

# Obligations and Permissions\*

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## Abstract

Utterances and statements concerning obligations and permissions are known as *deontic* expressions. They can present something of a challenge when it comes to formulating their meaning and behaviour. The content of these expressions can appear to support entailment relations similar to those of classical propositions, but such behaviour can sometimes lead to counter-intuitive outcomes. Historically, much of the descriptive work in this area has been philosophical in outlook, concentrating on questions of morality and jurisprudence. There have been some additional contributions from computer science, in part due to the need to specify normative behaviour. There are a number of formal proposals that seek to account for obligations and permissions, such as *Standard Deontic Logic*. There has also been discussion of various conundrums and dilemmas that need to be resolved, such as *the Good Samaritan*, *the Knower*, *the Gentle Murderer*, *Contrary to Duty Obligations*, *Ross's Paradox*, *Jørgensen's Dilemma*, *Sartre's Dilemma*, and *Plato's Dilemma*. Even so, there still appears to be no definite consensus about how these kinds of expressions should be analysed, or how all the deontic dilemmas should be resolved. It is possible that obligations themselves, as opposed to their satisfaction criteria, do not directly support a conventional logical analysis. It is also possible that a linguistically informed analysis of obligations and permissions may help to resolve some of the deontic dilemmas, and clarify intuitions about how best to formulate a logic of deontic expressions.

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# 1 Introduction

Work on the formal semantics of natural language has often focused on the propositional interpretation of indicative sentences. Such sentences can be analysed in terms of their truth conditions. This is achieved by translating sentences into propositions expressed in some form of classical logic. The logic may be enriched to make it easier to deal with phenomena such as anaphora and propositional attitudes, like *belief* and *knowledge*, and the modalities of *necessity* and *possibility*.

Of course, many linguistic utterances are not indicative in form. There are also *questions*, *answers*, *commands*, *obligations* and *permissions*, for example. Any comprehensive analysis of language needs to take these different kinds of expressions into account. This article is concerned with aspects of the interpretation of *obligations* and *permissions*. Statements and utterances pertaining to obligations and permissions are called *deontic expressions*. Logical systems that set out to capture the inferential behaviour of such expressions are referred to as *deontic logic*.

## 1.1 Deontic Expressions

Basic examples of deontic expressions include those given in (1). Some more complex cases will be considered later (for example, in §4).

- (1) (a) “*Peter must close the door.*”
- (b) “*Mary is obliged to find a job.*”
- (c) “*You must pay your taxes.*”
- (d) “*You can walk on the grass.*”
- (e) “*You are permitted to delay payment for up to three months.*”
- (f) “*Evan may go to the beach.*”

Deontic expressions do not necessarily include words directly related to “*obligation*” or “*permission*”, but instead can employ a modal verb, such as “*must*”, “*should*”, “*can*”, “*may*”, among others. There can be some ambiguity in the precise nature of the meaning of such modal expressions. For example, “*can*” may be used to express a physical ability (2) rather than permission.

- (2) “*John can run very fast.*”

And “*should*” and “*ought*” may be used to express *epistemic* claims (3), claims that make predictions based on our knowledge and belief.

(3) (a) “*The coin should fall when released.*”

(b) “*John ought to be in a lot of pain,*”

in the context where John has just suffered an injury.

The focus of this article is on determining how deontic expressions may best be interpreted in the context of formal semantics. We are not so concerned with analysing the ways in which obligations and permissions can be formulated in natural language

Some uses of deontic expressions may be intended to describe which obligations and permissions are currently in operation. Other uses of deontic expression may actually bring such obligations and permissions into being, as with a *proclamation*, or *performative* utterance (see Kamp, 1973, 1979; Kempson, 1977; Lemmon, 1962b, for example). We will not consider the analysis of these kinds of speech acts here.

## 1.2 Formal semantics of natural language

In the paradigm of formal semantics, the objective is to put the interpretation of language on a systematic footing. Usually this is achieved using a systematic translation of natural language into a formal language that has a rigorously defined syntax and behaviour. The translation process and formalisation are often targeted at particular aspects of meaning, and usually do not attempt to deal with the full complexity of meaning in all its richness. This can be seen as a form of *abstraction*.

In this paradigm, we have to have a clear understanding of our intuitions concerning how a given expression should be interpreted. We need to consider whether the behaviour of interest should be captured by the translation process, or the formal language, and whether there may be confounding influences from some other aspect of meaning and interpretation.

