# Stochastic Conjugate Gradient based Multi-user Constant Modulus Algorithm for use in Multiuser DS-CDMA Environment

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***Abstract:-* A Stochastic Conjugate Gradient Algorithms (SCGA) is proposed for the solving nonlinear optimization problem associated with the multi-user constant modulus algorithm (CC-CMA) for DS-CDMA receivers in a multi-user environment. The algorithm referred to as the stochastic conjugate gradient CC-CMA (SCGCC-CMA). Simulations show that the SCGCC-CMA algorithm preserves the fast convergence rate of the block-shanno cross correlation constant modulus algorithm (BSCC-CMA) [1], can be configured to perform similar to the recursive least squares (RLS) version of the CC-CMA algorithm, and can outperform the conventional CC-CMA for less cost. The proposed algorithms can also be used in DS/CDMA systems to solve the problem of joint blind channel equalization and blind source separation in a single-user and multi-user environment. Alternatively, a window sliding parameter may be adjusted to trade off between performance and computation to match system requirements. We will also propose a convergence analysis for the proposed algorithms.**

## 1. INTRODUCTION

In a communication system, channel equalisation/interference cancellation is required to eliminate intersymbol interference (ISI) and multiple access interference (MAI). Blind algorithms, which do not require a training sequence, have the potential to increase channel bandwidth efficiency. Among all blind equalisation algorithms, the constant modulus algorithm (CMA) [7], which exploits the constant modulus property of the signal, appears to be the algorithm of choice due to its computational simplicity [2]. For single-user and multi-user environments with zero mean, circularly symmetric sources, provided that the so-called zero and length condition are satisfied, the CMA algorithm converges globally to a solution where ISI and MAI are removed [7]. To prevent the same source being repeatedly retrieved in a multi-user environment, a multi-user CMA was proposed in [4]. However, when the adaptation of the CC-CMA algorithm is realised with normal stochastic gradient type algorithm, the rate of convergence to such a solution can be too slow. To mitigate this limitation, in [1] we employed an approximation of the Hessian matrix to represent the local curvature of the CC-CMA cost function. An order of magnitude improvement in convergence rate is observed. The problem with such approaches is that considerable memory/computational requirements are required for storing, updating, and inverting an estimate of the associated Hessian. In [1], we also proposed an alternative type of modified Newton algorithm, Block-Shanno cross correlation constant modulus algorithm (BSCC-CMA), that implicitly computes a positive definite approximation to the inverse of the Hessian of the objective function using computational/memory requirements which are roughly $O(M)$ as opposed to $O(M^2)$. The block cross-correlation constant modulus objective function is developed to implement a modified Newton's algorithm. The resulting algorithm is based on Shanno's technique to circumvent much of the computation associated with the line search of conjugate gradient techniques [2], [3]. The objective in the present paper is to extend these ideas to develop a low cost block adaptive filter that converges to a optimum value of the CC-CMA cost function significantly faster.

For every block of data, the BSCC-CMA algorithm updates the filter tap weights along specified search directions until a stopping criterion is reached. Then it produces a block of outputs and proceeds to the next block of data. The BSCC-CMA algorithm has superior convergence properties to the RLS- type CC-CMA and Newton-like CC-CMA as reported in [1]. However it can be very expensive to let the BSCC-CMA converge all the way before proceeding to the next block of data. In this paper, we propose a stochastic conjugate gradient method to reduce the computational complexity while preserving the power of BSCC-CMA [1]. The algorithm developed in this paper can be summarised as follows. For every $K$ samples, a data block of size $N$ is formed, a search direction is determined (using Shanno's Technique [1], [2], [3], [5]), and an appropriate step size is determined based on the current block of data. Next the filter tap weights are updated once, and a block of $K$ outputs is produced. Now instead of converging on each block, we find one search direction per block, and span the search direction across blocks. This eliminates costly inner loops, and significantly reduces complexity. Also data blocks can be overlapped in such a way that as more overlap is added, convergence speed increases at the cost of an increased computation. In section 2, we will discuss the BSCC-CMA developed in [1], in section 3, we will develop SCGCC-CMA algorithm, in section 4, we will analyse the convergence of SCGCC-CMA in terms of their macro quantities and finally simulations and conclusions will be summarised.

