. Note that the normalized a_p and b_p are identical and that k_{1p} is almost independent of grain size.

not to overload the figure, we deleted the internal loops. The minor deviation between the two models arises from the specific shape of the Lorentzian PDF and from the fact that the Preisach model is a dipole model, which leads to specific magnetization curves that cannot be precisely the same as those from the JA model. Figure 3 gives the normalized variation of the Preisach parameters a_p , b_p and k_{1p} (normalized with respect to their maximum). The parameter c_p remains constant ($c_p = 12.88$) when varying the grain size. The parameter k_{2p} is slightly increasing from 0.100 08 to 0.120 07 when the grain size increased from 8 to 40 μm .

Next, the dislocation density ζ_d was varied in the range $(3\text{--}36) \times 10^{10} \text{ m}^{-2}$, while the grain size ϕ was fixed at 20 μm . Again, the JA magnetization loops were calculated by considering equations (9a) and (9b). The parameters in the Lorentzian PDF (see equation (8)) were identified in order to obtain the best fit between the JA loops and the Preisach magnetization loops according to the procedure described above. Figure 4 depicts the variation of the magnetization loops due to the variation of the dislocation density ζ_d and confirms the correspondence between the JA loops and the Preisach magnetization loops. Figure 5 gives the normalized variation of the Preisach parameters a_p , b_p , k_{1p} and k_{2p} (again normalized with respect to their maximum). The parameter c_p remained constant ($c_p = 12.88$) as before, when varying the dislocation density.

Taking into account the results given in figures 3 and 5, one may rewrite the parameters a_p and b_p , which define the position of the peak and the shape of the peak of the PDF in the Preisach model, as a function of ϕ and ζ_d :

$$a_p = a_{p0} \left(G_1 + \frac{G_2}{\phi} \right) \sqrt{\zeta_d} \quad (10)$$

and

$$b_p = b_{p0} \left(G_1 + \frac{G_2}{\phi} \right) \sqrt{\zeta_d}. \quad (11)$$

Notice the correspondence with equations (9a) and (9b).

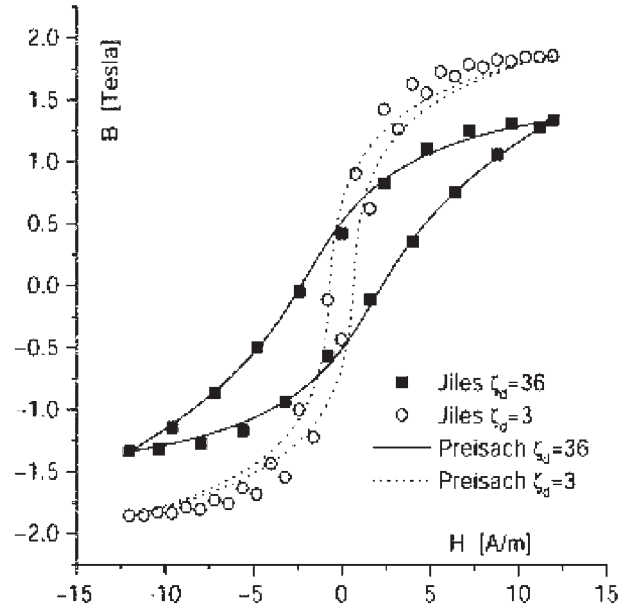


Figure 4. Variation of the magnetization loops with the dislocation density ζ_d .

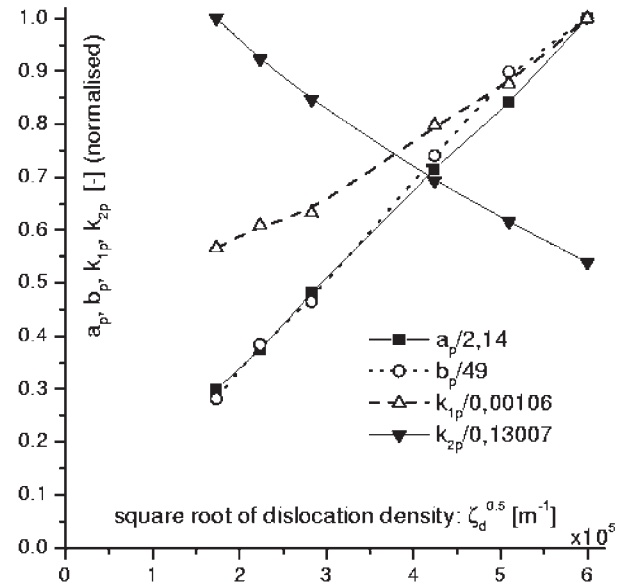


Figure 5. Normalized a_p , b_p , k_{1p} and k_{2p} as functions of the square root of the dislocation density at a constant grain size. The normalized values for a_p and b_p are identical.