## 1.3 Truth conditions and inference

Propositional theories of indicative sentences are concerned primarily with truth. This can involve determining the *truth conditions* of indicatives, or the legitimate

patterns of *reasoning* from truth to truth. Such patterns of reasoning do not automatically apply when it comes to the *content* of deontic expressions. Although we may consider the truth of whether there is an obligation in force, or that some permission has been granted, it does not necessarily follow that the content of a deontic expression supports the very same notions of equivalence and entailment that are supported by classical propositions.

There are cases where the content of deontic expressions appears to support logical patterns of entailment and equivalence akin to those of classical propositions. Many would agree that from (4) we should be able to infer (5), even though the content of (4) “*You sit down and eat the cheese*” is not simply being asserted as a proposition.

(4) “*You should sit down and eat the cheese.*”

(5) “*You should sit down.*”

This can lead to questions about the sense in which the content of such expressions can have a logic. This conundrum was raised by Jørgensen (1937–38) in connection with imperatives, and is known as *Jørgensen’s Dilemma*. Arguably such concerns are more salient in the case of imperatives: while we can argue that a complete deontic expression is a proposition, there is no obvious sense in which we describe an imperative as being true, or false.

For practical purposes, it seems this dilemma can be ignored provided we seek to formalise valid patterns of behaviour for deontic expressions in a way that avoids assuming the truth of the *content* of such expression. We might consider the validity of claims of the form given in (6).

(6) (a) A deontic statement follows from another deontic statement, for example from “*You are obliged to mow the lawn and prune the tree*” we may infer “*You are obliged to mow the lawn*”.

(b) Two deontic expressions are incompatible with each other, for example “*You must eat all the food*” is incompatible with “*You must leave some cake for Mary*”.

(c) The satisfaction of an obligation is possible. Consider “*You must ensure that  $1 + 1 = 3$* ”.

(d) The satisfaction of an obligation implies the satisfaction, or absence of satisfaction, of some other obligation: for example satisfying “*You must eat bread and cheese*” also satisfies “*You must eat bread*”.

(e) A particular state of affairs, or action, satisfies an obligation, or is consistent with a permission.

As we shall see in §4 the view that the content of deontic expressions supports classical patterns of entailment is not uncontroversial. It could be said that this assumption leads to a number of dilemmas, such as those described in §4.3.2 in the case of examples like those given in (4) and (6a).

#### 1.4 Obligation to and Obligation that

When it comes to the kind of things that may satisfy an obligation, obvious candidates are actions—where an obligation is *an obligation to do something*—and outcomes, or states—where an obligation is *an obligation that something be the case* (Jackson, 1985).

In some analyses, the intended interpretation is not made explicit. Furthermore, the boundary between the two characterisations may be somewhat artificial. An action in itself could be characterised by the state of affairs that results from its successful completion.

One approach to this question is simply to ignore it; provided we assume that there is some way of expressing the satisfaction conditions of an obligation, we can go on to consider facets of their analysis without making specific commitments as to their basic nature. Such agnosticism may not always be appropriate: consider (7).

(7) “... *surviving being shot is not something that Kennedy ought to have done, though it is something that ought to have been.*” (Jackson, 1985, p179)

#### 1.5 Scope of this article

Here we consider some existing approaches to formalising deontic statements, including syntax, logical rules and semantic interpretation. We follow the practice of many working in the field of assuming plausible representations for natural language examples, rather than attempting a rigorous and highly systematic interpretation.

While allowing us to focus on the logical and formal details, there is admittedly some danger in this approach: there may be other aspects of interpretation that confound the proposed analysis, or which, if properly analysed, cast light on apparently problematic examples. As we shall see, it is not unusual for formal accounts to ignore issues such as quantification, predicates and relations, effectively stripping things down to a propositional logic for obligations and permissions.

## 2 Standard Deontic Logic

The formalisation known as *Standard Deontic Logic* (SDL) represents a classic approach to formalising deontic statements. SDL extends classical propositional logic (see Chapter 5 of Allwood *et al.*, 1977, for example) by adding *modal operators* (Lemmon and Scott, 1977) for “*obligation*” and “*permission*”, together with rules and axioms that govern the behaviour of these new entities (von Wright, 1953). In brief, if  $p$  is a proposition, then  $OB(p)$  means that  $p$  is obligatory, and  $PE(p)$  that  $p$  is permitted.<sup>1</sup>

### 2.1 Axioms and Rules for SDL

SDL is conventionally presented using rules and axioms as given in (8), where “ $a$ ” and “ $b$ ” are propositions, “ $\wedge$ ” represents logical conjunction (*and*), “ $\vee$ ” is disjunction (*or*), “ $\rightarrow$ ” is material implication (*if... then...*) and “ $\neg$ ” is negation (*not*). The expression “ $\vdash a$ ” means that  $a$  is a tautology: the truth of  $a$  follows from the rules and axioms of the logic.<sup>2</sup>

