A survey on pickup and delivery problems

Part I: Transportation between customers and depot

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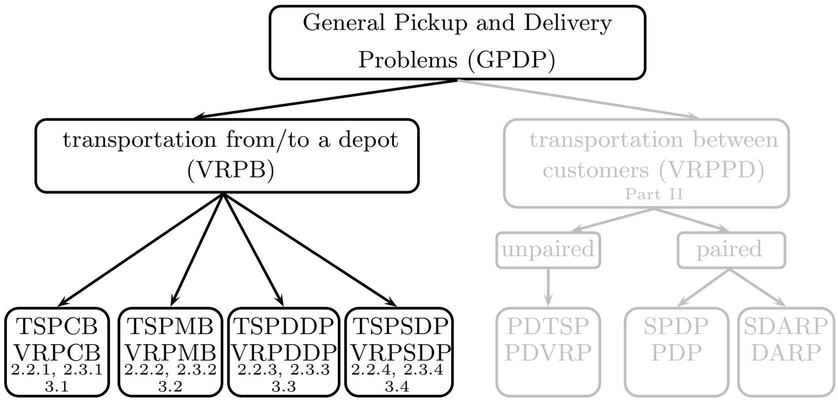
Abstract

This paper is the first part of a comprehensive survey on pickup and delivery problems. Basically, two problem classes can be distinguished. The first class, discussed in this paper, deals with the transportation of goods from the depot to linehaul customers and from backhaul customers to the depot. This class is denoted as Vehicle Routing Problems with Backhauls (VRPB). Four subtypes can be considered, namely the Vehicle Routing Problem with Clustered Backhauls (VRPCB - all linehauls before backhauls), the Vehicle Routing Problem with Mixed linehauls and Backhauls (VRPMB - any sequence of linehauls and backhauls permitted), the Vehicle Routing Problem with Divisible Delivery and Pickup (VRPDDP - customers demanding delivery and pickup service can be visited twice), and the Vehicle Routing Problem with Simultaneous Delivery and Pickup (VRPSDP - customers demanding both services have to be visited exactly once). The second class, dealt with in the second part of this survey, refers to all those problems where goods are transported between pickup and delivery locations. These are the Pickup and Delivery Vehicle Routing Problem (PDVRP - unpaired pickup and delivery points), the classical Pickup and Delivery Problem (PDP - paired pickup and delivery points), and the Dial-A-Ride Problem (DARP - passenger transportation between paired pickup and delivery points and user inconvenience taken into consideration). Single as well as multi vehicle versions of the mathematical problem formulations are given for all four VRPB types, the corresponding exact, heuristic, and metaheuristic solution methods are discussed.

1 Motivation and basic definitions

Over the past decades extensive research has been dedicated to modeling aspects as well as optimization methods in the field of vehicle routing. Especially freight transportation involving both, pickups and deliveries, has received considerable attention. This is mainly due to the need for improved efficiency, as the traffic volume increases much faster than the street network grows (compare Eurostat, 2004, 2006, for data on the European situation). Thus, given the current efficiency, this may eventually lead to a breakdown of the system. However, with rapidly increasing computational power intelligent optimization methods can be developed and used to increase the efficiency in freight transportation and alleviate the above mentioned problem. Moreover, along with the increasing use of geographical information systems, companies seek to improve their transportation networks in order to tap the full potential of possible cost reduction. The rapidly growing body of research has led to a somewhat confusing terminology used to describe the various problem types arising in

Figure 1: Pickup and delivery problems. The numbers indicated refer to the sections covering the respective problems.



this context. Indeed, the same problem types are denoted by various names and different problem classes are referred to by the same denotations. The aim of this two-part survey is to provide a clear classification scheme as well as a comprehensive survey covering all pickup and delivery routing problems and their variants.

In the field of pickup and delivery problems we distinguish between two problem classes. The first class refers to situations where all goods delivered have to be loaded at one or several depots and all goods picked up have to be transported to one or several depots. Problems of this class are usually referred to as Vehicle Routing Problems with Backhauls (VRPB). The second class will be considered in part II of this survey. It comprises all those problems where goods (passengers) are transported between pickup and delivery customers (points) and will be referred to as Vehicle Routing Problems with Pickups and Deliveries (VRPPD).

The two pickup and delivery problem classes as well as their subclasses are depicted in Figure 1. The black part refers to all problems discussed in the remainder of this paper, the gray part to those that are considered in the second part. The numbers specify the sections covering the respective problems. The first two indicators in each of the boxes correspond to the mathematical modeling sections. The third indicators refer to the sections dealing with the various solution methods. A more detailed description of the different subclasses as well as the limitations of this survey are given in the following.

1.1 VRPB subclass definitions

The VRPB can be subdivided into four subclasses. In the first two subclasses, customers are either delivery or pickup customers but cannot be both. In the last two subclasses, each customer requires a delivery and a pickup. The first subclass is characterized by the requirement that the group or cluster of delivery customers has to be served before the first pickup customer can be visited. Delivery customers are also denoted as linehaul customers, pickup customers as backhaul customers. In the literature this problem class is denoted as Vehicle Routing Problem (VRP) with Backhauls (VRPB), a term coined by Goetschalckx and Jacobs-Blecha (1989). This naming is also used, however, for the case where linehaul and backhaul customers can be served in arbitrary order (see Casco et al., 1988). In order to avoid further confusions we will refer to the all linehauls before backhauls version as Vehicle Routing Problem with Clustered Backhauls (VRPCB). The single vehicle case will be denoted as Traveling Salesman Problem (TSP) with Clustered Backhauls (TSPCB).

The second VRPB subclass does not consider a clustering restriction. Mixed visiting sequences are explicitly allowed. Mosheiov (1994) denotes the single vehicle case as Traveling Salesman Problem with pickup and Delivery (TSPD), Anily and Mosheiov (1994) as TSP with Delivery and Backhauls (TSPDB), and Baldacci et al. (2003) as TSP with Deliveries and Collections (TSPDC). Its multi vehicle variant has also been referred to as Pickup and Delivery Problem (PDP) (Mosheiov, 1998), as Mixed Vehicle Routing Problem with Backhauls (MVRPB) (Salhi and Nagy, 1999, Ropke and Pisinger, 2006), and as Vehicle Routing Problem with Backhauls with Mixed load (VRPBM) (e.g. Dethloff, 2002). In the following we will denote this problem class as VRP with Mixed linehauls and Backhauls (VRPMB) in the multi vehicle case, and TSP with Mixed linehauls and Backhauls (TSPMB) in the single vehicle case.

The third VRPB subclass describes situations where customers are associated with both a linehaul and a backhaul quantity but, in contrast to subclass four, it is not required that every customer is only visited once. Rather, two visits, one for delivery and one for pickup are possible. In this case, so called lasso solutions can occur, in which first a few customers are visited for delivery service only, in order to empty the vehicle partially. Then, in the "loop of the lasso", customers are visited where goods are delivered and picked up. At the end, the pickups are performed for the customers initially visited for delivery. Gribkovskaia et al. (2007) denote the single vehicle version as Single Vehicle Routing Problem with Pickups and Deliveries (SVRPPD). Most often however, no clear distinction between this type of problem and subclass four has been made. We will refer to the single vehicle case as TSP with Divisible Delivery and Pickup (TSPDDP) and to the multi vehicle case as VRP with Divisible Delivery and Pickup (VRPDDP) in order to emphasize that a customer can either be visited once for both pickup and delivery or twice, first for delivery and then for pickup. A further extension to this class would be to also allow the splitting of the delivery or the pickup service. This so-called "split delivery" case was studied, e.g., by Archetti et al. (2006b,a) within the context of the classical VRP but could be extended to the VRPDDP.