## 2. BLOCK SHANNO CROSS CORRELATION-CMA ALGORITHM (BSCC-CMA).

Given an objective function $f : \Re^{M} \to \Re$, the classical Newton's algorithm [1, 214-215] updates the filter tap weights as

$$w_j = w_{j-1}\, H_{newton}^{-1}\left(w_{j-1}\right) g\left(w_{j-1}\right) \qquad (1)$$

where $H_{newton}\left(w_{j-1}\right)\Re \in \Re^{M\times M}$ is the Hessian of the objective function $f$, $g\left(w_{j-1}\right)\Re \in \Re^{M}$ is the gradient of the objective function $f$, evaluated at $w_{j-1}$ and $j$ is an iteration index. Shanno's algorithm is a type of modified Newton's algorithm that approximates the inverse of the Hessian $H_{newton}^{-1}\left(w_{j-1}\right)$ by the Shanno approximation, which is a positive definite matrix denoted by $H^{-1}(w_{j-1})$. This matrix is implicitly determined by the gradients of the two most recent iterations, a search direction, and a step-size based on a previous iteration. The complexity of the implicit calculation of $H^{-1}(w_{j-1})$ is of $O(M)$, as opposed to $O(M^2)$, for the calculation of $H^{-1}_{newton}(w_{j-1})$. Shanno's algorithm builds up a trajectory of $w_0, w_1, w_2, \ldots$ iteratively according to

$$w_j = w_{j-1} + \gamma_j d_j \tag{2}$$

where $d_t$ is the search direction, written as

$$d_j = -H^{-1}\left(w_{j-1}\right)g\left(w_{j-1}\right) \tag{3}$$

Shanno's approximation of the inverse of the Hessian is given by [1, 218-220].

$$H^{-1}(w_{j-1}) = I - \frac{u_j d_{j-1}^H + d_{j-1}u_j^H}{d_j^H u_{j+1}} + c_j \frac{d_{j-1}d_{j-1}^H}{d_{j-1}^H u_j} \tag{4}$$

Suppose $\gamma_j$ is positive real number that satisfies the step size constraints

$$f\left(w_{j-1} + \gamma_j d_j\right) \le f\left(w_{j-1}\right) + \alpha\gamma_j g\left(w_{j-1}^H\right) d_j \tag{5}$$

$$g\left(w_{j-1} + \gamma_j d_j\right)^H d_j \ge \beta g\left(w_{j-1}\right)^H d_j \tag{6}$$

where $0 < \alpha < \beta < 1$, [1]. A procedure for calculating the step size is outlined in [1]. If $\gamma_j$ satisfies the above constraints, the algorithm is guaranteed to converge to a set of critical points [1], [3].

### 2.1. BLOCK SHANNO CC-CMA FORMATION AND GRADIENT CALCULATION

In order to apply Shanno's algorithm, we first extend the CC-CMA objective function to admit block processing. Let a block of data be defined as:

$$D_i = \left[x\left(\left(i-1\right)N\right) \vdots x\left(\left(i-1\right)N+1\right) \vdots \ldots \vdots x\left(iN-1\right)\right] \in C^{M\times N} \tag{7}$$

and define a block CC-CMA objective function

$$f(w) = \frac{1}{4N}\sum_{\Omega=0}^{N-1}\left[\left|w_c^H x_{l_1}((i-1)N) + \Omega\right|^2 - 1\right]^2 \tag{8}$$
$$+ \frac{\kappa}{N^2}\sum_{\delta=-M}^{M}\sum_{l_2=l_1}\left|\sum_{\Omega=0}^{N-1} w_c^H x_{l_1}((i-1)N+\Omega) x_{l_2}((i-1)N+\Omega-\delta) w_c\right|^2$$

