Low-frequency fluctuations in solids: 1/*f* noise

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An astonishing variety of systems show properties that fluctuate with approximately $1/f$-shaped spectral densities. In this review we deal with selected topics regarding $1/f$ fluctuations (or noise) in the resistance of simple condensed matter systems, especially metals. We find that considerable experimental and conceptual progress has been made, but specific physical processes mostly remain to be identified.

CONTENTS

INTRODUCTION

> It is probably fair comment to say that to many physicists the subject of fluctuations (or "noise" to put it bluntly) appears rather esoteric and perhaps even pointless; spontaneous fluctuations seem nothing but an unwanted evil which only an unwise experimenter would encounter!

This quote, from MacDonald's 1962 text, *Noise and Fluctuations*, probably represents an accurate assessment of present-day thinking about electrical noise. This point of view, while popular, is indeed surprising given the interest over the past fifteen years in fluctuations associated with second-order phase transitions. While we can describe the physical consequences of order parameter fluctuations in intricate detail, we have comparatively little knowledge about the microscopic origins of voltage fluctuations in a simple resistor.

In this review we shall be concerned with fluctuations which have spectral densities varying approximately as $1/f$ over a large range of frequency, f. Fluctuations with such spectra have been observed in a tremendous variety of dissimilar physical systems. The great difficulties encountered in reasonably explaining the shape of the spectrum and in ascribing a physical origin to the noise in each system (let alone a universal explanation) have kept $1/f$ noise in the forefront of unsolved problems in condensed matter physics. Given the diversity of systems which contain $1/f$ fluctuations, it is obvious that the physical origin of the noise cannot be universal. The detailed mechanics for generating noise in traffic flow (Musha and Higuchi, 1977), for example, are certainly unrelated to those generating noise in a carbon resistor. Still, because of the ubiquity of the phenomenon, it is tempting to search for universality in the underlying equations of motion. In light of the current status of the field, we see no reasonable prospect that such searches will be successful: The most general models are poorly correlated with experiments, whereas the most successful theories are the ones with the most specific applications. Restricting one's attention to the less grandiose problem of voltage noise in condensed matter systems, one can again ask: Is there a universality in the underlying equations which leads to $1/f$ noise in many apparently unrelated systems? Unfortunately, the existing experimental information is not sufficient to answer this question, although we will present some evidence of conceptual progress in this direction.

In this review we shall be concerned mainly with $1/f$ fluctuations in the resistance of "simple" condensed matter systems, with emphasis on the noise in metals. Other areas, such as vacuum tubes, solid state devices, carbon resistors, contacts, etc., are covered in a recent and wide-ranging review by Van der Ziel (1979). The reader is also directed to this article and to Press (1978) for discussions of many other issues neglected or insufficiently dealt with in the present paper.

We begin this review with an elementary description (Sec. I) of how $1/f$ noise is observed and what its basic features are. Section II discusses some general questions regarding the nature of the noise; this is followed by a description (Sec. III) of the main classes of theories that attempt to model $1/f$ noise. The temperature-fluctuation model, one that has achieved considerable success, is dealt with in detail in its own section (Sec. IV). In Sec. V we discuss the growing weight of evidence that the temperature-fluctuation model does not explain the properties of the noise except in special cases where the coupling of the temperature to the sample voltage is quite large, and in Sec. VI we describe recent progress towards identification of the types of processes that actually cause resistance fluctuations in normal bulk solids.

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A word of apology regarding notation: The use of f for frequency is traditional in the $1/f$ noise field, whereas most physicists are more comfortable with ω. We have used the original author's notation as far as possible, and elsewhere used whichever symbol reduces the number of explicit 2π's.

I. DESCRIPTION OF THE PHENOMENON

The circuit typically used to measure voltage noise is illustrated in Fig. 1. Let $V(t)$ equal the instantaneous voltage drop across the sample with resistance R_s. In the steady state ($I_{DC}=\text{const}$) the sample voltage is observed to fluctuate about its average value $\langle V\rangle \equiv V_{DC}$. The spectral density (or power spectrum) of $V(t)$ is defined [see, for example, Van Kampen (1964)] as the cosine transform of the voltage-voltage autocorrelation function $C_v(\tau)$:

$$S_v(f)\equiv 4\int_0^\infty C_v(\tau)\cos(2\pi f\tau)d\tau\,,$$

$$C_v(\tau)=\langle V(\tau)V(0)\rangle-\langle V\rangle^2=\int S_v(f)\cos(2\pi f\tau)df\,.$$

When $I_{DC}=0$, $V_{DC}=0$ and the fluctuations in $V(\tau)$ are known as Johnson or Nyquist noise. In this limit the fluctuations $S_v(f)$ can be related to the real part of the impedance $R(f)$ through a fluctuation-dissipation theorem (Callen and Welton, 1951). The result for $k_BT \gg hf$ is

$$S_v(f)=4k_BTR(f)\,. \tag{1}$$

Since for most conductors $R(f)=R(0)$ for $f<10^{10}$ Hz, the noise is frequency independent (or white) at low frequencies. Johnson or Nyquist noise is well understood and will not be further discussed here.

In the steady-state condition $I_{DC}=\text{const}\neq 0$, the fluctuations in V are observed to increase over the equilibrium value given by Eq. (1). There are two frequently observed sources of current-induced noise. The first of these is shot noise, which is proportional to I_{DC}. Shot noise arises because of the finite size of the electrical charge which leads to current pulses at the electrodes of the sample. As a fraction of the dc power, shot noise is larger at low currents where the discreteness of electrical charge is more important. As with Johnson noise, this noise is white at low frequencies

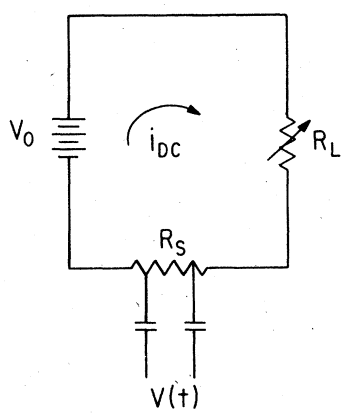

FIG. 1. Basic circuit diagram.

and is relatively well understood. At sufficiently low frequencies, however, the so-called "flicker" or "$1/f$" noise is the dominant form of excess noise. It is this extra low-frequency noise that is the subject of this review.

It should be pointed out in passing that the shape of the power spectrum uniquely characterizes the process only if it is stationary and Gaussian (all higher-order correlations are zero). In other cases, information is being lost whenever the customary procedure of recording only the power spectrum is followed. We shall discuss nonstationary and non-Gaussian processes only briefly (Sec. II); see, however, Nelkin and Tremblay (1980).

Many of the features of flicker noise are illustrated by the phenomenological equation due to Hooge (1969), which we write in the form

$$S_v(f)=\gamma\frac{V_{DC}^{2+\beta}}{N_c f^\alpha}\,. \tag{2}$$

Here α, β, and γ are constants ($\beta\approx 0$), N_c is the number of charge carriers in the sample, and f is the frequency. Note that γ is dimensionless only if $\alpha=1$ and $\beta=0$. The inverse dependence on N_c was postulated by Hooge to unify the noise processes in metals and semiconductors with $\gamma\simeq 2\times 10^{-3}$.

According to this equation, the spectral density is independent of temperature and material parameters and is a power law at all frequencies. Indeed, Hooge (1969) has tabulated a vast amount of data on both metals and semiconductors which show room-temperature noise in approximate agreement with Eq. (2). Over the years, many other papers have attempted to show that one system or another shows flicker noise that is consistent with the "magic number" $\gamma\simeq 2\times 10^{-3}$ (e.g., Hooge, 1976; Vandamme, 1974; Hooge and Kleinpenning, 1975; Kleinpenning, 1976; Strocken and Kleinpenning, 1976). If this were indeed true, Eq. (2) would be a striking statement providing a powerful key to the understanding of $1/f$ noise. Unfortunately, the strong dependence of the noise in semiconductors on the oxidation state of the surface (McWhorter, 1957) and the magnitude of the noise in the semimetal bismuth (Voss and Clarke, 1976) are in strong disagreement with Eq. (2). Many of the measurements by Hooge and collaborators are on contacts; not only is the relevant N_c in a contact area difficult to estimate, but the universal magnitude of noise measured under such conditions may demonstrate the universality of contact noise rather than of fluctuations in bulk materials. In the case of manganin, for example, contact noise measurements (Hooge, 1977) yield close to the Hooge value for γ, whereas bulk measurements (Voss and Clarke, 1976) yield $\gamma<10^{-4}$. In the case of such metals as Cu, Ag, and Au, the noise in bulk samples at room temperature is indeed in order-of-magnitude agreement with Eq. (2); however, the noise is temperature dependent in ways that are characteristic of each metal (Eberhard and Horn, 1978) and therefore cannot even be described by a universal function $\gamma(T)$. Since a considerable amount of old and new material exists in the literature in support of the "magic number," the reader should also be forewarned that Eq. (2) is entirely a postulate and there is no theoretical reason to expect any underlying tendency toward such

a universal equation. More will be said about this later in the review.

Excess noise from different sources can have very different origins and properties, even when the spectra are apparently similar. Therefore, in any attempt to characterize and tabulate the properties of flicker noise there will be exceptions. However, it is reasonably well established for "simple" systems such as bulk metals that:

(a) $S_v \propto \langle V \rangle^2$. We shall see that this implies that the current does not drive the fluctuation but merely makes resistance fluctuations in the sample visible through Ohm's law (see Sec. II.B.). One exception is $1/f$ noise due to turbulent convection in ionic solutions (Lifson *et al.*, 1978).

(b) In any given material, the noise is inversely proportional to the sample volume ($S_v \propto N^{-1}$). This implies that the noise is a bulk rather than a surface effect (see Sec. II.A). Noise due to surface traps (McWhorter, 1957) is an obvious exception.

(c) $S_v \propto f^{-\alpha}$ with $0.9 \lesssim \alpha \lesssim 1.4$. In almost all cases to be discussed here α does not change within the frequency range observed. The experimentally accessible frequency range is limited by many factors. At high frequencies excess noise becomes too small to be seen, becoming lost in the Johnson or amplifier background noise. Typical upper limits are from 100 to 10^4 Hz, depending on the magnitude of noise in the sample. At low frequencies drifts in parameters such as sample temperature and the patience of the experimentalist limit the accuracy of measurements; a typical lower limit is 10^{-2} Hz, although careful experiments under ideal conditions can extend down to 10^{-7} Hz or below (Dukelow, 1974). A power-law spectrum with a constant α is most often found.

It is exactly this power-law (or scale-invariant) nature of excess noise that makes the problem both interesting and difficult. Specifically, $\int_0^\infty f^{-\alpha} df = \infty$ for all α; for $\alpha = 1$, the integral diverges at both ends. Thus, if $S_v(f) \propto f^{-\alpha}$, the zero-time autocorrelation function $C_v(0)$ diverges and $\langle V \rangle$ becomes undefined. If the process is stationary (a reasonable assumption wherever there is no overwhelming evidence to the contrary), it is necessary that "rolloffs" should exist, i.e., that $\alpha < 1$ at sufficiently low frequencies and $\alpha > 1$ for sufficiently high frequencies. Thus it has been aptly pointed out by M. Weissman (private communication) that although $1/f$ noise is usually viewed as a low-frequency phenomenon, in any theory of stationary processes the $1/f$ law can only appear as an intermediate frequency limit. In any theory involving a process with upper and lower limits to the characteristic times, the existence of these rolloffs is indisputable. The conceptual problem in understanding flicker noise is at low frequencies where the failure to observe rolloffs leads to the need to postulate implausibly long time scales.

A related problem caused by the apparent power-law nature of the observed frequency spectra is that we cannot extrapolate to find the magnitude of the total fluctuation. Any quantitative comparison to a theory which yields a power-law spectrum over some region is, therefore, sensitive to where the (unobserved) rolloff frequencies are postulated to be.

II. BASIC QUESTIONS

The phenomenological formula of Hooge [Eq. (2)], although not universally applicable, can be used to provide a crude estimate of the noise magnitude in metals. Taking Hooge's value of 2×10^{-3} for γ, the magnitude of the noise at 20 Hz for a typical metal is about

$$S_v(\mathrm{V^2\,Hz^{-1}}) \simeq 3 \times 10^{-38} J^2(\mathrm{amp^2\,cm^{-4}}) L/A(\mathrm{cm^{-1}}),$$

where J is the current density and L and A are the length and cross section of the sample. For low impedances ($<100\ \Omega$) one can readily measure $10^{-19}\ \mathrm{V^2\,Hz^{-1}}$ using a conventional amplifier with a moderate bandpass. Thus the noise in metals is usually only visible for

$$J^2(\mathrm{amp^2\,cm^{-4}}) L/A(\mathrm{cm^{-1}}) \simeq 10^{18},$$

which shows why small samples ($L/A \gtrsim 10^6\ \mathrm{cm^{-1}}$) and large current densities ($J \gtrsim 10^5\ \mathrm{amp\,cm^{-2}}$) must be used in noise experiments. Therefore, it is an *a priori* possibility that the noise arises from the large surface-to-volume ratio or is generated by nonlinear effects due to the driving current. In Sec. II.A we present the arguments for and against a bulk rather than a surface origin of the noise. In Sec. II.B we discuss the effects of the driving current. Sections II.C and II.D deal generally with the evidence for and against considering the noise to be generated by stationary and linear processes.