The fourth VRPB subclass covers situations where every customer is associated with a linehaul as well as a backhaul quantity. It is imposed that every customer can only be visited exactly once. In the literature this problem class has been first referred to as VRP with simultaneous delivery and pickup points by Min (1989). Gendreau et al. (1999) denote the single vehicle variant as TSP with Pickup and Delivery (TSPPD). Angelelli and Mansini (2002) refer to the multi vehicle case as VRP with simultaneous pickup and delivery. In Nagy and Salhi (2005) the same problem is called simultaneous VRP with Pickup and Delivery (simultaneous VRPPD), in Dell'Amico et al. (2006) VRP with Simultaneous Distribution and Collection (VRPSDC). We will denote this problem class as VRP with Simultaneous Delivery and Pickup (VRPSDP), its single vehicle version as TSP with Simultaneous Delivery and Pickup (TSPSDP).

1.2 Limitations

In the field of transportation two problem classes can be distinguished: full-truck-load problems and less-than-truck-load problems. Full-truck-load problems deal with vehicles of unit capacity as well as unit demand or supply at every customer location. In a backhauling situation a vehicle route can only comprise one delivery and one pickup location between two stops at the depot. Consequently, full-truck-load dispatching approaches will not be covered in this article.

1.3 Structure of the survey

This article is organized as follows. First, model formulations for the single as well as the multi vehicle case are presented in order to clearly define the different pickup and delivery subclasses. Then, for each problem class an overview of the different solution methods proposed in the literature is given. This is followed by the description of the benchmark instances used. Finally, a conclusion section provides some hints on the currently best approaches for each problem class and gives directions for future research.

2 Mathematical problem formulation

In the following section a consistent mathematical problem formulation will be presented. First, the notation used throughout the paper is given. Then, two basic problem formulations are introduced and extended to all versions of the VRPB. The aim of this section is to clearly define the different problem types regardless their computational complexity. All variants considered are NP-hard as they generalize the well-known TSP (Garey and Johnson, 1979).

2.1 Notation

n ... number of pickup vertices
$\tilde{n}$ ... number of delivery vertices
P ... set of backhauls or pickup vertices, $P = \{1, \dots, n\}$
D ... set of linehauls or delivery vertices, $D = \{n+1, \dots, n+\tilde{n}\}$
K ... set of vehicles
q_i ... demand/supply at vertex i; pickup vertices are associated with a positive value, delivery vertices with a negative value; at the start depot 0 and the end depot $n+\tilde{n}+1$ the demand/ supply is zero, $q_0 = q_{n+\tilde{n}+1} = 0$
e_i ... earliest time to begin service at vertex i
l_i ... latest time to begin service at vertex i
d_i ... service duration at vertex i
c_{ij}^k... cost to traverse arc or edge (i, j) with vehicle k
t_{ij}^k... travel time from vertex i to vertex j with vehicle k
C^k... capacity of vehicle k
T^k... maximum route duration of vehicle/route k

This notation is valid for the symmetric as well as for the asymmetric case. In the symmetric case $t_{ij}^k = t_{ji}^k$ and $c_{ij}^k = c_{ji}^k$, arc (i, j) and arc (j, i) could thus be modeled by one edge. Consequently, fewer variables would be needed. However, since we focus on problem definition and not on computational efficiency we refrain from presenting these variants here. VRPB are modeled on complete graphs $G = (V, A)$ where V is the set of all vertices $V = \{0, n+\tilde{n}+1\} \cup P \cup D$ and A is the set of all arcs. For practical reasons the arc set can be reduced to $A = \{(i, j) : i, j \in V, i \neq n+\tilde{n}+1, j \neq 0, i \neq j\}$.

During the optimization process some or all of the following decision variables are determined, depending on the problem considered.

$$x_{ij}^k \; \dots = \begin{cases} 1, & \text{if arc } (i,j) \text{ is traversed by vehicle } k \\ 0, & \text{else} \end{cases}$$

Q_i^k ... load of vehicle k when leaving vertex i
B_i^k ... beginning of service of vehicle k at vertex i

Note that vehicle dependent start as well as end vertices can easily be introduced into the model. However, for the sake of simplicity we will not consider this extension in our formulation.

In the single vehicle problem formulation the superscript k can be omitted, resulting in the parameter coefficients t_{ij}, c_{ij}, C, T and the decision variables x_{ij}, Q_i, B_i.

2.2 Single vehicle pickup and delivery problem formulations

The single vehicle formulation for the different pickup and delivery problem classes is based on an open TSP formulation. Open refers to the fact that the resulting route is not a cycle but a path since the depot is denoted by two different indices. The basic open TSP formulation is

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{i:(i,j)\in A} x_{ij} = 1 \qquad \forall j \in V \setminus \{0\}, \tag{2}$$

$$\sum_{j:(i,j)\in A} x_{ij} = 1 \qquad \forall i \in V \setminus \{n+\tilde{n}+1\}, \tag{3}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A. \tag{4}$$

Subtour elimination constraints,

$$\sum_{(i,j)\in A(S,\bar{S})} x_{ij} \geq 1 \qquad \forall S \subseteq V \setminus \{n+\tilde{n}+1\}, S \neq \emptyset, \tag{5}$$

with $A(S,\bar{S}) = \{(i,j) \in A : i \in S, j \notin S\}$, or

$$x_{ij} = 1 \Rightarrow \quad B_j \geq B_i + d_i + t_{ij} \qquad \forall (i,j) \in A. \tag{6}$$

The objective function (1) minimizes total routing cost. Equalities (2) and (3) ensure that each vertex is visited exactly once.

Constraints (5) and (6) present two alternative possibilities to avoid subtours and thus guarantee route-connectivity. The first option is to append inequalities (5). This formulation uses subsets to ensure connectivity. At least one routed arc has to leave every non-empty subset $S \subseteq V \setminus \{n+\tilde{n}+1\}$. The solution to a linear programming relaxation of a formulation involving these constraints gives a good lower bound (cf. Toth and Vigo, 2002a). However, the cardinality of this set of constraints grows exponentially with $|V|$.

For the second option, given in (6), additional time variables B_i, referring to the beginning of service at vertex i, have to be introduced. Given that $t_{ij} > 0$ or $(t_{ij} + d_i) > 0$ for all $(i,j) \in A$, every vertex i is associated with a different value of B_i and subtours are avoided. This option should be chosen whenever other time related constraints, such as time windows, are considered, see Section 2.2.5. The linear programming relaxation of this option provides weaker lower bounds; however, this set of constraints has a polynomial cardinality (cf. Toth and Vigo, 2002a).

A third option (not presented here) is the addition of the traditional Miller-Tucker-Zemlin (MTZ) subtour elimination constraints (cf. Miller et al., 1960), to the above model. These are given by (6) and $e_i \leq B_i \leq l_i$, when defining the artificial sum of travel and service time as $(t_{ij} + d_i) = 1$, and the artificial time window boundaries as $e_i = 1$ and $l_i = |V|$ for all $i \in V$.

Note that precedence constraints as well as additional constraints, such as e.g. a Last-In-First-Out (LIFO) loading rule, maximum ride time and route duration restrictions, can also be guaranteed by means of infeasible path inequalities. However, constraints of this type decrease the readability of the model. Since the aim of this section is a clear definition of the different VRPB subclasses, this option will not be considered in the following.

2.2.1 TSPCB

The TSPCB can be modeled on the basis of a clustered TSP with two clusters, one corresponding to all linehaul customers, and one corresponding to all backhaul customers, with the additional condition that the backhaul cluster has to be visited after the linehaul cluster. This is ensured by adding,

$$x_{ij} = 0 \qquad \forall i \in P, j \in D, \tag{7}$$

to the above formulation (1) – (5). Equalities (7) state that no arc can be routed from the backhaul to the linehaul customer set. Consequently, only one arc from the linehaul to the backhaul customer set can be used. Note that these constraints become redundant if appropriate graph pruning techniques are applied prior to solving the mathematical program (cf. Toth and Vigo, 1997, 2002b).