- (8) (a) All the axioms and rules of classical logic.
- (b)  $OB(a \rightarrow b) \rightarrow (OB(a) \rightarrow OB(b))$  (OB-K)
- (c)  $OB(a) \rightarrow \neg OB(\neg a)$  (OB-D)
- (d) If  $\vdash a$  then  $\vdash OB(a)$  (OB-NEC)

Rule (8b) says that obligation distributes across implication; (8c), that if something is obligatory, then you cannot also maintain that it is not obligatory, (8d) that all tautologies of the logic are obligatory.

When taken together, it can be shown that if  $b$  follows from  $a$ , then  $OB(b)$  follows from  $OB(a)$ . That is, if  $a$  is obligatory, then so is  $b$ . This allows the theorems

given in (9) to be derived, among other things.

(9) (a) If  $\vdash a \rightarrow b$  then  $\vdash \text{OB}(a) \rightarrow \text{OB}(b)$  (OB-RM)

(b)  $\text{OB}(a \wedge b) \rightarrow (\text{OB}(a) \wedge \text{OB}(b))$  (OB-M)

(c)  $\text{OB}(a) \rightarrow \text{OB}(a \vee b)$

It is conventional to define *permission* as the ‘dual’ of *obligation*, as in (10).

(10)  $\text{PE}(p) =_{\text{def}} \neg \text{OB}(\neg p)$

SDL is not uncontentious. It does not impose constraints on what kinds of proposition can be obligatory, or permitted. Furthermore, concerns have been expressed that it is too strong, leading to counterintuitive conclusions and dilemmas (see §4, and McNamara, 2006a,b, for example). Many authors have expressed concern about (9a) and also (8c), for their role in creating deontic paradoxes and ruling out deontic conflicts, respectively (see Goble, 1990a,b, 1991, 1993; Hansson, 1988, 1990, 2001; Jackson, 1985; Schotch and Jennings, 1980, for example. Some of these issues surrounding (9a) are also discussed by van der Torre, 1997).

## 2.2 A Possible Worlds Model for SDL

In addition to a system of rules and axioms, it is useful to consider whether there is a *model* that can provide a *consistent* interpretation of the rules. This can help demonstrate that the proposed rules and axioms are formally coherent. Like many modal logics, SDL can be given a possible worlds interpretation. (Kripke, 1959, 1963; von Wright, 1951, 1953). In the standard account, for  $p$  to be an obligation in the current world, it must be true in all accessible *ideal* worlds, where an ideal world is one in which all obligations have been satisfied. For  $p$  to be permitted, it must be true in some such worlds.

## 3 Other Approaches

SDL is not the only approach. Here we sketch a small selection of alternative proposals. Additional proposals are discussed in §5.

### 3.1 The Andersonian-Kangerian reduction

An alternative approach is to say that a proposition is obligatory if some bad thing, a *sanction*, arises whenever that proposition is false, or that this sanction is avoided if the proposition is true. This sanction can be represented by a distinguished proposition  $S$ .

This approach is proposed by Prior (1958) and developed by Anderson (1958). Kanger (1971) gives an equivalent alternative in which the distinguished proposition represents the absence of a sanction. The sanction is fixed, and does not indicate which obligations are unsatisfied.

A variant of this approach, combined with *dynamic deontic logic* (§3.3) is proposed by Wyner (2008), but where there are propositions that indicate both compliance and non-compliance, and the obligation involved.

### 3.2 Input-Output Logic

Another alternative to SDL that is founded on different conceptual assumptions is *input-output logic* (Makinson and van der Torre, 2000, 2001, 2003a,b). Essentially this takes the perspective of an agent that determines what obligations hold on the basis of facts about the state of the world. On this view, a deontic system is an input-output *transducer* from states to obligations. Natural language deontic statements could be interpreted as *specifications* of this transducer.

### 3.3 Dynamic Deontic Logic

The final alternative that we will mention here is where obligations, and their satisfaction, are expressed in terms of actions in the framework of *dynamic logic* (Harel, 1984). We can model actions as things that bring about a state of affairs. Assuming that an action  $\alpha$  can be carried out (i.e. its *preconditions* are satisfied), then we can write  $[\alpha]P$  to indicate that proposition  $P$  is true following the execution of action  $\alpha$ . Propositions and actions can be combined in various ways.