The results of equations (10) and (11) should also be compared to results for which the PDF parameters were obtained directly from experimental hysteresis loops for specimens having varying grain size [7]. The result of equation (10) for a_p is consistent with previous work in which a_p was found to vary as $A_1 + A_2/\phi$. Parameters c_p and k_{2p} were previously found to be independent of grain size [7], just as in this paper. On the other hand, in the previous work [7], parameters b_p and k_{1p} were found to be independent of grain size, whereas here only k_{1p} appears to show approximately such behaviour. However, this can be clarified as follows.

In the previous study, materials were considered for which the a_p values (position of peak, see figure 1 in [7]) are always

larger than the b_p values (shape of peak, see figure 2 in [7]), while in this paper, $a_p < b_p$ (see the normalizing constants for a_p (1.4) and b_p (18.7) in figure 2). The consequences are: (1) when $a_p > b_p$, we have more or less a symmetric distribution along the line $\alpha_p = -\beta_p$ in the Preisach plane; (2) when $a_p < b_p$, we have an asymmetric distribution along the line $\alpha_p = -\beta_p$ as the PDF is only defined for $\alpha_p > \beta_p$. Therefore, the law, according to which b_p is varying as a function of grain size, may be completely different for the two cases just mentioned (symmetric or asymmetric distribution). The law for b_p (see (11)), which we obtained here, i.e. linear with respect to the inverse of the grain size, is in accordance with the observations in [10], where a distribution function was used for which the authors enforced the peak to be positioned in the origin of the Preisach plane, i.e. $a_p = 0$ (asymmetric distribution). Notice that [10] also proposes the JA parameter k_j to vary with ϕ according to $k_j = k_{j0} (G_1 + G_2/\phi)$, as in equation (9a). However, in that paper, equation (9b) for a_j is not derived and no dislocation density effects are discussed.

4. Physical interpretation

One may come to the following observations.

- The second term on the right-hand side of equation (8), which is proportional to $\delta_{\alpha_p \beta_p}$, introduces a reversible behaviour. Since c_p and k_{2p} are reversible parameters, they do not change with grain size because they are lattice dependent (the lattice does not change with grain size).
- On the contrary, k_{2p} exhibits a dependence on the dislocation density. If k_{2p} is a lattice-dependent parameter, then the presence of dislocations must tend to destroy the lattice symmetry and thus reduce the relative contribution of the reversible part. Consequently, k_{2p} should decrease. This decrease in k_{2p} is observed.
- In figure 5, k_{1p} does not show the same apparent oscillatory relationship with the dislocation density as with the grain size. In fact, there is a monotonic increase of k_{1p} with dislocation density, which is in agreement with the decreasing behaviour of k_{2p} . The lattice-dependent contribution to magnetization (proportional to k_{2p}) is decreasing, while the microstructurally driven irreversible part (proportional to k_{1p}) is increasing due to increasing dislocation density.

5. Conclusions

In order to have a good agreement between the two models (Preisach and JA) and experimental data, it is sufficient to have simultaneously the following.

- The parameters k_j and a_j , which in the JA model define the pinning effects and the anhysteretic curve respectively, must vary both linearly with $1/\phi$ and must both be proportional to $\sqrt{\zeta_d}$. The remaining parameters M_{sj} , c_j and α_j must remain constant.
- The parameters a_p and b_p , which define position and shape of the peak in the PDF, vary also linearly with $1/\phi$ and are

also both proportional to $\sqrt{\zeta_d}$. The parameter c_p may not depend on grain size and dislocation density. The parameters k_{1p} and k_{2p} are not strongly influenced by ϕ ; they are, respectively, increasing and decreasing functions of the dislocation density.

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