In (8), $x((i-1)N+\Omega) \in C^M$ represents the received signal vector corresponding to the $n^{th}$ transmitted symbol interval, where $n = (i-1)N+\Omega$, $i \ge 0$, $0 \le \Omega < N$, and the length of $x$, i.e. $M$, is equal to the length of the filter tap. Here, $w_c$ is a complex filter tap weight vector for the $l^{th}$ equaliser. Since the Shanno algorithm requires the input data to be real, we develop an alternative form of the block objective function to accommodate complex input signals involved with complex channels. First, we define

$$w = \begin{bmatrix} w_c^r \\ w_c^i \end{bmatrix} \tag{9}$$

and then define two kinds of real vectors containing complex data,

$$x_f(n) = \begin{bmatrix} x^r(n) \\ -x^i(n) \end{bmatrix} \quad \text{and} \quad x_b(n) = \begin{bmatrix} x^i(n) \\ -x^r(n) \end{bmatrix} \tag{10}$$

Next, we define the real objective function and its gradient with respect to w as

$$f(w) = \frac{1}{4N}\sum_{n=0}^{N-1}\left[w^T X_1(n) w - 1\right]^2 + \frac{\kappa}{N^2}\sum_{\delta=-M}^{M}\sum_{l_1\neq l_2}\left[\sum_{n=0}^{N-1} w^T X_2(n) w\right]^2 \tag{11}$$

$$\nabla f(w) = g(w) = \frac{1}{N}\sum_{n=0}^{N-1}\left[w^T X_1(n) w - 1\right] X(n) w + \frac{\kappa}{N^2}\sum_{\delta=-M}^{M}\sum_{l_1\neq l_2}\left\{\left[\sum_{n=0}^{N-1} w^T X_2(n) w\right]\left[\sum_{n=0}^{N-1} X_2(n) w\right]\right\} \tag{12}$$

where

$$X_1(n) = x_{l_1,f}(n) x_{l_1,f}^T(n) + x_{l_1,b}(n) x_{l_1,b}^T(n) \tag{13}$$

$$X_2(n) = x_{l_1,f}(n) x_{l_2,f}^T(n-\delta) + x_{l_1,b}(n-\delta) x_{l_2,b}^T(n) \tag{14}$$

These equations are used in the following sections for the development of the SCGCC-CMA algorithm.

## 3. STOCHASTIC CONJUGATE GRADIENT BASED CC-CMA ALGORITHM (SCGCC-CMA)

### 3.1 SCGCC-CMA Algorithm Description:

To facilitate analysis, we introduce the quantities that we call "macro" quantities, which is denoted by superscript "$k$". The macro data block, which is an ensemble of $\{D_k, D_{k-1}, \ldots, D_{k-T+1}\}$ is denote by $D^k$. Note that $\{D_{k-\tau}\}$ are the complex input data blocks of the form defined in equation (8), where $\tau = 0, 1, \ldots, T-1$. Here $T$ is the search direction reset interval and $\tau$ is the search direction reset counter. The *macro filter tap weight vector* is defined as

$$w^k = \begin{bmatrix} w_k \\ w_{k-1} \\ \vdots \\ w_{k-T+1} \end{bmatrix} = \begin{bmatrix} w_0^k \\ w_1^k \\ \vdots \\ w_{T-1}^k \end{bmatrix} \tag{15}$$

and the *macro search direction* and *gradient vector* can defined as

$$d^k = \begin{bmatrix} d_k \\ d_{k-1} \\ \vdots \\ d_{k-T+1} \end{bmatrix} = \begin{bmatrix} d_0^k \\ d_1^k \\ \vdots \\ d_{T-1}^k \end{bmatrix} \quad \text{and} \quad g^k = \begin{bmatrix} g_k \\ g_{k-1} \\ \vdots \\ g_{k-T+1} \end{bmatrix} = \begin{bmatrix} g_0^k \\ g_1^k \\ \vdots \\ g_{T-1}^k \end{bmatrix}$$