Using this paradigm, we can follow Meyer (1988), and express obligations in terms of actions, so  $OB(\alpha)$  means that action  $\alpha$  is obligatory.<sup>3</sup> It is claimed that this approach can account for problematic examples, such as *Contrary-to-Duty* obligations (§4.3.4), although there may be other problems with this approach (see Anglberger, 2008, for example).



## 4 Common Issues and Difficulties

There are many problematic examples which present difficulties for formalisations such as SDL. These may be due to (i) foundational issues concerning whether obligations must be coherent and fulfillable (§4.1), (ii) the use of representations for natural language which have inappropriate consequences (§4.2), and (iii) inappropriate inferential behaviour in the representation language (§4.3). The precise nature of these categories may be subjective and open to dispute. They are not entirely independent, and some examples may have aspects that fall into more than one category. Nevertheless this categorisation helps to provide some structure to the exposition.

### 4.1 Foundational Issues

Any account of obligations and permissions has to address the possibility of conflict, either between obligations, and permissions, or between obligations and our understanding of how the world is.

#### 4.1.1 Conflicting obligations

Examples (11) and (12) indicate two cases where there may be conflicting obligations (Lemmon, 1962a).<sup>4</sup>

(11) (a) “*You are obliged to have dinner with your friend.*”

(b) “*You are obliged to rush your choking child to hospital.*”

(12) (a) “*You are obliged to return the knife.*”

(b) “*You are obliged to avoid giving a knife to someone who will commit murder.*”

Resolving such conflicts may require some way of prioritising or ordering the obligations. It could be argued this is *moral* rather than a logical question (Bonevac, 1998, p43). Either way, any formal theory of obligations should be able to accommodate conflicts without resulting in inconsistency of the logic itself. This is one motivation for considering alternatives to SDL (§5).

#### 4.1.2 Unfulfillable obligations

We may also question whether all felicitous obligations should be individually fulfillable. Some obligations, such as (13) under a literal interpretation, are clearly unreasonable.

(13) “*You are obliged to fly me to the moon.*”

Others, such as (14), are not possible.

(14) “*Mary must ensure that  $2 + 2$  is 5.*”

The view that such obligations are infelicitous is characterised by “*Kant’s Law*”, namely that “*ought*” implies “*can*”. This view is not universally accepted (see Martin, 2009, for example). Some argue that it is a conversational implicature rather than a logical rule (Sinnott-Armstrong, 1984).

In general we must account for obligations that conflict with each other, or with the world as we understand it to be, and we should be able to do so without giving rise to a formal inconsistency in the semantic theory itself. This problem of conflict is not confined to deontic expressions.

## 4.2 Representational Issues

Some seemingly straightforward representations of deontic expressions can have unfortunate consequences. This issue can arise when there is some propositional content—perhaps a relative clause or some propositional condition—that intuitively should lie outside the scope of any obligation.

Some problems might be avoided if the given representations behaved differently, for example if obligations did not distribute to constituent parts (unlike SDL, §2). Even so, there is still an underlying question about how such examples should be represented.

### 4.2.1 The Good Samaritan

Given one of the obligations in (15), we do not wish to infer that there is an obligation to rob a man in order to then help him, and thus satisfy the obligation to help a robbed man (Prior, 1958).

- 285 (15) (a) “*You are obliged to help a man who has been robbed.*”  
 286 (b) “*You are obliged to help a robbed man.*”

287 Such examples are similar to conditional obligations (16a, 16b). Indeed, some argue  
 288 that all forms of *the Good Samaritan* are essentially disguised conditionals (Castañeda,  
 289 1981; Tomberlin, 1981).

- 290 (16) (a) “*If a man has been robbed, then you should help him.*”  
 291 (b) “*There is an obligation such that if a man is robbed, you help him.*”

292 Generally, (16a) and (16b) can be formulated in SDL-like languages by an expression  
 293 of the form (17a) and (17b), respectively, where  $p$  corresponds to “*a man has been*  
 294 *robbed*” and  $a$  is “*you help him*”.

- 295 (17) (a)  $p \rightarrow \text{OB}(a)$   
 296 (b)  $\text{OB}(p \rightarrow a)$

297 It is not clear whether (17b) really expresses what is desired. From this, SDL would  
 298 allow us to infer (18).

- 299 (18)  $\text{OB}(p) \rightarrow \text{OB}(a)$

300 This seems odd; we are only obliged to help in the event that there is an obligation  
 301 to rob. In the case of (17a), the original conditional obligation (16a) will then be  
 302 judged “true” in the event that a man has not been robbed.

