Elimination of Harmonics in a Multilevel Converter for HEV Applications

Leon M. Tolbert[1,2], John Chiasson[1], Keith McKenzie[1] and Zhong Du[1]
[1]ECE Department, The University of Tennessee, Knoxville, TN 37996-2100
tolbert@utk.edu, chiasson@utk.edu, kmc18@utk.edu, zdu1@utk.edu
[2]Power Electronics and Electric Machinery Research Center
Oak Ridge National Laboratory, NTRC, 2360 Cherahala Boulevard, Knoxville, TN 37932

Abstract— **One promising technology to interface battery packs in electric and hybrid electric vehicles are multilevel converters. In the work presented here, it is shown how the switching times (angles) in a multilevel inverter can be chosen to achieve a required fundamental voltage and not generate specific higher order harmonics. The method gives a complete solution to the problem in that all possible solutions are found.**

Keywords— **Hybrid Electric Vehicles, Multilevel Converters, Harmonic Elimination, Resultants**

I. Introduction

Designs for heavy duty hybrid-electric vehicles (HEVs) that have large electric drives such as tractor trailers, transfer trucks, or military vehicles will require advanced power electronic inverters to meet the high power demands (> 100 kW) required of them. Development of large electric drive trains for these vehicles will result in increased fuel efficiency, lower emissions, and likely better vehicle performance (acceleration and braking). One promising technology to interface battery packs in electric and hybrid electric vehicles are multilevel converters. Transformerless multilevel inverters are particularly suited for this application because of the high VA ratings possible with these inverters [6]. The multilevel voltage source inverter's unique structure allows it to reach high voltages with low harmonics without the use of transformers or series-connected, synchronized-switching devices. The general function of the multilevel inverter is to synthesize a desired voltage from several levels of dc voltages. For this reason, multilevel inverters can easily provide the high power required of a large electric traction drive. For parallel-configured HEVs, a cascaded H-bridges inverter can be used to drive the traction motor from a set of batteries, ultracapacitors, or fuel cells. The use of a cascade inverter also allows the HEV drive to continue to operate even with the failure of one level of the inverter structure [8][11][10].

In the work presented here, it is shown how the switching times (angles) in a multilevel inverter can be chosen to achieve a required fundamental voltage and not generate specified higher order harmonics. The method gives a complete solution to the problem in that all possible solutions are found.

II. Cascaded H-bridges

The cascade multilevel inverter consists of a series of H-bridge (single-phase full-bridge) inverter units. The general function of this multilevel inverter is to synthesize a desired voltage from several separate dc sources (SDCSs), which may be obtained from batteries, fuel cells, or ultracapacitors in a HEV. Figure 1 shows a single-phase structure of a cascade inverter with SDCSs [6].Each SDCS is connected

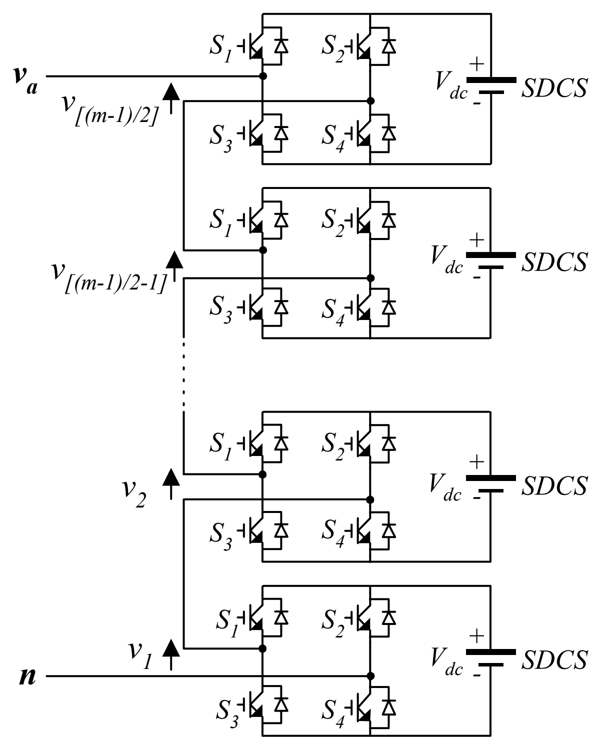

Fig. 1. Single-phase structure of a multilevel cascaded H-bridges inverter.

