EE 423 – Power System Analysis

[Section 2 – Power System Faults]

Learning Objectives

To be able to perform analysis on power systems with regard to load flow, faults and system stability

Outline Syllabus

1. **Power Flow Analysis: (8 hrs)**

   Analogue methods of power flow analysis: dc and ac network analysers

   Digital methods of analysis: Power Flow algorithms and flow charts, analysis using iterative techniques.

2. **Power system faults (8 hrs)**

   Causes and effects of faults. Review of per unit system and symmetrical components.

   Symmetrical three-phase faults.

   Asymmetrical faults, short circuit and open circuit conditions. Introduction to simultaneous faults.

3. **Power System Stability: (8 hrs)**

   Steady state stability: Power angle diagram, effect of voltage regulator, swing equation

   Transient stability: Equal area criterion, stability under fault conditions, step by step solution of swing equation

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2 Power System Fault Analysis – Prof J Rohan Lucas

2.0 Introduction

The fault analysis of a power system is required in order to provide information for the selection of switchgear, setting of relays and stability of system operation. A power system is not static but changes during operation (switching on or off of generators and transmission lines) and during planning (addition of generators and transmission lines). Thus fault studies need to be routinely performed by utility engineers (such as in the CEB).

Faults usually occur in a power system due to either insulation failure, flashover, physical damage or human error. These faults, may either be three phase in nature involving all three phases in a symmetrical manner, or may be asymmetrical where usually only one or two phases may be involved. Faults may also be caused by either short-circuits to earth or between live conductors, or may be caused by broken conductors in one or more phases. Sometimes simultaneous faults may occur involving both short-circuit and broken-conductor faults (also known as open-circuit faults).

Balanced three phase faults may be analysed using an equivalent single phase circuit. With asymmetrical three phase faults, the use of symmetrical components help to reduce the complexity of the calculations as transmission lines and components are by and large symmetrical, although the fault may be asymmetrical.

Fault analysis is usually carried out in per-unit quantities (similar to percentage quantities) as they give solutions which are somewhat consistent over different voltage and power ratings, and operate on values of the order of unity.

In the ensuing sections, we will derive expressions that may be used in computer simulations by the utility engineers.

2.1 Equivalent Circuits - Single phase and Equivalent Single Phase Circuits

In a balanced three phase circuit, since the information relating to one single phase gives the information relating to the other two phases as well, it is sufficient to do calculations in a single phase circuit. There are two common forms used. These are (i) to take any one single phase of the three phase circuit and (ii) to take an equivalent single phase circuit to represent the full three phase circuit.

2.1.1 *Single Phase Circuit*

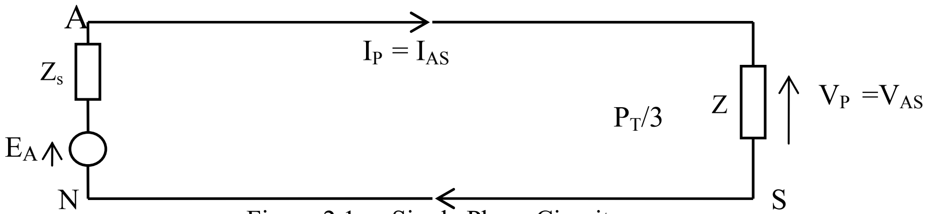


Figure 2.1 - Single Phase Circuit

Figure 2.1 shows one single phase "***AN***" of the three phase circuit ***"ABC N"***. Since the system is balanced, there is no current in the neutral, and there is no potential drop across the neutral wire. Thus the star point **"S"** of the system would be at the same potential as the neutral point **"N"**. Also, the line current is the same as the phase current, the line voltage is √3 times the phase voltage, and the total power is 3 times the power in a single phase.

$$I = I_P = I_L,\ \ V = V_P = V_L/\sqrt{3}\ \ \text{and}\ \ S = S_P = S_T/3$$

Working with the single phase circuit would yield single phase quantities, which can then be converted to three phase quantities using the above conversions.

2.1.2 *Equivalent Single Phase Circuit*

Of the parameters in the single phase circuit shown in figure 2.1, the Line Voltage and the Total Power (rather than the Phase Voltage and one-third the Power) are the most important quantities. It would be useful to have these quantities obtained directly from the circuit rather than having conversion factors of √3 and 3 respectively. This is achieved in the Equivalent Single Phase circuit, shown in figure 2.2, by multiplying the voltage by a factor of √3 to give Line Voltage directly.

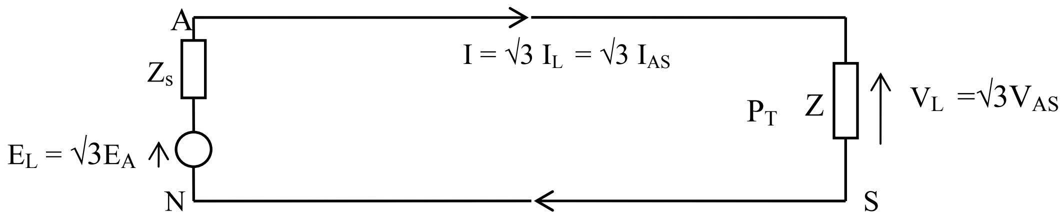


Figure 2.2 - Equivalent Single Phase Circuit

The Impedance remains as the per-phase impedance. However, the Line Current gets artificially amplified by a factor of √3. This also increases the power by a factor of $(\sqrt{3})^2$, which is the required correction to get the total power.

Thus, working with the Equivalent single phase circuit would yield the required three phase quantities directly, other than the current which would be $\sqrt{3}\ I_L$.

2.2 Revision of Per Unit Quantities

Per unit quantities, like percentage quantities, are actually fractional quantities of a reference quantity. These have a lot of importance as per unit quantities of parameters tend to have similar values even when the system voltage and rating change drastically. The per unit system permits multiplication and division in addition to addition and subtraction without the requirement of a correction factor (when percentage quantities are multiplied or divided additional factors of 0.01 or100 must be brought in, which are not in the original equations, to restore the percentage values). Per-unit values are written with "pu" after the value.

For power, voltage, current and impedance, the per unit quantity may be obtained by dividing by the respective base of that quantity.

$$S_{pu} = \frac{S}{S_{base}} \qquad V_{pu} = \frac{V}{V_{base}} \qquad I_{pu} = \frac{I}{I_{base}} \qquad Z_{pu} = \frac{Z}{Z_{base}}$$

Expressions such as Ohm's Law can be applied for per unit quantities as well. Since Voltage, Current, Impedance and Power are related, only two Base or reference quantities can be independently defined. The Base quantities for the other two can be derived there from. Since Power and Voltage are the most often specified, they are usually chosen to define the independent base quantities.

2.2.1 Calculation for Single Phase Systems

If VA_{base} and V_{base} are the selected base quantities of *power* (complex, active or reactive) and *voltage* respectively, then

Base current $\qquad I_{base} = \frac{V_{base} I_{base}}{V_{base}} = \frac{VA_{base}}{V_{base}}$

Base Impedance $\qquad Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V^2{}_{base}}{I_{base} V_{base}} = \frac{V^2{}_{base}}{VA_{base}}$

In a power system, voltages and power are usually expressed in *kV* and *MVA*, thus it is usual to select an MVA_{base} and a kV_{base} and to express them as

Base current $\qquad I_{base} = \frac{MVA_{base}}{kV_{base}}$ in *kA*, $\qquad [\because 10^6/10^3 = 10^3]$

Base Impedance $\qquad Z_{base} = \frac{kV^2_{base}}{MVA_{base}}$ in Ω, $\qquad [\because (10^3)^2/10^6 = 1]$

In these expressions, all the quantities are single phase quantities.

2.2.2 Calculations for Three Phase Systems

In three phase systems the line voltage and the total power are usually used rather than the single phase quantities. It is thus usual to express base quantities in terms of these.

If $VA_{3\phi base}$ and V_{LLbase} are the base three-phase power and line-to-line voltage respectively,

Base current $\qquad I_{base} = \frac{VA_{base}}{V_{base}} = \frac{3VA_{base}}{3V_{base}} = \frac{VA_{3\phi base}}{\sqrt{3}V_{LLbase}}$

Base Impedance $\qquad Z_{base} = \frac{V^2{}_{base}}{VA_{base}} = \frac{\left(\sqrt{3}\right)^2 V^2{}_{base}}{3VA_{base}} = \frac{V^2{}_{LLbase}}{VA_{3\phi base}}$

and in terms of $MVA_{3\phi base}$ and kV_{LLbase}

Base current $\quad I_{base} = \dfrac{MVA_{3\phi base}}{\sqrt{3}kV_{LLbase}} \quad$ in kA

Base Impedance $\quad Z_{base} = \dfrac{kV_{LLbase}^2}{MVA_{3\phi base}} \quad$ in Ω

It is to be noted that while the base impedance for the three phase can be obtained directly from the $VA_{3\phi base}$ and V_{LLbase} (or $MVA_{3\phi base}$ and kV_{LLbase}) without the need of any additional factors, the calculation of base current needs an additional factor of √3. However this is not usually a problem as the value of current is rarely required as a final answer in power systems calculations, and intermediate calculations can be done with a variable $\sqrt{3}I_{base}$.

Thus in three phase, the calculations of per unit quantities becomes

$$S_{pu} = \frac{S_{actual}(MVA)}{MVA_{3\phi base}},$$

$$V_{pu} = \frac{V_{actual}(kV)}{kV_{LLbase}},$$

$$I_{pu} = I_{actual}(kA).\frac{\sqrt{3}kV_{LLbase}}{MVA_{3\phi base}} \qquad \text{and}$$

$$Z_{pu} = Z_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV_{LLbase}^2}$$

P and *Q* have the same base as *S*, so that

$$P_{pu} = \frac{P_{actual}(MW)}{MVA_{3\phi base}}, \quad Q_{pu} = \frac{Q_{actual}(M\text{var})}{MVA_{3\phi base}}$$

Similarly, *R* and *X* have the same base as *Z*, so that

$$R_{pu} = R_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV_{LLbase}^2}, \quad X_{pu} = X_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV_{LLbase}^2}$$

The *power factor* remains unchanged in *per unit*.

2.2.3 Conversions from one Base to another

It is usual to give data in per unit to its own rating [ex: The manufacturer of a certain piece of equipment, such as a transformer, would not know the exact rating of the power system in which the equipment is to be used. However, he would know the rating of his equipment]. As different components can have different ratings, and different from the system rating, it is necessary to convert all quantities to a common base to do arithmetic or algebraic operations. Additions, subtractions, multiplications and divisions will give meaningful results only if they are to the same base. This can be done for three phase systems as follows.

$$S_{puNew} = S_{puGiven}.\frac{MVA_{3\phi baseGiven}}{MVA_{3\phi baseNew}}, \quad V_{puNew} = V_{puGiven}.\frac{kV_{LLbaseGiven}}{kV_{LLbaseNew}}, \quad \text{and}$$

$$Z_{pu} = Z_{puGiven}.\frac{MVA_{3\phi baseNew}}{MVA_{3\phi baseGiven}}.\frac{kV_{LLbaseGiven}^2}{kV_{LLbaseNew}^2}$$

Example:

A 200 MVA, 13.8 kV generator has a reactance of 0.85 p.u. and is generating 1.15 pu voltage. Determine (a) the actual values of the line voltage, phase voltage and reactance, and (b) the corresponding quantities to a new base of 500 MVA, 13.5 kV.