303 Various questions can be raised about these representations, such as: the desir-  
 304 ability of using material implication to represent conditional obligations (cf. §4.3.4  
 305 & §5.2)<sup>5</sup>; whether obligations should distribute to constituent parts (§5.4); and  
 306 whether such inferences should be defeasible (§5.5). The difficulty of analysing  
 307 complex obligations involving conditionals and other constructs arises in other  
 308 contexts.

#### 309 4.2.2 The Knower

310 Most moral people would argue that from (19) we should not infer (20), given a  
 311 deontic interpretation of “*ought*”.

- 312 (19) “*It ought to be the case that A knows his wife is committing adultery.*”

313 (20) It ought to be the case that *A*'s wife is committing adultery.

314 There appears to be a risk of such entailments in some formulations that combine  
315 obligation with knowledge (Åqvist, 1967; Jones and Pörn, 1985). This is sometimes  
316 called *the Paradox of Epistemic Obligation*.

#### 317 4.2.3 The Gentle Murderer

318 Following Forrester (1984), if we were to utter (21) we probably mean that in the  
319 unfortunate event that John murder's his wife, he ought to do so gently. From this  
320 we should not be able to infer (22).

321 (21) "*John ought to murder his wife gently.*"

322 (22) John ought to murder his wife.

323 Other modalities also appear not to distribute into adverbial expressions (Jack-  
324 son, 1985). It is unlikely that anyone would claim 24 follows from (23).

325 (23) "*I want to die a painless death.*"

326 (24) I want to die.

327 Jackson (1985) argues that interpretation must be relative to a set of alternatives  
328 (see §5.1) as in (25).

329 (25) Given *A* ("*you murder your wife*") it ought to be the case that *B* ("*you do so*  
330 *gently*").

#### 331 4.2.4 The Hygienic Cook

332 Some of the previous conundrums might be avoided if distributive inferences did  
333 not apply when faced with contrary obligations (§5.5). But there are examples  
334 where such a proposal does not seem entirely appropriate, as in the morally neutral  
335 example (26) (Fox, 2010).

336 (26) "*You are obliged to use a clean knife.*"

337 This may give rise to an obligation for the knife to be clean, in contrast to the  
338 behaviour (15). Furthermore, it could be claimed there is no obligation to use a knife

(clean or not), only that in the event we use a knife, it ought to be clean, echoing (25).

The different readings available for (26) lend weight to the view that obligations be interpreted with respect to relevant alternatives, as has been proposed for the analysis of the pragmatic notions of *topic* and *focus* (Rooth, 1993).<sup>6</sup> This appears to correspond to the subjunctive thesis, with relativised interpretation, as discussed in §5.1.

### 4.3 Behavioural Issues

Finally in this section, we consider examples that raise questions about the basic behaviour of representations of deontic expressions.

#### 4.3.1 Free choice permission

The issue of *free choice* interpretations arises with deontic expressions involving disjunction (Kamp, 1973; Ross, 1941), such as (27).

(27) “*You may go to the beach or watch television.*”

Under the free-choice interpretation (28) the subject can choose which permission to take advantage of.

(28) You may go to the beach or watch television (or neither), the choice is yours.

Such free-choice permission may be *exclusive* (29); if you go to the beach, you may no longer have permission to watch television.

(29) You may *either* go to the beach or watch television (or do neither), the choice is yours.

Free choice permission appears to indicate a space of possibilities—the “paths” that a subject can take without fear of retribution (Dignum *et al.*, 1996). This interpretation could be captured by considering the consistency (or coherence) of a set of obligations. In particular, (27) would be inconsistent with (30) and perhaps even with (31). This is problematic for SDL, where  $PE(a \vee b)$  follows from  $PE(a)$ .

(30) (a) “*You are obliged not to go to the beach.*”

(b) “*You are obliged not to watch television.*”

367 (31) “*You may go to the beach and you may watch television.*”

#### 368 4.3.2 Conjunctive commitments

369 In some cases it may seem that the force of an obligation should distribute across  
370 conjunction. Given (32) it seems reasonable to conclude both (33a) and (33b).

371 (32) “*You ought to have a shower and go to bed.*”

372 (33) (a) You ought to have a shower.

373 (b) You ought to go to bed.

374 But consider (34).

375 (34) “*You are obliged to jump off the bridge and land on the train.*”

376 It might be unfortunate if a subject were then to infer (35).

377 (35) You are obliged to jump off the bridge.

378 Indeed, (34) is presumably consistent with (36).

379 (36) “*It is not permitted for you to jump off the bridge and not land on the train.*”

380 Distributive behaviour is enforced by SDL (9b), but is not supported by other  
381 accounts (see Goble, 1990a; Jackson, 1985; Jones and Pörn, 1985; Lewis, 1973, for  
382 example).

