http://dx.doi.org/10.5540/DINCON.2011.001.1.0113

DINCON 2011
10ª Conferência Brasileira de Dinâmica, Controle e Aplicações
28 de agosto a 1º de setembro de 2011

STUDY OF ENERGY DISSIPATION IN BEAMS SUBJECTED TO MOVING LOADS

Renata M. Soares [1], *Zenon J. G. N. del Prado* [2], *Paulo B. Gonçalves* [3]

[1] Federal University of Goiás, Goiânia, Brazil, msrenata@gmail.com.br
[2] Federal University of Goiás, Goiânia, Brazil, zenon@eec.ufg.br
[3] Pontifical University of Rio de Janeiro, Rio de Janeiro, Brazil, paulo@puc-rio.br

Abstract: In this work the vibrations control and energy dissipation of a simply supported beam subjected to moving loads is studied. The beam is controlled by a moving vibration absorber that acts as energy dissipator. The absorber is described as a linear spring-mass-damper system. The results show the importance of the position of the damper on the energy dissipator.

Keywords: energy dissipation, beam oscillations, engineering applications.

1. INTRODUCTION

The study of bridge oscillations and control is a problem that has been the object of interest of engineers and scientist over the last century [1-3]. For example, Den Hartog [4] derived the optimum parameters of the absorber for suppressing the dynamic response of a single degree-of-freedom spring-mass system. Here, only a few investigations will be cited.

Greco and Santini [2], using an extension of the complex mode superposition method, analyzed the dynamic problem of a continuous beam with two end rotational viscous dampers under a single moving load. They concluded that the damper's effectiveness is strongly dependant on the load velocity and proved that, in the relevant range of velocities, a considerable reduction of the dynamic response of the beam is to be expected if the damper's constants are selected properly.

Wu [5] proposed the use of helical absorbers to reduce the vibrations of beams subjected to moving loads. To study the behavior of the beam, the governing equations were reduced to the first modal coordinate and, following Den Hartog's approach [4], this simplified model was used to obtain optimal values for the stiffness and damping ratio of the absorber. The possibility of reduction of the resonant vibration of simple beams under moving loads by increasing the structural damping with passive energy dissipation devices was evaluated by Museros and Martinez-Rodrigo [3]. The authors used a linear viscous damper (FVDs) to connect the main beam, which carries the loads, to an auxiliary beam placed underneath the main one. The results show that the resonant response of the main beam can be drastically reduced with this type of device.

Moreover, the damper attached to the beam can act as passive absorber of vibration energy. Georgiades and Vakakis [6] showed numerically that an appropriately designed and placed nonlinear mass-spring absorber, designated nonlinear energy sink (NES) can passively absorb and locally dissipate a major portion of the shock energy of the beam, a result that paves the way for the implementation of the NES concept to flexible systems. Recently, Samani and Pellicano [7] analyzed the effectiveness of a dynamic vibration absorber applied to a beam excited by moving loads. The performance of the dynamic dampers in vibration reduction was estimated through the maximum amplitude of vibration and by the amount of energy dissipated by the dynamic damper.

In this work, the Euler-Bernoulli linear beam theory is used to study the vibrations control and energy dissipation of simply supported beams subjected to moving loads and controlled by a fixed or moving absorber. The beam is considered as a linear elastic continuous system and the absorber is described as a linear spring-mass-damper system moving with a specified velocity along the beam. A modal expansion with five modes is used to model the lateral displacements of the beam and the Galerkin method is used to obtain a set of discretized equations of motion which are, in turn, solved by the Runge-Kutta method. The rate of input energy dissipation is obtained by the ratio between the energy absorbed by the damper and internal energy of the beam due to loading. The obtained results show the importance of position and velocity of the damper on the vibration control and in energy dissipation of the beam. This can be used for engineers to optimize the position of vibration absorbers.

2. PROBLEM FORMULATION

Consider a simply supported elastic beam with length L, Young's modulus E, inertia I, distributed mass $\overline{m}$ and damping coefficient c subjected to a moving load $F(x,t)$ with velocity V_L as shown in Figure 1. The beam is connected to an absorber represented by a small mass m_A, a linear spring with stiffness k_A and a linear viscous damper with damping coefficient λ. It is assumed that the absorber can either be fixed or move along the beam with a velocity V_A.