(a) Line voltage = $1.15 * 13.8 = 15.87$ kV

Phase voltage = $1.15 * 13.8/\sqrt{3} = 9.16$ kV

Reactance = $0.85 * 13.8^2/200 = 0.809\ \Omega$

(b) Line voltage = $1.15 * 13.8/13.5 = 1.176$ pu

Phase voltage = $1.15 * (13.8/\sqrt{3})/(13.5/\sqrt{3}) = 1.176$ pu

Reactance = $0.85 * (13.8/13.5)^2/(500/200) = 0.355$ pu

2.2.4 Per Unit Quantities across Transformers

When a transformer is present in a power system, although the power rating on either side of a transformer remains the same, the voltage rating changes, and so does the base voltage across a transformer. [This is like saying that full or 100% (or 1 pu) voltage on the primary of a 220kV/33 kV transformer corresponds to 220 kV while on the secondary it corresponds to 33 kV.] Since the power rating remains unchanged, the impedance and current ratings also change accordingly.

While a common $MVA_{3\phi base}$ can and must be selected for a power system to do analysis, a common V_{LLbase} must be chosen corresponding to a particular location (or side of transformer) and changes in proportion to the nominal voltage ratio whenever a transformer is encountered. Thus the current base changes inversely as the ratio. Hence the impedance base changes as the square of the ratio.

For a transformer with turns ratio $N_P{:}N_S$, base quantities change as follows.

Quantity	**Primary Base**	**Secondary Base**
Power (S, P and Q)	S_{base}	S_{base}
Voltage (V)	V_{1base}	$V_{1base} \,.\, N_S/N_P = V_{2base}$
Current (I)	$S_{base}/\sqrt{3}V_{1base}$	$S_{base}/\sqrt{3}V_{1base} \,.\, N_P/N_S = S_{base}/\sqrt{3}V_{2base}$
Impedance (Z, R and X)	V_{1base}^2/S_{base}	$V_{1base}^2/S_{base} \,.\, (N_S/N_P)^2 = V_{2base}^2/S_{base}$

Example :

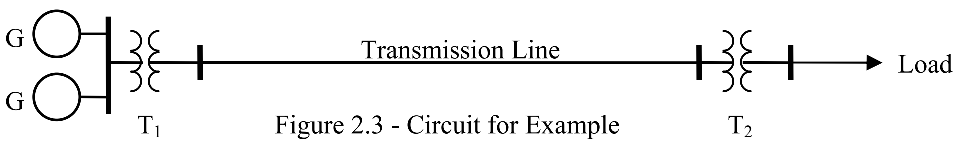


Figure 2.3 - Circuit for Example

In the single line diagram shown in figure 2.3, each three phase generator G is rated at 200 MVA, 13.8 kV and has reactances of 0.85 pu and are generating 1.15 pu. Transformer T_1 is rated at 500 MVA, 13.5 kV/220 kV and has a reactance of 8%. The transmission line has a reactance of 7.8 Ω. Transformer T_2 has a rating of 400 MVA, 220 kV/33 kV and a reactance of 11%. The load is 250 MVA at a power factor of 0.85 lag. Convert all quantities to a common base of 500 MVA, and 220 kV on the line and draw the circuit diagram with values expressed in pu.

Solution:

The base voltage at the generator is (220*13.5/220) 13.5 kV, and on the load side is (220*33/220) 33 kV. [Since we have selected the voltage base as that corresponding to the voltage on that side of the transformer, we automatically get the voltage on the other side of the transformer as the base on that side of the transformer and the above calculation is in fact unnecessary.

Generators G

Reactance of 0.85 pu corresponds 0.355 pu on 500 MVA, 13.5 kV base (see earlier example)

Generator voltage of 1.15 corresponds to 1.176 on 500 MVA, 13.5 kV base

Transformer T_1

Reactance of 8% (or 0.08 pu) remains unchanged as the given base is the same as the new chosen base.

Transmission Line

Reactance of 7.8 Ω corresponds to $7.8 * 500/220^2 = 0.081$ pu

Transformer T_2

Reactance of 11% (0.11 pu) corresponds to $0.11 * 500/400 = 0.1375$ pu

(voltage base is unchanged and does not come into the calculations)

Load

Load of 250 MVA at a power factor of 0.85 corresponds to 250/500 = 0.5 pu at a power factor of 0.85 lag (power factor angle = 31.79°)

$\therefore$ resistance of load = 0.5 * 0.85 = 0.425 pu

and reactance of load = 0.5 * sin 31.79° = 0.263 pu

The circuit may be expressed in per unit as shown in figure 2.4.

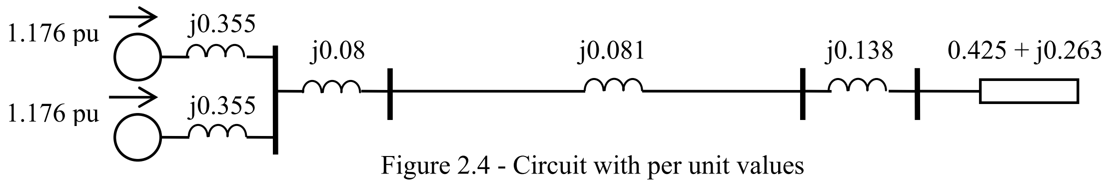


Figure 2.4 - Circuit with per unit values

2.3 Symmetrical Three Phase Fault Analysis

A three phase fault is a condition where either (a) all three phases of the system are short-circuited to each other, or (b) all three phase of the system are earthed.

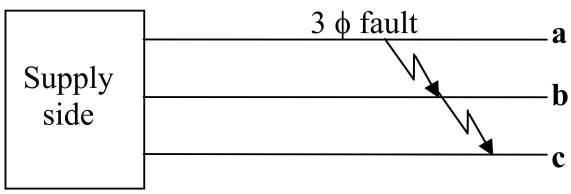


Figure 2.5a – Balanced three phase fault

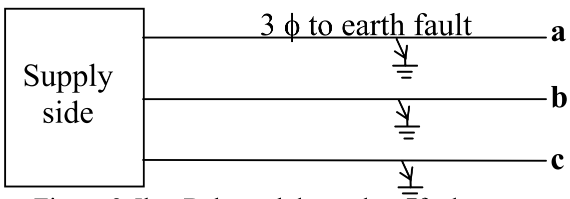


Figure 2.5b – Balanced three phase fault to

This is in general a balanced condition, and we need to only know the positive-sequence network to analyse faults. Further, the single line diagram can be used, as all three phases carry equal currents displaced by 120°.

Typically, only 5% of the initial faults in a power system, are three phase faults with or without earth. Of the unbalanced faults, 80 % are line-earth and 15% are double line faults with or without earth and which can often deteriorate to 3 phase fault. Broken conductor faults account for the rest.

2.3.1 Fault Level Calculations

In a power system, the maximum the fault current (or fault MVA) that can flow into a zero impedance fault is necessary to be known for switch gear solution. This can either be the balanced three phase value or the value at an asymmetrical condition. The Fault Level defines the value for the symmetrical condition. The fault level is usually expressed in MVA (or corresponding per-unit value), with the maximum fault current value being converted using the nominal voltage rating.

$$MVA_{base} = \sqrt{3} \text{ . } Nominal\ Voltage(kV) \text{ . } I_{base}\ (kA)$$

$$MVA_{Fault} = \sqrt{3} \text{ . } Nominal\ Voltage(kV) \text{ . } I_{sc}\ (kA)$$

where

MVA_{Fault} – Fault Level at a given point in MVA

I_{base} – Rated or base line current

I_{sc} – Short circuit line current flowing in to a fault

The per unit value of the Fault Level may thus be written as

$$Fault\ Level = \frac{\sqrt{3}\text{ . Nominal Voltage . }I_{sc}}{\sqrt{3}\text{ . Nominal Voltage . }I_{base}} = \frac{\sqrt{3}I_{sc}}{\sqrt{3}I_{base}} = I_{sc,pu} = \frac{V_{Nominal,pu}}{Z_{pu}}$$

The per unit voltage for nominal value is unity, so that

$$Fault\ Level\ (pu) = \frac{1}{Z_{pu}},$$

$$Fault\ MVA = Fault\ Level\ (pu) \times MVA_{base} = \frac{MVA_{base}}{Z_{pu}}$$

The Short circuit capacity (SCC) of a busbar is the fault level of the busbar. The strength of a busbar (or the ability to maintain its voltage) is directly proportional to its SCC. An infinitely strong bus (or Infinite bus bar) has an infinite SCC, with a zero equivalent impedance and will maintain its voltage under all conditions.

Magnitude of short circuit current is time dependant due to synchronous generators. It is initially at its largest value and decreasing to steady value. These higher fault levels tax Circuit Breakers adversely so that current limiting reactors are sometimes used.

The Short circuit MVA is a better indicator of the stress on CBs than the short circuit current as CB has to withstand recovery voltage across breaker following arc interruption.

The currents flowing during a fault is determined by the internal emfs of machines in the network, by the impedances of the machines, and by the impedances between the machines and the fault.

Figure 2.6 shows a part of a power system, where the rest of the system at two points of coupling have been represented by their Thevenin's equivalent circuit (or by a voltage source of 1 pu together its fault level which corresponds to the per unit value of the effective Thevenin's impedance).

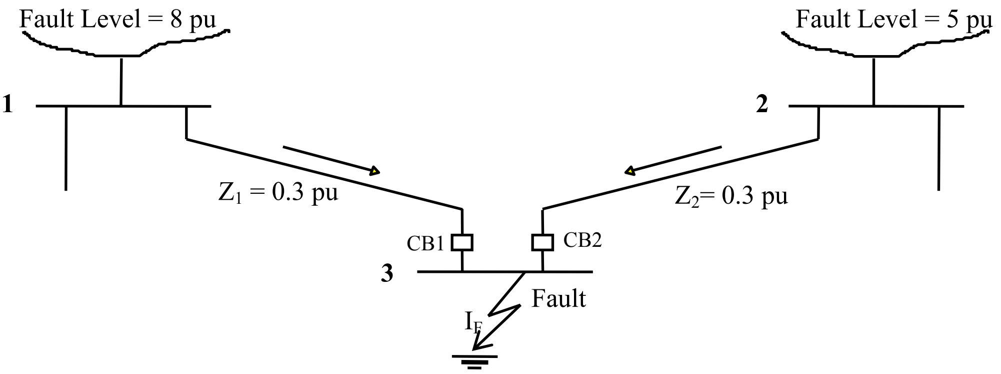


Figure 2.6 – Circuit for Fault Level Calculation

With CB1 and CB2 open, short circuit capacities are

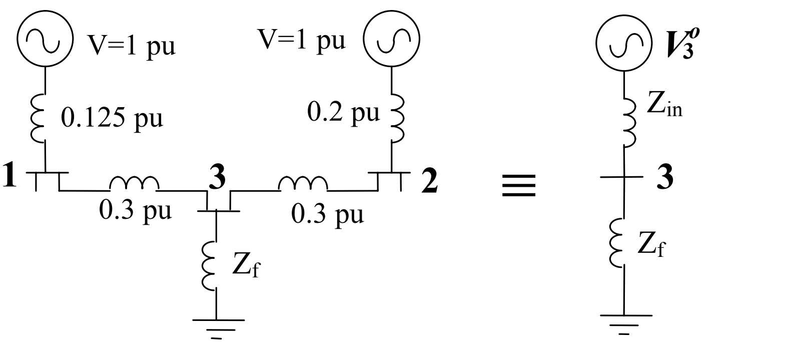
SCC at bus 1 = 8 p.u. gives $Z_{g1} = 1/8 = 0.125$ pu

SCC at bus 2 = 5 p.u. gives $Z_{g2} = 1/5 = 0.20$ pu

Each of the lines are given to have a per unit impedance of 0.3 pu.