to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs, $+V_{dc}, 0$ and $-V_{dc}$ by connecting the dc source to the ac output side by different combinations of the four switches, S_1, S_2, S_3 and S_4. The ac output of each level's full-bridge inverter is connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. The number of output phase voltage levels in a cascade multilevel inverter is then $2s+1$, where s is the number of dc sources. An example phase voltage waveform for an 11-level cascaded multilevel inverter with five SDCSs ($s = 5$) and five full bridges is shown in Figure 2. The output phase voltage is given by $v_{an} = v_{a1} + v_{a2} + v_{a3} + v_{a4} + v_{a5}$.With

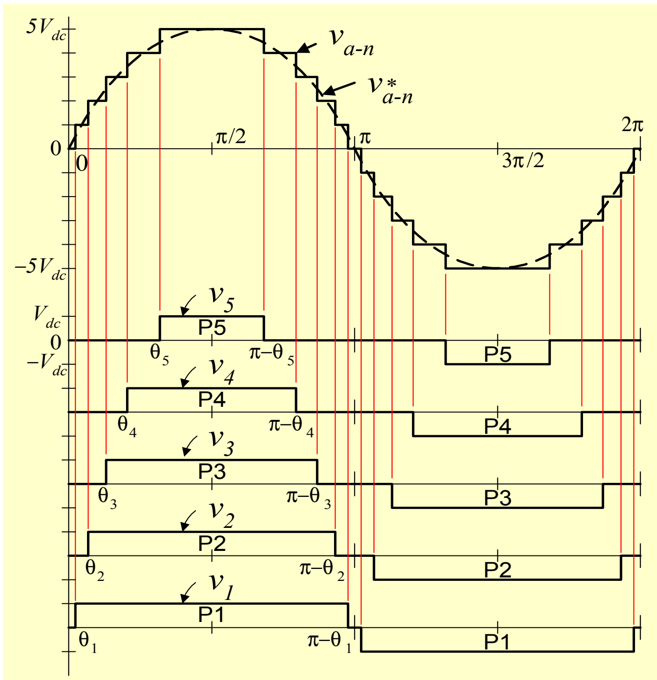

Fig. 2. Output waveform of an 11-level cascade multilevel inverter.

enough levels and an *appropriate* switching algorithm, the multilevel inverter results in an output voltage that is almost sinusoidal.

III. Switching Algorithm for the Multilevel Converter

The Fourier series expansion of the (stepped) output voltage waveform of the multilevel inverter as shown in Figure 2 is [2][8][11][10]

$$V(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{n\pi}\left(\cos(n\theta_1) + \cos(n\theta_2) + \cdots + \cos(n\theta_s)\right)\sin(n\omega t) \tag{1}$$

where s is the number of dc sources. Ideally, given a desired fundamental voltage V_1, one wants to determine the switching angles $\theta_1, \cdots, \theta_n$ so that (1) becomes $V(\omega t) = V_1 \sin(\omega t)$. In practice, one is left with trying to do this approximately. The approach here is to eliminate the lower dominant harmonics and filter the output to remove the higher residual frequencies. Specifically, the goal here is to choose the switching angles $0 \le \theta_1 < \theta_2 < \cdots < \theta_s \le \pi/2$ so as to make the first harmonic equal to the desired fundamental voltage V_1 and specified higher harmonics of $V(\omega t)$ equal to zero. As the application of interest here is a three-phase motor drive, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Consequently, the desire here is to cancel the $5^{th}, 7^{th}, 11^{th}, 13^{th}$ order harmonics as they dominate the total harmonic distortion.

