

# Matched Bandstop Resonator with Tunable K-Inverter

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**Abstract** — This paper demonstrates a matched bandstop resonator with a tunable K inverter. Two separated  $\lambda/4$  resonators are coupled through a tunable impedance inverter which consists of a varactor loaded microstrip T-shunt stub. In contrast to conventional K-inverter realizations, which possess narrowband properties, this paper demonstrates a physical realization of a novel tunable K-inverter using a varactor diode. Theoretical analysis together with the experimental results is presented in this paper. The optimum coupling value can be easily achieved by independently tuning the varactor where an enhanced notch response with 64dB attenuation is measured and shown in this paper.

**Index Terms** — Notch filter, immittance inverter.

## I. INTRODUCTION

New developments in the design of band reject filters are essential to meet the ever increasing demands on suppression of unwanted signals and miniaturization of microwave communications systems. A compact design can be achieved through the implementation of planar microstrip technology. Conventional electronically tunable bandstop filters suffer performance degradation due to the finite unloaded Q of the resonators and also the loss associated with the tuning elements [1]. Recently a new filter topology using lossy resonators has been introduced where the topology can be used to partially compensate for the loss [2]-[4]. A frequency-agile bandstop filter based on this topology was presented [4], but such filters as well as conventional tunable bandstop filters encounter performance degradation in terms of tuning bandwidth and stopband bandwidth due to the frequency-dependant losses and couplings. In this paper, theory describing a microstrip T-shunt stub impedance inverter is presented that validates the suitability of the inverter for narrowband filter design. Such an impedance inverter is made tunable to provide the optimum inter-resonator coupling using a varactor diode. The tunability of the inverter properties can be used to compensate for the frequency dependant couplings in the conventional tunable bandstop filters to retain the optimum stopband bandwidth or loaded Q while tuning the center frequencies, thus maximizing the available tuning bandwidth. Reconfigurable filter with relative bandwidth tuning was presented [5]-[6], where the bandwidth tuning is accomplished by changing the coupling among filter

resonators. The filters were realized with some additional transmission line segments with the attached varactor diodes or switch elements, in order to achieve the inter-resonator coupling tuning. The work reported here offers an alternative realization of a compact tunable inter-resonator coupling circuit, which is also useful for relative filter bandwidth tuning. Work building upon the matched-notch concept [2] is presented, including the compact design prototype of the matched notch filter with tunable K-inverter.

## II. THEORY

### A. Microstrip T-Shunt Stub Impedance Inverter

Consider the microstrip T-junction shown in Fig. 1 and its equivalent circuit which consists of a junction capacitance and an ideal transformer. The junction capacitor arises from the residual charge at the junction discontinuity and hence the susceptance B is positive.

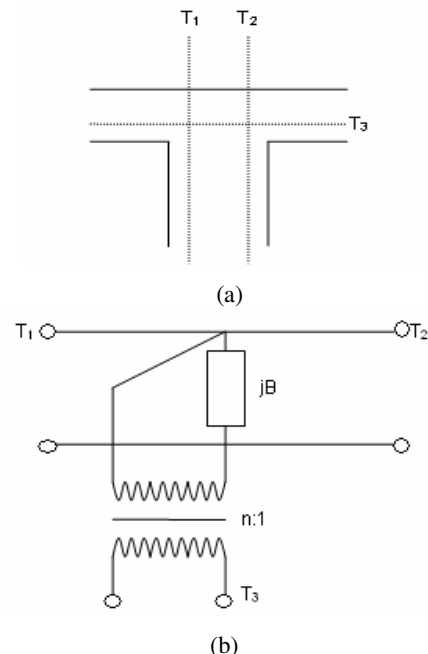


Fig. 1. Microstrip T-junction (a) structure and (b) equivalent circuit.