2.2.2 TSPMB

In the TSPMB the order of linehauls and backhauls is only restricted by the amount of goods the vehicle is able to transport. In addition to the basic model (1) – (4),

$$Q_0 = -\sum_{i \in D} q_i, \tag{8}$$

$$x_{ij} = 1 \;\Rightarrow\; Q_j \geq Q_i + q_j \qquad \forall (i,j) \in A, \tag{9}$$

$$\max\{0, q_i\} \leq Q_i \leq \min\{C, C + q_i\} \qquad \forall i \in V, \tag{10}$$

are required. Equality (8) guarantees that the vehicle starts with a load equal to the total amount to be delivered. Inequalities (9) and (10) ensure that the vehicle's capacity limit is respected at all vertices. In the traditional VRP, constraints similar to (9) and (10) can be used to ensure route-connectivity. To avoid subtours here either (5) or (6) have to be used. If the vehicle's capacity is greater than or equal to the sum of the total linehaul and the total backhaul amount the problem reduces to the simple TSP.

2.2.3 TSPDDP

For the TSPDDP the same formulation as for the TSPMB can be used. The only difference is that all customers are associated with both a linehaul as well as a backhaul quantity and that customers can be visited twice, once for pickup and once for delivery service. This can be achieved by modeling every customer as two vertices, one for the linehaul and one for the backhaul amount. In this sense the TSPDDP can be seen as a special case of the TSPMB since it can be transformed into the latter.

2.2.4 TSPSDP

To model the simultaneous pickup and delivery case the basic formulation (1) - (4), either (5) or (6) to avoid cycles and the capacity constraints (9) and (10) are required. The only difference between the TSPDDP and the TSPSDP is the way customers that are both, linehaul and backhaul customers, are treated. In the former case those customers are modeled as if they were two customers, one linehaul and one backhaul customer. In the latter version every customer can only be visited exactly once ($\tilde{n} = 0$). Let q_i^+ denote the backhaul amount and $q_i^- \geq 0$ the linehaul amount at customer i, an equality ensuring that the vehicle starts its tour with the total amount of goods to be delivered,

$$Q_0 = \sum_{i \in P} q_i^-, \tag{11}$$

has to be appended. Then, in (9) and (10) only the net demand of every customer is considered. It is positive whenever the backhaul amount exceeds the linehaul amount:

$q_i = q_i^+ - q_i^- \ldots$ difference between backhaul and linehaul amount at vertex i.

2.2.5 Time window constraints

A last class of constraints that can be added to all of the above models are Time Windows (TW),

$$e_i \leq B_i \leq l_i \qquad \forall i \in V. \tag{12}$$

In case of TW, constraint set (6) has to be used.

Note that the above formulations are not linear due to constraints (6) and (9). However, these constraints can easily be reformulated in a linear way by utilizing the big M formulation (cf. Cordeau, 2006).

2.3 Multi vehicle pickup and delivery problem formulations

The basic model for multi vehicle pickup and delivery problems is an adapted three index VRP formulation of the one proposed by Cordeau et al. (2002, p. 158f.) for the VRPTW.

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{13}$$

subject to:

$$\sum_{k \in K} \sum_{j:(i,j) \in A} x_{ij}^k = 1 \qquad \forall i \in P \cup D, \tag{14}$$

$$\sum_{j:(0,j) \in A} x_{0j}^k = 1 \qquad \forall k \in K, \tag{15}$$

$$\sum_{i:(i,n+\tilde{n}+1) \in A} x_{i,n+\tilde{n}+1}^k = 1 \qquad \forall k \in K, \tag{16}$$

$$\sum_{i:(i,j) \in A} x_{ij}^k - \sum_{i:(j,i) \in A} x_{ji}^k = 0 \qquad \forall j \in P \cup D, k \in K, \tag{17}$$

$$x_{ij}^k = 1 \Rightarrow \quad B_j^k \geq B_i^k + d_i + t_{ij}^k \qquad \forall (i,j) \in A, k \in K, \tag{18}$$

$$x_{ij}^k = 1 \Rightarrow \quad Q_j^k = Q_i^k + q_j \qquad \forall (i,j) \in A, k \in K, \tag{19}$$

$$\max\{0, q_i\} \leq Q_i^k \leq \min\left\{C^k, C^k + q_i\right\} \qquad \forall i \in V, k \in K, \tag{20}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j) \in A, k \in K. \tag{21}$$

The objective function (13) minimizes routing cost over all vehicles. Equalities (14) guarantee that every vertex is served exactly once. Constraints (15) and (16) ensure that every vehicle starts at the depot and returns to the depot at the end of its route. Note that this does not mean that every vehicle has to be used. A vehicle can use arc $(0, n+\tilde{n}+1)$ which represents no tour. Equalities (17) refer to the usual flow conservation. Time variables are introduced in constraints (18) to eliminate subtours, given that $(t_{ij}^k + d_i) > 0$ for all $(i,j) \in A, k \in K$. As in the single vehicle case inequalities like (5) or the multi vehicle variant of the MTZ constraints can also be used instead of (18) to avoid subtours. However, for simplicity we refrain from writing them down explicitly. Inequalities (19) and (20) ensure that a vehicle's capacity is not exceeded throughout its tour. Whenever the initial load of the vehicle is set to the total amount to be delivered, or all $q_j \geq 0$, or only paired pickups and deliveries occur, constraints (19) can be relaxed to $Q_j \geq Q_i + q_j$.

Non-linear constraints, given in (18) and (19), can be linearized using a big M formulation (cf. Cordeau, 2006).

2.3.1 VRPCB

24 The VRPCB requires the introduction of an additional set of constraints that guarantees that no arc from a backhaul to a linehaul customer can be used. Thus, ensuring that every vehicle first visits all linehaul customers belonging to its route,

$$x_{ij}^k = 0 \qquad \forall i \in P, j \in D, k \in K. \tag{22}$$

As mentioned above, constraints (22) become redundant if the arc set of model (13) – (21) is appropriately defined.

2.3.2 VRPMB

The VRPMB does not require constraints (22). It can be solved by simply applying the basic version of the problem formulation, given in (13) – (21), and equalities (23) ensuring that every vehicle starts with a load equal to the total amount to be delivered,

$$Q_0^k = -\sum_{j \in D} q_j \sum_{i \in V} x_{ij}^k \qquad \forall k \in K. \tag{23}$$

2.3.3 VRPDDP

To model the VRPDDP the same formulation as for the VRPMB can be used. In contrast to the VRPMB, every customer is associated with a linehaul as well as a backhaul quantity and every customer can be visited at most twice, first for delivery and second for pickup service. Thus, modeling every customer that demands both services as two separate customers, one for pickup and one for delivery, suffices to accommodate this problem class.

2.3.4 VRPSDP

The fourth class of VRPB deals with situations of simultaneous delivery and pickup requirements. Each customer is both, a linehaul and a backhaul customer. In contrast to the

VRPDDP, where customers belonging to both sets are modeled as two separate customers, every customer can only be visited exactly once ($\tilde{n} = 0$).

Again, let q_i^+ denote the backhaul amount and $q_i^- \geq 0$ the linehaul amount at customer i, a constraint ensuring that each vehicle starts its tour with the total amount of goods to be delivered,

$$Q_0^k = \sum_{j \in P} q_j^- \sum_{i \in V} x_{ij}^k \qquad \forall k \in K, \tag{24}$$

has to be added to the basic model (13) – (21). Then, in (19) and (20) the net demand of every customer is considered which is positive whenever the backhaul amount exceeds the linehaul amount. Thus, the definition of q_i becomes,

$q_i = q_i^+ - q_i^- \ldots$ difference between backhaul and linehaul amount at vertex i.