The absorber can passively absorb a major portion of the vibration energy of the beam induced by the load, thus acting as an energy dissipator. Furthermore, the targeted energy transfer from the linear beam to the absorber can be optimized by appropriate design and placement of the absorber [6].

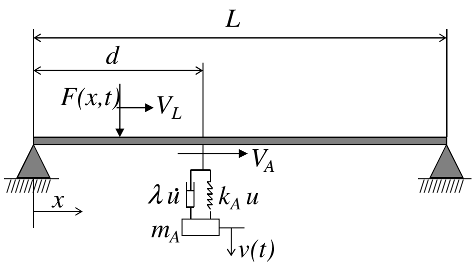


Figure 1 - The controlled beam model

In this work the mathematical formulation will follow that previously presented by Samani and Pellicano [7]. The partial differential equations of motion governing the flexural behavior of a simply supported beam using the linear Euler-Bernoulli theory and absorber can be found in the works Yang, Yau and Hsu [1], Greco and Santini [2]; and Muserosa and Martinez-Rodrigo [3] and are written as:

$$EI\frac{\partial^4 y}{\partial x^4}+\overline{m}\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}+\left[k_A u+\lambda\frac{\partial u}{\partial t}\right]G(x,t)=F(x,t) \quad (1)$$

$$m_A\frac{\partial^2 v}{\partial t^2}-k_A u+\lambda\frac{\partial u}{\partial t}=0 \qquad x\in(0,L)\ \ t>0 \quad (2)$$

where $y(t)$ represents the transversal displacement field of the beam; $v(t)$ is the absolute position of the mass m_A; d is the absorber position and $u(t) = y(d, t)-v(t)$.

In Eq. (1) the term $\left[k_A u+\lambda\partial u/\partial t\right]G(x,t)$ is the force exerted by the absorber and $G(x, t)$ is given by:

$$G(x,t)=\delta(x-d) \text{ For a fixed absorber;} \quad (3)$$

$$G(x,t)=\delta(x-V_A t)\,H\left(\frac{L}{V_A}-t\right) \text{ For a moving absorber;} \quad (4)$$

where $x = d$ represents the location of the damper in the beam at time t; V_A is the absorber velocity; δ is the Dirac delta function which defines the location of the dynamic damper while $H(t)$ is the Heaviside function.

The external force $F(x, t)$ is a moving load given by:

$$F(x,t)=F_o\,\delta(x-V_L t)\,H\left(\frac{L}{V_L}-t\right). \quad (5)$$

The attached mass is small compared to the beam mass. In this work, the lumped mass of the absorber is taken to be 5% of the total mass of the beam [5]. The equations of motion of the system, represented by Eqs. (1) and (2), are analyzed after projecting the partial differential Eq. (1) into a complete orthonormal basis [7]. The eigenfunctions of the simply supported beam with no attachments can be used as interpolating functions. They are given by:

$$\phi_r(x)=\sin\left(\frac{r\pi x}{L}\right) \qquad r = 1, 2, 3 \ldots. \quad (6)$$

The natural frequency of the beam for the r^{th} mode is given by $\omega_r=(\pi r)^2\sqrt{EI/\overline{m}L^4}$ and the transverse vibration of the beam can be assumed as [3, 7]:

$$y(x,t)=\sum_{r=1}^{\infty}A_r(t)\,\phi_r(x) \quad (7)$$

where $A_r(t)$ are the unknown functions of time and $\phi_r(x)$ is given by Eq. (6).

Substituting Eq. (7) into Eqs. (1) and (2), applying the Galerkin method and using the orthonormality conditions, the following system of equations is obtained:

$$\frac{\overline{m}L}{2}\ddot{A}_p(t)+\xi_p\omega_p\overline{m}L\,\dot{A}_p(t)+\frac{\omega_p^{2}\overline{m}L}{2}A_p(t)+ +\left\{\begin{matrix} k_2\left[\sum_{r=1}^{\infty}A_r(t)\,\phi_r(D)-v(t)\right] \\ \lambda\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\,\phi_r(D)-\dot{v}(t)\right]\end{matrix}\right\}\phi_p(D)\,G(t)= F_o\phi_p(V_L t)\,H\left(\frac{L}{V_L}-t\right) \quad (8)$$

$$m_A\ddot{v}(t)-k_A\left[\sum_{r=1}^{\infty}A_r(t)\phi_r(D)-v(t)\right] -\lambda\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(D)-\dot{v}(t)\right]=0 \quad (9)$$

where $\dot{A}_p(t)=dA_p/dt$; $D = d$ and $G(t) =1$ for fixed absorber; $D = V_A t$ and $G(t) = H\ (1/V_A - t)$ for moving absorber.