$Z_1 = Z_2 = 0.3$ p.u.

With CB1 and CB2 closed, what would be the SCCs (or Fault Levels) of the busbars in the system ?



System Equivalent Circuit Thevenin's Equivalent at 3

Figure 2.7a Determination of Short circuit capacities

This circuit can be reduced and analysed as in figure 2.7b.

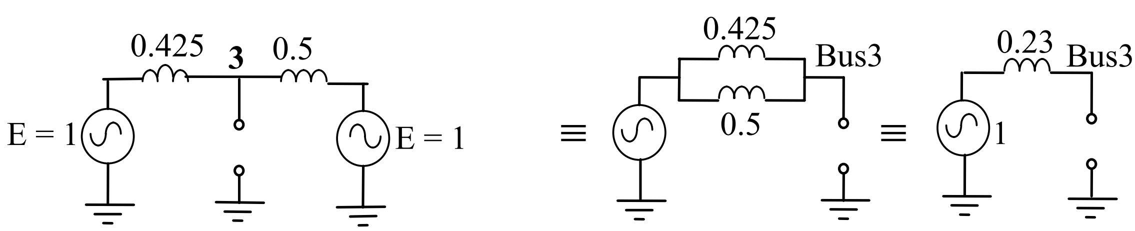


Figure 2.7b Determination of Short circuit capacity at Bus 3

Thus, the equivalent input impedance is given by to give Z_{in} as 0.23 pu at bus 3, so that the short circuit capacity at busbar 3 is given as

$$|\ SCC3\ | = 1/0.23 = 4.35 \text{ p.u}$$

The network may also be reduced keeping the identity of Bus 1 as in figure 2.7c.

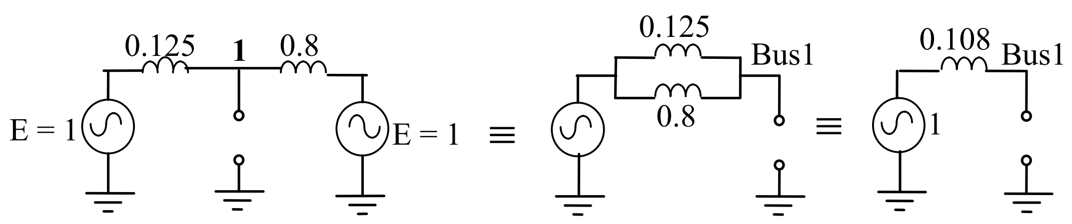


Figure 2.7c Determination of Short circuit capacity at Bus 1

Thus, the equivalent input impedance is given by to give Z_{in} as 0.108 pu at bus 1, so that the short circuit capacity at busbar 1 is given as

$$|\,SCC1\,| = 1/0.108 = 9.25 \text{ p.u}$$

This is a 16% increase on the short circuit capacity of bus 1 with the circuit breakers open.

The network may also be reduced keeping the identity of Bus 2. This would yield a value of Z_{in} as 0.157 pu, giving the short circuit capacity at busbar 2 as

$$|\,SCC2\,| = 1/0.157 = 6.37 \text{ p.u}$$

This is a 28% increase on the short circuit capacity of bus 2 with the circuit breakers open.

Typical maximum values of short circuit capacities at substations in Sri Lanka in 2000 are shown in table 2.1. Actual fault currents are lower than these values due to the presence of fault impedance in the circuit.

Ampara	206	Kelanitissa	434	Puttalam GS	366
Anuradhapura	223	Kiribathkumbura	440	Rantembe	257
Anuradhapura	183	Kolonnawa	623	Ratmalana	680
Badulla	434	Kolonnawa	543	Sapugaskanda	572
Balangoda	177	Kosgama	457	Sapugaskanda	274
Biyagama	503	Kotugoda	474	Seetawaka	463
Bolawatte	543	Kurunegala	234	Thulhiriya	429
Deniyaya	194	Madampe	354	Trincomalee	183
Embilipitiya	160	Matara	286	Ukuwela	423
Galle	189	Matugama	286	Wimalasurendra	509
Habarana	314	Nuwara Eliya	417		
Inginiyagala	160	Oruwela	63	**Average SCC**	**377 MVA**
Kelanitissa	549	Panadura	389	Std deviation	166 MVA
Kelanitissa	646	Pannipitiya	697	**Average Isc**	**6.6 kA**

Table 2.1 – Maximum 3ϕ Fault Levels at 33kV Substations in Sri Lanka in 2000

2.4 Fault Currents in synchronous machines

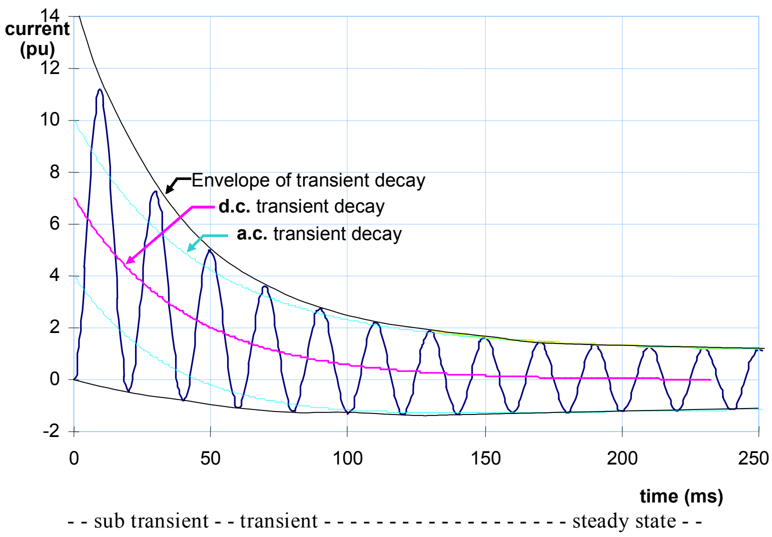


Figure 2.8 – Transient decay of current in synchronous generator

As mentioned earlier, the currents flowing in the power system network during a fault is dependant on the machines connected to the system. Due to the effect of armature current on the flux that generates the voltage, the currents flowing in a synchronous machine differs immediately after the occurrence of the fault, a few cycles later, and under sustained or steady-state conditions.

Further there is an exponentially decaying d.c. component caused by the instantaneous value at the instant of fault occurring. These are shown in figure 2.8.

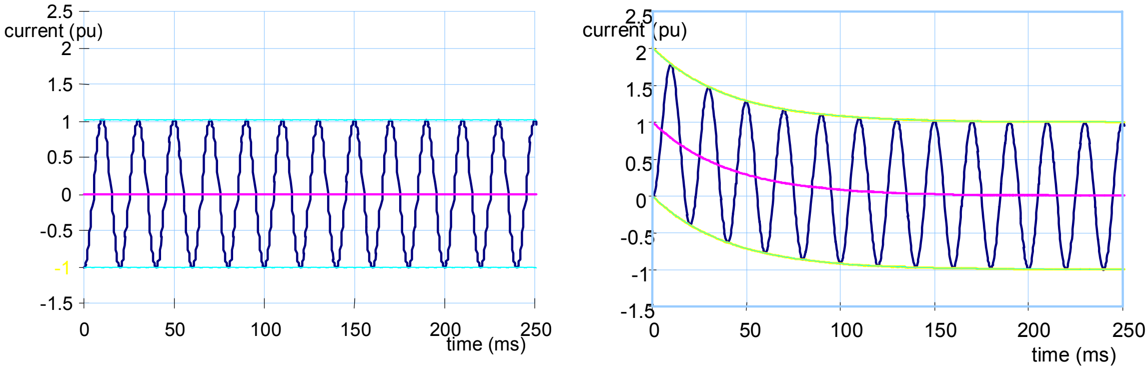


Figure 2.9a & b – Steady state and Transient current

Figure 2.9a and 2.9b show the steady state current waveform, and the transient waveform of a simple R-L circuit, to show the decay in the d.c. component. In addition to this, in the synchronous machine, the magnitude of the a.c. current peak also changes with time as shown in figure 2.9c, with the unidirection component of the transient waveform removed.

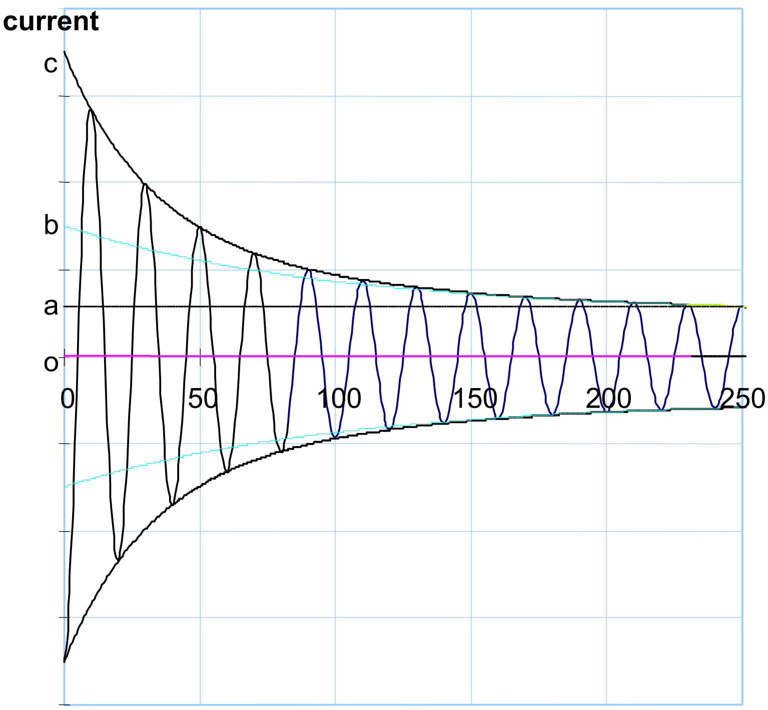


Figure 2.9c–Synchronous machine transient

Due to the initial low back emf at the instant of fault resulting in high current, the effective impedance is very low. Even when the d.c. transient component is not present, the initial current can be several times the steady state value. Thus three regions are identified for determining the reactance. These are the sub-transient reactance x_d'' for the first 10 to 20 ms of fault, the transient reactance x_d' for up to about 500 ms, and the steady state reactance x_d (synchronous reactance).