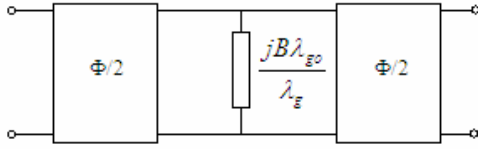


Fig. 2. Embedded shunt capacitive element in a uniform microstrip transmission line section of electrical length  $\Phi$ .

The transfer matrix of the shunt capacitor is given by,

$$[T] = \begin{bmatrix} 1 & 0 \\ \frac{jB\lambda_{go}}{\lambda_g} & 1 \end{bmatrix} \quad (1)$$

where  $\lambda_{go}/\lambda_g$  corresponds to the normalized frequency. By symmetrically embedding the capacitive element in a uniform section of transmission line of electrical length  $\Phi$  as shown in Fig. 2, the transfer matrix is given by:

$$[T] = \begin{bmatrix} \cos\left(\frac{\Phi}{2}\right) & j\sin\left(\frac{\Phi}{2}\right) \\ j\sin\left(\frac{\Phi}{2}\right) & \cos\left(\frac{\Phi}{2}\right) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{jB\lambda_{go}}{\lambda_g} & 1 \end{bmatrix} \begin{bmatrix} \cos\left(\frac{\Phi}{2}\right) & j\sin\left(\frac{\Phi}{2}\right) \\ j\sin\left(\frac{\Phi}{2}\right) & \cos\left(\frac{\Phi}{2}\right) \end{bmatrix} \quad (2)$$

hence,

$$[T] = \begin{bmatrix} \cos(\Phi) - \frac{B\lambda_{go}}{2\lambda_g} \sin(\Phi) & j \left[ \sin(\Phi) - \frac{B\lambda_{go}}{\lambda_g} \sin^2\left(\frac{\Phi}{2}\right) \right] \\ j \left[ \sin(\Phi) + \frac{B\lambda_{go}}{\lambda_g} \cos^2\left(\frac{\Phi}{2}\right) \right] & \cos(\Phi) - \frac{B\lambda_{go}}{2\lambda_g} \sin(\Phi) \end{bmatrix} \quad (3)$$

Let the electrical length

$$\Phi = \frac{\Phi_o \lambda_{go}}{\lambda_g} \quad (4)$$

and an inverter with the transfer matrix

$$[T] = \begin{bmatrix} 0 & \frac{j\lambda_g}{K\lambda_{go}} \\ \frac{jK\lambda_{go}}{\lambda_g} & 0 \end{bmatrix} \quad (5)$$

where K is the characteristic admittance when  $\lambda_g = \lambda_{go}$ .

By equating (3) and (5), the T-junction becomes an inverter where:

$$\cos(\Phi_o) - \frac{B}{2} \sin(\Phi_o) = 0 \quad (6)$$

and therefore:

$$\Phi_o = \tan^{-1}\left(\frac{2}{B}\right) \quad (7)$$

As the susceptance B is positive,  $\Phi_o$  must be a positive length. Now consider the A parameter ( $T_{11}$ ) in (3),

$$A = \cos\left(\frac{\Phi_o \lambda_{go}}{\lambda_g}\right) - \frac{\lambda_{go}}{\tan(\Phi_o) \lambda_g} \sin\left(\frac{\Phi_o \lambda_{go}}{\lambda_g}\right) \quad (8)$$

and differentiating A parameter with respect to  $\lambda_g$  gives:

$$\frac{dA}{d\lambda_g} = \frac{\Phi_o \lambda_{go}}{\lambda_g^2} \sin\left(\frac{\Phi_o \lambda_{go}}{\lambda_g}\right) + \frac{\lambda_{go}}{\tan(\Phi_o) \lambda_g^2} \sin\left(\frac{\Phi_o \lambda_{go}}{\lambda_g}\right) + \frac{\lambda_{go}^2 \Phi_o}{\tan(\Phi_o) \lambda_g^3} \cos\left(\frac{\Phi_o \lambda_{go}}{\lambda_g}\right) \quad (9)$$

With  $\Phi_o$  relatively small (9) is reduced to:

$$\frac{dA}{d\lambda_g} \approx \frac{\Phi_o^2 \lambda_{go}^2}{\lambda_g^3} + \frac{2\lambda_{go}^2}{\lambda_g^3} \quad (10)$$

Therefore it is shown that the inverter approximation is only valid for narrow bandwidths.