The portion of the input energy dissipated by the viscous damper at time t_1 is computed by the expression [6, 7]:

$$\eta=\frac{E_{ED}}{E_{IN}}=\frac{\int_0^{t_1}\lambda\left[\dot{v}(t)-\sum_{r=1}^{n}\dot{A}_r(t)\,\phi_r(D)\right]^2 dt}{\int_0^{t_0}F_o\left[\sum_{r=1}^{n}\dot{A}_r(t)\,\phi_r(V_L t)\right]^2 dt} \quad (10)$$

E_{ED} is the energy passively absorbed and locally dissipated by the absorber, E_{IN} represents the total energy input of the beam due to the load and t_0 is the load duration in the beam.

3. NUMERICAL RESULTS

For the numerical analysis, consider a beam with Young's modulus $E = 206.8$ GPa, mass density $\rho = 7820$ Kg/m^3, cross-sectional area 0.03 m x 0.03 m and $F_o = 9.8$ N. Three different lengths are considered say beam $L4$, $L5$ and $L6$ and are chosen only to extend the work by Samani and Pellicano [7]. They are shown in Table 1 together with the associated system parameters.

DINCON 2011
10ª Conferência Brasileira de Dinâmica, Controle e Aplicações
28 de agosto a 1º de setembro de 2011

The optimal control parameters were found using the Den Hartog's theory [4] where the ratio between the masses of absorber and principal system is $\mu = 0.1$, the optimum tuned mass is $\alpha_o = 0.909$ and the optimal damping ratio is found to be $\xi_{2ot} = 0.18464$.

Table 1 – Parameters of the beam and absorber with $\mu = 0.1$; $\alpha_o = 0.909$ and $\xi_{2ot} = 0.18464$.

	L4	*L5*	*L6*
L (m)	4	5	6
$\overline{m}$ (Kg/m)	7.038	7.038	7.038
ω_0 (rad/s)	27.471	17.582	12.209
m_A (Kg)	1.4076	1.7595	2.1114
k_A (N/m)	877.92	494.45	286.11
λ (Ns/m)	12.98	10.89	9.076

3.1. *Fixed absorber and moving load*

To check the accuracy of the present model, consider the system shown in Figure 1 with a fixed absorber ($V_A(t) = 0$ and $d = 0.5L$) and external force with constant amplitude (F_o) and constant velocity $V_L(t) \neq 0$. Figure 2 shows the maximum displacement at the mid-span of the $L4$ beam with and without damper and increasing values of load velocity. As can be observed, for this beam the maximum displacement occurs for a velocity $V_L = 21.2$ m/s. The inclusion of the absorber reduces the maximum displacement of the beam up to 5%.

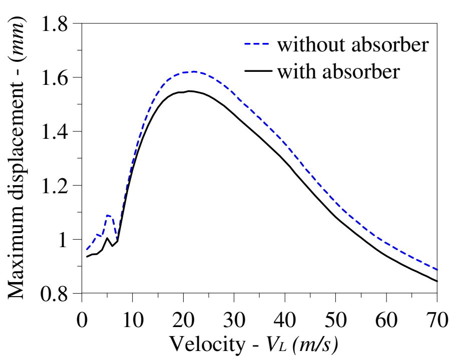


Figure 2 - Maximum displacement at mid-span of the beam ($x = 0.5L$). Beam *L4* with and without absorber.

The optimization of the dynamic damper is focused on the minimization of the maximum beam displacement. Then, the location of the dynamic absorber can be varied to find the optimum absorber position. For each beam, the absorber position is varied considering the load velocities V_L that generates the maximum displacement at the beam. The obtained results are displayed in Table 2 where the ratio $l_{máx}/L$ indicates the position at which the maximum displacement occurs and d/L indicates the best position of absorber.