383 Questions about distributive inferences arise with other logical connectives. The  
384 identification of an appropriate representation for natural language constructs is  
385 dependent on the presence or absence of such inferences (§4.2).

#### 386 4.3.3 Disjunctive Obligations and Ross’s Paradox

387 Theories that import all valid inferences of classical logic into deontic contexts, like  
388 SDL (§2), allow (38) to be inferred from (37).

389 (37) “*You are obliged to post the letter.*”

390 (38) You are obliged to post the letter or burn the letter.

391 One way to satisfy (38) is to satisfy (39).

392 (39) You are obliged to burn the letter.

393 If these notions of validity and satisfaction were conflated, then (39) would fol-  
394 low from (37) (Ross, 1945). This counter-intuitive outcome is referred to as *Ross's*  
395 *Paradox*.<sup>7</sup>

396 The conclusion we can draw from this ‘paradox’ is that the notion of validity  
397 (*which obligations follow from existing obligations*) should not be conflated with the  
398 notion of satisfaction (*which other putative obligations may be satisfied when satisfying*  
399 *a given obligation*).

400 Even so, we may wonder whether it is appropriate to be able to infer the  
401 obligation (38) from the obligation (37), just as we may question whether the  
402 existence of an *utterance* (or *belief*) of the form “ $a \vee b$  is the case” can be inferred  
403 from an utterance (belief) that “ $a$  is the case”. One argument against unrestricted  
404 *disjunction introduction*—exemplified by the move from (37) to (38)—is that there are  
405 free-choice connotations associated with the disjunction which may not be intended.

#### 406 4.3.4 Contrary to Duty Obligations

407 Difficulties can arise in analysing obligations that specify how we should make  
408 amends, or compensate, for a failure to satisfy other obligations. A classic example  
409 (40) is due to Chisholm (1963).<sup>8</sup>

410 (40) (a) “*It ought to be that a certain man go to the assistance of his neighbours.*”

411 (b) “*It ought to be that if he does go, he tell them he is coming.*”

412 (c) “*If he does not go then he ought not to tell them he is coming.*”

413 (d) “*He does not go.*”

414 From these we should be able to conclude (41).

415 (41) He ought not to tell them he is coming.

416 It turns out that regardless of whether conditional expressions (40b) and (40c)  
417 are represented in the form  $OB(a \rightarrow b)$ , or  $a \rightarrow OB(b)$  (cf. §4.2.1), then apparently  
418 faithful representations in SDL are either inconsistent, or one of the obligations  
419 follows from another. Both of these outcomes are counter-intuitive. Some proposed  
420 solutions are mentioned in §3.3 and §5.1.

## 5 Alternative Formalisations

Some of the issues mentioned in §4 have motivated alternative proposals for representing and reasoning with deontic expressions.

In general, given a straightforward interpretation of deontic statements, SDL appears to allow conclusions to be drawn which are counter-intuitive or contradictory. To avoid this, we may reconsider the nature of the interpretation of natural language examples (§5.1 and §5.2), prioritise obligations (§5.3) or weaken the logic in some way (§5.4 and §5.5).

### 5.1 Relativisation of interpretation

It may be possible to avoid inappropriate patterns of entailment for *the Good Samaritan* (§4.2.1), *the Knower* (§4.2.2) and *the Gentle Murderer* (§4.2.3) by evaluating the meaning of deontic expressions with respect to a *context*. The obligations to “*help*” (15), “*know*” (19), or “*murder gently*” (21) arise in those contexts in which it is given that there has been (or will be) robbery, adultery, and murder, respectively.

Such ‘relativised’ interpretations have been proposed by Jackson (1985); Kratzer (1981); Prakken and Sergot (1996), for example.<sup>9</sup> Carmo and Jones (2002) disagree with the need to relativise interpretation of deontic expressions in this way, and Zvolenszky (2002) shows there are problems with the relativised account of Kratzer (1981).

### 5.2 Dyadic modality

The use of *dyadic* modal operators has been proposed to deal with the conditional forms or interpretations of *the Good Samaritan* (§4.2.1) (see van Fraassen, 1972, for example), and the *Contrary-to-Duty* obligations (§4.3.4) (Chisholm, 1963). Instead of “overloading” the notation for material implication, we borrow from the notation for conditional probability, and express the obligation to *b* given that *a* is the case by writing  $OB(b|a)$ , and avoid the use of the material implication as in  $OB(a \rightarrow b)$  or  $a \rightarrow OB(b)$  (van Fraassen, 1972; Hansson, 1969; Spohn, 1975; von Wright, 1957). Appropriate patterns of behaviour have to be attributed to dyadic conditionals (Anderson, 1959; Chellas, 1980; van Fraassen, 1972, 1973; von Wright, 1961, 1962), such as (42).