. The sub-transient must usually be used in fault analysis.

oa - peak value of steady state short-circuit current

ob - peak value of transient short-circuit current

oc - peak value of sub-transient short-circuit current

The r.m.s. values of current are given by

$$|I| = \frac{oa}{\sqrt{2}} = \frac{|E|}{X_d}$$

$$|I'| = \frac{ob}{\sqrt{2}} = \frac{|E|}{X'_d}$$

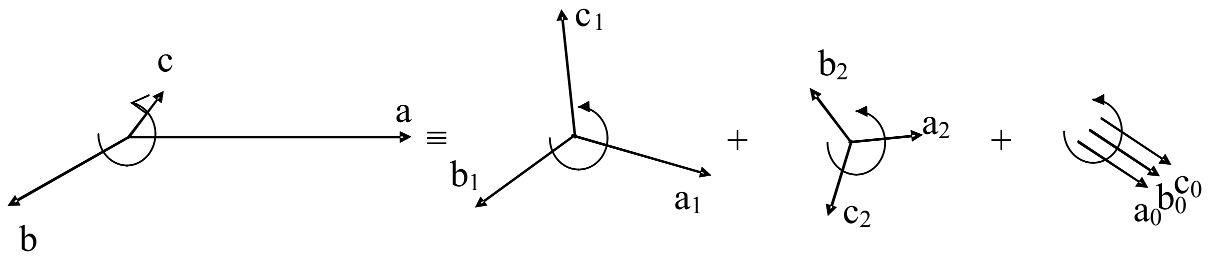
$$|I''| = \frac{oc}{\sqrt{2}} = \frac{|E|}{X''_d}$$

	subtransient reactance	transient reactance	steady-state reactance
turbo-generator	10-20 %	15-25 %	150-230 %
salient-pole generator	15-25 %	25-35 %	70-120 %

The typical generator reactance values are given above for reference.

2.5 Revision of Symmetrical Component Analysis

Unbalanced three phase systems can be split into three balanced components, namely Positive Sequence (balanced and having the same phase sequence as the unbalanced supply), Negative Sequence (balanced and having the opposite phase sequence to the unbalanced supply) and Zero Sequence (balanced but having the same phase and hence no phase sequence). These are known as the Symmetrical Components or the Sequence Components and are shown in figure 2.10.



Unbalanced system	**Positive Sequence**	**Negative Sequence**	**Zero Sequence**
3 unknown magnitudes	1 unknown magnitude	1 unknown magnitude	1 unknown magnitude
3 unknown angles	1 unknown angle	1 unknown angle	1 unknown angle

Figure 2.10 – Symmetrical Components of unbalanced 3 phase

The phase components are the addition of the symmetrical components and can be written as follows.

$$\mathbf{a} = \mathbf{a_1} + \mathbf{a_2} + \mathbf{a_0}$$

$$\mathbf{b} = \mathbf{b_1} + \mathbf{b_2} + \mathbf{b_0}$$

$$\mathbf{c} = \mathbf{c_1} + \mathbf{c_2} + \mathbf{c_0}$$

The unknown unbalanced system has three unknown magnitudes and three unknown angles with respect to the reference direction. Similarly, the combination of the 3 sequence components will also have three unknown magnitudes and three unknown angles with respect to the reference direction.

Thus the original unbalanced system effectively has 3 complex unknown quantities **a**, **b** and **c** (magnitude and phase angle of each is independent), and that each of the balanced components have only one independent complex unknown each, as the others can be written by symmetry. Thus the three sets of symmetrical components also have effectively 3 complex unknown quantities. These are usually selected as the components of the first phase **a** (i.e. $\mathbf{a_0}$, $\mathbf{a_1}$ and $\mathbf{a_2}$) . One of the other phases could have been selected as well, but all 3 components should be selected for the same phase.

Thus it should be possible to convert from either sequence components to phase components or vice versa.

2.5.1 Definition of the operator α

When the balanced components are considered, we see that the most frequently occurring angle is 120^0.

In complex number theory, we defined $\boldsymbol{j}$ as the complex operator which is equal to $\sqrt{-1}$ and a magnitude of unity, and more importantly, when operated on any complex number rotates it anti-clockwise by an angle of 90^0.

i.e. $j = \sqrt{-1} = 1 \angle 90^0$

In like manner, we define a new complex operator α which has a magnitude of unity and when operated on any complex number rotates it anti-clockwise by an angle of 120^0.

i.e. $\alpha = 1 \angle 120^0 = -0.500 + j\,0.866$

Some Properties of α

$$\alpha = 1\angle 2\pi/3 \text{ or } 1\angle 120^0$$

$$\alpha^2 = 1\angle 4\pi/3 \text{ or } 1\angle 240^0 \text{ or } 1\angle -120^0$$

$$\alpha^3 = 1\angle 2\pi \text{ or } 1\angle 360^0 \text{ or } 1$$

i.e. $\boldsymbol{\alpha^3 - 1 = (\alpha - 1)(\alpha^2 + \alpha + 1) = 0}$

Since α is complex, it cannot be equal to 1, so that $\boldsymbol{\alpha - 1}$ cannot be zero.

$\therefore \quad \boldsymbol{\alpha^2 + \alpha + 1 = 0}$

This also has the physical meaning that the three sides of an equilateral triangles must close as in figure 2.11.

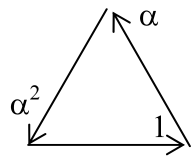


Figure 2.11 Phasor Addition

Also $\alpha^{-1} = \alpha^2$ and $\alpha^{-2} = \alpha$

2.5.2 Analysis of decomposition of phasors

Let us again examine the sequence components of the unbalanced quantity, with each of the components written in terms of phase **a** components, and the operator α, as in figure 2.12.

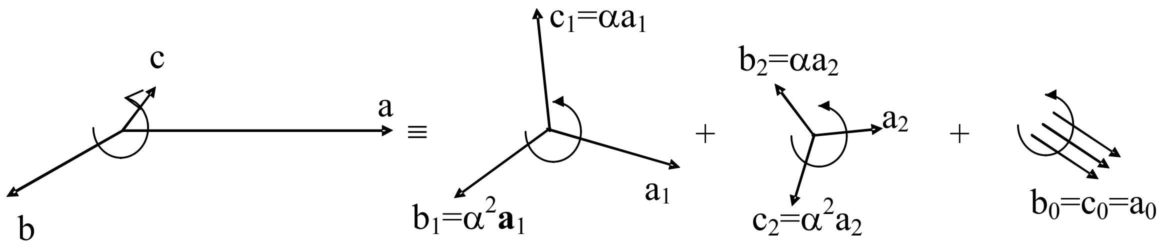


Figure 2.12 – Expressing components in terms of phase **a**

We can express all the sequence components in terms of the quantities for **A** phase using the properties of rotation of 0^0, 120^0 or 240^0.

Thus

$$\mathbf{a} = \mathbf{a_0} + \mathbf{a_1} + \mathbf{a_2}$$

$$\mathbf{b} = \mathbf{a_0} + \alpha^2\,\mathbf{a_1} + \alpha\,\mathbf{a_2}$$

$$\mathbf{c} = \mathbf{a_0} + \alpha\,\mathbf{a_1} + \alpha^2\,\mathbf{a_2}$$

This can be written in matrix form.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\underline{Ph} \qquad [\Lambda] \qquad \underline{Sy}$$

This gives the basic symmetrical component matrix equation, which shows the relationship between the phase component vector $\underline{Ph}$ and the symmetrical component vector $\underline{Sy}$ using the symmetrical component matrix [Λ]. Both the phase component vector $\underline{Ph}$ and the symmetrical component vector $\underline{Sy}$ can be either voltages or currents, but in a particular equation, they must of course all be of the same type. Since the matrix is a [3×3] matrix, it is possible to invert it and express $\underline{Sy}$ in terms of $\underline{Ph}$.

2.5.3 Decomposition of phasors into symmetrical components

Now let us invert the symmetrical component matrix [Λ].

$$[\Lambda]^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^{-1} = \frac{1}{\Delta} \begin{bmatrix} \alpha^4 - \alpha^2 & -(\alpha^2 - \alpha) & \alpha - \alpha^2 \\ -(\alpha^2 - \alpha) & \alpha^2 - 1 & 1 - \alpha \\ \alpha - \alpha^2 & 1 - \alpha & \alpha^2 - 1 \end{bmatrix}$$

$$= \frac{1}{\Delta} \begin{bmatrix} \alpha - \alpha^2 & \alpha - \alpha^2 & \alpha - \alpha^2 \\ \alpha - \alpha^2 & \alpha^2 - 1 & 1 - \alpha \\ \alpha - \alpha^2 & 1 - \alpha & \alpha^2 - 1 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \alpha(1-\alpha) & \alpha(1-\alpha) & \alpha(1-\alpha) \\ \alpha(1-\alpha) & -(1-\alpha)(\alpha+1) & 1-\alpha \\ \alpha(1-\alpha) & 1-\alpha & -(1-\alpha)(\alpha+1) \end{bmatrix}$$

and the discriminent $\Delta = 3(\alpha - \alpha^2) = 3\alpha\,(1-\alpha)$

Substituting, the matrix equation simplifies to give

$$[\Lambda]^{-1} = \frac{1}{3\alpha} \begin{bmatrix} \alpha & \alpha & \alpha \\ \alpha & -(\alpha+1) & 1 \\ \alpha & 1 & -(\alpha+1) \end{bmatrix}$$

Since $\alpha^{-1} = \alpha^2$, $\alpha^{-2} = \alpha$ and $\mathbf{1 + \alpha + \alpha^2 = 0}$, the matrix equation further simplifies to

$$[\Lambda]^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

It is seen that α is the complex conjugate of α^2, and α^2 is the complex conjugate of α.

Thus the above matrix $[\Delta]^{-1}$ is one-third of the complex conjugate of $[\Delta]$.

i.e. $\quad [\Lambda]^{-1} = \frac{1}{3}[\Lambda]^*$

This can now be written in the expanded form as

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\underline{Sy} \qquad [\Lambda] \qquad \underline{Ph}$$

2.5.4 Sequence Impedances

Consider how the impedance appears in sequence components.