A. Is the noise a property of the bulk material?

Hooge (1969) has measured the noise in gold film samples of varying thickness but otherwise identical geometry. He found that the noise is proportional to the resistance, which, in turn, is inversely proportional to the thickness t, so that

$$S_v(\omega) \propto t^{-1}, \tag{3}$$

which is consistent with

$$S_v(\omega) \propto N^{-1}$$

as in Eq. (2) and is indicative of a bulk phenomenon. (The factor of N^{-1} is a familiar one for statistical fluctuations in the square of any quantity that is proportional to N.)

Celasco, Fiorello, and Masoero (1979) have suggested that the noise in continuous metal films in fact originates at the interface between the film and the substrate. Here the surface layer is uneven and can be considered as a discontinuous layer shunted by the bulk of the film. Noise in very thin discontinuous films has been extensively studied by these authors (see, for example, Celasco *et al.*, 1978). If varying the thickness of the film served only to change the fraction of current passing through the "discontinuous" layer, the noise would depend on the thickness according to

$$S_v(\omega) \propto t^{-2}. \tag{4}$$

Unfortunately, the limited accuracy of the measurements to date, and the limited range of thicknesses over which the noise can be measured, makes it difficult to conclusively rule out Eq. (4), although Eq. (3) is a much better fit to the data. Celasco *et al.* (1979)

further point out that the temperature dependence of the noise is somewhat similar in continuous and discontinuous films. Unfortunately, the noise in whiskers, which have no substrate, also has a similar temperature dependence (Dutta and Horn, unpublished) so that the analogies yield no conclusion. According to the theory of Celasco *et al.* (1978) the noise is a universal feature of the metal-substrate interface. In metal films, however, the noise is a characteristic property of the metal being studied (Eberhard and Horn, 1978) and is known not to depend on the substrate except in a special case (Dutta, Eberhard, and Horn, 1978). The weight of the evidence seems, therefore, to suggest that the noise is a bulk effect, and the small sample sizes required are explained by the N^{-1} factor rather than by the need to increase surface area. A completely different situation exists in semiconductors, however, where the surface oxidation state is often an important parameter in determining the noise in the sample. This is why Hooge's empirical equation [Eq. (2)] unifying the noise in metals and semiconductors with $S_v \propto N_c^{-1}$ is untenable. One way the surface could affect the noise is through the modulation of carrier density by surface traps—this was proposed by McWhorter (1957) (see Sec. III.A). The evidence seems to indicate, however, that even in semiconductors the noise can be a combination of bulk and surface effects and is not created at the surface alone.

B. Is the noise present in equilibrium?

In the introduction we discussed the phenomenological formula of Hooge (Eq. 3), which states that the noise power $S_v(\omega) \propto \langle V \rangle^2 = I^2 R^2$, where I is the driving current and R is the sample resistance. Indeed, quite often in $1/f$ noise experiments it is found that $S_v(\omega) \propto \langle V \rangle^{2+\beta}$ where $\beta > 0$. Consider for the moment the case where $\beta = 0$ as in the Hooge formula. In this case, $S_v \propto I^2$ can be thought of as a linear response in the same way that Ohm's law represents linear response. In the case of Ohm's law, the current does not generate the resistance but is necessary to measure it. Similarly, in the case of $1/f$ noise, the current does not generate the resistance fluctuations but is necessary to have an observable noise voltage.

The validity of the above picture, which describes the current-induced noise as a fluctuation in the resistance, is a basic issue which must be resolved before any modeling of the noise process can begin. Unfortunately, typical experiments (especially in metals) utilize quite large current densities. In these cases, if the power-law behavior $S_v \propto V^{2+\beta}$ is assumed, values of β greater than zero are obtained even after the effects of increasing sample temperature are taken into account. One might suggest that nonzero values of β thus measured are actually a manifestation of the effects of terms of higher order than V^2 in a perturbation expansion for the noise. If this were the case, the voltage dependence of the noise would be represented more properly by a power series in V^2 rather than by a power law. In the former case, $\lim_{V\to 0}[dS_v(f)/d(V^2)]$ is constant, i.e., the noise is at least partly a resistance fluctuation, while in the latter $\lim_{V\to 0}[dS_v(f)/d(V^2)] = 0$. Unfortunately, most data at small voltages are not accurate enough to distinguish between these two possibilities.

A fundamental experiment attempting to clarify the situation has been performed by Voss and Clarke (1976) on semiconductor and metal films and repeated by Beck and Spruit (1978) on carbon resistors. Loosely, the experiments test the expectation that, since the Johnson noise power is proportional to R, fluctuations in R at zero current will show up as fluctuations in the Johnson noise power. In practice, the Johnson noise is "filtered", i.e., band limited to the range $\omega_0 \pm \Delta\omega$, averaged over an interval $\Delta t > 1/\omega_0$, and analyzed for $1/f$ fluctuations over time scales larger than $1/\Delta\omega$. It is found that the spectrum consists of

(a) a white background ascribed to statistical fluctuations resulting from the finite bandwidth, and

(b) a $1/f$ component that is not seen in reference samples that do not show $1/f$ noise when current is passed (e.g., thick metal film resistors, which were used to determine and subtract the white background). Typical data are shown in Fig. 2.

These results show that in these systems there exists a $1/f$ mechanism that is intrinsic to the sample and is not generated by the driving current. Nelkin and Tremblay (1980) have shown that the existence of a non-white component in this type of experiment requires that the fluctuation in V contain a non-Gaussian component. If this component is small, it may not appreciably distort the Gaussian shape of the probability distribution of fluctuations in a normal (current $\neq 0$) $1/f$ noise measurement; however, such measurements and the experiment described in this section probe somewhat different aspects of the situation. Nelkin and Tremblay (1980) have developed a model in which a small nonlinearity in the equations of motion leads to non-Gaussian behavior visible in the zero-current situation, and at the same time to a Gaussian behavior in leading order in the nonequilibrium situation. Experimentally, the fact that thick metal films show no $1/f$ spectrum in either experiment makes plausible the supposition that there is a single, slightly non-Gaussian, equilibrium mechanism in samples showing $1/f$ noise. One hopes that similar experiments will be carried out in many other systems; it should also be added that it has not been proved that current *cannot* generate $1/f$ noise. However, in our search for fundamental processes, we shall henceforth ignore mechanisms that rely intrinsically on the driving current (e.g., turbulence, electromigration, etc.).

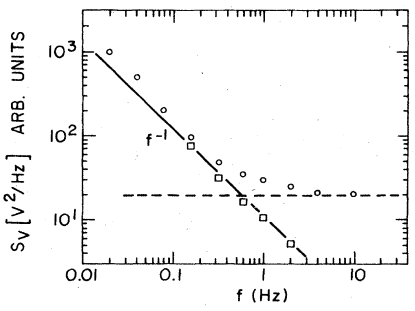

FIG. 2. $1/f$ noise in the variance of the Johnson noise: open circles, observed noise: dashes, white background from a metal-film resistor as a reference; open boxes, $1/f$ noise corrected for the background. Source: Beck and Spruit (1978).

At this stage a word about terminology is in order. Voss's experiment shows only that $1/f$ noise is an equilibrium phenomenon insofar as it is *not generated by the applied current*; it does not imply that the sample itself is in thermal equilibrium (Nelkin and Tremblay, 1980). In glasses, for example, the anomalous low-temperature thermal properties are associated with the nonequilibrium nature of the state. At this stage, we cannot rule out the possibility that $1/f$ noise is generated by nonequilibrium effects such as frozen-in disorder. More will be said about these effects in Sec. V.

C. Is the process stationary?

A spectacular low-frequency noise measurement was made by Calloyannides (1974) on an operational amplifier, where he found the noise to go as $f^{-1.23}$ with no evidence of flattening off down to 10^{-6} Hz. It is generally believed that a turnover must exist in all cases; however, if indeed $fS_v(f)$ does not go to zero as $f \to 0$, the process in question is a nonstationary one. [This follows from the $f \to 0$ contribution to $c_v(\tau = 0) = \int f S_v(f) df/f$.] Tandon and Bilger (1976) have shown that if $\langle V(t)V(t+\tau)\rangle$ is a function of t, it is not necessarily true that the power spectrum, averaged over a sufficiently large time, will change with time; however, they were able to obtain a stable f^{-1} spectrum only with a suitably chosen correlation function that is an *oscillatory* function of t. There is some experimental evidence that the short-time average of the $1/f$ noise power, especially in carbon resistors, shows statistical fluctuations larger than expected for stationary signals (Brophy, 1968, 1969, 1970; Purcell, 1972; Dell, 1973, etc.). However, other measurements (e.g., Stoisiek and Wolf, 1976) find the noise in the same systems to have the same statistical properties as stationary Gaussian noise, and the current trend of opinion is to ascribe the discrepancies to spurious effects such as burst noise.

The only samples producing $1/f$ noise in which macroscopic nonstationary behavior has been observed are evaporated Bi films (~1500 Å thick) studied by Tandon and Bilger (1977). They report that the resistances of their samples drifted at the rate of about 0.01% per hour. Noise was measured in the same samples with a frequency spectrum of $1/f^{\alpha}$ $(1.3 \lesssim \alpha \lesssim 1.5)$. The authors therefore attribute the noise to a nonstationary stochastic process manifested also in the resistance drift. One should note that if the noise does in fact originate from a slow drifting of the resistance, and if over the short time scales in a noise experiment the resistance drift is approximately linear with time, the Fourier transform of the drift goes as $1/f^2$ rather than $1/f$. Furthermore, for the specific case of bismuth films, Dimon, Dutta, and Horn (1979) find that the resistance change is not uniform; it is highest in a freshly prepared sample and unobservable after about two weeks. Also, a film that is heated to about 150 °C and then cooled to room temperature shows no resistance drift. Since the melting point of Bi is 545 K, it seems that the resistance drift may well be due to slow annealing of the film. There is, however, absolutely no difference in the noise from a film whether it is freshly prepared, aged, or rapidly annealed, so that the noise and the drift clearly do not have the same origin.

As is customary in statistical physics, we shall henceforth assume that the noise process is stationary, for simplicity, and in the absence of overwhelming evidence to the contrary. A fuller discussion of nonstationarity and its consequences for the spectrum has been given by Van der Ziel (1979).

D. Is the mechanism producing noise linear?

The search for the mechanisms underlying $1/f$ noise has led to many measurements of the statistical properties of the noise (see, for example, Hooge and Hoppenbrowers, 1969; Brophy, 1969; Purcell, 1972). One important question is whether the mechanism, or in other words the equation governing the fluctuation process, is linear. Voss (1978) looked for a type of linearity by measuring the "conditional mean" defined as follows: $\langle V(t) \mid V(0) = V_0 \rangle \equiv$ average of $V(t)$ over all members of the ensemble for which $V(t=0) = V_0$. If

$$\frac{\langle V(t) \mid V(0) = V_0 \rangle}{V_0} = \phi(t), \tag{5}$$

where $\phi(t)$ does not depend on V_0, the average time-evolution of the system is always the same except for a linear dependence on the "initial condition" V_0. Intuition suggests that this "linear response" indicates that the microscopic dynamics leading to the fluctuations are linear.

The conditional mean was determined from five different $1/f$ noise sources by digitizing their noise signals. Figure 3 shows the distribution of the fluctuations for each sample. Figure 4 shows $\langle V(t) \mid V(0) = V_0 \rangle / V_0$ for different values of V_0. Clearly, Eq. (5) is applicable to sources A, B, and C; for source D the function $\langle V(t) \mid V(0) = V_0 \rangle / V_0$ depends very slightly on V_0, and for source E it depends strongly on V_0.