The mathematical statement of these conditions is then

$$\begin{aligned}
\frac{4V_{dc}}{\pi}\left(\cos(\theta_1) + \cos(\theta_2) + \cdots + \cos(\theta_s)\right) &= V_1 \\
\cos(5\theta_1) + \cos(5\theta_2) + \cdots + \cos(5\theta_s) &= 0 \\
\cos(7\theta_1) + \cos(7\theta_2) + \cdots + \cos(7\theta_s) &= 0 \\
\cos(11\theta_1) + \cos(11\theta_2) + \cdots + \cos(11\theta_s) &= 0 \\
\cos(13\theta_1) + \cos(13\theta_2) + \cdots + \cos(13\theta_s) &= 0.
\end{aligned} \tag{2}$$

This is a system of 5 transcendental equations in the unknowns $\theta_1, \theta_2, \cdots, \theta_s$ so that at least 5 steps are needed ($s = 5$) if there is to be any chance of a solution. One approach to solving this set of nonlinear transcendental equations (2) is to use an iterative method such as the Newton-Raphson method [3][8][11][10]. The correct solution to the conditions (2) would mean that the output voltage of the $11-$level inverter would not contain the $5^{th}, 7^{th}, 11^{th}$ and 13^{th} order harmonic components. However, such iterative techniques do not find all solutions; they find only one solution or do not converge. In contrast, the method presented here will determine any and all solutions to the problem. In particular, it will be shown below that a solution exists for only specific ranges of the modulation index[1] $m_a \triangleq V_1/\left(s4V_{dc}/\pi\right)$ and for some ranges there are more than one solution set. This method is based on the theory of resultants of polynomials [2].

To proceed, let $s = 5$, and substitute $x_1 = \cos(\theta_1), x_2 = \cos(\theta_2), x_3 = \cos(\theta_3), x_4 = \cos(\theta_4), x_5 = \cos(\theta_5)$ into (2) which upon using some trigonometric identities, (2) becomes

$$\begin{aligned}
p_1(x) &\triangleq x_1 + x_2 + x_3 + x_4 + x_5 - m = 0 \\
p_5(x) &\triangleq \sum_{i=1}^{5}\left(5x_i - 20x_i^3 + 16x_i^5\right) = 0 \\
p_7(x) &\triangleq \sum_{i=1}^{5}\left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0 \\
p_{11}(x) &\triangleq \sum_{i=1}^{5}\left(-11x_i + 220x_i^3 - 1232x_i^5 + \right. \\
&\qquad \left. 2816x_i^7 - 2816x_i^9 + 1024x_i^{11}\right) = 0 \\
p_{13}(x) &\triangleq \sum_{i=1}^{5}\left(13x_i - 364x_i^3 + 2912x_i^5 - 9984x_i^7\right. \\
&\qquad \left. + 16640x_i^9 - 13312x_i^{11} + 4096x_i^{13}\right) = 0
\end{aligned} \tag{3}$$

where $x = (x_1, x_2, x_3, x_4, x_5)$ and $m \triangleq V_1/\left(4V_{dc}/\pi\right) = sm_a$. This is a set of five equations in the five unknowns

[1] Each inverter has a dc source of V_{dc} so that the maximum output voltage of the multilevel inverter is sV_{dc}. A square wave of amplitude sV_{dc} results in the maximum fundamental output possible of $V_{1\,\max} = 4sV_{dc}/\pi$. The modulation index is therefore $m_a \triangleq V_1/V_{1\,\max} = V_1/\left(s4V_{dc}/\pi\right)$.

x_1, x_2, x_3, x_4, x_5. The interest here is to find solutions x for $m \in [0, s]$ which satisfy $0 \le x_5 < \cdots < x_2 < x_1 \le 1$. This development has resulted in a set of polynomial equations rather than trigonometric equations. Though the degree is high, the theory of resultants of polynomials [4][5] provides a systematic way to determine *all* the zeros of the set of polynomials (3). This theory is summarized next.

A. Resultants

Given the two polynomials $a(x_1, x_2)$ and $b(x_1, x_2)$

$$\begin{aligned} a(x_1, x_2) &= a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1) \\ b(x_1, x_2) &= b_3(x_1)x_2^3 + b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1) \end{aligned}$$

how does one find their common zeros? That is, the values (x_{10}, x_{20}) such that

$$a(x_{10}, x_{20}) = b(x_{10}, x_{20}) = 0.$$

A consequence of the famed *Nullstellensatz* theorem of Hilbert [4] is that the polynomials $a(x_1, x_2)$ and $b(x_1, x_2)$ do *not* have a common zero if and only if there exists another pair of polynomials