(42) If  $OB(a|p)$  then  $OB(a|p \wedge q)$

We may model dyadic obligation by saying that  $OB(b|a)$  holds if  $b$  is true in the “best” worlds in which  $a$  is true.<sup>10</sup> The *monadic* expression  $OB(b)$  is then equivalent to  $OB(b|\top)$ , where  $\top$  is a tautology.

### 5.3 Prioritised obligation

Some dilemmas could be avoided if obligations had different *priorities*, where higher-level priorities over-rule lower-level priorities (Åqvist, 1967). This could resolve *conflicting* obligations (§4.1.1), and *Contrary-to-Duty* obligations (§4.3.4). The issue then becomes how to determine priorities, and indeed whether there should be fixed priorities within the logic. As discussed in §5.5, there are alternatives for resolving conflicts that may not need to appeal directly to a fixed priority assignment.

In general we may question whether it is the responsibility of a *linguistic* theory of meaning to account for such behaviour, or whether this falls within the realms of general, non-linguistic reasoning. The problem of conflicts is a general one that also arises with non-deontic utterances.

### 5.4 Weaker logic

Many deontic dilemmas and conflicts could be resolved by weakening the logic in various ways (see Goble, 1999, 2001, 2004; Routley and Plumwood, 1989, for example). For instance, difficulties with some apparently problematic inferences—like *the Good Samaritan* (§4.2.1) and conjunction (§4.3.2)—might be resolved if obligations did not distribute across logical connectives such as conjunction (see Jackson, 1985; Jones and Pörn, 1985, for example)

If a logic has OB-RM (9a) as a theorem, as is the case with SDL, then obligations will distribute across conjunction; and disjunction introduction within deontic contexts will also follow (§4.3.3). Given that such inferences are seen as problematic, some propose weakening the logic so that OB-RM does not follow (Goble, 1990a,b, 1991, 1993; Hansson, 1988, 2001; Jackson, 1985).

Others defend OB-RM on the grounds that it captures the idea of an agent taking moral responsibility for the logical consequences of her commitments Nute and Yu (1997); Schotch and Jennings (1989). But to argue that agents need to understand the

consequences of their obligations does not mean that OB-RM must necessarily be supported (Jackson, 1985).<sup>11</sup>

## 5.5 Weaker inference

An alternative to weaker rules and axioms is to adopt a weaker notion of inference. With this approach, we can still allow obligations to distribute, for example, but take any problematic entailments to be *defeasible* (Bonevac, 1998; Makinson and van der Torre, 2003b; Nute, 1997). For *the Good Samaritan* (15) a prior obligation not to rob overrides the default inference to rob, and for *the Gentle Murderer* (21), a prior obligation not to murder overrides the default inference to murder.<sup>12</sup> This may be appropriate if distributive inferences are thought appropriate in “normal” circumstances, and the main issue with *the Good Samaritan*, and similar examples, is viewed as residing in a conflict between primary obligations and derived obligations. Arguably this is related to proposals to stratify deontic statements into different levels of priority (§5.3). There may be both logical and moral issues to resolve in determining the relative priority of obligations.

In the case of deontic conflicts, it is also possible to consider *paraconsistency*, where reasoning is performed with respect to maximal consistent collections of obligations (da Costa, 1988; da Costa and Carnielli, 1986; Loparic and Puga, 1986).

## 5.6 Logic-free obligations

An alternative approach sketched by Fox (2009) is to allow entailments between satisfaction conditions but not between distinct obligations. If an obligation is unsatisfied, then a *transgression* has occurred. For those obligations that have been satisfied, we may wish to record the subject’s *compliance*. Transgressions, and compliance, can be specific to the obligation in question (cf. Wyner, 2008), unlike the notion of a sanction (Anderson, 1958; Prior, 1958).

This allows for partial fulfilment, including partial fulfilment of contradictory and unfulfillable systems of obligations, as well as *Contrary-to-Duty* obligations. In such cases, if an agent fails to comply with any compensating obligations then there are simply more unfulfilled obligations (or transgressions). The satisfaction of obligations, conflicting or not, and the transgressions to avoid, can then be

511 considered a question of moral judgement, rather than logical inference (cf. Bonevac,  
512 1998, p43).