However, while these experiments provide an interesting class of statistical information, it has been pointed out by Nelkin and Tremblay (1980) that the validity of Eq. (5) does not imply linearity of the microscopical dynamics. Any stationary Gaussian process,

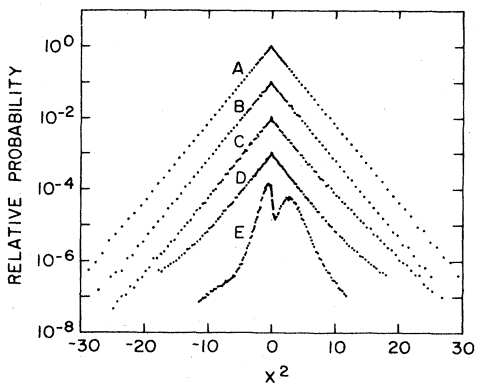

FIG. 3. Relative probability of occurrence versus X^2, where $X \equiv \Delta V / V_{\mathrm{RMS}}$. The samples are: A, metal-oxide semiconductor field-effect transistor near threshold; B, carbon resistor; C, base-collector junction of reverse-biased NPN transistor; D, output of NPN transistor configured as common emitter amplifier; E, reverse-biased PN diode (sample E showed substantial burst noise). Source: Voss (1979).

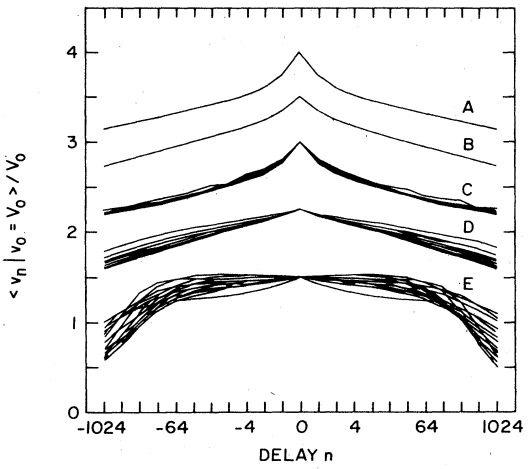

FIG. 4. Superposition of $\langle V(n)|V(0)=V_0\rangle/V_0$ versus delay n for many different V_0. Samples A–E are described in caption of Fig. 3.

i.e., one for which correlation functions of order higher than second are zero, satisfies the condition

$$\langle V(t)|V(0)=V_0\rangle/V_0=\langle V(t)V(0)\rangle/\langle V^2\rangle$$

(see, for example, the review by Wang and Uhlenbeck, 1945). The other characteristic of a Gaussian process is the well-known Gaussian distribution of the probabilities of occurrence of fluctuation amplitudes:

$$P(V)=[2\pi\langle V^2\rangle]^{1/2}\exp[-V_1^2/2\langle V^2\rangle]. \tag{6}$$

It can be seen, in fact, from Voss's data that the systems that satisfy Eq. (5) (A, B, and C) are also the ones with Gaussian distributions. Gaussian behavior, however, "typically arises via the central limit theorem by linear superposition of independent events" (Nelkin and Tremblay, 1980) and does not tell us about the microscopic dynamics of individual events. Only in cases where the fluctuating variable is not formed by superposition of events, but itself obeys a stochastic differential equation, can the "linearity" condition of Voss provide information on the linearity of this equation. This is, of course, not expected to be the case for systems such as metals and semiconductors.

III. EXPLAINING A 1/f SPECTRUM: SOME THEORETICAL APPROACHES

In most cases, the observed excess noise spectra have a frequency dependence of the form $f^{-\alpha}$, where $\alpha\approx 1$ and is constant throughout the frequency region studied. One cannot, of course, study the spectrum continuously to zero frequency; one therefore has a choice of (a) taking the apparent time- (and hence energy-) scale invariance as real; or (b) proposing that the observed spectrum is a $1/f$ region of a non-scale-invariant spectrum, in which case the theory must explain both the large frequency region over which $f^{-\alpha}$ noise is observed in some systems, and its ubiquity in so many dissimilar sytems.

In a recent paper, Nelkin and Tremblay (1980) have given an excellent discussion of scale similarity. Several basic problems in statistical physics can be described in terms of scale similarity. For example, scale similarity has led to discussions relating the $1/f$ noise problem to the mathematics of fractal dimensionalities (see, for example, Voss, 1979). While the analogy is amusing and perhaps useful from a pedagogical point of view, it has until now not been fruitful in generating detailed predictions for the physical origin of the noise. Another attractive example is the length-scale invariance at the critical point of a second-order phase transition, for which a probabilistic approach has been developed by Cassandro and Jona-Lasinio (1978). The spectrum of the density fluctuations goes as $K^{\eta-2}$ where K is the wave number, and the spectrum of energy fluctuations goes as K^{-z}, where z and η are critical exponents. A second example discussed by Nelkin and Tremblay (1981) is hydrodynamic turbulence (Monin and Yaglom, 1975). The curious contrast to low-frequency voltage fluctuations is that in the above cases the physics is well established even if experimental data on scale similarity is limited, whereas although $f^{-\alpha}$ noise is documented over many decades, the lack of any theoretical reason to expect scale similarity leads us instead to models where the observed scale invariance is an artifact.

We describe below a few types of theories that attempt to generate a $1/f$ region in the spectrum. A particularly important member of the second group is developed further in Sec. IV. No attempt has been made to produce a comprehensive list of theories; but we warn the reader that the physical assumptions behind the many attempts to mathematically generate $1/f$ spectra vary rather widely in their plausibility.

A. Activated random processes

A random process with a characteristic time τ has a Debye-Lorentzian spectrum

$$S(\omega)\propto\frac{\tau}{\omega^2\tau^2+1}. \tag{7}$$

Any spectrum may be generated by postulating an appropriate distribution $D(\tau)$ of the characteristic times within the sample. This could arise if, for example, the sample was inhomogeneous. Then

$$S(\omega)\propto\int\frac{\tau}{\omega^2\tau^2+1}D(\tau)d\tau. \tag{8}$$

In particular, if

$$D(\tau)\propto\tau^{-1}\quad\text{for }\tau_1\lesssim\tau\lesssim\tau_2,$$

then

$$S(\omega)\propto\omega^{-1}\quad\text{for }\tau_2^{-1}\ll\omega\ll\tau_1^{-1}.$$

Bernamont (1937) has pointed out that if, as in many physical processes, τ is thermally activated,

$$\tau=\tau_0\exp(E/kT),$$

then the required energy distribution is

$$D(E)=\text{const}\quad\text{for }kT\ln(\tau_1/\tau_0)\lesssim E\lesssim kT\ln(\tau_2/\tau_0).$$

The problem of justifying a $1/f$ spectrum has now been shifted to one of motivating the required energy

distribution. McWhorter (1957) has proposed that the noise in semiconductors is due to modulation of the carrier density by trapping and detrapping from surface states. This causes fluctuations in the resistance and is exactly the sort of activated random process postulated above, with the activation energy being the depth of the trap. Then if, over a given range, all trap depths are equally probable, the fluctuation spectrum $\propto 1/\omega$ over the corresponding range of ω (see Fig. 5).

However, McWhorter's theory requires that the noise magnitude be proportional to the temperature; in semiconductors the temperature dependence of the noise magnitude is usually weaker. In other systems (such as metals) it is not always easy to find a random process that can have a flat distribution of activation energies; moreover, the noise magnitude in metals is not proportional to T either. See Sec. VII for a discussion of a major extension of this model that recognizes the implications of nonlinear temperature dependences.

B. Diffusion

The existence of long time scales in $1/f$ noise has repeatedly led to theories involving the diffusion equation (e.g., Richardson, 1950; see also the review by van Vliet and Fasset, 1964). For example one may assume that the resistance is coupled to a diffusing parameter $P(\mathbf{x}, t)$ through a coupling term $g(\mathbf{x})$

$$R(t) = \int g(\mathbf{x}) P(\mathbf{x}, t) d^3x$$

where $P(\mathbf{x}, t)$ obeys a diffusion equation

$$\frac{\partial P}{\partial t}(\mathbf{x}, t) = D\nabla^2 P(\mathbf{x}, t) + \mathrm{f}(\mathbf{x}, t) ,$$

$\mathrm{f}(\mathbf{x}, t)$ being a random driving term. This gives, for the Fourier transforms of P and f,

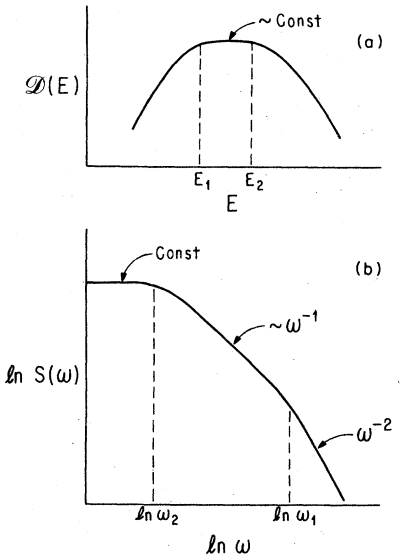

FIG. 5. (a) "flat" energy distribution. (b) frequency spectrum resulting from this distribution according to model described in Sec. III.A.

$$P(\mathrm{q}, \omega) = \frac{\mathrm{f}(\mathrm{q}, \omega)}{i\omega + Dq^2} .$$

The resulting spectrum of R in d dimensions is

$$S_R(\omega) \propto \int |g(q)|^2 \frac{\langle |\mathrm{f}(q, \omega)|^2 \rangle}{\omega^2 + (Dq^2)^2} d^d q \tag{9}$$

and will depend on the nature of the coupling $g(\mathbf{x})$ and the driving term $\mathrm{f}(\mathbf{x}, t)$. The explicit frequency denominator $[\omega^2 + (Dg^2)^2]$, however, is of little help in obtaining an explicit $1/f$ spectrum. Only when $|g(q)|^2$ or $\langle |\mathrm{f}|^2 \rangle$ has an anomalous q dependence and/or the spatial dimensionality differs from $d=3$ does Eq. (9) yield a $1/f$ spectrum. For example, Richardson (1950) derived a $1/f$ spectrum using rather special constraints on $g(\mathbf{x})$ for $d=2$. Unfortunately, for a problem with homogeneous current flow, the necessary $g(\mathbf{x})$ is difficult to rationalize physically. Alternatively, one particularly simple manner of getting the necessary $g(\mathbf{x})$ was illustrated by Weissman (1975), who considered the case of contact (or spreading) resistors. Without reproducing his derivation in detail, we note that for diffusion-controlled fluctuations [Eq. (9)] the nonuniform current flow pattern in the vicinity of a circular contact of radius a leads to a noise $S_R(\omega) \sim (1/a^3)(1/\omega)$ for $\omega \gg D/a^3$. This result represents a natural explanation for noise in point contacts. $1/\omega$ noise at low frequencies requires the resistance to be coupled to a very slowly diffusing variable; impurity diffusion is one possibility. Unfortunately the extension of these ideas to explain the observed noise in bulk metals, for example, is not possible. Lundstrom, McQueen, and Klason (1973) have shown how $1/f$ noise can be obtained for a two-dimensional system if the free energy fluctuations couple to the *gradient* of $P(\mathbf{x}, t)$, again an unphysical assumption. Many similar model calculations exist; one (Voss and Clarke, 1976) has an especially clear-cut connection to physical processes and will be discussed in detail in the following section.

C. Noise from bremsstrahlung

By far the most ambitious theory of $1/f$ noise is that proposed by Handel (1975, 1977). In this model, electrons in a metal or semiconductor emit low-frequency photons when scattered by an impurity potential. Since the energy of the emitted photon can be arbitrarily small, the energy of the electrons in the solid can fluctuate at low frequencies. These energy fluctuations lead to voltage fluctuations which, according to Handel, have a spectral density which exactly agrees with Hooge's phenomenological formula in both magnitude and frequency dependence. However, the agreement between theory and experiment is undoubtedly an accident. This is because Handel's model is a zero-temperature theory for a beam of electrons which can emit low-energy photons. However, most electrons in a metal, for example, cannot emit low-energy photons because all the nearby states are occupied. At $T=0$ less than $1/10^{18}$ of the total free electrons are available to emit photons of frequency 10^{-4} Hz. Handel's theory does not consider the equilibrium distribution of charged carriers in any way, and hence the magnitude prediction cannot be relevant for experiments in either metals or semiconductors.

Furthermore, Handel's result is universal, as exemplified by Hooge's formula in Eq. (2). Thus the noise is independent of the number of impurity sites for scattering and hence has a magnitude which is independent of whether or not a bremsstrahlung event has ever occured.

Quite apart from noise in solids, one can ask the question: What is the current noise in a beam of charge carriers? Tremblay and Martin (private communication) have attempted to answer this question. They find in a complicated diagrammatic calculation that the noise is more likely to go as f rather than $1/f$ at low frequencies. Thus neither the magnitude prediction nor the $1/f$ spectral density are well established.