The *macro gradient difference vector* is defined as

$$u^k = \begin{bmatrix} u_k \\ u_{k-1} \\ \vdots \\ u_{k-T+1} \end{bmatrix} = \begin{bmatrix} u_0^k \\ u_1^k \\ \vdots \\ u_{T-1}^k \end{bmatrix} = \begin{bmatrix} g_0^k - g_1^k \\ g_1^k - g_2^k \\ \vdots \\ g_{T-1}^k - g_T^k \end{bmatrix} \tag{16}$$

The *macro objective function* is defined as

$$f(w, D^k) = \frac{1}{T}\sum_{\tau=0}^{T-1} f_k(w_\tau, D_{k-\tau}) \tag{17}$$

where $f_k(\bullet)$ is the block CC-CMA objective function defined in (8) for data block $D_k$. The SCGCC-CMA objective function is defined as

$$f(w) = \mathrm{E}_{D^k}\{f(w, D^k)\} \tag{18}$$

In the macro quantity notation, the SCGCC-CMA algorithm may be described as follows:

**ALGORITHM 1: SCGCC-CMA:** Let $\alpha_k$ satisfy

$$\sum_{k=0}^{\infty}\alpha_{kT} = \infty \qquad \text{and} \qquad \sum_{k=0}^{\infty}\alpha_{kT}^2 < \infty$$

For $k = 0, 1, 2, ......$ update according to

$$w^k = w^{k-1} + \alpha_k z(w^{k-1}, D^k) \tag{19}$$

where

$$z(w^{k-1}, D^k) = \begin{bmatrix} \gamma_k d_0^k \\ \gamma_{k-1} d_1^k \\ \vdots \\ \gamma_{k-T+1} d_{T-1}^k \end{bmatrix} \tag{20}$$

*Assume there exists a continuous function $\Gamma_t$ such that $\gamma_{t-1} = \Gamma_t(w^{k-1}, D^k)$ is the properly chosen stepsize defined in (6) and (7),*

$$d_t^k = \begin{cases} -g_o^k & \text{if } t = 0 \\ \begin{cases} -g_t^k + a_t^k u_t^k + (b_t^k - c_t^k a_t^k) d_{t-1}^k \\ a_t^k = \frac{[d_{t-1}^k]^T g_t^k}{[d_{t-1}^k]^T u_t^k} \\ b_t^k = \frac{[u_t^k]^T g_t^k}{[d_{t-1}^k]^T u_t^k} \\ c_t^k = \gamma_{t-1}^k + \frac{|u_t^k|^2}{[d_{t-1}^k]^T u_t^k} \end{cases} & \end{cases}$$

Algorithm 1 is developed so as to conform to the framework of bounded discrete-time stochastic dynamic systems. These definitions will be later used in the SCGCC-CMA convergence analysis. The practical implementation version of the algorithm updates the filter tap weight vector once per data block $D^k$ using the Shanno update. A computer implementation version that does not rely on macro-quantities can be found in the following section.

### 3.2. SCGCC-CMA ALGORITHM: Simulation Version

Let: $i$ - *(Initially set to 0) index for data blocks,* $f(w)$ - *block CC-CMA objective function defined in (8) evaluated at* $w_i^j$, $g(w_i^j)$ - *gradient of block CC-CMA defined in (12) evaluated at* $w_i^j$;

*Step 1: Form a block of data $D_i$ (of size $M \times N$, where $N$ is the block size) from the received signal using (7). Set $j = 0$. Initialise the filter tap weight vector $w_0^0$ to all elements being zero except one element being unity.*