#### B. Practical Realization of coupled $\lambda/4$ Resonators with T-Shunt Stub Impedance Inverter

Consider the microstrip structure shown in Fig. 3 and its equivalent circuit. Let  $\theta_1 = \lambda/4$  and  $\theta_2 < \lambda/4$  which gives the input admittance of the stub,

$$Y_{in} = jY_2 \tan \theta_2 \quad (11)$$

where  $\tan(\theta_2)$  is positive. Hence  $B_T$  is positive where  $B_T$  refers to the total susceptance of the shunt capacitor plus the transformed  $Y_{in}$  due to the presence of an ideal transformer. The equivalent circuit of the coupled  $\lambda/4$  resonators in Fig. 3 may be shown as Fig.4. The resonators at both sides have an electrical length of  $\lambda/4 - \Phi_o/2$ .

#### C. Novel Tunable K-Inverter Realization

It is already shown in previous section that the microstrip T-shunt stub is useful for a narrowband impedance inverter approximation. In contrast with the conventional inverters, a novel tunable K-inverter could be realized using the T-shunt stub, leading to practical broadband inverter realization.

Now let C parameter ( $T_{21}$ ) in (3) and (5) to be equal,

$$K \frac{\lambda_{go}}{\lambda_g} = \sin\left(\frac{\Phi_o \lambda_{go}}{\lambda_g}\right) + \frac{B\lambda_{go}}{\lambda_g} \cos^2\left(\frac{\Phi_o \lambda_{go}}{2\lambda_g}\right) \quad (12)$$

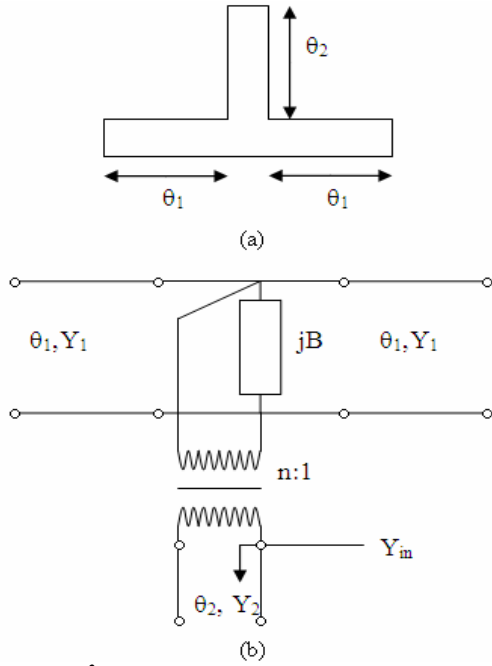


Fig. 3. Coupled  $\lambda/4$  resonators with T-shunt stub impedance inverter (a) layout and (b) equivalent circuit.

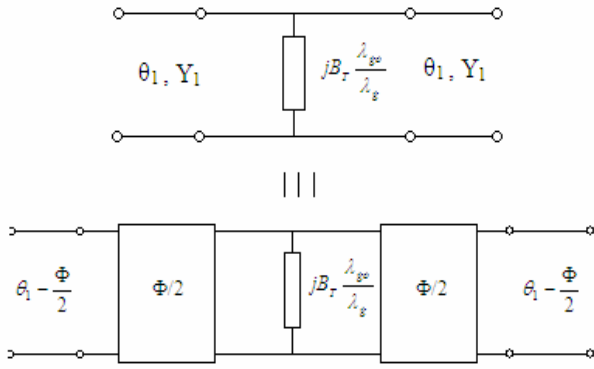


Fig. 4. Transformed equivalent circuit of coupled  $\lambda/4$  resonators with T-shunt stub impedance inverter.