To do this we must first look at the impedance matrix in phase components.

$$\underline{V}_p = [Z_p].\underline{I}_p$$

Substituting for $\underline{V}_p$ and I_p in terms of the symmetrical components we have

$$[\Lambda]\,\underline{V}_s = [Z_p].\,[\Lambda]\,\underline{I}_s$$

pre-multiplying equation by $[\Lambda]^{-1}$ we have

$$\underline{V}_s = [\Lambda]^{-1}.[Z_p].\,[\Lambda]\,\underline{I}_s$$

This gives the relationship between the symmetrical component voltage $\underline{V}_s$ and the symmetrical component current $\underline{I}_s$, and hence defines the symmetrical component impedance matrix or Sequence Impedance matrix.

Thus $$[Z_s] = [\Lambda]^{-1}.[Z_p].\,[\Lambda] = \tfrac{1}{3}\,[\Lambda]^{*}.[Z_p].\,[\Lambda]$$

In a similar manner, we could express the phase component impedance matrix in terms of the symmetrical component impedance matrix as follows.

$$[Z_p] = [\Lambda].[Z_s].\,[\Lambda]^{-1} = \tfrac{1}{3}\,[\Lambda].[Z_s].\,[\Lambda]^{*}$$

The form of the sequence impedance matrix for practical problems gives one of the main reasons for use of symmetrical components in practical power system analysis.

If we consider the simple arrangement of a 3 phase transmission line (figure 2.13), we would have the equivalent circuit as

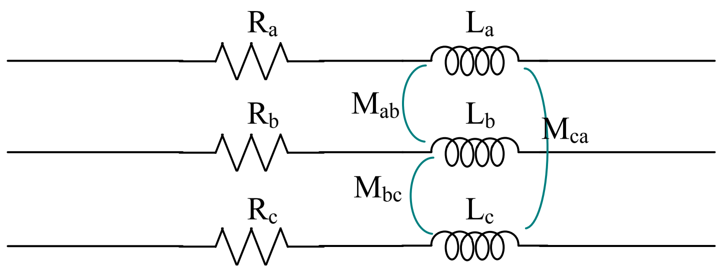


Figure 2.13 – 3 phase transmission line

If we think of an actual line such as from Victoria to Kotmale, we would realise that all 3 phase wires would have approximately the same length (other than due to differences in sagging) and hence we can assume the self impedance components to be equal for each phase.

i.e. $R_a = R_b = R_c$ and $L_a = L_b = L_c$

When a current passes in one phase conductor, there would be induced voltages in the other two phase conductors. In practice all three phase conductors behave similarly, so that we could consider the mutual coupling between phases also to be equal.

i.e. $M_{ab} = M_{bc} = M_{ca}$

In such a practical situation as above, the phase component impedance matrix would be fully symmetrical, and we could express them using a self impedance term z_s and a mutual impedance term z_m.

Thus we may write the phase component impedance matrix as

$$[Z_p] = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix}$$

We may now write the symmetrical component impedance matrix as

$$[Z_s] = \tfrac{1}{3}[\Lambda]^*.[Z_p][\Lambda] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} z_s + 2z_m & z_s + (\alpha+\alpha^2)z_m & z_s + (\alpha+\alpha^2)z_m \\ z_s + 2z_m & \alpha^2 z_s + (1+\alpha)z_m & \alpha z_s + (1+\alpha^2)z_m \\ z_s + 2z_m & \alpha z_s + (1+\alpha^2)z_m & \alpha^2 z_s + (1+\alpha)z_m \end{bmatrix}$$

This can be simplified using the property $1+\alpha+\alpha^2 = 0$ as follows

$$[Z_s] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} z_s + 2z_m & z_s - z_m & z_s - z_m \\ z_s + 2z_m & \alpha^2(z_s - z_m) & \alpha(z_s - z_m) \\ z_s + 2z_m & \alpha(z_s - z_m) & \alpha^2(z_s - z_m) \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3(z_s + 2z_m) & 0 & 0 \\ 0 & (1+\alpha^3+\alpha^3)(z_s - z_m) & 0 \\ 0 & 0 & (1+\alpha^3+\alpha^3)(z_s - z_m) \end{bmatrix}$$

i.e. $$[Z_s] = \begin{bmatrix} (z_s + 2z_m) & 0 & 0 \\ 0 & (z_s - z_m) & 0 \\ 0 & 0 & (z_s - z_m) \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

We see an important result here. While the phase component impedance matrix was a full matrix, although it had completely symmetry, the sequence component impedance matrix is diagonal. The advantage of a diagonal matrix is that it allows decoupling for ease of analysis.

2.6 Power associated with Sequence Components

With phase components, power in a single phase is expressed as

$P_{phase} = V\ I \cos\phi$

Thus in three phase, we may either write $P = \sqrt{3}\ V_L\ I_L \cos\phi = 3\ V_p\ I_p \cos\phi$ for a balanced three phase system. However, with an unbalanced system this is not possible and we would have to write the power as the addition of the powers in the three phases.

Thus Apparent Complex Power S = $V_a I_a^* + V_b I_b^* + V_c I_c^*$

The active power P is obtained as the Real part of the complex variable S.

This equation may be re-written in matrix form as follows.

$$S = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} = \underline{V}_p^T . \underline{I}_p^*$$

Let us now convert it to symmetrical components, as follows.

$$S = \underline{V}_p^T . \underline{I}_p^* = \left[[\Lambda]\underline{V}_s\right]^T . \left[[\Lambda]\underline{I}_s\right]^*$$

which may be expanded as follows.

$$S = \underline{V}_s^T [\Lambda]^T . [\Lambda]^* . \underline{I}_s^* = \underline{V}_s^T [\Lambda] . 3 [\Lambda]^{-1} . \underline{I}_s^* = 3\, V_s^T . I_s^*$$

i.e. $S = 3\,(V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^*)$

This result can also be expected, as there are 3 phases in each of the sequence components taking the same power.

Thus $P = 3\,(V_{a0} I_{a0} \cos\phi_0 + V_{a1} I_{a1} \cos\phi_1 + V_{a2} \cos\phi_2)$

2.7 Asymmetrical Three Phase Fault Analysis

2.7.1 Assumptions Commonly Made in Three Phase Fault Studies

The following assumptions are usually made in fault analysis in three phase transmission lines.

- All sources are balanced and equal in magnitude & phase
- Sources represented by the Thevenin's voltage prior to fault at the fault point
- Large systems may be represented by an infinite bus-bars
- Transformers are on nominal tap position
- Resistances are negligible compared to reactances
- Transmission lines are assumed fully transposed and all 3 phases have same Z
- Loads currents are negligible compared to fault currents
- Line charging currents can be completely neglected

2.7.2 Basic Voltage – Current Network equations in Sequence Components

The generated voltages in the transmission system are assumed balanced prior to the fault, so that they consist only of the positive sequence component V_f (pre-fault voltage). This is in fact the Thevenin's equivalent at the point of the fault prior to the occurrence of the fault.

$$V_{a0} = 0 - Z_0 I_{a0}$$

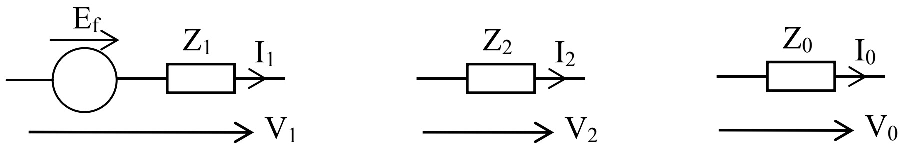
$$V_{a1} = E_f - Z_1 I_{a1}$$

$$V_{a2} = 0 - Z_2 I_{a2}$$

This may be written in matrix form as

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

These may be expressed in Network form as shown in figure 2.14.



Positive Sequence Network   Negative Sequence Network   Zero Sequence Network

Figure 2.14 – Elementary Sequence Networks

2.8 Analysis of Asymmetrical Faults

The common types of asymmetrical faults occurring in a Power System are single line to ground faults and line to line faults, with and without fault impedance. These will be analysed in the following sections.

2.8.1 Single Line to Ground faults (L – G faults)

The single line to ground fault can occur in any of the three phases. However, it is sufficient to analyse only one of the cases. Looking at the symmetry of the symmetrical component matrix, it is seen that the simplest to analyse would be the phase **a**.

Consider an L-G fault with zero fault impedance as shown in figure 2.15.

Since the fault impedance is 0, at the fault

$V_a = 0, I_b = 0, I_c = 0$

since load currents are neglected.

These can be converted to equivalent conditions in symmetrical components as follows.

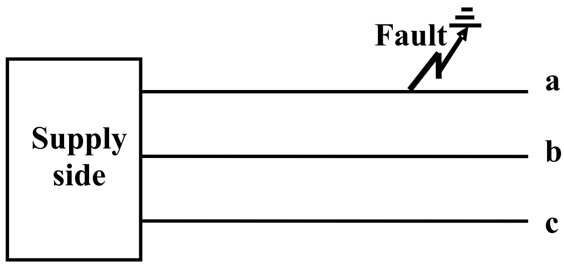


Figure 2.15 – L-G fault on phase **a**

$V_a = V_{a0} + V_{a1} + V_{a2} = 0$

and $\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b = 0 \\ I_c = 0 \end{bmatrix}$, giving $I_{a0} = I_{a1} = I_{a2} = I_a/3$

Mathematical analysis using the network equation in symmetrical components would yield the desired result for the fault current $I_f = I_a$.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} = I_a/3 \\ I_{a1} = I_a/3 \\ I_{a2} = I_a/3 \end{bmatrix}$$

Thus $V_{a0} + V_{a1} + V_{a2} = 0 = - Z_0.I_a/3 + E_f - Z_1.I_a/3 - Z_2.I_a/3$

Simplification, with $I_f = I_a$, gives

$$I_f = \frac{3E_f}{Z_1 + Z_2 + Z_0}$$

Also, considering the equations

$V_{a0} + V_{a1} + V_{a2} = 0$, and

$I_{a0} = I_{a1} = I_{a2}$ indicates that the three networks (zero, positive and negative) must be connected in series (same current, voltages add up) and short-circuited, giving the circuit shown in figure 2.16.

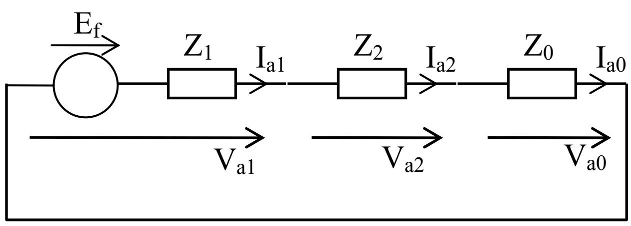


Figure 2.16 – Connection of Sequence Networks for L-G fault with $Z_f = 0$

In this case, I_a corresponds to the fault current I_f, which in turn corresponds to 3 times any one of the components ($I_{a0} = I_{a1} = I_{a2} = I_a/3$). Thus the network would also yield the same fault current as in the mathematical analysis. In this example, the connection of sequence components is more convenient to apply than the mathematical analysis.

Thus for a single line to ground fault (L-G fault) with no fault impedance, the sequence networks must be connected in series and short circuited.

Consider now an L-G fault with fault impedance Z_f as shown in figure 2.17.