Also other alternative formulations for the VRPSDP exist. We refer the interested reader, e.g., to Desaulniers et al. (1998, p.71).

2.3.5 Additional constraints

Finally two more sets of constraints can be added to all of the above problem classes. These correspond to time window and maximum route duration restrictions,

$$e_i \leq B_i^k \leq l_i \qquad \forall i \in V, k \in K, \tag{25}$$

$$B_{n+\tilde{n}+1}^k - B_0^k \leq T^k \qquad \forall k \in K. \tag{26}$$

The latter are motivated by labor regulations, concerning the amount of hours a driver is allowed to drive per day.

3 Solution approaches for VRPB

The development of VRPB was motivated by the fact that significant cost reduction can be achieved by combining linehaul with backhaul tours; as this results in less empty hauls. Beullens (2001) compares and analyzes the various backhauling strategies, i.e. the VRPMB, the VRPCB as well as an on call backhaul strategy. The impacts of reverse logistics are also subject to investigation in Beullens et al. (2004), Dethloff (2001), Fleischmann et al. (1997). Van Breedam (1995) gives an overview of VRP with side constraints, covering the VRP with mixed as well as with clustered backhauls. A recent survey on pickup and delivery problems that was developed in parallel to this survey can be found in Berbeglia et al. (2007).

While we will not describe general solution methods in detail, we provide the interested reader with some references. Information on local search methods can be found, e.g., in Aarts and Lenstra (1997). Neighborhood based methods are discussed, e.g. in Bräysy and Gendreau (2005), Funke et al. (2005), metaheuristics in Hoos and Stützle (2005). Methods used in the context of exact solution algorithms, such as additive bounding, branch and cut, and branch and price are described, e.g., in Barnhart et al. (1998), Desaulniers et al. (2005), Fischetti and Toth (1989), Padberg and Rinaldi (1991).

In the following different solution methods for the various VRPB are discussed. First, the all linehauls before backhauls version is presented, followed by the mixed linehauls and backhauls, the divisible delivery and pickup, and the simultaneous delivery and pickup case. Within each problem class the solution approaches are discussed in subsections devoted to exact, heuristic and metaheuristic methods. The benchmark instances mentioned are described in Section 4. Whenever it was prohibitive with respect to the length of this paper to describe all methods proposed in detail, an overview is given in tabular form. Only contributions we considered more important due to their recency or originality (marked by an asterisk in the respective tables) are described in further detail.

3.1 All linehauls before backhauls (TSPCB, VRPCB)

The TSPCB can be viewed as a special case of the Clustered Traveling Salesman Problem (CTSP), where only two clusters are considered. The CTSP was first formulated in Chisman (1975). Already in Lokin (1978) an application of the CTSP to a backhaul problem is suggested. Thus, all solution algorithms for CTSP, namely those presented in Gendreau et al. (1996b), Jongens and Volgenant (1985), Laporte et al. (1996), Potvin and Guertin (1996, etc.), are also valid solution techniques for the TSPCB with the additional constraint that the set of linehaul customers is visited first. A survey on solution methods can be found in Toth and Vigo (2002b).

3.1.1 Exact approaches

The first exact approach for the VRPCB is introduced in Yano et al. (1987). The authors use a branch and bound algorithm to generate an optimal routing plan with up to four linehaul and four backhaul customers per route, considering opening and closing times at destinations, maximal driving time as well as vehicle capacity concerning weight and volume.

In Gélinas et al. (1995) a branch and bound strategy for the VRPCB with TW that branches on time intervals is presented. Problem instances from Solomon (1987) for the VRP with TW are adapted to the backhaul case. The largest instance solved consists of 100 customers.

In Toth and Vigo (1997) a mathematical problem formulation for the VRPCB is presented. Based on various relaxations lower bounds are computed. The developed branch and bound algorithm generates optimal solutions to most of the benchmark instances provided in Goetschalckx and Jacobs-Blecha (1989), Toth and Vigo (1996, 1999).

Another exact algorithm as well as lower bounding procedures are described in Mingozzi et al. (1999). Mingozzi et al. introduce a new (0-1) integer problem formulation. Variable reduction via pricing allows for solving the reduced problem. Benchmark instances with up to 100 customers (Goetschalckx and Jacobs-Blecha, 1989, Toth and Vigo, 1996) are solved to optimality.

3.1.2 Heuristics

Quite a lot of research has been conducted in the field of heuristic methods for VRPCB. An overview of existing work is given in Table 1 in chronological order, divided into single and multi vehicle approaches. In the first column the respective references are listed. Column two refers to the objective function used, column three states additional constraints or the problem type considered. In column four the proposed algorithm is sketched and in column five either the benchmark instances, as described in Table 8, used to test the respective algorithm or the size of the largest problem instance solved are reported. All methods described in further detail below are marked by an asterisk.

Only little research has been conducted in the area of heuristics for the single vehicle case. Gendreau et al. (1996a) adapt six different construction-improvement heuristics to the TSPCB. Three different versions of the GENeralized Insertion (GENI) construction and Unstringing and Stringing (US) improvement heuristic, originally developed in Gendreau et al. (1992) for the TSP, are investigated and compared to three other construction-improvement

Table 1: Heuristics for the VRPCB

Reference	Obj.	Con./Type	Algorithm	Bench./Size
The single vehicle case				
* Gendreau et al. (1996a)	min. RC	-	3 GENIUS based heur.; Cheapest Insertion (CI) - Unstringing Stringing (US); GENI - Or-opt; CI - Or-opt	GHL96
Gendreau et al. (1997)	min. RC	-	3/2-approximation algorithm, based on Christofides (1975)	-
The multi vehicle case				
Deif and Bodin (1984)	min. RC	RL	savings based construction heur.	up to 300 cust. (10-50% bh.)
* Goetschalckx and Jacobs-Blecha (1989)	min. RC	-	space filling curves construction heur., 2-opt, 3-opt	GJ89
Min et al. (1992)	min. RC	MD	cluster first route second	up to 161 cust.
Derigs and Metz (1992).	min. RC	TW, HV, splitting	approximate solutions based on set partitioning formulation, matching	up to 160 cust.
Goetschalckx and Jacobs-Blecha (1993)	min. RC	-	general assignment based heur.	up to 150 cust. (20-50% bh.)
* Toth and Vigo (1996, 1999)	min. RC	-	cluster via Lagr. relaxation, routing, inter/intra route optimization	GJ89, TV96, TV99
* Thangiah et al. (1996)	min. NV, min. RC	TW	push-forward insertion; improved by λ-interchanges, 2-opt* exchanges	GDDS95, TPS96
* Anily (1996)	min. RC	-	circular regional partitioning with delivery and bh. heur.	-

Bench. = Benchmark, bh. = backhauls, Con. = Constraints, cust. = customers, HV = Heterogeneous Vehicles, heur. = heuristic, Lagr. = Lagrange, MD = Multi Depot, NV = Number of Vehicles, Obj. = Objective(s), RC = Routing Cost, RL = Route Length, TW = Time Windows; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

heuristics, i.e. GENI construction with Or-opt improvement (Or, 1976), Cheapest Insertion (CI) construction with US improvement and CI construction with Or-opt improvement. GENI is based on the idea that whenever a new vertex is inserted into a route it is connected to the vertices closest to it, even if these were not connected previous to this insertion. The US operator refers to removing a vertex from the route and inserting it back in. The GENI idea is used for vertex insertion and its reversal for vertex removal.

The multi vehicle case of the VRPCB has received considerably more attention. One of the earliest heuristic methods was developed by Goetschalckx and Jacobs-Blecha (1989). The clustering as well as the routing part are solved by means of a spacefilling curve heuristic, i.e. generating a continuous mapping of the unit circle onto the unit square preserving closeness across vertices. Results for its combination with 2-opt and 3-opt improvement heuristics (Lin, 1965) are discussed.