$$\begin{aligned} \alpha(x_1, x_2) &= \alpha_2(x_1)x_2^2 + \alpha_1(x_1)x_2 + \alpha_0(x_1) \\ \beta(x_1, x_2) &= \beta_2(x_1)x_2^2 + \beta_1(x_1)x_2 + \beta_0(x_1) \end{aligned}$$

such that

$$\alpha(x_1, x_2)a(x_1, x_2) + \beta(x_1, x_2)b(x_1, x_2) = 1.$$

Let

$$\begin{aligned} a(x_1, x_2) &= a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1) \\ b(x_1, x_2) &= b_3(x_1)x_2^3 + b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1) \end{aligned}$$

and

$$\begin{aligned} \alpha(x_1, x_2) &= \alpha_2(x_1)x_2^2 + \alpha_1(x_1)x_2 + \alpha_0(x_1) \\ \beta(x_1, x_2) &= \beta_2(x_1)x_2^2 + \beta_1(x_1)x_2 + \beta_0(x_1). \end{aligned}$$

Equating powers of x_2, the equation

$$\alpha(x_1, x_2)a(x_1, x_2) + \beta(x_1, x_2)b(x_1, x_2) = 1$$

may be rewritten in matrix form as

$$S_{a,b}(x_1) \begin{bmatrix} \alpha_0(x_1) \\ \alpha_1(x_1) \\ \alpha_2(x_1) \\ \beta_0(x_1) \\ \beta_1(x_1) \\ \beta_2(x_1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$S_{a,b}(x_1) \triangleq \begin{bmatrix} a_0(x_1) & 0 & 0 & b_0(x_1) & 0 & 0 \\ a_1(x_1) & a_0(x_1) & 0 & b_1(x_1) & b_0(x_1) & 0 \\ a_2(x_1) & a_1(x_1) & a_0(x_1) & b_2(x_1) & b_1(x_1) & b_0(x_1) \\ a_3(x_1) & a_2(x_1) & a_1(x_1) & b_3(x_1) & b_2(x_1) & b_1(x_1) \\ 0 & a_3(x_1) & a_2(x_1) & 0 & b_3(x_1) & b_2(x_1) \\ 0 & 0 & a_3(x_1) & 0 & 0 & b_3(x_1) \end{bmatrix}.$$

The polynomials $a(x_1, x_2)$and $b(x_1, x_2)$ do *not* have a common zero iff the *resultant polynomial*

$$r(x_1) \triangleq \det S_{a,b}(x_1) \neq 0 \text{ for all } x_1.$$

The polynomials $\{a(x_1, x_2), b(x_1, x_2)\}$ have a common zero at (x_{10}, x_{20}) only if $r(x_{10}) \triangleq \det S_{a,b}(x_{10}) = 0$ [1][5]. (For an arbitrary pair of polynomials $\{a(x), b(x)\}$ of degrees n_a, n_b in x respectively, the matrix $S_{a,b}$ is of dimension $(n_a + n_b) \times (n_a + n_b)$.)

Procedure to compute the common zeros:
1. Compute the roots x_{1k}, $k = 1, ..., n_{r_1} = \deg_{x_1}\{r_1(x_1)\}$ of $r(x_1) = 0$
2. Substitute these roots into $a(x_1, x_2)$.
3. For $k = 1, ..., n_{r_1}$ solve $a(x_{1k}, x_2) = 0$ to get the roots $x_{2k\ell}$ for $\ell = 1, ..., n_{a2} = \deg_{x_2}\{a(x_{1k}, x_2)\}$.
4. The common zeros of $\{a(x_1, x_2), b(x_1, x_2)\}$ are then those values of $(x_{1k}, x_{2k\ell})$ that satisfy $b(x_{1k}, x_{2k\ell}) = 0$.