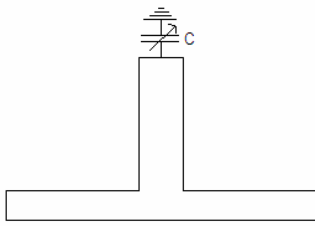


Fig. 5. Coupled  $\lambda/4$  resonators with tunable impedance inverter.

For relatively small  $\Phi_o$ ,

$$K \approx B_T \quad (13)$$

where  $B_T$  is the total susceptance. Therefore the tuning of K-inverter can be achieved by changing the effective length of

the shunt stub. This can be done by incorporating a tunable capacitor at the end of the stub.

### III. PERFECT NOTCH CONCEPT USING LOSSY RESONATORS

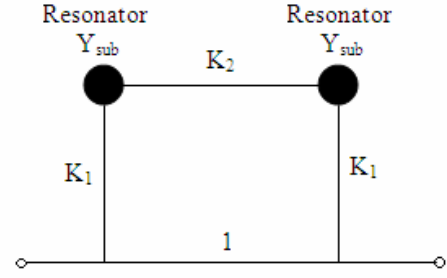


Fig. 6. Generalized coupled resonator model of matched notch filter.

It has been shown that a class of second order networks could be used to realize a perfectly matched bandstop filters which exhibit an ideal bandstop at resonance [2]-[4]. Practical implementation of this filter is based on the coupled-resonator model shown in Fig. 6 where the even- and odd-mode admittance may be shown as:

$$Y_e(p) = -j + \frac{K_1^2}{Y_{sub} + jK_2} \quad (14)$$

and

$$Y_o(p) = \frac{1}{Y_e(p)} \quad (15)$$

### IV. EXPERIMENTS

A matched notch filter with a novel tunable impedance inverter was constructed as shown in Fig. 7. The inter-resonators coupling  $K_2$  is realized using the proposed T-shunt stub impedance inverter shown in Fig. 5 with a grounded varactor attached to the end of the stub. The varactor being used in the design prototype is Metelics SMTD3001-SOD323 which has a capacitance ratio of 3 and a Q factor of 142 at 1GHz with  $V_{br}=4V$ . The coupling  $K_1$  is implemented with parallel microstrip coupled lines. The through line gives a  $90^\circ$  phase shift between the coupled resonators. The circuit is fabricated on Rogers Duroid 5880 with  $\epsilon_r$  of 2.2, substrate thickness of 787 $\mu m$  and metal thickness of 17.5 $\mu m$ . The  $K_2$  coupling values was tuned experimentally by tuning the bias voltage. The measured responses for a bias voltage tuning range of 2.2V to 4.2V are shown in figure 8. It was successfully demonstrated that the novel tunable K-inverter gave an enhancement to the inverter bandwidth properties as a broad range of coupling values can be easily obtained by tuning the varactor. The independent control of the notch attenuation without shifting the center frequency is also demonstrated. The optimum inter-resonator coupling is easily achieved by tuning the bias voltage to 3.19V which gave a

64dB rejection and 45 dB of return loss. The center frequency is 983.6MHz and the loaded Q is found to be 76.

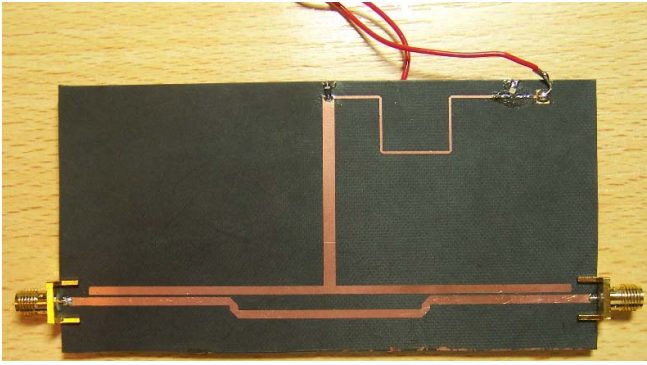


Fig. 7. Microstrip circuit prototype of matched notch filter with tunable impedance inverter.