Thangiah et al. (1996) present another construction-improvement heuristic. The construction phase consists of a push-forward insertion algorithm improved by λ-interchange (Osman, 1993) and 2-opt*-exchange heuristics (Potvin and Rousseau, 1995).

In Toth and Vigo (1996, 1999) a cluster first route second algorithm is proposed. Toth and Vigo establish a clustering algorithm that uses the solution of the relaxed VRPCB as input. k linehaul as well as k backhaul clusters are obtained from the clustering step. Each of the linehaul clusters is matched with a backhaul cluster. A farthest insertion procedure is applied to the TSPCB instances followed by an intra-route improvement phase. Final refinements on the whole routing plan are achieved by the application of an inter-route improvement phase, using inter-route 1- and 2-exchanges.

Another cluster first route second algorithm is described in Anily (1996). The clustering phase is accomplished by a modified circular partitioning heuristic; followed by the construction of traveling salesman tours through the different clusters (each cluster contains either only linehaul or only backhaul customers). Then, linehaul clusters are assigned to backhaul clusters (Kuhn, 1955). Finally, a route generation phase is initiated determining the optimal connections between depot, linehaul and backhaul clusters.

3.1.3 Metaheuristics

In this section the different metaheuristic approaches for the VRPCB are discussed. Among these a further distinction applies. On the one hand, there are metaheuristic approaches that are population based or related to population based methods, such as genetic algorithms or ant colony optimization, and, on the other hand, there are methods that are based on different local search neighborhoods, such as tabu search, variable neighborhood search, or simulated annealing. Table 2 provides an overview of existing work in this field. For each reference the objectives considered, additional constraints used, a sketch of the proposed algorithm as well as the benchmark instances solved are given.

The first metaheuristic solution approach for the VRPCB is introduced by Potvin et al. (1996). They present a genetic algorithm that is combined with a greedy route construction heuristic. Customers are inserted one by one based on an a priori ordering determined by the genetic algorithm.

A tabu search heuristic for the VRPCB with TW is proposed by Duhamel et al. (1997). The developed method uses 2-opt*, Or-opt, and swap neighborhoods. At each iteration the neighborhood searched is selected randomly, in order to enlarge the size of the neighborhood actually explored and to introduce diversification.

Osman and Wassan (2002) present a reactive tabu search that uses an additional operator controlling diversification and intensification phases on the basis of the detection of repeated solutions. Based on the λ-interchange operator, neighborhoods of different size are explored.

Zhong and Cole (2005) develop a cluster first route second approach. They use a guided local search algorithm to improve the solution obtained from an adapted sweep construction heuristic in the clustering phase. In the routing phase an algorithm, called section planning, is executed. It inserts new routes to achieve feasibility and arranges customers within routes such that travel distances are decreased.

Another tabu search algorithm is introduced in Brandão (2006). Three different procedures are applied to generate the initial solution. The first is based on open VRP solutions. The second two are based on a K-tree relaxation of the VRP. The tabu search sequentially applies three phases using neighborhoods defined by insert and swap moves. An intra-route repair operator is applied if the precedence constraint (linehauls before backhauls) is violated.

Ropke and Pisinger (2006) propose a unified heuristic based on a large neighborhood search algorithm. It can be used to solve three VRPB classes, namely the VRPCB, the VRPMB, and the VRPSDP with and without TW, by transforming them into rich PDP with TW. The term *rich* refers to the additional features that need to be considered, such as pickup and delivery precedence numbers, in order to accommodate the different characteristics of

Table 2: Metaheuristics for the VRPCB

Reference	**Obj.**	**Con.**	**Algorithm**	**Benchmark**
The single vehicle case				
Mladenovic and Hansen (1997)	min. RC	-	variable neighborhood search	GHL96
Ghaziri and Osman (2003)	min. RC	-	neural network, self-organizing feature maps	GHL96
The multi vehicle case				
* Potvin et al. (1996)	min. RC	TW	greedy insertion genetic algorithm	GDDS95
* Duhamel et al. (1997)	min. NV, min. RC	TW	tabu search algorithm	GDDS95, adapted Solomon (1987)
Hasama et al. (1998)	min. NV, min. RC	TW	simulated annealing	TPS96
Crispim and Brandão (2001)	min. RC	-	a) reactive tabu search b) variable neighborhood search	GJ89, TV96
Reimann et al. (2002)	min. NV, min. RC	TW, RL	insertion based ant system	GDDS95
* Osman and Wassan (2002)	min. RC	-	reactive tabu search	GJ89, TV96
* Zhong and Cole (2005)	min. NV, min. RC	TW	adapted sweep heur., improved by guided local search, section planning	GDDS95
Ghaziri and Osman (2006)	min. RC	-	extension of Ghaziri and Osman (2003)	TV96
* Brandão (2006)	min. RC	-	tabu search algorithm	GJ89, TV96
* Ropke and Pisinger (2006)	min. RC, max. NRS	TW	heur. based on large neighborhood search	GJ89, TV96; TW: GDDS95, TPS96
* Ganesh and Narendran (2007)	min. RC	-	construction heur., genetic algorithm (CLOVES)	GJ89, TV96

Con. = Constraints, heur. = heuristic, NRS = Number of Requests Served, NV = Number of Vehicles, Obj. = Objective(s), RC = Routing Cost, RL = Route Length, TW = Time Windows; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

the various problem types. The developed heuristic uses different removal and insertion algorithms. At every iteration a certain number of requests is removed from the routes by means of either random, Shaw (Shaw, 1998), worst request, cluster or history based removal. The free requests are then inserted using a basic or a regret insertion heuristic. The choice of removal and insertion procedure is determined by a learning and monitoring layer. It reports how often a certain removal or insertion procedure contributed to the construction of a new accepted solution.

Ganesh and Narendran (2007) investigate a variation of the VRPCB arising in the context of blood distribution to hospitals. Instead of two sets of customers three sets are considered, consisting of either pure linehaul customers (hospitals), pure backhaul customers (blood camps) or those that are both (usually also hospitals). All customers that require a delivery service have to be visited first. Ganesh and Narendran (2007) develop a cluster-and-search heuristic (CLOVES), that clusters vertices based on spatial proximity, determines their orientation by means of a shrink wrap algorithm and then assigns them to vehicles. A genetic algorithm is used to improve the solution found by the construction heuristic.

3.1.4 Summary

Over the years with increasing computational power a shift from simple heuristic methods towards more sophisticated metaheuristic solution procedures can be observed. Thus, recent state-of-the-art methods in the field of VRPCB predominantly belong to the metaheuristic domain. Comparison can be done by looking at the different results achieved for the same set of benchmark instances. In case of the VRPCB without TW, the benchmark instances most often used are the ones of Goetschalckx and Jacobs-Blecha (1989) (GJ89) and Toth and Vigo (1996) (TV96). The largest instance solved to optimality of the GJ89 data set comprises 90 customers and for the TV96 data set 100 customers (see Mingozzi et al., 1999). The latest new best results for these data sets are reported in Ropke and Pisinger (2006) and Brandão (2006). In case of the VRPCB with TW the data set proposed in Gélinas et al. (1995) (GDDS95) is the prevalent one. The largest instance solved to optimality within this data set consists of 100 customers (cf. Gélinas et al., 1995). Most recent new best results have also been reported in Ropke and Pisinger (2006).