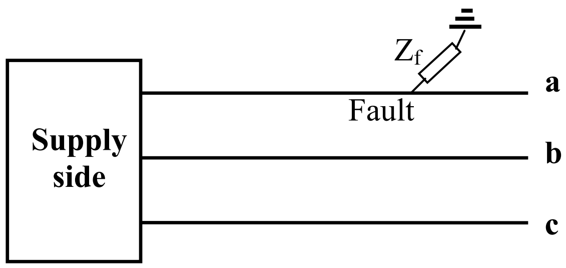


Figure 2.17 – L-G fault on phase **a** with Z_f

at the fault

$$V_a = I_a Z_f,\ \ I_b = 0,\ \ I_c = 0$$

These can be converted to equivalent conditions in symmetrical components as follows.

$$V_{a0} + V_{a1} + V_{a2} = (I_{a0} + I_{a1} + I_{a2}).Z_f$$

and $\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b = 0 \\ I_c = 0 \end{bmatrix}$,

giving $I_{a0} = I_{a1} = I_{a2} = I_a/3$

Mathematical analysis using the network equation in symmetrical components would yield the desired result for the fault current I_f as

$$I_f = \frac{3V_f}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

Similarly, considering the basic equations,

$$I_{a0} = I_{a1} = I_{a2} = I_a/3,$$

and

$$V_{a0} + V_{a1} + V_{a2} = 3I_{a0}.Z_f$$

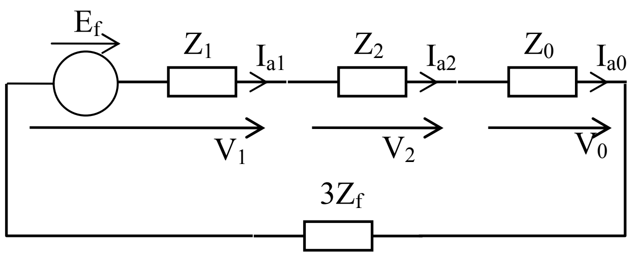


Figure 2.16 – Connection of Sequence Networks for L-G fault with Z_f

or $V_{a0} + V_{a1} + V_{a2} = I_{a0}.3Z_f$, would yield a circuit connection of the 3 sequence networks in series an in series with an effective impedance of 3 Z_f.

2.8.2 Alternate Methods of Solution

The addition of the fault impedance can be treated in two alternate methods as follows. These methods are also applicable for other types of asymmetrical three phase faults.

(a) Z_f considered as part of earth path impedance

The fault impedance Z_f in the L-G fault, is effectively in the earth path. Both the positive sequence and the negative sequence being balanced and being 120^0 apart will always add up to zero and would never yield a current in the earth path. On the other hand, the zero sequence currents in the three phase are balanced but in phase giving an addition of 3 times the zero sequence current (3 I_{a0}) in the earth path. This would give a voltage drop in the earth path (or zero sequence circuit) of $3I_{a0}.Z_f$ or mathematically equal to $I_{a0}.3Z_f$, giving an increase of the zero sequence impedance of $3Z_f$, giving the circuit shown in figure 2.17 which is identical to that of figure 2.16, except that V_0 now incorporates the effects of $3Z_f$ as well.

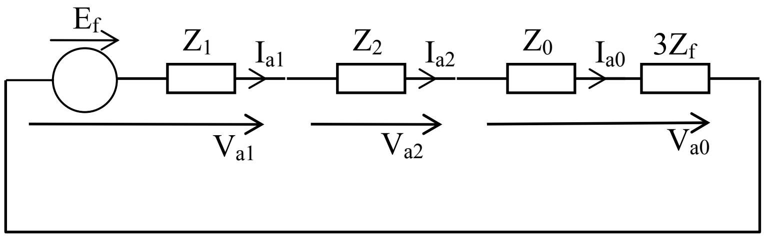


Figure 2.17 – Alternate method for L-G fault with Z_f in ground path

(b) ***Z_f considered as part of each line impedance***

The fault impedance Z_f in the L-G fault, is effectively in the path of phase **a**. Since the other two phases are having zero currents (load currents neglected), addition of an impedance in series to either of these lines would not cause any voltage drop or other change in circuit conditions. Thus the problem can also be considered as each line having an additional line impedance of Z_f, and a zero impedance L-G fault at its end.

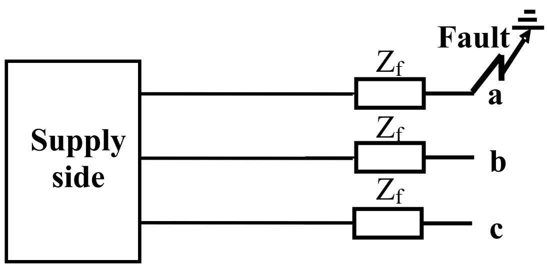


Figure 2.18 – L-G fault with Z_f

This would yield a sequence connection of networks, with each of the sequence impedances increased by an amount of Z_f, as shown in figure 2.19. This result too is identical to that of figure 2.16, except that V_{a0}, V_{a1}, V_{a2} all now incorporates the effect of Z_f as well.

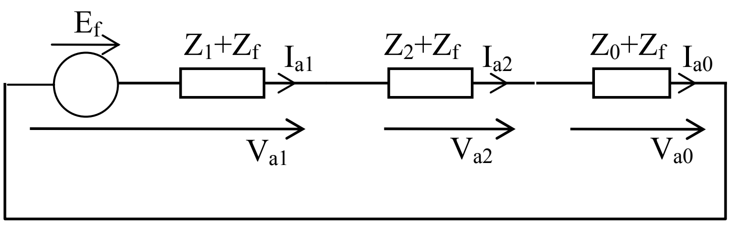


Figure 2.19 – Alternate method for L-G fault with Z_f in line path

The two alternate methods described are useful when analysing faults which are somewhat complication in connection.

It is also to be noted, that while the mathematical solution method will always work for any type of fault, a connection of networks need not always be available.

2.8.3 Line to Line faults (L – L faults)

Line-to-Line faults may occur in a power system, with or without the earth, and with or without fault impedance.

(a) L-L fault with no earth and no Z_f

Solution of the L-L fault gives a simpler solution when phases **b** and **c** are considered as the symmetrical component matrix is similar for phases **b** and **c**. The complexity of the calculations reduce on account of this selection. At the fault,

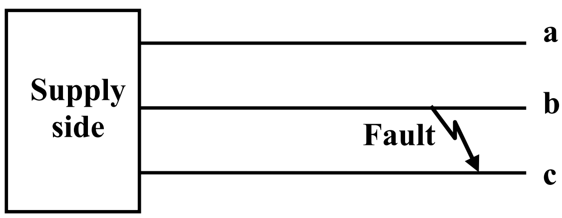


Figure 2.20 – L-L fault on phases **b-c**

$I_a = 0$, $V_b = V_c$ and $I_b = -I_c$

Mathematical analysis may be done by substituting these conditions to the relevant symmetrical component matrix equation. However, the network solution after converting the boundary conditions is more convenient and will be considered here.

$I_a = 0$ and $I_b = -I_c$ when substituted into the matrix equation gives

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a = 0 \\ I_b \\ I_c = -I_b \end{bmatrix}$$

which on simplification gives $I_{a0} = 0$, and $I_{a1} = -I_{a2}$ or $I_{a1} + I_{a2} = 0$

and $V_b = V_c$ on substitution gives

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c = V_b \end{bmatrix}$$

which on simplification gives

$V_{a1} = V_{a2}$

The boundary conditions

$I_{a0} = 0$, $I_{a1} + I_{a2} = 0$, and $V_{a1} = V_{a2}$ indicate a solution where the two networks positive and negative are in parallel and the zero sequence on open circuit, as given in figure 2.21.

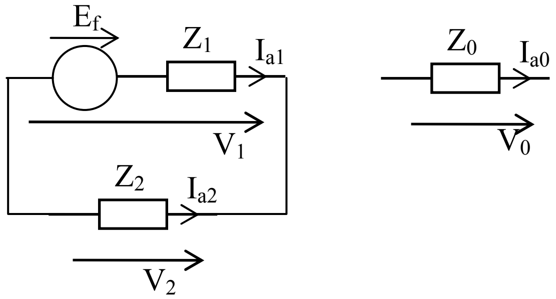


Figure 2.21 – Connection of Sequence Networks for L-L fault

(b) L-L-G fault with earth and no Z_f

At the fault,

$I_a = 0$, $V_b = V_c = 0$

gives

$I_{a0} + I_{a1} + I_{a2} = I_a = 0$

and the condition

$V_{a0} = V_{a1} = V_{a2}$ (can be shown)

These conditions taken together, can be seen to correspond to all three sequence networks connected in parallel.

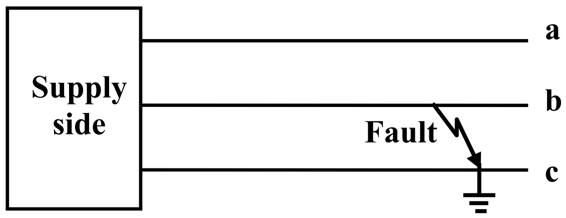


Figure 2.22 – L-L fault on phases **b-c**

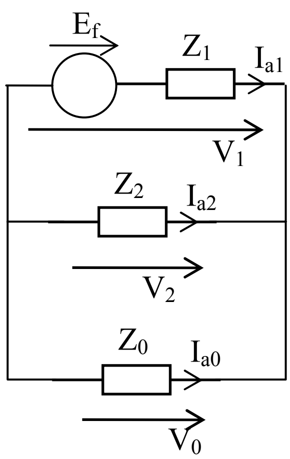


Figure 2.23 – Connection for L-L-G fault

(c) L-L-G fault with earth and Z_f

If Z_f appears in the earth path, it could be included as $3Z_f$, giving $(Z_0 + 3Z_f)$ in the zero sequence path.

(d) L-L fault with Z_f and no earth

If Z_f appears in the fault path, between phases **b** and **c**, it could be included as ½ Z_f in each of **b** and **c**. Inclusion of ½ Z_f in **a** havin zero current would not affect it, so that in effect, ½ Z_f can be added to each of the three phases and hence to each of the 3 sequence networks as (Z_1+½ Z_f), (Z_2+½ Z_f) and (Z_0+½ Z_f). The normal circuit analysis would have yielded the positive and negative sequence networks in parallel with a connecting impedance of Z_f, which is effectively the same.

2.9 Derivation of Sequence Networks

2.9.1 Sequence impedances of network components

The main network components of interest are the transmission lines, transformers, and synchronous machines.

(a) The conductors of a transmission line, being passive and stationary, do not have an inherent direction. Thus they always have the same positive sequence impedance and negative sequence impedance. However, as the zero sequence path also involves the earth wire and or the earth return path, the zero sequence impedance is higher in value.

(b) The transformer too, being passive and stationary, do not have an inherent direction. Thus it always has the same positive sequence impedance, negative sequence impedance and even the zero sequence impedance. However, the zero sequence path across the windings of a transformer depends on the winding connections and even grounding impedance.