B. Seven Level Case

To illustrate the procedure of using the theory of resultants to solve the system (3), the seven level case is considered. The conditions are

$$\begin{aligned} p_1(x) &\triangleq x_1 + x_2 + x_3 - m = 0, \quad m \triangleq \frac{V_1}{4V_{dc}/\pi} = sm_a \\ p_5(x) &\triangleq \sum_{i=1}^{3} \left(5x_i - 20x_i^3 + 16x_i^5\right) = 0 \qquad (4) \\ p_7(x) &\triangleq \sum_{i=1}^{3} \left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0. \end{aligned}$$

Substitute $x_3 = m - (x_1 + x_2)$ into p_5, p_7 to get

$$\begin{aligned} p_5(x_1, x_2) &= 5x_1 - 20x_1^3 + 16x_1^5 + 5x_2 - 20x_2^2 + 16x_2^5 \\ &\quad +5(m - x_1 - x_2) - 20(m - x_1 - x_2)^3 \\ &\quad +16(m - x_1 - x_2)^5 \end{aligned}$$

$$\begin{aligned} p_7(x_1, x_2) &= -7x_1 + 56x_1^3 - 112x_1^5 + 64x_1^7 - 7x_2 \\ &\quad +56x_2^3 - 112x_2^5 + 64x_2^7 - 7(m - x_1 - x_2) \\ &\quad +56(m - x_1 - x_2)^3 - 112(m - x_1 - x_2)^5 \\ &\quad +64(m - x_1 - x_2)^7 \end{aligned}$$

The goal here is to find solutions of

$$\begin{aligned} p_5(x_1, x_2) &= 0 \\ p_7(x_1, x_2) &= 0. \end{aligned}$$

For each fixed x_1, $p_5(x_1, x_2)$ can be viewed as a polynomial of (at most) degree 5 in x_2 whose coefficients are polynomials of (at most) degree 5 in x_1. A pair (x_{10}, x_{20}) is a simultaneous solution of $p_5(x_{10}, x_{20}) = 0, p_7(x_{10}, x_{20}) = 0$, if and only if the corresponding resultant polynomial $r_{5,7}(x_{10}) = 0$. Consequently, finding the roots of the resultant polynomial $r_{5,7}(x_1) = 0$ gives candidate solutions for x_1 to check for common zeros of $p_5 = p_7 = 0$. Here, the resultant polynomial $r_{5,7}(x_1)$ of the pair $\{p_5(x_1, x_2), p_7(x_1, x_2)\}$ was found with the use of the software package MATHEMATICA using the `Resultant` command. The polynomial $r_{5,7}(x_2)$ turned out to be a 22^{nd} order polynomial. The algorithm is as follows:

Algorithm for the 7 Level Case

1. Given m, find the roots of $r_{5,7}(x_1) = 0$.
2. Discard any roots that are less than zero, greater than 1 or that are complex. Denote the remaining roots as $\{x_{1i}\}$.
3. For each fixed zero x_{1i} in the set $\{x_{1i}\}$, substitute it into p_5 and solve for the roots of $p_5(x_{1i}, x_2) = 0$.
4. Discard any roots (in x_2) that are complex, less than zero or greater than one. Denote the pairs of remaining roots as $\{(x_{1j}, x_{2j})\}$.
5. Compute $m - x_{1j} - x_{2j}$ and discard any pair (x_{1j}, x_{2j}) that makes this quantity negative or greater than one. Denote the triples of remaining roots as $\{(x_{1k}, x_{2k}, x_{3k})\}$.
6. Discard any triple for which $x_{3k} < x_{2k} < x_{1k}$ does not hold. Denote the remaining triples as $\{(x_{1l}, x_{2l}, x_{3l})\}$. The switching angles that are a solution to the three level system (4) are

$$\{(\theta_{1l}, \theta_{2l}, \theta_{3l})\} = \{(\cos^{-1}(x_{1l}), \cos^{-1}(x_{2l}), \cos^{-1}(x_{3l}))\}.$$

The results are summarized in Figure 3 which shows the switching angles $\theta_1, \theta_2, \theta_3$ vs m for those values of m in which the system (4) has a solution. The parameter m was