IV. THERMAL FLUCTUATION MODEL OF VOSS AND CLARKE

The most extensive development of a model where resistance fluctuations are caused by equilibrium thermal fluctuations is due to Voss and Clarke (1976). The model is based on the energy fluctuations of a small sample considered as a canonical ensemble. Although the temperature of a canonical ensemble is, strictly speaking, the same as that of the thermal bath (and is fixed), it is useful to define the *temperature of the sample* as a fluctuating variable through the relationship $\Delta E = C_v \Delta T$, where C_v is the specific heat of the sample. We should emphasize at this point that, strictly speaking there is no such thing as a "temperature fluctuation" in a canonical ensemble since the temperature is defined by the thermal reservoir. More correctly the model of Voss and Clarke is a model for spontaneous, equilibrium enthalpy fluctuations (Weissman, 1975). However, defining the temperature of the sample as a fluctuating quantity is a physically intuitive concept and we therefore choose to retain the Voss-Clarke terminology.

Temperature fluctuations lead to resistance fluctuations through dR/dT, the temperature coefficient of resistance of the sample. Thus for the voltage fluctuations in the presence of a current,

$$\langle \Delta V^2 \rangle = V_{DC}^2 R^{-2} \langle \Delta R^2 \rangle = V_{DC}^2 R^{-2} (dR/dT)^2 C_v^{-2} \langle \Delta E^2 \rangle .$$

For a canonical ensemble,

$$\langle \Delta E^2 \rangle = k_B T^2 C_v ,$$

thus, defining $\beta \equiv R^{-1}(dR/dT)$,

$$\langle \Delta V^2 \rangle = V_{DC}^2 \beta^2 k_B T^2 C_v^{-1} . \tag{10}$$

Since this is by definition a resistance fluctuation, $\langle \Delta V^2 \rangle \propto V_{DC}^2$ as in the empirical formula of Hooge (1969). Moreover, since $C_v \propto N$ the noise has the proper inverse volume dependence. Given this $1/N$ dependence, it is clear that the above model cannot apply to semiconductors where the noise is large in macroscopic samples. However, it might be appropriate to metals, especially in situations where dR/dT is large.

Since the energy is a conserved quantity, the frequency spectrum of the above model is that given by a standard diffusion equation approach like that in the models considered by Richardson (1950). However, the significance of the model is sufficient to warrant reproducing the derivation of the frequency spectrum. Following Voss and Clarke we write a Langevin diffusion equation for the local temperature $T(\mathbf{x}, t)$:

$$\frac{\partial T}{\partial t} = D\nabla^2 T + c^{-1}\nabla \cdot \mathbf{F} , \tag{11}$$

where D = thermal diffusivity, c = specific heat, and $\mathbf{F}$ is an uncorrelated random driving term:

$$\langle \mathbf{F}(\mathbf{x}+\mathbf{s}, t+\tau) \cdot \mathbf{F}(\mathbf{x}, T) \rangle \propto \delta(\mathbf{s})\delta(t) .$$

The temperature-fluctuation spectrum is

$$S_T(\omega) = \int \langle T(t+\tau)T(t) \rangle \cos(\omega\tau) d\tau ,$$

where $T(t)$ is the sample average of $T(\mathbf{x}, t)$ and the magnitude of F is determined by the normalization condition

$$\int S_T(\omega) d\omega = \langle \Delta T^2 \rangle = k_B T^2 C_v^{-1} .$$

Voss and Clarke find that for a sample with dimensions $l_1 \times l_2 \times l_3$, where $l_1 \gg l_2 \gg l_3$, four frequency regions can be identified, separated by the three frequencies $\omega_i = D/2l_i^2$. The shapes of the spectra in these regions are (see Fig. 6)

For $\omega \gg \omega_3$:

$$S_T(\omega) \propto \omega^{-3/2}$$

For $\omega_3 \gg \omega \gg \omega_2$:

$$S_T(\omega) \propto \omega^{-1/2}$$

For $\omega_2 \gg \omega \gg \omega_1$:

$$S_T(\omega) \propto (\text{const} - \ln\omega)$$

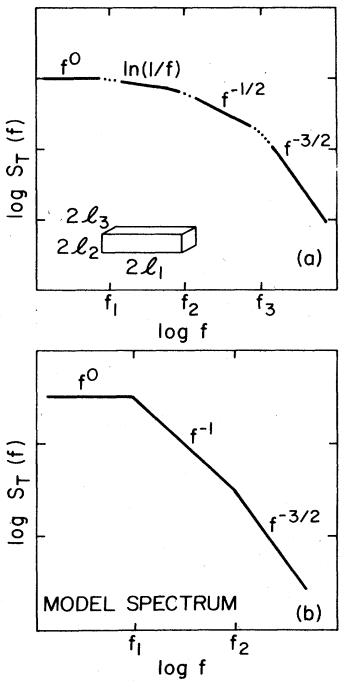

FIG. 6. (a) Spectrum derived from diffusion equation [Eq. (11)] for sample with $l_1 \gg l_2 \gg l_3$. (b) Model spectrum with $1/f$ region. Source: Voss Clarke (1976).

For $\omega_1 \gg \omega$:

$$S_T(\omega) \propto \text{const}.$$

Thus, as anticipated above, the standard Langevin diffusion equation yields no explicit $1/f$ region. However, one can introduce a $1/f$ region in the spectrum in a number of different *ad hoc* ways. These ways usually involve postulating the existence of anomalous behavior at long wavelengths in either the coupling constant of the voltage to the fluctuating quantity or in the mean square amplitude for fluctuations. Voss and Clarke motivate the insertion of a $1/f$ region by the following exercise: The Langevin equation is replaced by

$$\frac{\partial T}{\partial t} = D\nabla^2 T + C^{-1}P(\mathbf{x}, t), \tag{12}$$

where $P(\mathbf{x}, t)$ is uncorrelated in space and time:

$$\langle P(\mathbf{x}+\mathbf{s}, t+\tau)P(\mathbf{x}, t)\rangle \propto \delta(\mathbf{s})\delta(\tau).$$

The $\nabla \cdot \mathbf{F}$ term represented a random flow of energy within the system; the $P(\mathbf{x}, t)$ term causes a fluctuation in the energy of the system. Equation (12) causes the temperature fluctuations to be spatially correlated with the mean square amplitude for temperature fluctuations $\langle |\Delta T(q)|^2\rangle$ divergent at long wavelengths: $\langle |\Delta T(q)|^2\rangle \sim 1/q^2$. In this case Voss and Clarke find that

$$S_T(\omega) \propto \omega^{-1} \quad \text{for } \omega_1 \ll \omega \ll \omega_2.$$

This is similar to the result obtained by Lundstrom, McQueen, and Klasen (1973), who have also shown that an explicit $1/f$ region can be obtained when the free energy is dominated by nonlocal terms such as $(\nabla T)^2$, again giving rise to spatially correlated fluctuations. Although the physical origin is completely different, much of this was anticipated by Richardson (1950) over twenty years earlier when he argued that an explicit $1/f$ region could obtain if the coupling constant $g(\mathbf{x})$ (between the resistance and the diffusing variable) was singular at long wavelengths $g(q) \sim 1/q^2$. For the particular case of temperature fluctuations, the coupling constant $\beta = (1/R)dR/dT$ is clearly nonsingular, so that a $1/f$ region only obtains if the mean square amplitude $\langle |\Delta T(q)|^2\rangle$ is singular.

Unfortunately, it is hard to rationalize Eq. (12) based on any microscopic model. The energy source term implies (Weissman, 1978) that the specific heat is not a well defined local quantity. Liu (1977) has suggested that the source term in Eq. (13) might represent the effect of heat flow from a metal film to a low thermal conductivity substrate. This is a physically pleasing picture which explains the strong substrate dependence of the noise near the superconducting transition (see below). However, Weissman (1978) has pointed out that such a model does not represent a bulk source of noise and that for a system in equilibrium, Liu's model explicitly violates conservation of energy. Indeed, to our knowledge, there is no microscopic justification that Eq. (12) is a good approximation in any limit. This is particularly disconcerting because, as we shall show below, there is good experimental evidence suggesting that temperature fluctuations can indeed lead to something approximating $1/f$ noise.

In light of the above results, Voss and Clarke construct a hybrid model spectrum (Fig. 6) that has a $1/\omega$ behavior between the frequencies ω_1 and ω_2, with $S_T(\omega) \propto f^{-3/2}$ for $\omega > \omega_2$ and $S_T(\omega) = \text{const}$ for $\omega < \omega_1$. If we retain the equilibrium normalization,

$$\int S_T(\omega)d\omega = k_B T^2 C_v^{-1},$$

then, in the $1/f$ region,

$$S_v(f) = \frac{V_{DC}^2 \beta^2 k_B T^2}{C_v[3 + 2\ln(l_1/l_2)]f}. \tag{13}$$

The applicability of the temperature-fluctuation model has been explicitly demonstrated in two types of systems (1) Josephson tunnel junctions (Clarke and Hawkins, 1976) and (2) tin films near the superconducting transition (Ketchen and Clarke, 1978; Clarke and Hsiang, 1976). In the first case $1/f$ noise was seen in the voltage across shunted Josephson junctions biased at a current greater than the critical current I_c. Equilibrium temperature fluctuations modulate I_c, which in turn causes the voltage to fluctuate; the modified prediction of the model in this case was shown to be (Clarke and Hawkins, 1976)

$$S_v(f) = \frac{(dI_c/dT)^2(\partial V/\partial I_c)_I^2 k_B T^2}{C_c[3 + 2\ln(l_1/l_2)]f}. \tag{14}$$

One of the niobium-niobium oxide-tin tunnel junctions studied turned out to have an anomalous temperature dependence of the critical current I_c. Small changes in T near 2 K changed dI_c/dT by a large factor, allowing direct experimental observation of the dependence of $S_v(f)$ on (dI_c/dT). Clarke and Hawkins found that $S_v(f)$ scales with $(dI_c/dT)^2$ as predicted by the model (see Fig. 7).

The other test case for the model is the noise in samples near the superconducting transition where dR/dT is high and varies sharply over a small range of temperature (and also changes when a magnetic field is applied). Clarke and Hsiang (1976) have studied tin and lead films at the superconducting transition and find that the noise (see Fig. 8) scales (a) as Ω^{-1}, where Ω is the sample volume, (b) as $(dR/dT)^2$, and (c) as $V^2 \equiv V_{DC}^2$. All this is consistent with Eq. (13). Further,

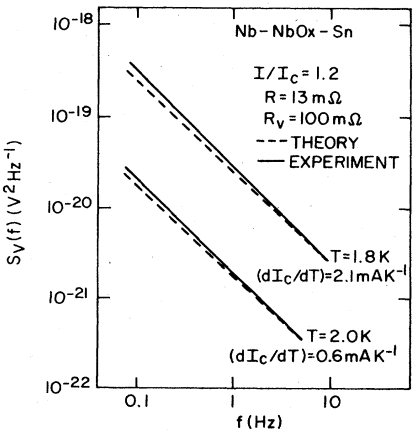

FIG. 7. Power spectra of the voltage fluctuations of an Nb-Nb-oxide-Sn tunnel junction at two temperatures, ie., two values of dI_c/dT. Source: Clarke and Hawkins (1976).

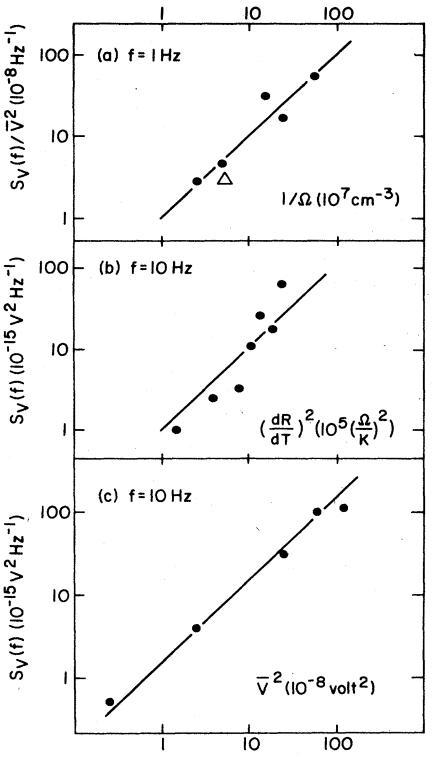

FIG. 8. Tin films near the superconducting transition. (a) $S_v(1\text{ Hz})/\bar{V}^2$ vs Ω^{-1}, where Ω is the sample volume: • 0.1-μm films; △, 0.6-μm film; (b) $S_v(10\text{ Hz})$ vs $(dR/dT)^2$; (c) $S_v(10\text{ Hz})$ vs $\bar{V}^2$. A line of unit slope has been fitted to each figure. Source: Clarke and Hsiang (1976).

when films are evaporated directly onto glass substrates (Type A), the noise spectra observed are $1/f$-like and consistent with Eq. (13) [Fig. 9, line (a)]. However, striking changes in the shape of the spectra are observed when the films are evaporated onto glass and sapphire substrates with a 5 nm aluminum underlay (Type B). These spectra are shown in Fig. 9. Separate experiments confirmed that the underlay decreased the thermal boundary resistance between the film and the substrate. These results make credible the general supposition that the thermal coupling to the substrate is relevant to the shape of the noise spectrum, although the mechanism by which increased thermal contact causes a flatter spectrum is unclear.