(c) The generator (or a synchronous machine), on the other hand, has a inherent direction of rotation, and the sequence considered may either have the same direction (no relative motion) or the opposite direction (relative motion at twice the speed). Thus the rotational emf developed for the positive sequence and the negative sequence would also be different. Thus the generator has different values of positive sequence, negative sequence and zero sequence impedance.

2.9.2 Single-line diagrams for network components

(a) Generator

The generator may, in general, be represented by the star-connected equivalent with possibly a neutral to earth reactance as shown in figure 2.24, together with the three phase diagrams for the positive sequence, negative sequence and zero sequence equivalent circuits. The neutral path is not shown in the positive and negative sequence circuits as the neutral current is always zero for these balanced sequences. Also, by design, the generator generates a balanced voltage supply and hence only the positive sequence will be present in the supply.

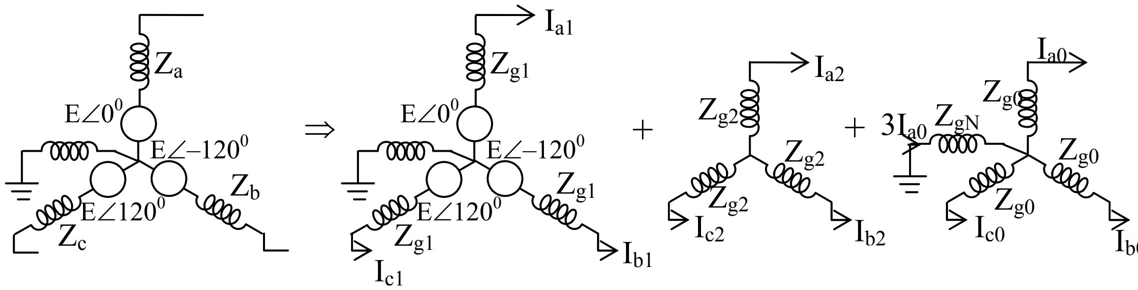


Figure 2.24 – Sequence component networks of generator

Since the 3 component networks are balanced networks, they may be represented by single-line diagrams in fault calculations.

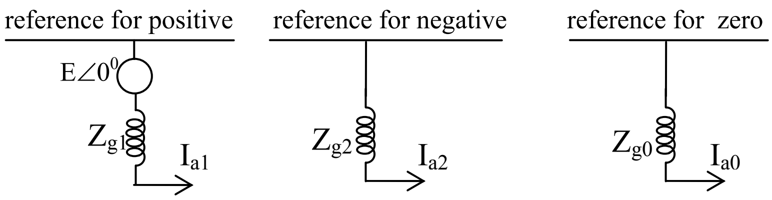


Figure 2.25 –single-line networks for sequences of generator

(b) Transmission lines and cables

The transmission line (or cable) may be represented by a single reactance in the single-line diagram.

Typically, the ratio of the zero sequence impedance to the positive sequence impedance would be of the order of 2 for a single circuit transmission line with earth wire, about 3.5 for a single circuit with no earth wire or for a double circuit line.

For a single core cable, the ratio of the zero sequence impedance to the positive sequence impedance would be around 1 to 1.25.

Transmission lines are assumed to be symmetrical in all three phases. However, this assumption would not be valid for long un-transposed lines (say beyond 500 km) as the mutual coupling between the phases would be unequal, and symmetrical components then cannot be used.

(c) Single windings

Consider each of the simple types of windings for the zero sequence path. These diagrams are shown, along with the zero sequence single line diagram in figure 2.25.

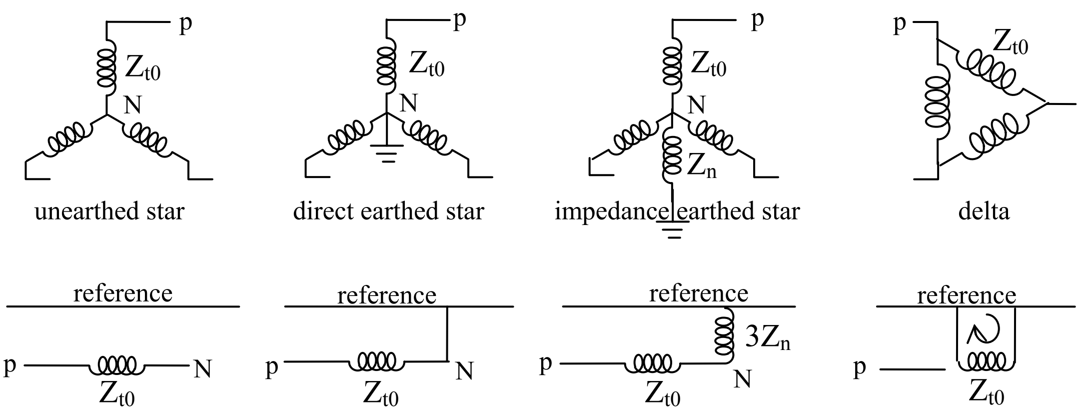


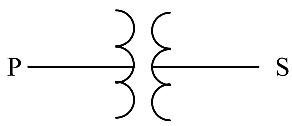
Figure 2.25 –single-line networks for sequences of generator

The unearthed star connection does not provide a path for the zero sequence current to pass across, and hence in the single line diagram, there is no connection to the reference. With an earthed star connection, the winding permits a zero sequence current to flow, and hence is shown with a direct connection to the reference. The earthed star with impedance, is similar except that 3 times the neutral impedance appears in the zero sequence path. The delta connection on the other hand does not permit any zero sequence current in the line conductors but permits a circulating current. This effect is shown by a closed path to the reference.

(d) Transformers

The equivalent circuit of the transformer would be a single reactance in the case of positive sequence and negative sequence for a two-winding transformer, but highly dependant on the winding connection for the zero sequence. The transformer would be a combination of single windings. The magnetising impedance is taken as open circuit for fault studies.

Two-winding transformers



Two winding (primary and secondary), three phase transformers may be categorised into (i) star-star, (ii) earthed star – star, (iii) earthed star – earthed star, (iv) delta – star, (v) delta – earthed star, (vi) delta – delta. There are also zig-zag windings in transformers which has not been dealt with in the following sections.

The figure 2.26 shows the zero-sequence diagrams of the transformers are drawn.

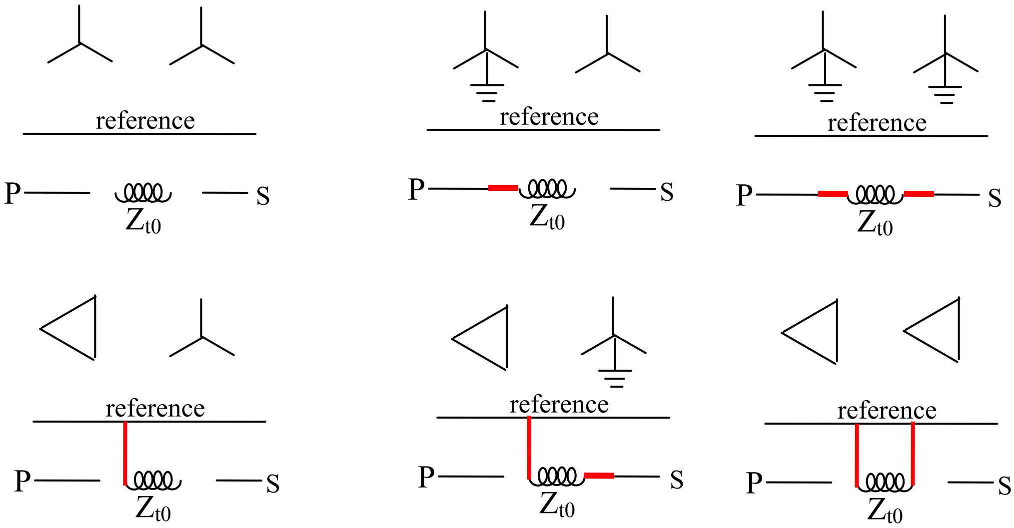
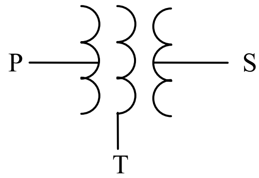


Figure 2.26 – single-line networks for sequences of two-winding transformers

Considering the transformer as a whole, it can be seen that the single-line diagrams indicate the correct flow of the zero-sequence current from primary to secondary.

Three-winding transformers



Three phase, three winding have an additional tertiary winding, and may be represented by a single line diagram corresponding to the ampere-turn balance, or power balance.

$$N_P\,I_P + N_S\,I_S + N_T\,I_T = 0 \quad \text{or} \quad V_P\,I_P + V_S\,I_S + V_T\,I_T = 0$$

which in per unit quantities would yield the common equation

$$I_{P,pu} + I_{S,pu} + I_{T,pu} = 0$$

This may be represented by three reactances connected in T, giving the general single line diagram for fault studies for the 3 winding transformer, as shown in figure 2.27.

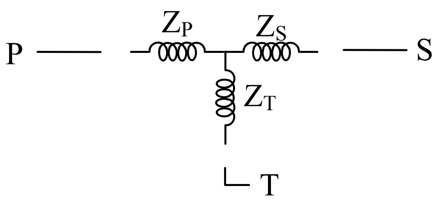


Figure 2.27 three-winding transformer

The positive sequence and negative sequence diagrams would have a direct connection to the T connection of reactances from P, S and T.

The zero sequence network would again be built up from the single winding arrangements described and would yield the single line diagrams given in the following section, and other combinations.