*Step 2: Do the following.*

$i$: $j = j + 1$,

$ii$: *Calculate the gradient of this block of data at $w_i^{j-1}$,*

$iii$: *If reset search direction flag is true (see Step 3);*

$$d_i^j = -g(w_i^{j-1}) + a_i^j u_i^j + (b_i^j - c_i^j a_i^j) d_i^{j-1} \tag{21}$$

$$\text{where} \quad u_i^j = g(w_i^{j-1}) - g(w_i^{j-2}), \; a_i^j = \frac{d_i^{j-1^T} g(w_i^{j-2})}{d^{j-1^T} u_i^j} \tag{22}$$

$$b_i^j = \frac{u_i^{j^T} g(w_i^{j-1})}{d^{j-1^T} u_i^j}, \; c_i^j = \gamma_i^{j-1} + \frac{|u_i^j|^2}{d^{j-1^T} u_i^j} \tag{23}$$

*iv: Update the filter tap weights $w_i^j = w_i^{j-1} + \gamma_i^j d_i^j$, where $\gamma_i^j$ is a positive real number that satisfies the constraints in (5) and (6).*

*v: If $|g(w_i^j)|$ is sufficiently small, or K blocks of data have been processed after the last search direction reset. Where K is an integer less that or equal to the dimension of the filter tap.*

*Step 3: Generate output for this block ($D_i$), and go to the Step 1 for the next block of data ($D_{i+1}$).*

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Let $f(w_j)$ denote the value of the objective function evaluated at $w_j$. We implement the so called *the back tracking line search* [2], [6] scheme to find the appropriate step size:

Given $\quad 0 < \alpha < 1/2, \quad 0 < \rho < 1,$

$(i)$ $\quad \gamma = 1;$

$(ii)$ $\quad$ *while* $f\left(w_j + \gamma_j d_j\right) > f\left(w_j\right) + \alpha \gamma_j g\left(w_j^H\right) d_j$

$\quad \gamma = \gamma \cdot \rho.$

From our simulation, we observed that an average of two iterations are needed for this *while* loop. The tendency is, the larger the block size, the smaller the number of iterations needed. For comparative purposes, we also compare the CC-CMA with two other block-based methods that are variants of the BSCC-CMA algorithm. In (21) if we set $a_i^j = 0$, a block conjugate gradient CC-CMA (BCGCC-CMA) algorithm results [1]. Furthermore, if we set $a_i^j$ and $b_i^j$ both equal to zero, then we obtain a block gradient descent CC-CMA (BGDCC-CMA) algorithm [1].

## 4. SCGCC-CMA CONVERGENCE ANALYSIS

With the SCGCC-CMA described in the foregoing form, we may now use a stochastic approximation theorem to prove convergence. The stochastic approximation theorem we use is discussed in [2, pp.169]. Similar stochastic approximation theorems are provided in [2], [3], [4]. The SCGCC-CMA convergence theorem makes use of the following definition for bounded discrete-time dynamical systems [2]. All subsequent analysis is centered around this class of systems.

**Definition 1:** *A stochastic sequence $D^0, D^1, D^2, ....,$ is a bounded stochastic sequence if there exists a finite real number $K$ such that $|D^k| < K$ with probability one for $k = 0, 1, 2...$*

**Definition 2:** *Let $D^0, D^1, D^2, ....,$ be a bounded stochastic sequence of independent and identically distributed M by N random matrices with the common probability mass (density) function $P_D = S \rightarrow (0, \infty)$, where $S \in \Re^{M \times N}$. Assume the sequence of strictly positive real numbers $\alpha_0, \alpha_1, ....$ satisfies*

$$\lim_{m \to \infty} = \left[\sum_{t=0}^{m} \alpha_t\right] = \infty \tag{24a}$$

$$\lim_{m \to \infty} = \left[\sum_{t=0}^{m} \alpha_t^2\right] < \infty \tag{24b}$$