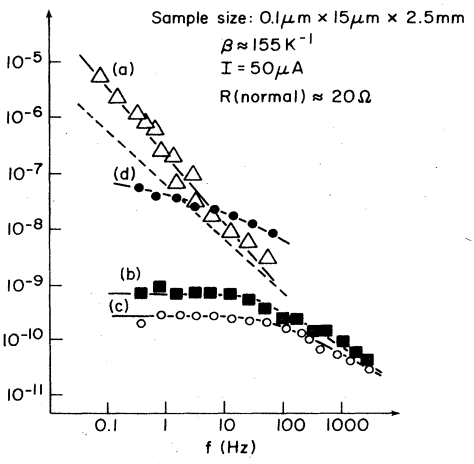

FIG. 9. Noise power spectra for tin films: (a) Type A (evaporated directly on glass). Dashed line is calculated from Eq. (13); (b) Type B (evaporated over a 5 nm aluminum underlay) on sapphire; (c) and (d) Type B on glass. Source: Clarke and Hsiang (1976).

Ketchen and Clarke (1978) measured the noise near the superconducting transition of tin films that were freely suspended (without a substrate). They find that these samples have noise spectra that flatten off at low frequencies and are distinctly steeper than f^{-1} at the highest frequencies observed (Fig. 10). The "knee" in the spectrum is near $f \approx D/L_1^2$ and the data are consistent with the prediction [Fig. 6(a)] of the diffusion equation with uncorrelated fluctuations [Eq. (11)]. Moreover, $S_v(0)$, the limiting level of the noise at low frequencies, scales with $(dR_F/dT)^2$, where R_F is the film resistance (Fig. 11).

These results lend credence to Eq. (11) as a description of temperature fluctuations in isolated systems, while systems on substrates (at least the Type A films of Clarke and Hsiang, 1976) bear out the semiempirical description of Voss and Clarke (1976) for noise that is affected by thermal coupling to a substrate [Eq. (12)].

A further piece of evidence in support of thermal fluctuations as the source of noise in superconducting films is the spatial correlation measurements of Clarke and Hsiang (1976). Figure 12 (inset) shows the experimental arrangement for measuring the noise from segments of a tin film a distance d apart. The noise signals were added and subtracted to get $S_+(f)$ and $S_-(f)$ and the fractional correlation defined as

$$C(f) \equiv [S_+(f) - S_-(f)]/[S_+(f) + S_-(f)].$$

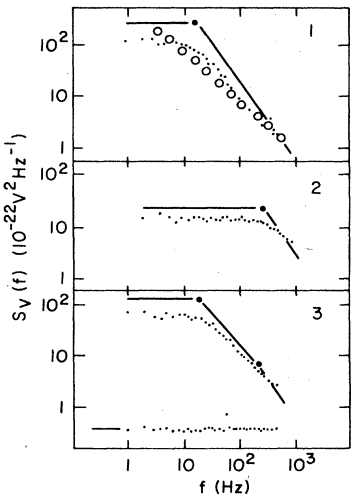

FIG. 10. Spectral densities for three undoped freely standing tin film samples. The solid lines represent the predictions of Eq. (11); the heavy dots represent the predicted values of the "knees" in the spectrum. For sample 1, the open circles represent the prediction of Eq. (13). For sample 3, the lower dotted line is the measured Johnson noise and the short solid line is the calculated Johnson noise. Source: Ketchen and Clarke (1978).

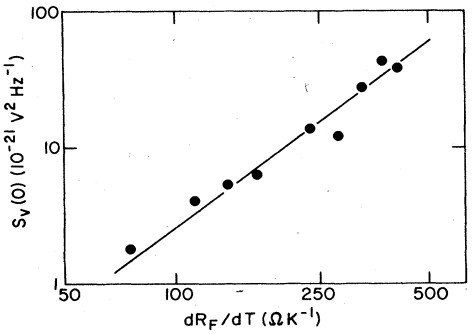

FIG. 11. Log-log plot of $S_v(0)$ vs dR_F/dT (R_F is the film resistance) for a freely standing tin film. Source: Ketchen and Clarke (1978). Solid line represents $S_v(0) \sim (dR_F/dT)^2$.

TABLE I. The measured temperature coefficient of resistance for several materials and the measured and calculated noise power at 10 Hz. Source: Voss and Clarke (1976).

Material	Measured β (K^{-1})	$S_v(f)/V^2$ measured at 10 Hz (10^{-16} Hz^{-1})	$S_v(f)/V^2$ calculated at 10 Hz (10^{-16} Hz^{-1})
Cu	0.0038	6.4	16
Ag	0.0035	6.4	2
Au	0.0012	0.6	0.76
Sn	0.0036	7.7	7.7
Bi	−0.0029	13	9.3
Manganin	$\lvert\beta\rvert < 10^{-4}$	$<7\times10^{-3}$	$<3.5\times10^{-3}$

$C(f)=1$ for identical signals and $C(f)=0$ for independent random signals. The thermal correlation length is $\lambda=(D/\pi f)^{1/2}$ so that thermal fluctuations should be correlated at low frequencies and uncorrelated at high frequencies, with the crossover in the region of $\lambda(f)\sim d$. The effective diffusivity was estimated to be $D\simeq 50$ cm^2 sec^{-1}. Figure 12 shows $C(f)$ for a Type A and a Type B sample. The Type A film has smaller correlations at low frequencies; this is attributed by the authors to the greater probability of a fluctuation's decaying into the substrate.

The cases of Josephson junctions and superconducting tin films are special in the sense that the voltage fluctuations are strongly coupled (through dI_c/dT or dR/dT) to the temperature fluctuations, thus ensuring that temperature-induced voltage fluctuations will dominate the noise. This is not necessarily true in the more commonly occurring sources of noise, e.g., metals and semiconductors. However, Voss and Clarke (1976) have measured the noise at room temperature in several metals and find considerable evidence to suggest that the noise in metals is also due to temperature fluctuations. They find general agreement (within a factor of 3, which is reasonable because of the large uncertainty in estimating the size of the sample) between the observed noise magnitudes and the magnitudes predicted from Eq. (13). Note that the only material-dependent parameter in Eq. (13) is β. (At room temperature $C_v \simeq 3Nk$ in all cases.) In particular, manganin, for which $\beta\approx 0$, shows no noise (see Table I).

These authors also report that bismuth films at room temperature show partial spatial correlations in the noise. The experimental details are similar to those for the correlation measurements on superconducting tin, and, as in that case, the noise is correlated at low frequencies with the correlation being lower for the sample with larger l (Fig. 13).

One other result due to these authors will be mentioned here. They show that the diffusion equation predicts that the temperature decay of a delta-function power input into a sample will have the same spectrum as the equilibrium fluctuations described by Eq. (11), whereas in the case of a step-function input the Fourier transform of the decay of the temperature will have the same shape as the spectrum derived from Eq. (12). They then perform an experiment to observe the dissipation of power inputs in the same samples in which $1/f$ noise is measured, and find that the measured noise is consistent in magnitude and frequency dependence with the prediction from the step-function response experiment, whereas the Fourier transform of the delta-function response is a much flatter spectrum (Fig. 14).

To summarize, the main points of evidence in support of the thermal fluctuation model are as follows:

(a) The proposed mechanism is a genuine equilibrium

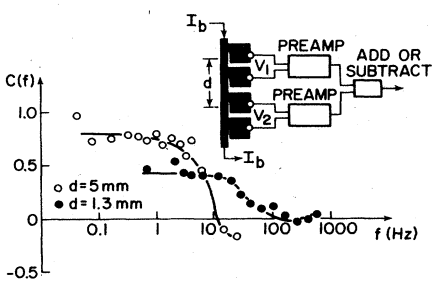

FIG. 12. $C(f)$ vs f: ○, type-A tin films on glass; •, type-B tin films on glass. Inset: experimental configuration. Source: Clark and Hsiang (1976).

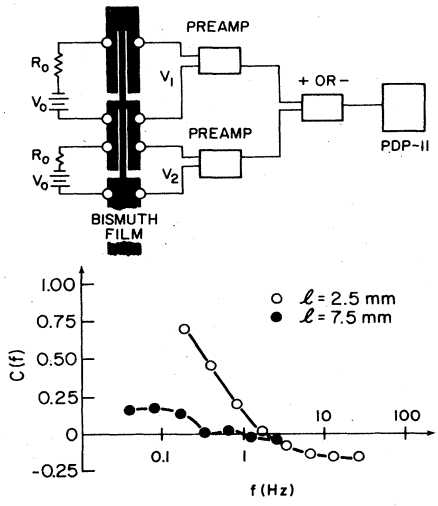

FIG. 13. Top: Experimental configuration. Bottom: fractional correlation for two bismuth film samples. Source: Voss and Clarke (1976).

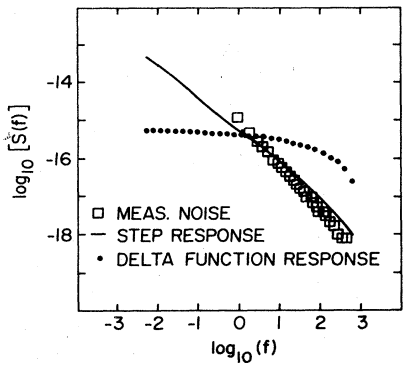

FIG. 14. Normalized $S(f)$ from cosine transform of temperature response to δ-function and step-function input of applied power; and measured noise spectrum on same sample. Source: Voss and Clark (1976).

phenomenon, as has been shown to be required in some systems (Voss and Clarke, 1976).

(b) The similar magnitudes of noise in Bi and such metals as Au, Cu, and Ag, when samples of equal volume are used, rules out an inverse dependence on the number of charge carriers (assuming the same mechanism to dominate) and suggests instead a dependence on the entire volume of the sample (as is the case with thermal fluctuations).

(c) The applicability of the model has been demonstrated in the case of special systems such as tin films at the superconducting transition, thereby making it plausible that, in other cases as well, the $1/f$ shape of the spectrum may be merely an artifact of the thermal coupling to the substrate.

(d) Low-frequency noise voltages, in two regions of a Bi film, were correlated up to a maximum frequency roughly consistent with the prediction based on thermal diffusion between the two regions. This room-temperature system, with a nonanomalous dR/dT, thus shows the same behavior as is seen in tin at the superconducting transition.

(d) The integrated magnitude of the noise in several metals at room temperature, estimated using postulated rolloffs, is comparable to the magnitude predicted by thermodynamics.

(e) Spectra derived from the dissipation of heat inputs have magnitudes and frequency dependences comparable to noise experiment results and theoretical predictions.

V. TEMPERATURE DEPENDENCE OF THE NOISE IN METALS: FAILURES OF THE THERMAL FLUCTUATION MODEL

It can be seen from Table I that the agreement between the room-temperature magnitudes of noise in various metals and the prediction of Eq. (13) is not exact, but only within about half an order of magnitude. This latitude is permissible because it is difficult to estimate accurately the volumes of the small samples in which the noise must be measured. Relative measurements of the magnitude in the same sample when other relevant parameters are varied are not subject to this inaccuracy, and this immediately suggests temperature dependence measurements as a more stringent probe of the nature of $1/f$ noise and test of Eq. (13). The temperature dependence in Eq. (13) is

$$S_v(f) \propto \beta^2 T^2 / C_v , \tag{15}$$

which for most metals is an extremely weak dependence in the 100–600 K region, since $\beta \sim T^{-1}$. Experiments by Eberhard and Horn (1978) explicitly contradict Eq. (15), however. Observations were made on 800 Å films of Ag (Fig. 15), Cu (Fig. 16), and Au (Fig. 17) over a range 100–600 K and in Ni (Fig. 18) over 100–700 K. In each figure the dotted line is the prediction of the temperature-fluctuation model and the box summarizes the data of Voss and Clarke (Table I) relative to the plotted data. Clearly, while the theory and the data of Voss and Clarke and the data of Eberhard and Horn agree within a factor of 3 at room temperature, the form of the temperature dependence is unmistakably different from that predicted by Eq. (13), with characteristic peak structures in some cases. At low temperatures, the observed noise magnitude is substantially *lower* than Eq. (13) predicts.

The noise in Ni (Fig. 18) presents an additional problem. At the Curie point, β (the temperature coefficient of resistance) peaks with a specific-heat-like exponent leading to a sharp peak in the predicted noise magnitude. In superconducting tin, where the situation is analogous, we have seen that the observed noise scales with dR/dT (Figs. 8 and 11). In Ni the noise shows slight structure at the temperature at which β peaks, but certainly does not scale with β^2.