513 A notion of *coherence* (cf. Makinson and van der Torre, 2003b) can be used  
514 in place of logical entailment. Instead of  $OB(a)$  following from  $OB(a \wedge b)$ , we  
515 can say that a *coherent* system of obligations will not combine  $OB(a \wedge b)$  with  
516  $OB(\neg a)$ , or indeed with any obligation whose satisfaction is at odds with the  
517 satisfaction of  $OB(a \wedge b)$ . If needed, *equivalence* and *subsumption* relationships  
518 between deontic systems can be formulated in terms of satisfaction conditions and  
519 coherence properties.

520 Coherence can be used to analyse permission. If  $a$  is permitted,  $PE(a)$ , then it  
521 would be incoherent for there to be obligations whose satisfaction is at odds with  
522  $a$ . For free-choice permission (§4.3.1), if “ $a$  or  $b$ ” is permitted,  $PE(a \vee b)$ , then it  
523 would be incoherent to have obligations that are at odds with  $a$ , or with  $b$ .<sup>13</sup>

524 By itself, this approach does not resolve how to identify the specific obligations  
525 imposed by *the Good Samaritan* (15), *the Gentle Murderer* (21) and *the Clean Knife*  
526 (26) examples. They may merit more analysis of the linguistic data, and the use  
527 contextualised interpretations (§5.1).

## 528 6 Further Reading

529 McNamara (2006a,b) describes SDL and other approaches, together with discussion  
530 of various paradoxes and conundrums and proposals for their resolution. McConnell  
531 (2002) discusses some moral dilemmas that any treatment of obligations and per-  
532 missions should consider. Hansen *et al.* (2007) presents key philosophical questions  
533 about deontic logic from the perspective of input/output logic. Other survey pa-  
534 pers include Åqvist (2002); Carmo and Jones (2002); Føllesdal and Hilpinen (1971);  
535 Hilpinen (1981a); Meyer and Wieringa (1993a).

## Notes

1. The precise syntax for the modal operators may vary.
2. Here the names of the various rules and axioms (OB-K, OB-D, OB-NEC, etc.) are derived from the conventional names for rules and axioms of these forms in modal logic (K, D, NEC, etc.). The provenance of these names is varied (K for *Kripke*, D for *deontic*, NEC for *necessitation*, etc.). Essentially they are given here as they are part of the vernacular of modern logical theories.
3. This kind of approach has been considered for the analysis of imperatives (Lascarides and Asher, 2004; Segerberg, 1990). We will not attempt to consider the relationship between deontic statements and imperatives in this article.
4. Examples of the form (11) and (12) are sometimes referred to, respectively, as *Satre's Dilemma*, from Sartre (1957/1946), and *Plato's Dilemma*, from Plato's *Republic*, Book I "...if a man borrows weapons from a sane friend, and if he goes mad and asks for them back, the friend should not return them, and would not be just if he did. Nor should anyone be willing to tell the whole truth to someone who is in such a state." (*Republic*, I, 331c). This example is used to counter the argument that "Justice is speaking the truth and repaying debts." (*Republic*, I, 331b–c).
5. If  $p$  is false, then material implication allows us to derive  $p \rightarrow \text{OB}(a)$  for any  $a$ . This may appear a counter-intuitive interpretation of conditionality. Some propose a distinct notation for conditional obligation, such as  $\text{OB}(a|p)$ , as sketched in §5.2. Others have used alternatives to material implication, such as *strong* implication (Prior, 1958, for example).
6. This approach has been considered explicitly by Wyner (2008, Section 2.7, pp69–74), in the analysis of *the Gentle Murderer*.
7. Ross's Paradox was originally described in the context of imperatives.
8. Tomberlin (1981) gives a detailed account of the problem of *Contrary-to-Duty* obligations, and some possible solutions.
9. The use of relativised interpretations for deontic expressions appears similar to proposals to use contextually relevant "comparison sets" in the pragmatic interpretation of discourse focus (Rooth, 1993).

- 566 10. Other model-theoretic interpretations of dyadic obligation are possible (Hansen,  
567 2008; Hansen *et al.*, 2007).
- 568 11. There are proposals for weaker logics that capture salient inferences between obliga-  
569 tions, such as the “weakened” OB-RM of Goble (2004), where if  $A$  implies  $B$ , then  
570  $OB(A)$  implies  $OB(B)$  provided  $A$  is permitted.
- 571 12. The idea of withdrawing conflicting conclusions does not seem to help determine  
572 the precise nature of the obligation imposed by *the Clean Knife* example (26), where  
573 there are no prior prohibitions on cleaning, or using, a knife.
- 574 13. In the case of exclusive free choice, it would be incoherent to combine  $PE(a \vee b)$   
575 with  $PE(a)$  and  $PE(b)$ .

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