Figure 3 shows the fraction of energy dissipated by the viscous damper, η, for different absorber positions for $t_1 = 1$ s and $t_1 = 30$ s and two different damping ratios of the principal system. As can be observed in both Table 2 and Figure 3, the best absorber position is located close to the mid-span of the beam.

Table 2 – Location of maximum beam displacement and optimal absorber position.

	V_L (m/s)	$l_{máx}/L$	d/L
L4	21.2	0.525	0.540
L5	16.7	0.520	0.544
L6	13.9	0.533	0.543

3.2. *Moving absorber and moving load*

Consider now the system of Figure 1, with a moving load and a moving absorber with velocity $V_A(t) =$ constant. Figure 4 shows the maximum displacement at the mid-span of the beam for the three adopted cases. Five different absorber velocities V_A are considered as a function of the load velocity V_L. As can be observed, all curves are rather similar but the maximum displacement depends of both load and absorber velocities. The maximum displacement reduction is obtained when V_A is near V_L.

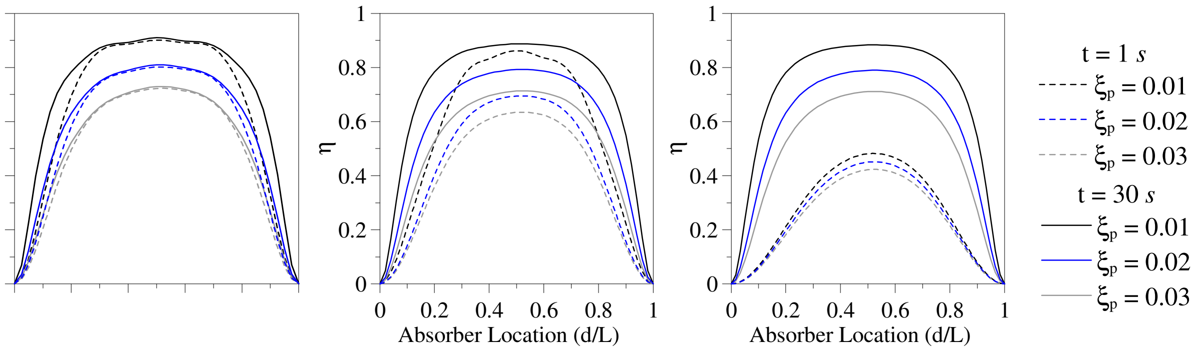
Figure 5 shows the fraction of energy dissipated, η, by the viscous damper for different load and absorber velocities. It is possible to observe that the maximum dissipated energy is obtained when the load velocity is close to the maximum velocities of Table 2 and, for low ratios between absorber and load velocities.

4. CONCLUSION

In this work the control vibration of simply supported beams subjected to moving loads was studied. Both, fixed and moving absorber were considered, for this the linear Euler-Bernoulli beam theory was used together with the Galerkin method to obtain a set of ordinary differential equations of dynamic equilibrium.

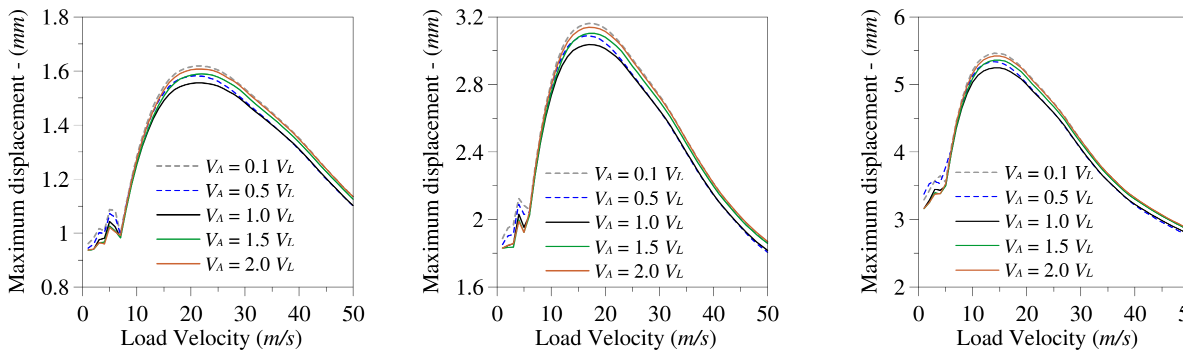
For fixed absorber, the obtained results show the influence of load velocity and absorber position on the reduction of the maximum beam displacement. Results show that, when the absorber position is close to the beam mid-span, there is a maximum transversal displacement reduction.