Dutta, Eberhard, and Horn (1978) have performed the pulse response measurements of Voss and Clarke (1976) as a function of temperature in Ag and Cu films on quartz and sapphire substrates. They find in Ag films [Fig. 19(a)] that (a) the temperature dependence of the pulse response spectra bears no relation to that of the observed voltage noise, and (b) the pulse response spectra vary according to whether the substrate is quartz or sapphire, whereas the voltage noise in Ag is independent of the substrate in the temperature range studied.

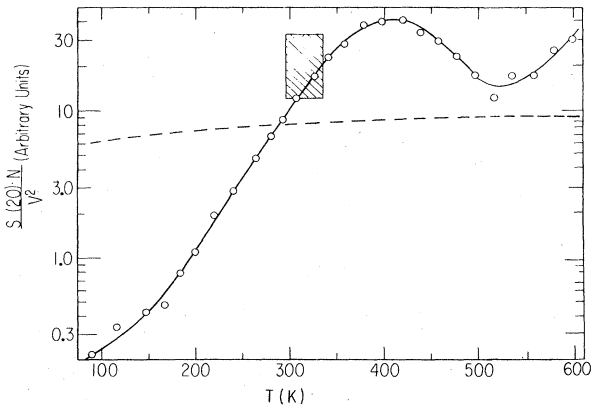

FIG. 15. Voltage noise measured at 20 Hz, $S_v(20)$. N/V^2 vs temperature for 800 Å Ag film on sapphire substrate. The dashed line is the prediction of the temperature-fluctuation model [Eq. (13)]; the box represents a summary of measurements by Voss and Clarke (1976). The solid line is a smooth curve through the data. Source: Eberhard and Horn (1978).

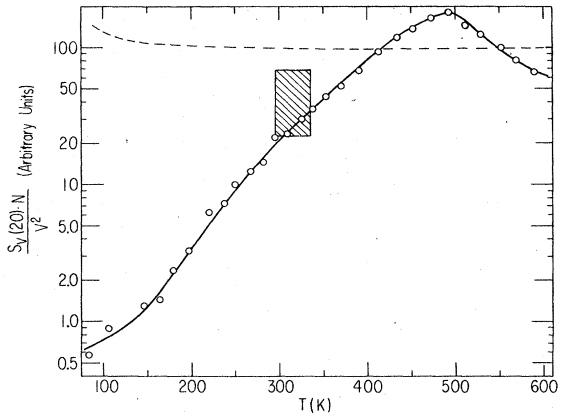

FIG. 16. Voltage noise measured at 20 Hz, $S_v(20)$. N/V^2, vs temperature for 800 Å Cu film on sapphire substrate. The dashed line is the prediction of the temperature-fluctuation model [Eq. (13)]; the box represents a summary of measurements by Voss and Clarke (1976). The solid line is a smooth curve through the data. Source: Eberhard and Horn (1978).

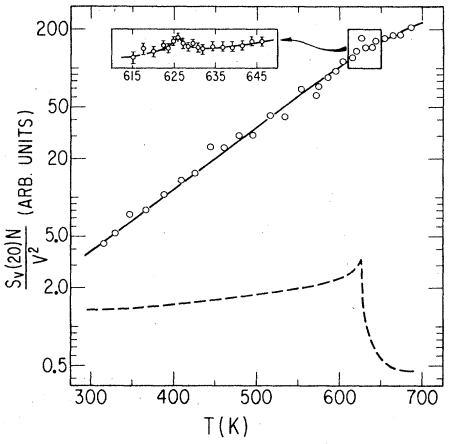

FIG. 18. $S_v(20)\ N/V^2$ vs temperature for 800 Å Ni film on sapphire substrate. The dashed line is the prediction of the temperature-fluctuation model [Eq. (13)]; the inset shows detailed measurements very near the Curie point. The solid line is a smooth curve through the data. Source: Eberhard and Horn (1978).

The discrepancies extend to the values for the slopes of the spectra (α) as a function of temperature. Eberhard and Horn have reported that at any given temperature there is no observable deviation from straight power laws $S_v(f) \propto f^{-\alpha}$ (α independent of f). However, α changes with temperature and is different in Ag and Cu (Fig. 20). The pulse spectra, however, yield values of α which, while also temperature-dependent, do not depend on the nature of the film; also, they do not match the reported values for voltage noise spectra (Fig. 20). More will be said about this later in this section.

Thus, while the applicability of the thermal fluctuation model is not in doubt in special cases with a strong coupling of the temperature to the resistance, the model is in serious trouble in other cases.

(a) The room-temperature agreement of magnitudes (Table I), which is dependent on the nature of the postulated model spectrum (Fig. 6), is fortuitous in light of the totally different temperature dependences predicted and observed.

(b) Scaling of the noise with $R^{-1}dR/dT$ fails in the case of nickel near the Curie point.

(c) Pulse response measurements do not support the model when performed as a function of temperature and on different substrates.

The experiments of Dutta, Eberhard, and Horn (1978) on Cu films [Fig. 19(b)] show that (a) above room tem-

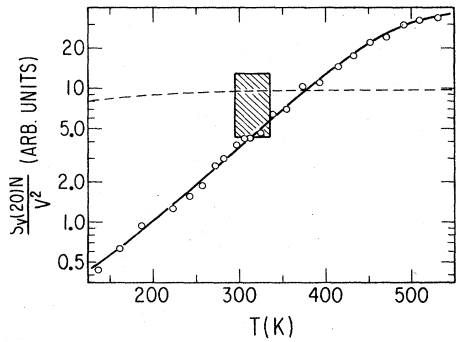

FIG. 17. Voltage noise measured at 20 Hz, $S_v(20)$. N/V^2, vs temperature for 800 Å Au film on sapphire substrate. The dashed line is the prediction of the temperature-fluctuation model [Eq. (13)]; the box represents a summary of measurements by Voss and Clark (1976). The solid line is a smooth curve through the data. Source: Eberhard and Horn (1978).

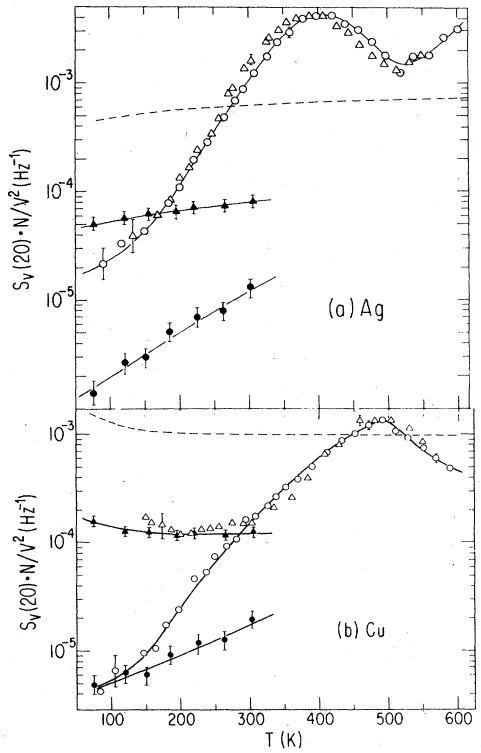

FIG. 19. $[S_v(20)N/V^2]$ vs temperature for sapphire ( ), and quartz (△) substrates; thermal-fluctuation voltage noise at 20 Hz, predicted from the thermal response of the samples to step-function power inputs for sapphire (●) and quartz (▲) substrates; dashed lines, prediction of the thermal-fluctuation model [Eq. (15)].

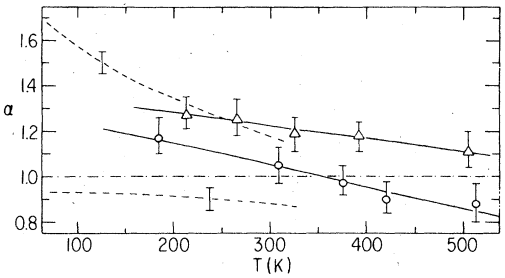

FIG. 20. (a) Frequency dependence of excess noise, α vs T: ○, 800 Å Ag films; △ 800 Å Cu film. Source: Eberhard and Horn (1978); (b) Frequency dependence of response to step-function thermal input: upper dashed line, trend of α vs T for Ag and Cu films on sapphire; lower dashed line, trend of α vs T for Ag and Cu films on quartz.

perature the noise in Cu, as in Ag, is not visibly affected by the substrate; (b) at lower temperatures, Cu on quartz has more noise, and a flatter temperature dependence of the noise, than Cu on sapphire. In conjunction with the fact that at room temperature the noise in Cu is about an order of magnitude lower than the noise in Ag, the curves in Fig. 19 are amenable to a plausible interpretation in terms of two types of noise existing simultaneously.

(a) Type A noise: a background that is weakly temperature dependent. Both its magnitude and its temperature dependence are affected by the nature of the substrate. It is possible that Type A noise is caused by temperature fluctuations modified by thermal conduction in the substrate (the thermal conductivity of quartz is almost constant between 100 and 300 K; the thermal conductivity of sapphire is a factor of 10 higher at 300 K, and almost a factor of 10^3 higher at 100 K).

(b) Type B noise: a strongly temperature-dependent fluctuation that is characteristic of the metal, independent (within the experimental accuracy) of the substrate, and dominant at high temperatures; it is not thermal fluctuation noise. We suggest that in Ag, Type B noise dominates at all temperatures. In Cu, however, the room-temperature noise is lower; thus in the case of Cu on quartz, it may be that the combination of the lower Type B noise in Cu and the higher Type A noise in films on quartz allows us to see a crossover from Type B to Type A noise as the temperature is lowered.

Several contradictions in previously existing data can be explained in terms of Type A and Type B noise:

(a) Measurements by Hooge and Hoppenbrowers (1969) have suggested that the temperature dependence of $1/f$ noise is weak. This can be explained as a result of (i) the use of Au films, in which Type B noise is low; (ii) the use of glass substrates; and (iii) the absence of observations above room temperature.

(b) The spatial correlations in Bi films on glass substrates (Voss and Clarke, 1976) may be explained if Type B noise is low in Bi so that thermal fluctuations dominate. If this is so, the amount of correlation seen should vary from metal to metal and with temperature.

(c) We have already pointed out the difference in behavior between two systems with similar resistive anomalies: Sn films near the superconducting transition and Ni films near the Curie point. This is now easily explained: At the high-temperature Curie point, Type B noise dominates and masks the behavior of any thermal fluctuation background. However, at the superconducting transition in tin, Type B noise is likely to be negligible.

However, Weissman (1978) and Weissman and Dollinger (1980), in attempting to determine the magnitude of the thermal fluctuation noise power at $f \to 0$, conclude that observable noise (such as Type A noise) in systems with $\beta T \sim 1$ cannot be caused by equilibrium thermal fluctuations. They show that if an external time-dependent current is applied to a resistance R and $P(t)$ is the power dissipation, and we write

$$R(t) = R_0 + \int_0^\infty P(t-\tau) G_{PR}(\tau) d\tau ,$$

then the same Green's function $G_{PR}(\tau)$ describes the autocorrelation function for resistance fluctuations:

$$\langle \delta R(t) \delta R(t+\tau) \rangle / R^2 = \beta k T^2 G_{PR}(\tau)/R .$$

These expressions relate the equilibrium resistance fluctuations to the response of the resistance to macroscopic time-varying power inputs of general shape; the authors describe an experimental verification of this relation. However, if $P(\tau)$ is constant, i.e., a constant V_{DC} is applied, the change in R due to Joule heating is just

$$\Delta R = \frac{V_{DC}^2}{R} \int_0^\infty G_{PR}(\tau) d\tau .$$

The voltage fluctuations observed on applying V_{DC} are just $\delta V = V_{DC} \delta R/R$, so that

$$\Delta R = \frac{V_{DC}^2}{R^2 \beta k_B T^2} \int_0^\infty \langle \delta V(t) \delta V(t+\tau) \rangle d\tau$$
$$= \frac{V_{DC}^2}{R^2 \beta k_B T^2} \frac{S_v(0)}{4} .$$

Thus the noise power at zero frequency is

$$S_v(0) = \beta T (4 k_B T R) \frac{\Delta R}{R} .$$

Since $4 k_B T R$ is the Johnson noise, low-frequency thermal fluctuations should be unobservable above the background as long as $\beta T \sim 1$ (which is true for normal metals, but not at the superconducting transition) and $\Delta R/R \ll 1$ (i.e., the resistance is not abnormally changed by Joule heating).

We should emphasize that the statistical mechanics result in Eq. (11) is not in dispute; disagreements regarding the magnitude of observable noise translate to disagreements regarding the shape of and method of normalizing the spectrum, not the total area under it. The magnitude predictions of Voss and Clarke involve a postulated spectrum [Fig. 6(b)]; the arguments of Weissman and Dollinger described above normalize $S_v(\omega)$ through $S_v(0)$ and avoid the question of the form of $G_{PR}(\tau)$.