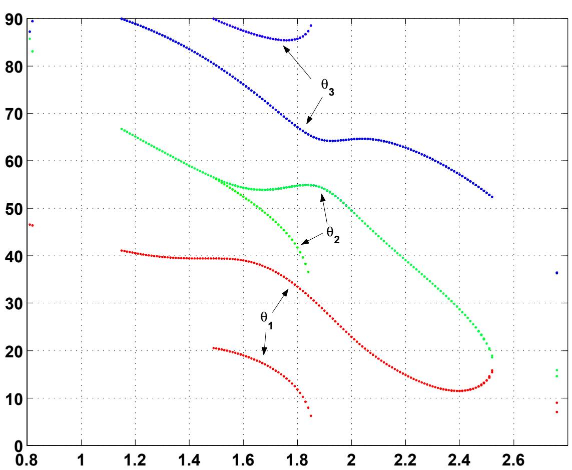

Fig. 3. The switching angles $\theta_1, \theta_2, \theta_3$ in degrees vs m

incremented in steps of 0.01. Note that for m in the range from approximately 1.49 to 1.85, there are two different sets of solutions that solve (4). On the other hand, for $m \in [0, 0.8]$, $m \in [0.83, 1.15]$ and $m \in [2.52, 2.77]$ there are *no* solutions to (4). Interestingly, for $m \approx 0.8, m \approx 0.82$ and $m \approx 2.76$ there are (isolated) solutions. In the range $m \in [1.49, 1.85]$ for which there are two sets of solutions, the solution which gives the smallest distortion due to the 11^{th} and 13^{th} harmonics is a good choice. This set of angles is given in Figure 4. As mentioned earlier, for $m \in [0, 0.8]$, $m \in [0.83, 1.15]$, $m \in [2.52, 2.77]$ and $m \in [2.78, 3]$ there are no solutions satisfying the conditions (4). Consequently, for these ranges of m, the switching angles were determined by

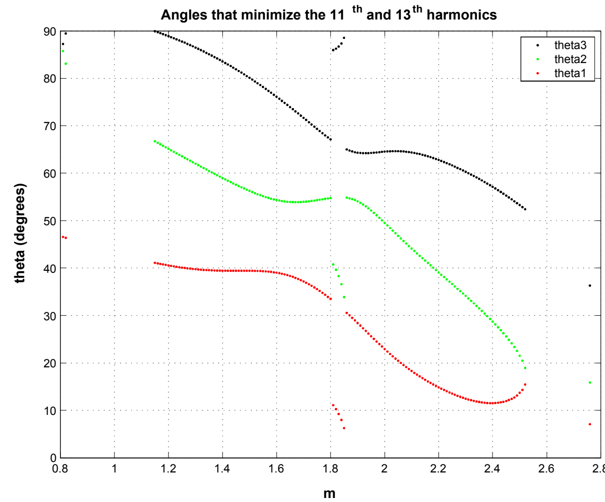

Fig. 4. Angles that give zero 3^{rd} and 5^{th} harmonics and the smallest 11^{th} and 13^{th} harmonics.

minimizing $\sqrt{(p_5/5)^2 + (p_7/7)^2}$. Figure 5 shows a plot of the resulting minimum error $\sqrt{(p_5/5)^2 + (p_7/7)^2}$ vs. m for these values of m. As Figure 5 shows, when $m \approx 0.81$