*Let $z = \Re^M \times \Re^{M \times N} \rightarrow \Re$ be a continuous function in both arguments on $\Re^M$ and $\Re^{M \times N}$. Assume the stochastic sequence of dimensional random matrices $w^0, w^1, .....$ is a bounded stochastic process such that for $k = 0, 1, ....$*

$$w^k = w^{k-1} + \alpha_k z(w^{k-1}, D^k) \tag{25}$$

*Then the stochastic sequence $w^0, w^1, .....$ is generated by a bounded discrete-time stochastic dynamical system with generator function $z$ with respect to the stochastic sequence $\{D^k\}$ and $\gamma_k$.*

In order to show the macro-tap weight vector $\{w^k\}$ will converge to the set of critical points of the macro-objective function, we make use of the stochastic approximation theorem [2], which is cited as follows:

**THEOREM 1:Stochastic Approximation Theorem :** *Suppose the stochastic process: $z = \Re^M \times \Re^{M \times N} \rightarrow \Re^M$ is generated by the bounded discrete-time stochastic dynamical system defined in (25), where is the generated function for the dynamical system with respect to the stochastic sequence $\{D^k\} \in \Re^{M \times N}$. Assume there exists $\overline{z} : \Re^M \times \Re^M$ such that for all $w \in \Re^M$ :*

$$\overline{z}\,(w) = E_{D^k}\left[z(w, D^k)\right] < \infty \tag{26}$$

*Let $f : \Re^M \times \Re$ be a continuous function on $\Re^M$. Assume the gradient of $f$, $g : \Re^M \times \Re^M$, and the Hessian of $f$, $H : \Re^M \times \Re^{M \times M}$, exists and are continuous on $\Re^M$. Assume that for all $w \in \Re^M$ :*

$$g(w)^T \overline{z}\,(w) \leq 0 \tag{27}$$

*Let*

$$\Omega = \{w \in \Re^M : g(w)^T \overline{z}\,(w) = 0\} \tag{28}$$

*Then $w \rightarrow \Omega$ with probability one as $k \rightarrow \infty$.*

**THEOREM 2: SCGCC-CMA Convergence Theorem 2 :** *Suppose a given data block set is a bounded stochastic sequence. Let be the macro filter tap weight vector generated by the bounded discrete-time dynamical system defined by (19), in which $M$ is the filter tap length, and $T$ is the algorithm reset time. Also suppose the generator function $z$ is defined in (20). Then $w$ converges to the set of critical points of the macro objective function defined in (18) with probability one.*

**Proof:** Since the stepsize is assumed to satisfy (24a) and (24b), the system defined in (25) is a bounded discrete-time dynamical system. We can prove the convergence for SCGCC-CMA algorithm by applying the Stochastic Approximation Theorem. Thus we need to verify that: (1) The SCGCC-CMA objective function $f(w)$ as defined in (19), its gradient and Hessian are continuous (2). $\overline{z} = E_{D^k}\{z(w, D^k)\}$ exists and $[\nabla f(w)]^T \overline{z} < 0$ is satisfied.

First, from (11), (12) and from its Hessians, it can be easily seen that the block CMA objective function $f_k(w)$, its gradient $\nabla f_w(k)$ and Hessian $\nabla^2 f_k(w)$ are continuous. Thus from (18), we know that the macro objective function, its gradient and Hessian are continuous. Finally, from (19), we can see that the SCGCC-CMA objective function, its gradient and Hessian are also continuous. Secondly, $\overline{z}$ exists because $z$ is a continuous function of its two arguments, and $\{D^k\}$ is a bounded stochastic sequence, thus

$$\overline{z}\,(w) = \int z(w, D^k) P(D^k) dD^k \tag{29}$$

is finite for a given $w$. Thirdly, let us verify condition 3.We can rewrite the Shanno search direction vector as the follows [2, p.218]:

$$d_t^k = -H_t^k g_t^k \tag{30}$$

where $g_t^k$ is the gradient for data block $D_t^k$ and $H_t^k$ is the Shanno estimate of the inverse of the Hessian matrix,