The noise in bismuth is currently a particularly vexing problem. The spatial correlations observed by Voss and Clarke are easily explained only in terms of thermal time scales. According to the above arguments, observable noise in bismuth ($\beta T \sim 1$) cannot be due to

equilibrium temperature fluctuations. It is, of course, possible that the agreement between the observed spatial correlations and thermal diffusion time scales is a fortuitous accident. However, a more interesting possibility is that the noise is due to temperature fluctuations driven by some nonequilibrium source with a magnitude much greater than $\langle(\Delta T)^2\rangle = k_B T^2 C_v^{-1}$. We hope future experiments will resolve this issue.

VI. MODEL FOR THE NOISE IN METALS

Once we have shown that the noise dominant in metals at room temperature is not temperature-fluctuation noise, the obvious question is: What causes the dominant (type B) noise? We have seen that it is apparently substrate independent and strongly temperature dependent, and that the temperature dependence, especially the peak structure, is characteristic of the metal being studied. The peaks in Ag and Cu are in the region $k_B T_p \sim$ a few hundredths of an electron volt, an unfamiliar energy region for plausible solid state processes. However, it has been recently shown by Dutta, Dimon, and Horn (1979) that a random-fluctuation model developed from that discussed in Sec. III.A can lead to plausible activation energies and account simultaneously for (a) the temperature dependence of the noise, (b) the observed $f^{-\alpha}$ nature of the noise (with $\alpha \approx 1$ and independent of frequency), and (c) the weak observed dependence of α with temperature.

We start by rewriting Eq. (8) as

$$S(\omega) \propto \int f(E,\omega)\mathfrak{D}(E)dE\,, \tag{16}$$

where, since $\tau = \tau_0 \exp(E/kT)$,

$$f(E,\omega) \equiv \frac{\tau_0 \exp(E/k_B T)}{1+\omega^2\tau_0^2 \exp(2E/k_B T)}\,. \tag{17}$$

$f(E,\omega)$ is a strongly peaked function of E with a width of the order of kT. Thus, although the conclusion reached in Sec. IV, i.e., $S(\omega) \propto \omega^{-1}$ only if $\mathfrak{D}(E) = \text{const}$, is still rigidly valid, we shall see that actually it is necessary only that $\mathfrak{D}(E)$ vary slowly compared to $k_B T$ for $S(\omega)$ to be "generic" $1/f$ noise (defined, following Voss and Clarke, 1976, as noise with $0.8 \lesssim \alpha \lesssim 1.4$). In fact, Eq. (16) can be integrated after expanding $\mathfrak{D}(E)$ in a Taylor series:

$$S(\omega,T) = \frac{k_B T}{\omega}\left[\mathfrak{D}(\tilde{E}) + \sum_{n=1}^{\infty} \frac{\mathcal{E}_n}{(2n)!}\left(\frac{\pi k_B T}{2}\right)^{2n} \frac{d^{2n}}{dE^{2n}}\mathfrak{D}(E)\Big|_{E=\tilde{E}}\right], \tag{18}$$

where $\tilde{E} \equiv -k_B T \ln(\omega\tau_0)$ is the value of E at which $f(E)$ peaks and $\mathcal{E}_n$ is the nth Euler number. When $\mathfrak{D}(E)$ varies slowly over any range $\Delta E \sim k_B T$, we retain only the first term in the expansion,

$$S(\omega,T) \propto \frac{k_B T}{\omega}\mathfrak{D}(\tilde{E})\,. \tag{19}$$

Clearly, if $\mathfrak{D}(E) = \text{const}$, $S(\omega) \propto \omega^{-1}$ as expected; but if $\mathfrak{D}(E) \neq \text{const}$, the dependence of $\tilde{E}$ on ω and T makes $S(\omega,T)$ a more complex function of both its arguments. This, as we have emphasized earlier, is also what experiments show. The question, then, is whether the independent experimental results on the T and ω dependences of $S(\omega,T)$ for any given material are consistent with each other, i.e., whether they support the model by being derivable from a single energy distribution. To show this we write Eq. (19) as

$$\mathfrak{D}(\tilde{E}) \propto \frac{\omega}{k_B T}\,S(\omega,T)\,,$$

which shows that if ω and T are varied so as to keep $\tilde{E} \equiv -k_B T \ln(\omega\tau_0)$ unchanged, $\omega S/k_B T$ is also unchanged. In other words there exists a ratio $\Delta\omega/\Delta T$ such that if

$$\left(\Delta\omega\,\frac{\partial}{\partial\omega} + \Delta T\,\frac{\partial}{\partial T}\right)[k_B T \ln(\omega\tau_0)] = 0$$

then,

$$\left(\Delta\omega\,\frac{\partial}{\partial\omega} + \Delta T\,\frac{\partial}{\partial T}\right)[\omega S(\omega,T)/k_B T] = 0\,.$$

Solving the two equations and defining α locally as $-\partial \ln S_v / \partial \ln\omega$ gives us

$$\alpha(\omega,T) = 1 - \frac{1}{\ln(\omega\tau_0)}\left(\frac{\partial \ln S(\omega,T)}{\partial \ln T} - 1\right). \tag{20}$$

We assume $\tau_0 = 10^{-14}$ sec, a typical attempt time (of the order of an inverse phonon frequency) for random processes in crystals. The results discussed below are quite insensitive to the exact value of τ_0 chosen, as long as $\omega \ll \tau_0^{-1}$. In that case,

(a) $|\ln(\omega\tau_0)| \gg 1$; as a result, $\alpha - 1$ will generally be small, giving rise to generic $1/f$ noise. α is temperature dependent in all cases except when the spectral density has a temperature dependence $S(T) \propto T^b$ (where b is a constant).

(b) $\omega \ll \tau_0^{-1}$; thus $\ln(\omega\tau_0)$ and hence α will be insensitive to changes in ω.

(c) The dependence of S on T is, however, not suppressed; its structure depends on the structure of $\mathfrak{D}(E)$ through Eq. (19).

Equation (20) can be tested quantitatively using the existing data on Ag due to Eberhard and Horn (1978). In Fig. 21 the smooth fit to the data for S_v vs T has been used to calculate α vs T. Not only is α always within the limits for generic $1/f$ noise, but the trends in the data are well reproduced.

Figure 21(c) shows the energy distribution for this system, calculated from Fig. 21(a) using Eq. (19). The distribution peaks at $E_p \sim 1$ eV and has a width of about 0.4 eV. Thus, although the width is large compared to $k_B T$, as needed for our approximations, it is *not* large compared to E_p; in fact $\mathfrak{D}(E)$ is a peaked function much more amenable to physical justification than the flat $\mathfrak{D}(E)$ heretofore thought to be essential in a random-fluctuation model. Further, $\mathfrak{D}(E)$ peaks at ~1 eV even though $k_B T_p \ll 1$ eV; the reason is that E_p and T_p are related by

$$E_p \approx -k_B T_p \ln(\omega\tau_0) \tag{21}$$

and $|\ln(\omega\tau_0)|$ is, of course, large. The electron-volt region is an attractive one: many random processes in crystals have activation energies in this region (see Sec. VII).

One prediction of this model follows from Eq. (21): Since E_p is a property of the material, T_p must be a function of ω, albeit a weak one (Voss, private com-

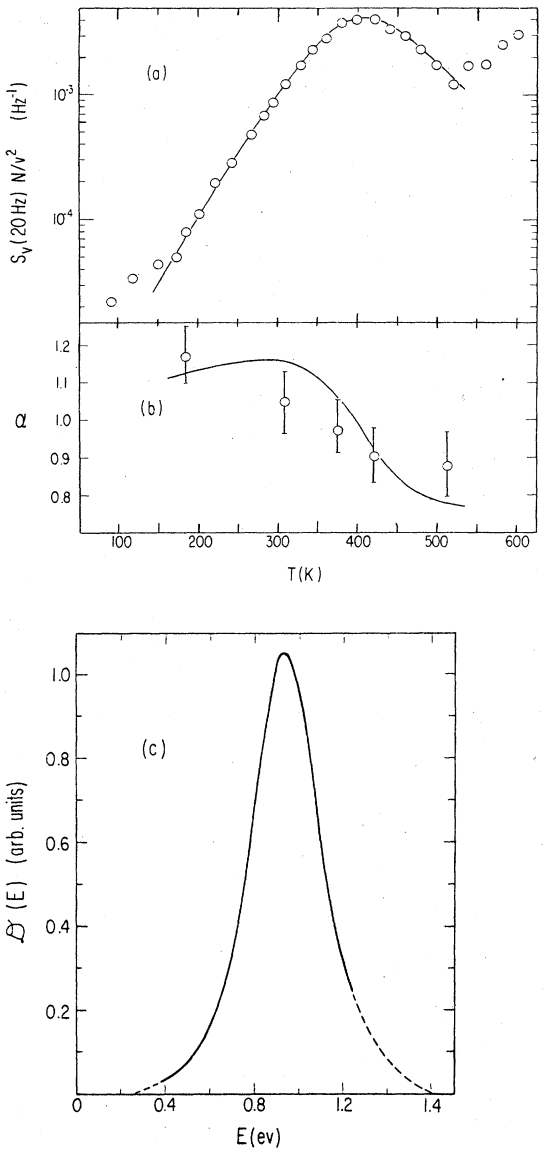

FIG. 21. (a) Noise magnitude at 20 Hz (corrected for dependence on voltage and sample volume) versus temperature for 800 Å Ag film (data from Fig. 15). The solid line is a smooth fit to the noise peak. (b) Frequency exponent vs temperature for 800 Å Ag film (data from Eberhard and Horn, 1978). The solid line is the prediction of Eq. (20) using the curve in Fig. 21(a). (c) Energy distribution calculated from the curve in Fig. 21(a). Dashed lines are extrapolations to estimate the area under the curve. Source: Dutta, Dimon, and Horn (1979).

munication; Martin, private communication). New measurements by Dutta and Horn (unpublished) confirm this. Figure 22 shows the structure of the noise-power-versus-temperature curve near the peak at two frequencies. Figure 23 shows the estimated locations of the peaks for frequencies 2, 5, 20, and 50 Hz. The data are not numerous or accurate enough to allow the calculation of accurate values of E_p and τ_0; for illustrative purposes we have drawn the theoretical curve using the values $E_p = 0.97$ eV, $\tau_0 = 10^{-14}$ sec. The agreement with the arguments above is obviously excellent.

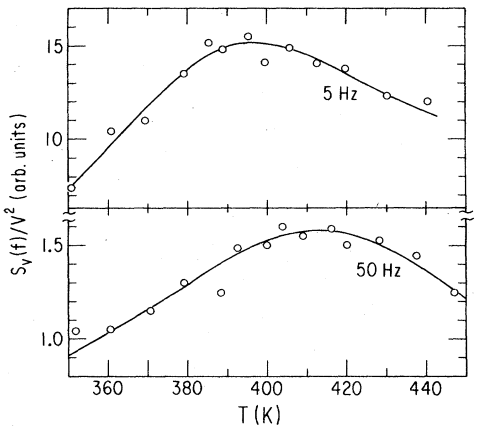

FIG. 22. Detail of noise peaks for 800 Å Ag film at 5 and 50 Hz.

Other measurements due to Dutta, Dimon, and Horn (1979) support this model. They find that the temperature dependence of the noise in Ag films changes slightly with changes in the thickness of the films (Fig. 24). The variation of α with T in any given film is consistent with the variation of S_v with T in the same film. Moreover, the data imply that $\mathcal{D}(E)$ has a sharper peak for thicker films, leading to the physically plausible picture that the width of the distribution is a result of sample inhomogeneity (whereas E_p may be a more fundamental property of the material). Indeed, the temperature dependence of the noise in Cu whiskers below room temperature (Dutta and Horn, unpublished) is considerably stronger than in Cu films in the same temperature region.

The temperature dependence of S_v has been measured not only in Ag, but also in Cu, Au, and Ni (Eberhard and Horn, 1978) and in Bi (Dimon, Dutta, and Horn, 1979). In each case the magnitude varies strongly with temperature, and in the case of Cu and Bi, peaks are seen within the observed temperature range. Thus segments of $\mathcal{D}(E)$ may be calculated for these materials as well. In all cases $\alpha(T)$ shows the same general trends as for Ag, so that there is qualitative agreement with Eq. (20).