3.2 Mixed linehauls and backhauls (TSPMB, VRPMB)

We now turn to the VRPMB where linehaul and backhaul customers can occur in any order along the route. The first solution methods proposed belong to the field of heuristics (compare, e.g., Casco et al., 1988, Golden et al., 1985). An analysis of different parameter settings in heuristic solution procedures for different VRP, including the VRPMB, is provided in van Breedam (2002). In van Breedam (2001) the performance of simulated annealing, tabu search as well as a descent heuristic applied to variants of the VRP, including the VRPMB, is studied. In the following the various exact, heuristic and metaheuristic solutions methods reported in the literature will be presented.

3.2.1 Exact methods

Exact solution methods for the VRPMB have only been developed for the single vehicle case. These are listed in Table 3. Süral and Bookbinder (2003) propose a new mathematical problem formulation using adaptations of the MTZ subtour elimination constraints for the TSP, ensuring feasibility of the vehicle load. Several tight LP relaxations are then considered. Medium-sized practical problems are solved. Baldacci et al. (2003) discuss valid inequalities and present lower bounds for the single vehicle case. These are embedded in a branch and cut algorithm.

3.2.2 Heuristics

Several heuristic algorithms have been developed to solve the single as well as the multi vehicle variant of the VRPMB. Table 4 gives an overview of the different methods, providing information with respect to the objectives used, additional constraints and the problem type considered, the algorithm proposed and the benchmark instances or the size of the largest instance used to test the respective algorithm.

Table 3: Exact methods for the VRPMB

Reference	Obj.	Con.	Algorithm	Benchmark/Size
The single vehicle case				
Tzoreff et al. (2002)	min. RC	-	linear time algorithm for tree graphs; polynomial time algorithms for cycle or warehouse graph [1]	-
* Süral and Bookbinder (2003)	min. RC	-	optional bh.; tight LP relaxations; exact solutions	up to 30 cust. (20, 30, 40% bh.)
* Baldacci et al. (2003)	min. RC	-	cutting plane approach	GLV99

bh. = backhaul(s), Con. = Constraints, cust. = customers, Obj. = Objective(s), RC = Routing Cost; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.
[1] i.e. two parallel corridors which are connected by at least two aisles.

Two heuristics for the single vehicle case are discussed in Mosheiov (1994). The first heuristic consists in constructing an (optimal) traveling salesman tour through all the customer vertices excluding the depot (by means of an exact algorithm or, if the problem instance is too large, a heuristic). Then the best starting point is chosen that makes the tour feasible with respect to capacity constraints. At this point the depot is inserted. The second heuristic extends the cheapest insertion heuristic for the TSP to the TSPMB. First, an optimal traveling salesman tour through all the linehaul points is constructed. Then, the backhaul customers are inserted along this tour with respect to the cheapest feasible insertion criterion, i.e. the capacity constraint has to be respected.

Another heuristic for the TSPMB is developed in Anily and Mosheiov (1994). It is based on the construction of a Minimal Spanning Tree (MST) through all the vertices (including the depot). Two copies of the MST are then used to construct a feasible pickup and delivery tour. They prove that the proposed heuristic has a worst case bound of 2.

An early construction heuristic for VRPMB is presented in (Casco et al., 1988). They introduce a procedure based on the Clarke-Wright algorithm (with reduced vehicle capacity) to construct initial linehaul tours. A load-based insertion criterion is used to insert the remaining backhaul customers, i.e. a delivery load after a pickup is penalized.

In (Mosheiov, 1998) the multi vehicle case is solved by means of two tour partitioning heuristics. Initially a giant TSP through all the vertices (except the depot) is constructed. The exhaustive iterated tour partitioning algorithm proceeds by identifying the longest segment that can be served by one vehicle from the first vertex. According to the length of this segment the tour is partitioned. This procedure is sequentially started from each vertex along the tour to identify the best partitioning with respect to the total distance traveled.

Salhi and Nagy (1999) propose an extension to the load based insertion procedure of (Golden et al., 1985). Instead of a single backhaul customer, clusters of backhaul customers are considered. For the first time also the multi depot case is tackled. This extension is accommodated by the notion of borderline customers, i.e. customers situated approximately half-way between two depots. The procedure divides the set of linehaul customers into borderline and non-borderline customers. First, the non-borderline customers are assigned to their nearest depot and the corresponding VRP are solved. Then, the borderline customers are inserted one-by-one into the routes.

Table 4: Heuristics for the VRPMB

Reference	Obj.	Con./Type	Algorithm	Benchmark/Size
The single vehicle case				
* Mosheiov (1994)	min. RC	-	2 algorithms. (1) pickup and delivery along optimal tour; (2) cheapest feasible insertion	up to 200 cust. (50% bh.)
* Anily and Mosheiov (1994)	min. RC	-	2MST heuristic. (based on 2 copies of a MST through all vertices)	up to 100 cust.
The multi vehicle case				
Golden et al. (1985)	min. RC	RL	Clarke-Wright[1] algorithm to schedule linehauls, cheapest insertion of bh.	Gal.85
* Casco et al. (1988)	min. RC	RL	Clarke-Wright algorithm to schedule linehauls, load-based insertion of bh.	61 cust. (18% bh.)
Halse (1992)	min. RC	-	cluster first route second (originally for VRPSDP)	Gal.85, GJ89
* Mosheiov (1998)	min. RC	-	2 heuristics. (1) exhaustive iterated tour partitioning; (2) full capacity iterated tour partitioning	up to 100 cust.
* Salhi and Nagy (1999)	min. RC	SD, MD	cluster insertion heuristic	SN99a
Wade and Salhi (2002)	min. RC	position of 1st bh.	VRP solution, insertion algorithm for bh.	GJ89, TV96
Dethloff (2002)	min. RC	-	application of algorithm of Dethloff (2001)	SN99a
* Nagy and Salhi (2005)	min. NV, min. RC	RL, SD, MD	heuristic algorithm. (1) construct weakly feasible VRPB (2) improvement (3) make strongly feasible (4) improvement	SN99a

bh. = backhauls, Con. = Constraints, cust. = customers, MD = Multi Depot, MST = Minimal Spanning Tree, NV = Number of Vehicles, Obj. = Objective(s), RC = Routing Cost, RL = Route Length, SD = Single Depot; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

[1] compare Clarke and Wright (1964)

Nagy and Salhi (2005) elaborate on an integrated construction-improvement heuristic for both, the VRPMB and the VRPSDP. The procedure departs from a weakly feasible VRPMB solution. A solution is weakly feasible if it does not violate the maximum route length, nor does the total load picked up or delivered exceed the vehicle's capacity. Strong feasibility is attained when the load constraint is respected at every arc. First, the weakly feasible solution is improved by local search procedures. Then, the improved solution is made strongly feasible and improved retaining strong feasibility.

3.2.3 Metaheuristics

Metaheuristics have not been applied as extensively to the VRPMB as to other problem types. In Table 5 an overview of the different metaheuristic approaches developed for the

Table 5: Metaheuristics for the VRPMB

Reference	**Obj.**	**Con.**	**Algorithm**	**Benchmark**
The multi vehicle case				
Kontoravdis and Bard (1995)	min. NV, min. RC	TW	greedy randomized adaptive search	KB95
Hasama et al. (1998)	min. NV, min. RC	TW	simulated annealing	TPS96
Wade and Salhi (2004)	min. RC	-	ant system	GJ89
Crispim and Brandão (2005)	min. NV, min. RC	-	hybrid algorithm (tabu search, variable neighborhood descent)	SN99a
* Zhong and Cole (2005)			see Table 2	KB95
Reimann and Ulrich (2006)	min. NV, min. RC	TW	insertion based ant system	GDDS95
* Ropke and Pisinger (2006)			see Table 2	SD: relaxed GJ89; SD, MD: SN99a; TW: KB95

Con. = Constraints, MD = Multi Depot, NV = Number of Vehicles, Obj. = Objective(s), RC = Routing Cost, SD = Single Depot, TW = Time Windows; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

VRPMB is provided. Information is given with respect to the objective function used, additional constraints applied, the type of algorithm developed as well as the benchmark instances used. We refer to the previous sections for a more detailed description of those entries that are marked by an asterisk.