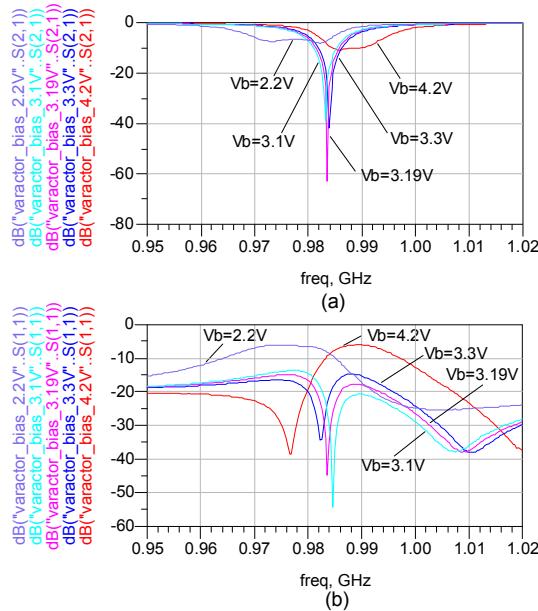


Fig. 8. Measured (a) transmission (b) reflection responses of the notch filter with varactor bias ( $V_b$ ) tuning from 2.2V to 4.2V.

## V. DISCUSSION

The matched notch filter prototype demonstrates that independent control of  $K_2$  coupling is achievable by tuning the varactor bias of the T-shunt stub inverter, and thus providing an enhanced notch response. Although the inverter properties are narrowband, it could be made tunable by changing the shunt stub length resulting in a practical broadband inverter realization. Circuit current distribution simulation is also performed for the notch filter without varactor loaded at the center frequency with the aid of Agilent ADS momentum as shown in Fig. 9. The result demonstrated that the currents in both  $\lambda/4$  resonators are  $90^\circ$  out of phase, indicating that the resonators are coupled by an ideal inverter.

## VI. CONCLUSION

A novel coupled resonator filter with tunable inverter is proposed and discussed. Theoretical analysis and experimental work are presented. The tunable of  $K_2$  coupling offers a great flexibility of achieving the optimum inter-resonator coupling which appreciably alleviate the needs of fine tuning. The tunability of the inverter properties can be used to compensate for the frequency dependant couplings in the conventional tunable bandstop filters while tuning the center frequencies, providing an optimum stopband bandwidth or loaded Q and to maximize the available tuning bandwidth. Potential future work includes implementing the compact tunable bandstop filter with tunable coupling as shown in Fig. 10.

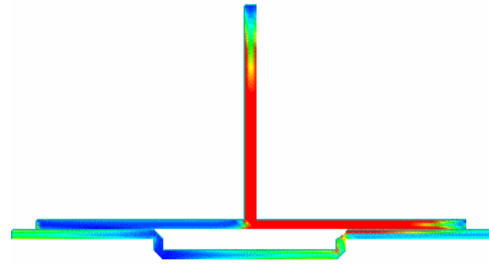


Fig. 9. Simulated current distribution of the notch filter.

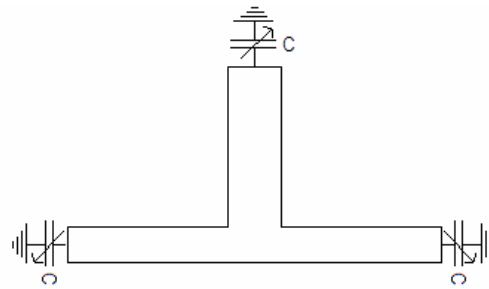


Fig. 10. Proposed circuit prototype of the tunable resonators with tunable coupling.

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