The noise in Cu whiskers has been measured by Dutta and Horn (unpublished). The data exist only in a small

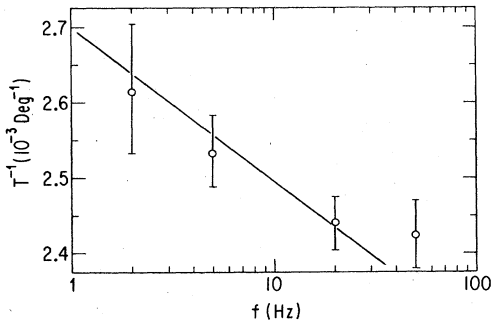

FIG. 23. Peak temperature T_p vs frequency. Straight line is an illustration of the prediction of Eq. (21) for the values $E_p = 0.97$ eV, $\tau_0 = 10^{-14}$.

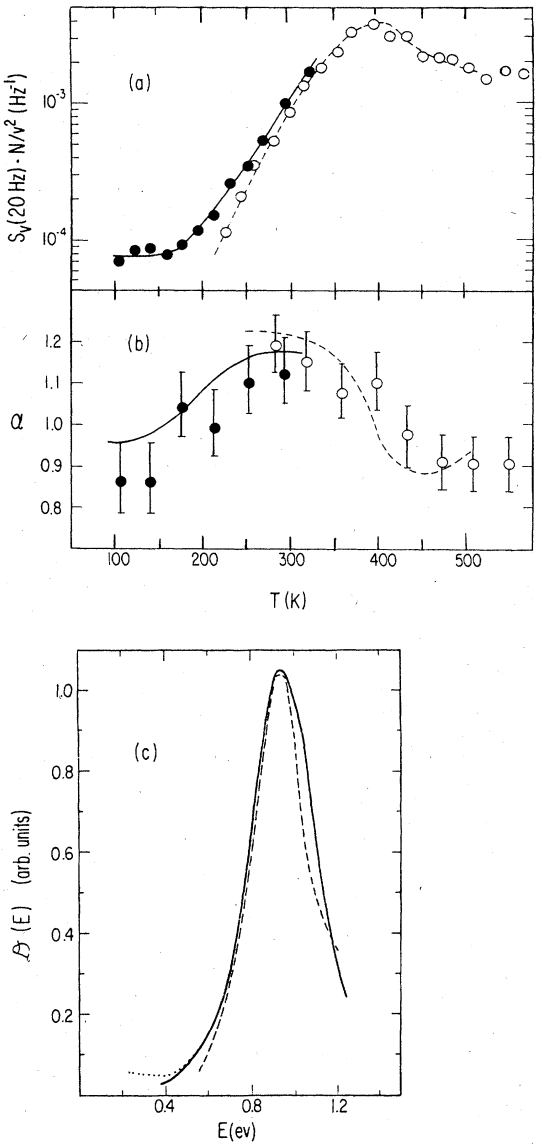

FIG. 24. (a) Normalized noise magnitude at 20 Hz vs temperature: •, 250 Å Ag film; ○, 1400 Å Ag film. Lines are smooth fits to data. (b) α vs T: •, 250 Å Ag film; ○, 1400 Å Ag film. Lines are predictions from Eq. (20) using the corresponding curves in Fig. 24(a). (c) Energy distributions: solid curve 800 Å film [same as Fig. 21(c)]; dotted curve 250 Å Ag film; dashed line, 1400 Ag film. Source: Dutta, Dimon and Horn (1979).

region below room temperature and show a noise that is larger at room temperature and has a stronger temperature dependence than noise in bulk Cu. It is tempting to speculate that in whiskers, which are single crystals, the broadening of $\mathcal{D}(E)$ is smaller than in evaporated films, so that the slope of $\mathcal{D}(E)$ is larger than for films. Another piece of information that may be correlated with this picture is that annealing a Ag film reduces the noise at room temperature by a factor of 2 to 6 compared to when it was freshly evaporated, and also makes the temperature dependence sharper (Eberhard and Horn, 1978). After the first anneal, the noise is completely reproducible. In our picture, this would be due to a sharpening-up of $\mathcal{D}(E)$ because of the reduction in inhomogeneity caused by annealing. We should point out that $\int \mathcal{D}(E)dE$ must be constant, but a sharpening-up of $\mathcal{D}(E)$ can either increase or decrease the noise at room temperature depending on the details of the change of shape.

The total noise power (integrated over all frequencies) cannot be estimated directly because $S_v(\omega) \sim \omega^{-\alpha}$ and much of the area underneath it is below the low-frequency experimental limit. However, Eq. (19) implies a relationship between the ω and T dependences of $S_v(\omega, T)$ through the functional $\tilde{E} \equiv -k_B T \ln(\omega \tau_0)$; in other words, either T or ω changes $\tilde{E}$ and probes the same energy distribution $\mathcal{D}(\tilde{E})$. The few decades of ω that are experimentally accessible hardly change $\tilde{E}$ at all, whereas by changing T we cover a large range of $\tilde{E}$. We therefore write

$$\langle \Delta V^2 \rangle \equiv \int S_v(\omega, T) d\omega$$

$$= \int \mathcal{D}(\tilde{E}) d\tilde{E}$$

$$= \omega \ln(\omega \tau_0) \int T^{-1} S_v(\omega, T) dT .$$

This last integral can be estimated over a wider range. Assuming $S_v(\omega, T)$ falls off faster than T at very low temperatures, and neglecting the contribution at very high temperatures, we find for silver,

$$\frac{\langle \Delta V^2 \rangle}{V^2} \gtrsim \frac{0.9}{N} , \tag{22}$$

where N is the number of atoms in the sample. Copper has an order of magnitude less noise than Ag at room temperature, but a broader noise peak; as a result, a similar number is found:

$$\frac{\langle \Delta V^2 \rangle}{V^2} \gtrsim \frac{0.3}{N} . \tag{23}$$

Note that the upturn in the noise magnitude in silver for $T > 500$ K has been neglected in the above estimate. This upturn could be caused either by a breakdown of the above model (due, for example, to the onset of a strongly temperature-dependent coupling constant) or by the existence of a second peak in $\mathcal{D}(E)$ at higher temperatures. In the latter case Eq. (20) implies that α should markedly increase above 500 K. At present there are no data on the spectral density of the noise for silver above 500 K. Clearly more experiments on this system are called for.

While these numbers are only approximate lower limits, the experimental results [Eqs. (22) and (23)] suggest that an acceptable model for the origin of the random fluctuations must yield a total magnitude $\int S(\omega) d\omega$ that is of the order of N^{-1}. At this point the reader should be reminded that the general conclusions in this section depend only on three basic assumptions:

(a) The excess noise is due to a superimposition of random processes whose characteristic times are thermally activated;

(b) The distribution of activation energies $\mathcal{D}(E)$ may have any shape as long as it varies slowly over any

$\Delta E \sim k_B T$;

(c) The "attempt frequency" τ_0^{-1} is much larger than the frequencies at which the noise is measured.

The results in this section have been obtained without specific reference to a type of random process. Processes such as vacancy-interstitial formation and recombination, diffusion of all types, and trapping and detrapping of charge carriers satisfy the criteria listed above. (See, for example, Yokota, 1980). We know of no existing experimental results that conclusively point to one process or another. It should be pointed out that in ascribing the noise to one such process one needs to postulate a plausible coupling to the resistance, which is after all the quantity being measured. The fact that we have self-consistently fitted the temperature dependence of the resistance noise with an $S(\omega, T)$ for the random process means that the coupling to the resistance can be, at most, weakly temperature dependent. For this reason it seems improbable that intrinsic defect mechanisms such as vacancy-interstitial creation-recombination is the source of the noise, since the number of such defects increases exponentially with temperature. However, a curious (perhaps coincidental) correlation (Fig. 25) between the peak temperature T_p for different metals and their cohesive energies suggests that this judgment may be too harsh. It can be seen from Eq. (21) that T_p is a measure of the energy scale for the distribution $\mathfrak{D}(E)$. We have tried to see if T_p is correlated with any of the known energy parameters in metals; the cohesive energy (which is related to the activation energy for vacancy-interstitial formation) is the only one that fits the bill. This may mean that the coupling to the resistance takes place in some complex way such that the coupling constant is temperature independent, or that the relevant energies scale with the cohesive energy for some yet-to-be-understood reason. In any case Fig. 25 is less than conclusive and is presented here merely as a curiosity.

VII. CONCLUSIONS

The physical origin of $1/f$ noise, even in metals, is very much an open question. Small samples near the superconducting transition [where $\beta = 1/R(dR/dT)$ is large] show excess noise which most probably has a temperature-fluctuation origin. The detailed mechanism by which temperature fluctuations lead, for superconducting films on substrates, to noise which is approximately $1/f$ remains a mystery. Equation (12) suggested by Voss and Clarke remains without any microscopic justification. In systems other than metals near the superconducting transition, the origin of excess noise is far less certain. For metal films at room temperature, the majority of evidence suggests that the bulk of the noise does not in fact originate from equilibrium temperature fluctuations. The absence of detailed knowledge of frequency cutoffs makes estimating the integrated noise power quite difficult and it may simply be that temperature-fluctuation noise is just too small to see for metals at room temperature (Weissmann and Dollinger, 1980). One of the most puzzling pieces of experimental data, in this regard, is the observed spatial correlation of the noise in Bi films. The temperature dependence of the magnitude of the noise in Bi films has the same qualitative shape as the predictions from the temperature-fluctuation model. Thus one might be tempted to say that Bi is somewhow different from most metals in that it does have noise dominated by equilibrium temperature fluctuations, but there is no additional evidence to support this conjecture. If Bi is not different from other metals, then we are left with a most puzzling situation. We know of no theoretical way of incorporating spatial correlations on thermal diffusion time scales, with defect-disorder models of the type suggested in the preceding section.

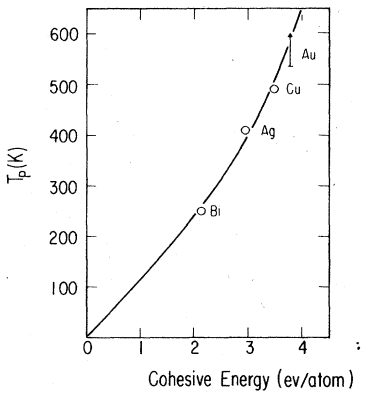

FIG. 25. Temperature T_p at which noise peaks versus cohesive energy, for various metals. In the case of Au only an approximate lower limit is known. Source: Dutta, Dimon, and Horn (1979).

In our opinion the relationship in metal films between $\alpha(T)$ and $S_v(20\text{ Hz})$ vs T represents rather compelling evidence for a model based on a distribution of relaxation times. Indeed, the simple fact that α is temperature dependent guarantees that the scale invariance inferred from the frequency dependence is an artifact of the finite frequency range covered in the experiment. Given a model based on a distribution of relaxation times, one must then ask what the actual noise source in metals is. At present there is no proven answer to this question; we can only say that any model must be consistent with the experimental result $\int S_v(f)df \sim N^{-1}$. It is interesting to note that for most metals, when $T > \Theta_D/4$ (where Θ_D is the Debye temperature), the integrated magnitude of the noise predicted by the temperature-fluctuation model is in agreement with this order-of-magnitude estimate. However, as we discussed above, this agreement might well be fortuitous.

We now return to the question of the universality of $1/f$ noise. We have seen that even in metals there may be more than one source of $1/f$ noise. Nonetheless, the existence of a distribution of relaxation times might well be a ubiquitous feature of condensed matter systems, and it is interesting to speculate regarding the universality of this particular noise mechanism. A search through the literature provides very little evidence either for or against such a noise source. One of the features of a distribution of relaxation times is that a power-law noise spectrum is an artifact of an experimental frequency range that is finite and located at low frequencies compared to τ_0^{-1}. For India ink resistors, the excess noise

has a frequency spectrum with $\alpha \simeq$ constant $\simeq 1.2$ over the range $10^{-5} \lesssim f \lesssim 10^{5}$ Hz (Voss, private communication). If this is rigidly true, it is easily shown that the model predicts that $S_v(20\text{ Hz}) \propto T$ from approximately 100 to 1000 K—a verifiable prediction. It is only when detailed measurements of $S_v(\omega)$ vs T and α vs T are compared that one has self-consistency checks, and nontautological statements can be made regarding distributions of relaxation times. We should stress that these are not particularly difficult experiments and the predictions based on a distribution of relaxation times are quite specific. Thus one could quite easily refute or support a distribution of relaxation times as the source of the excess noise in particular systems.

We should like to conclude by expressing the hope that future experiments will deal less with extracting $1/f$ noise from exotic systems and more with fundamental issues which make contact with theoretical efforts. Even widely accepted characteristics of the noise, such as the N^{-1} dependence, have less than perfect experimental confirmation. In addition to more detailed studies of temperature dependence and spectral shape (mentioned above), two areas seem to us particularly ripe for further investigation:

(a) statistical properties of the noise other than its power spectrum; for example the zero-current studies described in Sec. II.B, which measure four point correlation functions and probe the non-Gaussian nature of the fluctuations.

(b) spatial correlation measurements, in more detail and in systems other than bismuth.

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