3.2.4 Summary

The largest TSPMB instance solved to optimality is reported in (Baldacci et al., 2003). It is a single vehicle instance of the data set provided in (Gendreau et al., 1999), containing 200 customers. The VRPMB instances most widely used are those proposed in (Salhi and Nagy, 1999). Most recent improved results for these instances are given in (Ropke and Pisinger, 2006), outperforming earlier results by Nagy and Salhi (2005).

3.2.5 Related work

Mosheiov (1995) discusses an extension of the VRPMB, namely the pickup and delivery location problem. All customers either demand transportation to or from the depot as in the classical VRPMB; however, the location of the depot is yet to be determined. Thus, the objective of the problem becomes the determination of the best location of the central depot. Mosheiov (1995) develops two heuristic solution methods; one is based on the heuristic proposed by Mosheiov (1994), the other on a ranking of the vertices according to their probability of demand and their respective distance from all other vertices.

Irnich (2000) considers a variation of the traditional VRPMB. There is a central hub where all requests have to be picked up or delivered to. In addition, a number of depots are considered; every vehicle has to end at the same depot where is started. In contrast to the traditional VRPMB, the loading and unloading point of the goods transported is not the same as the vehicle's depot. Irnich (2000) solves this problem by a three step heuristic algorithm. It consists (1) in the model generation phase, (2) in solving the set covering model, and (3) in a postprocessing phase. Instances with up to 130 pickups and 112 deliveries are solved.

Delivery systems of overnight carriers are investigated in (Hall, 1996). On the morning delivery tours, early pickups might equally occur, resulting in VRPMB problem situations.

3.3 Divisible delivery and pickup (TSPDDP, VRPDDP)

This problem class is a mixture of the previously described VRPMB and the VRPSDP, subject to review in the next section. In contrast to the VRPMB, every customer can be associated with a pickup and a delivery quantity. However, these customers do not have to be visited exactly once. They can be visited twice, once for pickup and once for delivery service. Only little research has been explicitly dedicated to this problem class. However, all the solution methods designed for the VRPMB can be applied to VRPDDP instances if every customer demanding pickup and delivery service is modeled as two separate customers.

Salhi and Nagy (1999) apply their cluster insertion algorithm to VRPSDP instances, resulting in solutions that may violate the one-single-visit-per-customer restriction. Thus, they actually solve a VRPDDP.

Halskau et al. (2001) on the other hand, explicitly relax the VRPSDP to the VRPDDP. The aim is to create so-called lasso solutions, i.e. customers along the spoke are visited twice (first for delivery and second for pickup service). Customers along the loop are only visited once.

Hoff and Løkketangen (2006) also study lasso solutions but restricted to the single vehicle case. They develop a tabu search algorithm on the basis of a 2-opt neighborhood. Solutions to the test instances of Gendreau et al. (1999) are reported.

An in-depth study of different solution shapes for TSPDDP is conducted in (Gribkovskaia et al., 2007); they consider lasso, Hamiltonian, and double-path solutions. The concept of "general solutions" is introduced. Their work is motivated by the fact that additional cost reductions can be realized when relaxing the VRPSDP to the VRPDDP. The proposed methods are classical construction and improvement heuristics and a tabu search algorithm. They are tested on instances containing up to 100 customers. The results show that the best solutions obtained are often non-Hamiltonian and may contain up to two customers that are visited twice.

3.4 Simultaneous delivery and pickup (TSPSDP, VRPSDP)

The difference between VRPDDP and VRPSDP refers to customers demanding pickup and delivery service. In case of the VRPSDP these customers have to be visited exactly once for both services. The VRPMB is a special case of the VRPSDP where every customer only demands a pickup or a delivery but not both. This problem class was first defined by Min (1989).

3.4.1 Exact methods

The only exact algorithm for the VRPSDP with TW is presented in (Angelelli and Mansini, 2002). Based on a set covering formulation of the master problem a branch and price approach is designed. The pricing problem is an elementary shortest path problem with TW and capacity constraints on two types of resource variables: one for the load picked up and the other for the maximum load carried at some point until the current vertex. In order to obtain integer solutions, a branch and bound procedure is employed. Angelelli and Mansini were the first to tackle the extension of the VRPSDP with TW. The largest instance solved to optimality contains 20 customers.

Dell'Amico et al. (2006) also propose a branch and price algorithm to solve the VRPSDP but without TW. They use a hierarchy based on five pricing procedures: four heuristics and one exact method. The exact procedure uses bidirectional labeling algorithms (Salani, 2005). An iterative approach based on state-space relaxation is applied to generate elementary paths. A 40-customer instance is the largest instance that is solved to optimality.

3.4.2 Heuristics

Several different heuristic methods have been applied to the VRPSDP. In Table 6 an overview of the developed procedures is given. References marked by an asterisk, that have not yet been depicted in previous sections, are described in detail below.

A heuristic algorithm for the single vehicle case, departing from a heuristically constructed TSP cycle, is proposed in (Gendreau et al., 1999). Based on the TSP cycle an exact cycle algorithm is run. Its result is improved by introducing shortcuts and local search arc exchanges. This algorithm is compared to a tabu search algorithm, which will be discussed in the section on metaheuristic approaches. Gendreau et al. (1999) also apply the cheapest feasible insertion, the pickup and delivery along the optimal tour (Mosheiov, 1994), and the MST algorithm (Anily and Mosheiov, 1994) to their test instances. These three methods were originally proposed for the TSPMB; however, they are also applicable to the TSPSDP without additional adaptations.

Alshamrani et al. (2007) present a composite algorithm for the stochastic, periodic TSPSDP. It first constructs a feasible traveling salesman tour. In a second step, this tour is improved using the Or-opt operator, considering penalties for backhaul quantities left at stop locations. Demand figures are only known probabilistically.

The multi vehicle case is considered in (Dethloff, 2001). He proposes an extension of the cheapest insertion heuristic. It does not only rely on the measure of travel distance but also on residual capacity and radial surcharge. The developed method is also used to solve the VRPMB (see Dethloff, 2002).

3.4.3 Metaheuristics

Metaheuristic solution methods have also been applied to the VRPSDP, see Table 7 for an overview.

The first metaheuristic for the TSPSDP is a tabu search algorithm using a 2-exchange neighborhood (Gendreau et al., 1999). Two different versions are implemented. The first departs from the heuristic based on a traveling salesman cycle, proposed in the same paper. The second uses four different departure solutions constructed by the cycle, the MST (Anily and Mosheiov, 1994), the pickup and delivery along the optimal tour, and the cheapest feasible insertion heuristic (Mosheiov, 1994).

Tang Montané and Galvão (2006) discuss a tabu search algorithm for the multi vehicle case. They combine the four construction methods used in (Gendreau et al., 1999) with a tour partitioning heuristic and an adapted sweep algorithm to generate an initial solution, resulting in eight different methods. Four different neighborhoods are implemented, a relocation, an interchange, a crossover, and a combined neighborhood. At every iteration the best feasible non-tabu solution of the neighborhood is chosen. The 2-opt operator is used to improve the solution found.