$$H_t^k = A_t^k + \frac{\gamma_{t-1}^k d_{t-1}^k [d_{t-1}^k]^T}{[d_{t-1}^k]^T u_t^k} + v_t^k [v_t^k]^T \tag{31}$$

where $A_t^k = I - \frac{u_t^k [u_t^k]^T}{|u_t^k|^2}, u_t^k = g_t^k - g_{t-1}^k$ and $v_t^k = |u_t^k| [\frac{d_{t-1}^k}{[d_{t-1}^k]^2 u_t^k} + \frac{u_t^k}{|u_t^k|^2}]$. Since the search direction is reset to $-g_t^k$ every $T$ steps, for case of $t = 0$:

$$d_0^k = -g_0^k \tag{32}$$

we have $[d_0^k]^T g_0^k < 0$. For case of $t = 1$, $d_1^k = -H_1^k g_1^k$. From (31), we can see that if $[d_0^k]^T u_1^k > 0$, then Hessian $H_1^k$ is positive definite, and $[d_1^k]^T g_1^k < 0$. By applying the step size constraint in (6), we have

$$[d_0^k]^T g_1^k - \gamma_1^k [d_0^k]^T g_0^k > 0 \tag{33}$$

thus

$$[d_0^k]^T (g_1^k - g_0^k) + (1 - \gamma_1^k) [d_0^k]^T g_0^k > 0 \tag{34}$$

that is,

$$[d_0^k]^T u_1^k > 0 \tag{35}$$

Therefore, from (31), we know that Hessian matrix $H_1^k$ is positive definite [2], and $[d_1^k]^T g_1^k < 0$. If we proceed with a similar derivation for $t = 2, 3, ...T$, we can show that is is semi-positive definite for all $t$, and $[d_t^k]^T g_t^k < 0$ for all $t$.

Thus if we start from $w_{t-1}^k$ which is generated from data block $D_{t-1}^k$, the SCGCC-CMA algorithm drives the filter tap weight vector to $w_t^k$ based on data block $D_t^k$ with the search direction vector $d_t^k$. Hence moving from $w_{t-1}^k$ to $w_t^k$ is a down-hill step with respect to the block CC-CMA objective function $f(w, D_t^k)$. Since every step that SCGCC-CMA moves down-hill, the averaged search direction $\overline{z}\,(w)$ leads down-hill of the SCGCC-CMA objective function of $f(w)$. Now by implementing the Stochastic Approximation Theorem, we have proved that the SCGCC-CMA algorithm converges to a set of critical points.

With the knowledge of the block CC-CMA objective function [1], [5], we know that the critical points that SCGCC-CMA converges to is actually the global minimum. In the proof, we assumed that stepsize $\alpha_k$ satisfies the conditions (24a) and (24b). However, in the computer implementation of the algorithm these constraints are relaxed by setting $\alpha_k$ to one, and a satisfactory convergence result is observed. Moreover, the stepsize constraints (5) and (6) for $\gamma_t^k$ can also be relaxed.

## 5. SLIDING WINDOW SCGCC-CMA

To provide more control over the balance between the computational complexity and performance, we further introduce the sliding-window type SCGCC-CMA. The sliding-window SCGCC-CMA also takes in $N$ data at a time, but generate only $K$ outputs, $K \leq N$, that is, $K - N$ data elements from the current block will be reused for the next data block. The sliding-window SCGCC-CMA scheme provides control over the balance between the computational complexity and performance. Such flexibility will be very useful in higher data rate system. By adding more overlap between blocks, we increase the convergence rate and lower the residual error floor. However, we also raise the computational complexity.