For moving absorber, it is possible to observe that the maximum reduction is obtained when the ratio between load and absorber velocity is close to one and the maximum energy dissipated is obtained for low V_A/V_L ratios which correspond to the load velocity that generates maximum displacement.



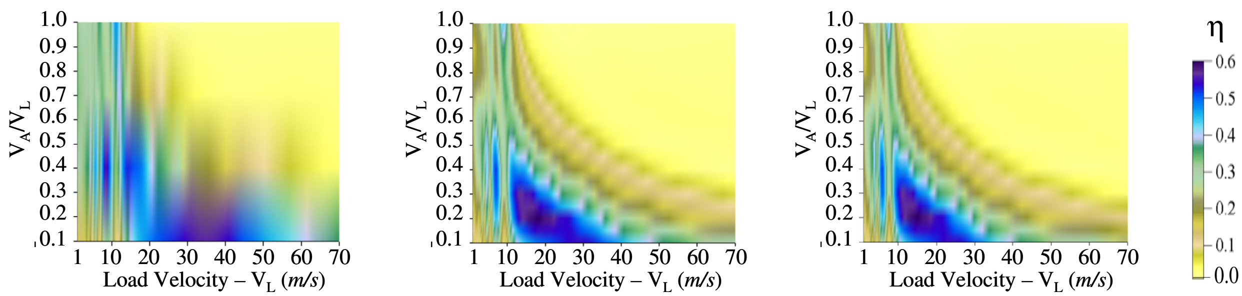
(a) *L4* beam (b) *L5* beam (c) *L6* beam

Figure 3 - Input energy dissipated versus absorber position. (a) beam *L4*, (b) *L5* and (c) *L6*.



(a) *L4* beam (b) *L5* beam (c) *L6* beam

Figure 4 - Maximum displacement at mid-span of the beams ($x = 0.5L$) versus load velocity (V_L). (a) beam *L4*, (b) *L5* and (c) *L6*.



(a) *L4* beam (b) *L5* beam (c) *L6* beam

Figure 5 - Portion of input energy dissipated as a function of Load velocity (V_L) and Absorber velocity (V_A) (a) beam *L4*, (b) *L5* and (c) *L6*.

ACKNOWLEDGMENTS

The authors acknowledge the financial support of the Brazilian research agencies CAPES, CNPq and FAPERJ.

REFERENCES

[1]DOI Yang Y. B., Yau J. D. Hsu L. C. "Vibration of simple beams due to trains moving at high speeds." Engineering Structures, Vol. 19, No 11 pp. 936-944, 1997.

[2]DOI Greco A., Santini A. "Dynamic response of a flexural non-classically damped continuous beam under moving loadings". Computers and Structures, Vol. 80; pp. 1945–1953, 2002.

[3]DOI Museros P., Martinez-Rodrigo M. D. "Vibration control of simply supported beams under moving loads using fluid viscous dampers. Journal of Sound and Vibration", Vol. 300, pp 292–315, 2007.

[4] Den Hartog J.P. Mechanical Vibrations. McGraw-Hill, 1956.

[5]DOI Wu J.J."Study on the inertia effect of helical spring of the absorber on suppressing the dynamic responses of a beam subjected to a moving load". Journal Sound and Vibration, Vol. 297, pp. 981-999, 2006.

[6]DOI Georgiades, F.,Vakakis, A. F. "Dynamics of a linear beam with an attached local nonlinear energy sink." Communications in Nonlinear Science and Numerical Simulation, Vol. 12, pp. 643-651, 2007.

[7]DOI Samani F.A., Pellicano F. "Vibration reduction on beams subjected to moving loads using linear and nonlinear dynamic absorbers". Journal Sound and Vibration, Vol. 325, pp. 742-754, 2009.