Bianchessi and Righini (2007) compare a tabu search algorithm to different construction and improvement heuristics. A combination of various arc-exchange (cross involving two or

Table 6: Heuristics for the VRPSDP

Reference	Obj.	Con./Type	Algorithm	Benchmark/Size
The single vehicle case				
* Gendreau et al. (1999)	min. RC	-	2 heuristics. (1) construct TSP cycle, apply exact cycle algorithm, use short cuts, local search improvements. (2) see Table 7	GLV99
* Alshamrani et al. (2007)	min. RC, min. PEN	periodic, stochastic	construction-improvement (Or-opt)	-
The multi vehicle case				
Min (1989)	min. RC	-	3 phase cluster first route second algorithm. (1) clustering (2) truck assignment (3) routing	Min89
Halse (1992)	min. RC	-	cluster first route second. (1) cluster by solution of assignment problem (2) routing plus improvement phase	Min89; up to 100 customers
* Dethloff (2001)	min. RC	-	cheapest insertion based algorithm	Min89, SN99b, Det01
* Nagy and Salhi (2005)			see Table 5	SD, MD: SN99b

Con. = Constraints, MD = Multi depot, Obj. = Objective(s), PEN = Penalty for backhaul quantities not picked up, RC = Routing Cost, SD = Single Depot; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.

Table 7: Metaheuristics for the VRPSDP

Reference	Obj.	Con.	Algorithm	Benchmark
The single vehicle case				
* Gendreau et al. (1999)	min. RC	-	tabu search; see also Table 6	GLV99
The multi vehicle case				
Crispim and Brandão (2005)			see Table 5	SN99b
Chen and Wu (2006)	min. RC	-	insertion based procedure, record-to-record heur.[1], tabu list	SN99b
* Tang Montané and Galvão (2006)	min. RC	RL	tabu search algorithm using different neighborhoods	Min89, SN99b, Det01
* Ropke and Pisinger (2006)			see Table 2	Min89, SN99b, Det01
* Bianchessi and Righini (2007)	min. RC	-	different local search heur., tabu search	Det01

Con. = Constraints, heur. = heuristic(s), Obj. = Objective(s), RC = Routing Cost, RL = Route Length; The respective benchmark instances are described in Section 4. Entries marked by an asterisk (*) are described in further detail in the text.
[1] compare (Dueck, 1993)

three routes), node-exchange (relocate, exchange) neighborhoods are tested. The tabu search algorithm uses two tabu lists (one for arc-based and one for node-based neighborhoods).

3.4.4 Summary

Summarizing this section on VRPSDP, again the same trend as for VRPMB can be observed. Early research favored simple heuristic algorithms whereas recent algorithms mostly belong to the field of metaheuristic solution procedures. The largest VRPSDP instance solved to optimality comprises 40 requests (Dell'Amico et al., 2006); however, no standard benchmark instance is considered. Two data sets have been most often referred to. These are those of Salhi and Nagy (1999) (SN99b) and Dethloff (2001) (Det01). The best pooled results for the SN99b instances hold Ropke and Pisinger (2006) and Nagy and Salhi (2005). Tang Montané and Galvão (2006) also report improved solutions, however, not the whole set is considered. Consequently, a direct comparison to the other two is impossible. For the Det01 data set Ropke and Pisinger (2006), Tang Montané and Galvão (2006), and Bianchessi and Righini (2007) obtain new best results of similar quality, but in different pooled formed, only comparing themselves to the results of Dethloff (2001). Whatever method produces the best results, all of them are metaheuristics, clearly indicating that these more sophisticated methods outperform straightforward heuristic procedures.

4 Benchmark instances for VRPB

In order to provide the interested researcher with some information on available benchmark instances, we decided to dedicate this section to a brief description of the data sets used in the VRPB literature. Table 8 provides the following information in chronological order. In the first column the according literature reference is given. Column two states the VRPB type the instances were designed for. Columns three and four give the size of the smallest and the largest instance, in terms of number of customers, and the number of instances provided, respectively. In column five a brief description of the instances can be found. Column six contains the abbreviations used in this survey.

In case of the VRPCB subclass the benchmark data sets most often used in the literature are GJ89 and TV96. The most recent new best results have been presented by Brandão (2006) and Ropke and Pisinger (2006), two metaheuristic approaches, outperforming earlier results by Osman and Wassan (2002).

Regarding the VRPMB and the VRPSDP, the single depot instances, SN99a and SN99b, have been most often solved in the literature. The most recent new best results for these two data sets are presented in (Ropke and Pisinger, 2006), and (Nagy and Salhi, 2005) for the second half of the SN99b data set. Also Tang Montané and Galvão (2006) report good results for parts of the SN99b instances.

5 Conclusion

Usually it is rather difficult to classify or even judge different heuristic and metaheuristic methods. According to Cordeau et al. (2005), a four dimensional evaluation scheme can be applied. The four dimensions are accuracy, speed, simplicity and flexibility. Traditionally, heuristics run faster than metaheuristic methods, whereas metaheuristic methods usually outperform simple heuristics with respect to solution quality. We thus come to the following conclusion. In terms of accuracy as well as flexibility the adaptive large neighborhood search of Ropke and Pisinger (2006) is the currently best method at hand. It is flexible since it can be applied to several versions of the VRPB and it is accurate since it provides new best solutions for different benchmark instances. In terms of simplicity and speed only a heuristic algorithm can be selected. Recent heuristic algorithms involve those of Nagy and Salhi (2005) and Dethloff (2001, 2002).

Table 8: Benchmark instances for VRPB

Literature Ref.	**Type**	**Cust.**	**#**	**Characteristics**	**Abbr.**
Golden et al. (1985)	VRPMB	55	1	based on instance 8 of Christofides and Eilon (1969), 10% bh.	Gal.85
Min (1989)	VRPSDP	22	1	real life instance	Min89
Goetschalckx and Jacobs-Blecha (1989)	VRPCB	25-150	62	25, 50 and 100% of the linehaul customers are bh.	GJ89
Gélinas et al. (1995)	VRPCB	25-100	45	TW, based on the first five problems proposed by Solomon (1987) for the VRPTW, 10, 30 50% bh.	GDDS95
Kontoravdis and Bard (1995)	VRPMB	100	27	based on the sets R2, C2 and RC2 (Solomon, 1987), $C^k = 250$, 50% bh.	KB95
Gendreau et al. (1996a)	TSPCB	100-1000	750	randomly generated points in the square $[0, 100]$, uniformly distributed, 10-50% bh.	GHL96
Toth and Vigo (1996)	VRPCB	21-100	33	based on VRP instances available at the TSPLIB library, 50, 66 and 80% bh.	TV96
Thangiah et al. (1996)	VRPCB	250-500	24	TW, based on the sets R1 and RC1 (Solomon, 1987), 10, 30 and 50% converted into bh.	TPS96
Toth and Vigo (1999)	VRPCB	33-70	24	asymmetric, adapted from the real world instances used by Fischetti et al. (1994)	TV99
Salhi and Nagy (1999)	VRPMB	20-249	SD:42 MD:33	based on SD instances (Christofides et al., 1979) and MD instances (Gillett and Johnson, 1976), adapted by defining 10, 25 and 50% of the customers as bh.	SN99a
	VRPSDP	20-249	SD:28 MD:22	same instances, adapted by splitting every customer's demand into a demand and a supply part	SN99b
Gendreau et al. (1999)	VRPSDP	6-261	1308	partly based on VRP instances from the literature, partly randomly generated	GLV99
Dethloff (2001)	VRPSDP	50	40	randomly generated, 2 geographical scenarios: (1) uniformly distributed customer locations over the interval $[0, 100]$, (2) more urban configuration; the pickup amount has at least half the size of the delivery amount	Det01

= number of instances, Abbr. = Abbreviation used, bh. = backhaul customers, Cust. = number of Customers per instance, MD = Multi Depot, SD = Single Depot

In our opinion, future research will be directed into several directions. First, researchers will attempt to adjust the simplified problems studied to real life problem situations (additional constraints, larger instances, etc.). Second, the incorporation of the effects of dynamism will be subject to future investigations. And last but not least, knowledge about the future, in terms of distributions of future demand and supply and the stochastics involved, will lead to additional research domains.

We hope that this survey will serve as a basis for future research in the area of vehicle routing involving pickups and deliveries.

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