## 6. SIMULATIONS

In a QPSK system with source alphabet (±1/sqrt(2) ±(1/sqrt(2))j), we assume $d = 3$ users, $R = 4$ sensors twelve random complex channels of order 1 ($M = 1$) and three 12 tap (i.e. $N = 2$ and R $(N + 1) = 12$) space-time equalisers adapted with the proposed algorithms. The block size for used for BSCC-CMA, SCGCC-CMA, BCGCC-CMA and BGDCC-CMA was 100. The channel convolution matrix is full rank square matrix of dimension of 12 and $\kappa = 4$. White Gaussian noise of SNR 30dB is present at the channel output. The forgetting factor $\lambda$ for the RLS version of CC-CMA [1] and the step size $\mu$ are set to 0.99 and 0.01 respectively. The constellation diagrams of the output of the equalizers are shown in Fig 1. (d), (e) and (f). After approximately 700 samples, all equalisers of RLSCC-CMA algorithm give open eye pattern. Comparing the residual error, which is defined as $\frac{\|h_i(k)\|_2^2 - max(|h_i(k)|)^2}{max(|h_i(k)|)^2}$, in Fig. 1 (a), (b), and (c). RLSCC-CMA algorithm shows much faster convergence than the conventional CC-CMA algorithm. In fact, for equalizer-2, clear open-eye constellation cannot be achieved even with 3900 samples. In Fig. 1. (g), (h) and (i), the combined channel + equalizer the three equalizers is given. The different position sections of the largest impulse confirm the retrieval all three sources. That is, equalizer-1, 2 and 3 retrieve sources with different delays. Fig. 2 (a), (b), (c) shows the ISI of proposed SCGCC-CMA, BSCC-CMA, BSCGCC-CMA and BGDCC-CMA. All of the proposed algorithms converge within the first 5 blocks. Fig. 2 (g), (h), (i) shows the combined channel + equalizer response for the proposed BSCC-CMA algorithm. Fig.2 (d), (e), (f) shows the output constellation for the BSCC-CMA algorithm.

## 7. CONCLUSIONS

To overcome the slow convergence of the conventional CC-CMA algorithm, several new quasi-Newton adaptive algorithms with rapid convergence property are proposed based upon the cross-correlation and constant modulus (CC-CM) criterion, namely the stochastic conjugate gradient CC-CMA (SCGCC-CMA) and variants of the BSCC-CMA algorithms or fast convergent quasi-Newton type cross-correlation and constant modulus algorithm (FCQN-CCCMA). Simulation shows the all of these proposed algorithms outperforms the conventional CC-CMA in terms of their super fast convergence and the compactness of their output constellations. We also studied the convergence analysis of the proposed algorithms.

## 8. REFERENCES

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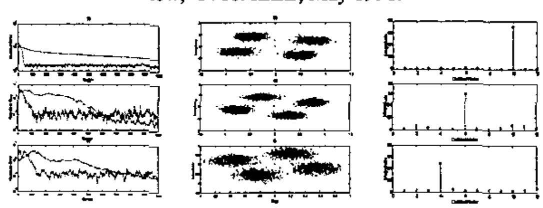

Figure 1. The Performance comparison between RLSCC-CMA and CC-CMA (a), (b) and (c) Residual error of EQ-1,2 and 3. (d), (e), and (f) show the Eye diagram of EQ-1, 2 and 3 after 500 samples. (g), (h) and (i) shows the combined channel + EQ-1 impulse response of h1,h2 and h3.

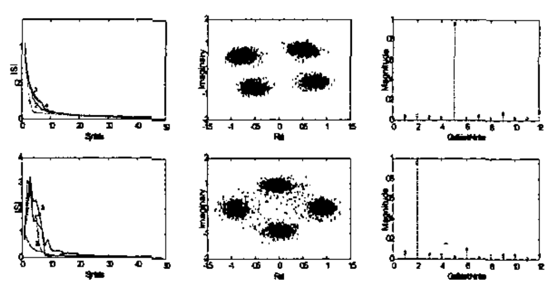

Fig. 2. Intersymbol Interference between the proposed SCGCC-CMA (2), BSCC-CMA (1), BCGCC-CMA (3), BGDCC-CMA (4) for EQ-1, EQ-2.