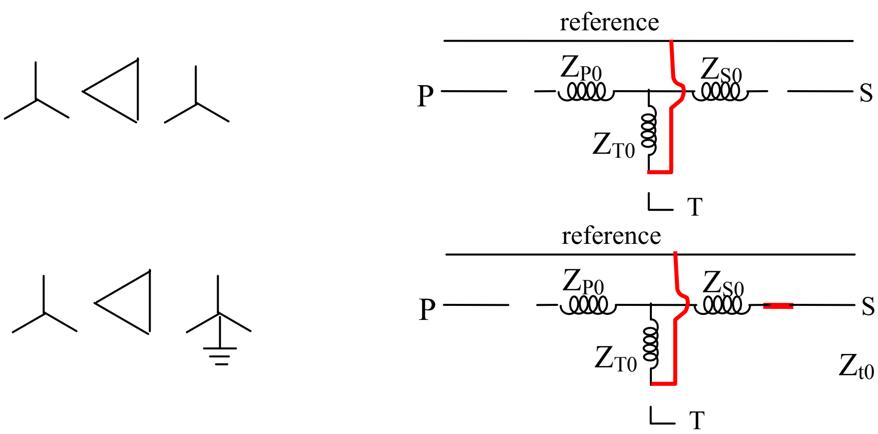


Figure 2.28a – single-line networks for sequences of three-winding transformers

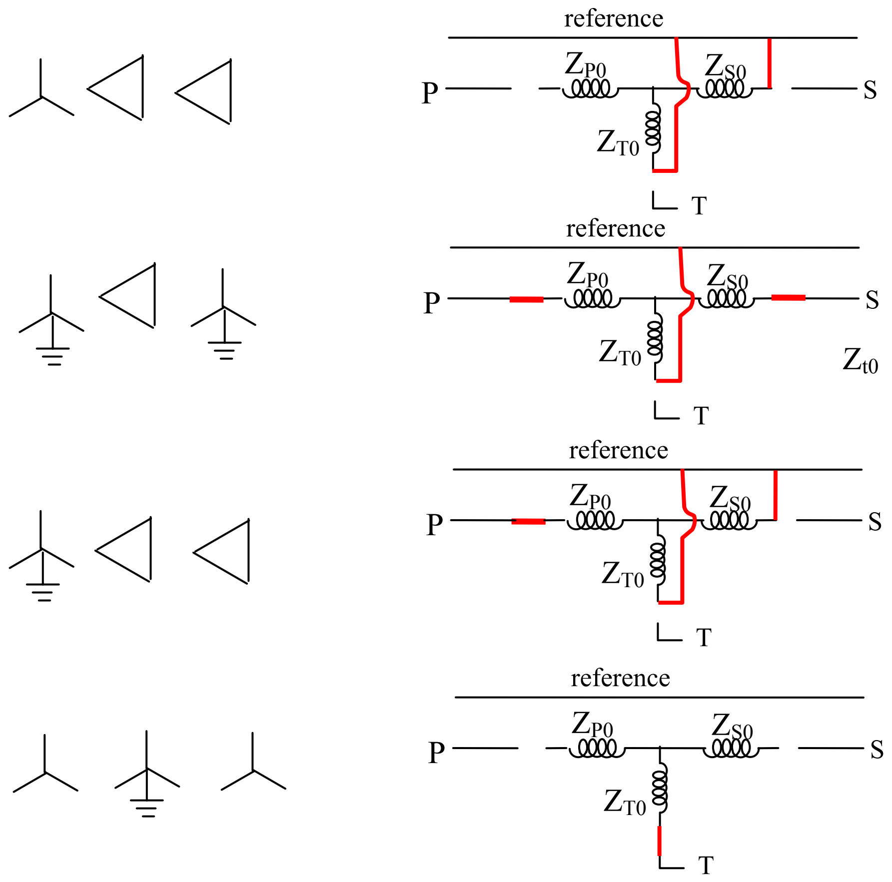


Figure 2.28b – single-line networks for sequences of three-winding transformers

A particular point to keep in mind is that what is generally available from measurements for a 3 winding transformer would be the impedances across a pairs of windings. (ie. Z_{PS}, Z_{PT}, and Z_{ST}), with the third winding on open circuit. Thus we could relate the values to the effective primary, secondary and tertiary impedances (Z_P, Z_S and Z_T) as follows, with reference to figure 2.27.

$$Z_{PS} = Z_P + Z_S, \qquad Z_{PT} = Z_P + Z_T, \qquad Z_{ST} = Z_S + Z_T,$$

The values of Z_P, Z_S and Z_T can then be determined as

$$Z_P = \tfrac{1}{2}\left(Z_{PS} + Z_{PT} - Z_{ST}\right), \qquad Z_S = \tfrac{1}{2}\left(Z_{PS} + Z_{ST} - Z_{PT}\right), \qquad Z_T = \tfrac{1}{2}\left(Z_{PT} + Z_{ST} - Z_{PS}\right)$$

As in the case of the 2 winding transformer, $3Z_n$ is included wherever earthing of a neutral point is done through an impedance Z_n.

<u>In Summary</u>

An unearthed star winding does not permit any zero sequence current to flow so that it could be represented in the single line diagram by a 'break' between the line terminal and the winding.

If the star point is solidly earthed, it could be represented by a solid connection across the break and for an earth connection through an impedance, by 3 times the earthing impedance across the break.

In the case of a delta winding, no current would flow from the line, but a current is possible in the winding depending on the secondary winding connections. This could be represented by a break in connection with the line but with the winding impedance being connected to the reference.

These diagrams are used as building blocks for obtaining the zero sequence networks for the two winding and 3 winding transformers.

Example:

Draw the three sequence networks for the transmission network shown in figure 2.30.

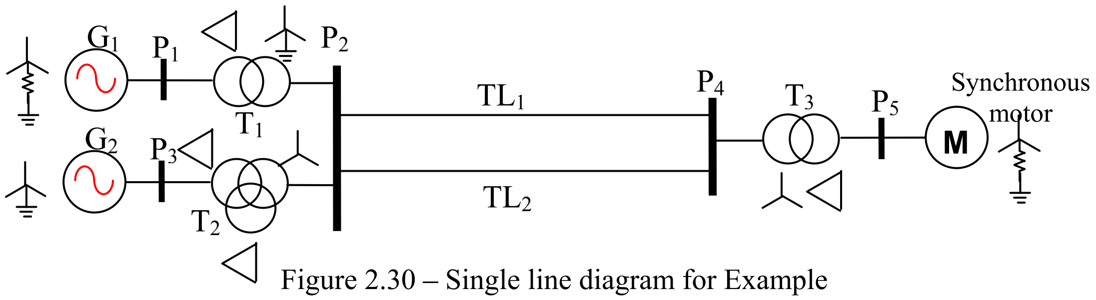


Figure 2.30 – Single line diagram for Example

The Positive sequence network is drawn similar to the single line diagram with the generator and the synchronous motor being replaced by their internal emf and impedance. This is shown in figure 2.31a.

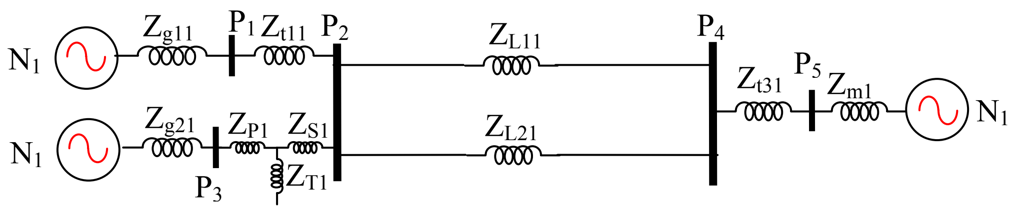


Figure 2.31a –Positive sequence diagram for Example

The negative sequence network is drawn as in figure 2.31b.

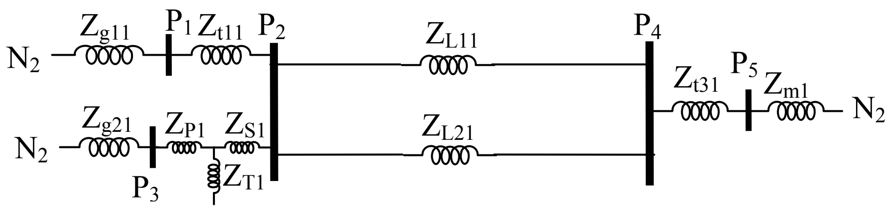


Figure 2.31b –Negative sequence diagram for Example

and the zero sequence network is drawn as in figure 2.31.c.

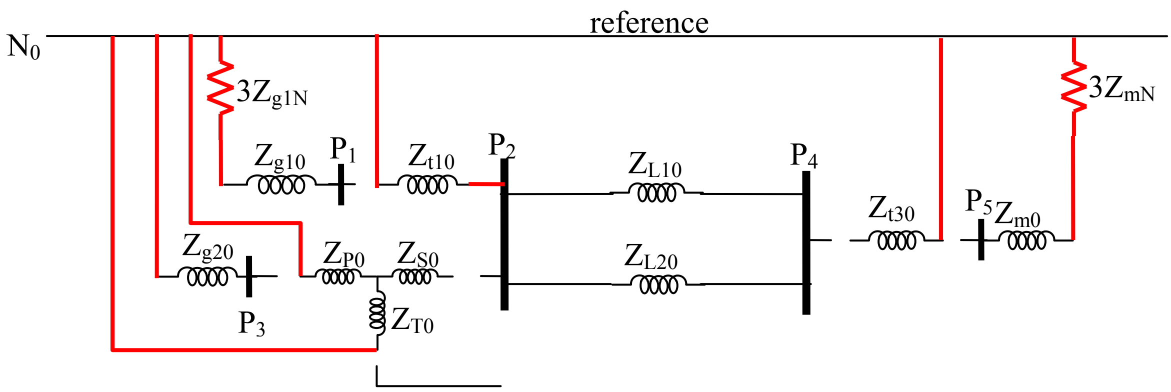


Figure 2.31c –Zero sequence diagram for Example

2.10 Broken conductor faults

In broken conductor (or open conductor) faults, the load currents cannot be neglected, as these are the only currents that are flowing in the network. The load currents prior to the fault are assumed to be balanced.

2.10.1 Single conductor open on phase "a"

In the case of open conductor faults, the voltages are measured across the break, such as a-a′.

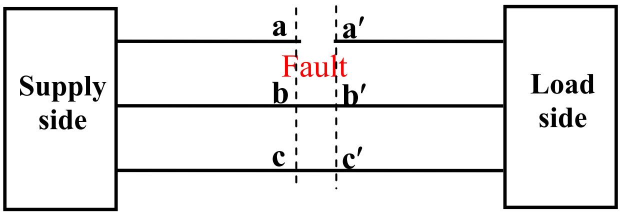


Figure 2. – Open conductor fault on phase **a**

For the single conductor broken on phase "**a**" condition, shown in figure 2. , the boundary conditions are

$I_a = 0, V_b = V_c = 0$

This condition is mathematically identical to the condition in the L-L-G fault in the earlier section, except that the voltages are measured in a different manner. The connection of sequence networks will also be the same except that the points considered for connection are different.

2.10.2 Two conductors open on phases "b" and "c"

For the two conductors broken on phases "**b**" and "**c**" condition, the boundary conditions are

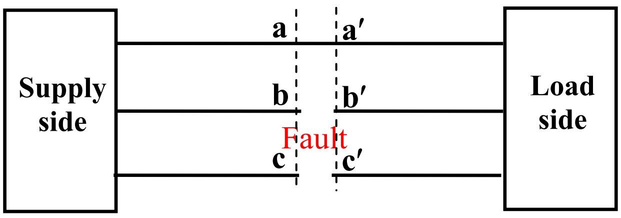


Figure 2. – Open conductor fault on phases **b** & **c**

$V_a = 0, I_b = I_c = 0$

This condition is mathematically identical to the condition in the L-G fault in the earlier section. The connection of sequence networks will also be the same except that the points considered for connection are different.

2.11 Simultaneous faults

Sometimes, more than one type of fault may occur simultaneously. These may all be short circuit faults, such as a single-line-to-ground fault on one phase, and a line-to-line fault between the other two phases. They may also be short-circuit faults coupled with open conductor faults.

Solution methods are similar, if the equations are considered, however they may not have an equivalent circuit to ease analysis. Sometimes, the constraints required cannot be directly translated to connections, but may also need ideal transformers to account for the different conditions.