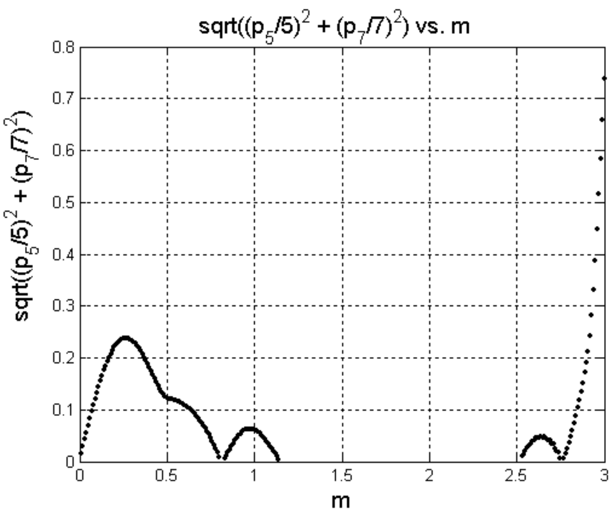

Fig. 5. Error $= \sqrt{(p_5/5)^2 + (p_7/7)^2}$ vs. m

and $m \approx 2.76$, the error is zero corresponding to the isolated solutions to (4) for those values of m. For $m = 1.15$ and $m = 2.52$, the error goes to zero because these values correspond to the boundary of the exact solutions of (4). However, note, e.g., when $m = 0.25$, the error is about 0.25, that is, the error is the same size as m. Other than close to the endpoints of the two intervals $[0, 0.8]$, $[2.78, 3]$ the minimum error $\sqrt{(p_5/5)^2 + (p_7/7)^2}$ is too large to make the corresponding switching angles for this interval of any use. Consequently, for m in this interval, one must use some other approach (e.g., PWM) in order to get reduced harmonics. For the other two intervals $[0.83, 1.15]$, $[2.52, 2.77]$, the minimum error $\sqrt{(p_5/5)^2 + (p_7/7)^2}$ is around 5% or less so that it might be satisfactory to use the corresponding switching angles for these intervals.

IV. Experimental Work

A prototype three-phase 11-level wye-connected cascaded inverter has been built using 100 V, 70 A MOSFETs as the switching devices [9]. A battery bank of 15 SDCSs of 48 Volts DC each feed the inverter (5 SDCSs per phase). In the experimental study here, this prototype system was configured as a 7-level (3 SDCSs per phase) converter with each level being 12 Volts. A 50 pin ribbon cable provides the communication link between the gate driver board and the real-time processor. In this work, the OpalRT real-time computing platform [7] was used to interface the computer (which generates the logic signals) to this cable. The OpalRT system allows one to write the switching algorithm in Simulink which is then converted to C code using RTW. The OpalRT software provides icons to interface the Simulink model to the digital I/O board and converts the C code into executables. Using the XHP (extra high performance) option in OpalRT as well as the multiprocessor option to spread the computation between two processors, an execution time of 16 microseconds was achieved.

Experiments were carried out to validate the theoretical results of section III-B. The value $m = 2$ is a case in which these harmonics can be eliminated. The frequency was set to 60 Hz in each case, and the program was run in real time with a 16 microseconds sample period, i.e., the logic signals were updated to the gate driver board every 16 microseconds.

The voltage was measured using a high speed data acquisition oscilloscope every $T = 5$ microseconds resulting in the data $\{v(nT), n = 1, ..., N\}$ where $N = 3(1/60)/\left(5 \times 10^{-6}\right) = 10000$ samples corresponding to three periods of the 60 Hz waveform. A fast Fourier transform was performed on this voltage data to get $\{\hat{v}(k\omega_0), k = 1, ..., N\}$ where the frequency increment is $\omega_0 = (2\pi/T)/N = 2\pi(20)$ rad/sec or 20 Hz. The number $\hat{v}(k\omega_0)$ is simply the Fourier coefficient of the k^{th} harmonic (whose frequency is $k\omega_0$ with $\omega_0 = \frac{2\pi}{N}\frac{1}{T}$) in the Fourier series expansion of the phase voltage signal $v(t)$. With $a_k = |\hat{v}(k\omega_0)|$ and $a_{\max} = \max_k \{|\hat{v}(k\omega_0)|\}$, the data that is plotted is the normalized magnitude $a_k/a_{\max}$.

Figure 6 is the plot of the phase voltage for $m = 2$. The corresponding FFT of this signal is given in Figure 7. Figure 7 shows 5^{th} (300 Hz) and 7^{th} (420 Hz)harmonics are zero as predicted in Figure 5. For $m = 1.83$, there are two possible set of solutions which generate zero 5^{th} and 7^{th} harmonics (See Figures 3 and 4). To compare the two sets of switching angles, Figure 8 shows the FFT of the data where the 5^{th} and 7^{th} harmonics are zero and the normalized 11^{th} and 13^{th} harmonics (at 660 Hz and 780 Hz, respectively) are both about 0.04 if the switching angles are chosen according to Figure 4. In contrast, Figure 9 is an FFT of the data ($m = 1.83$) in which the other set of switching angles is chosen. In this case, Figure 9 shows the 5^{th} and 7^{th} harmonics are zero, but the normalized 11^{th} and 13^{th} harmonics are about 0.06 and 0.04, respectively.

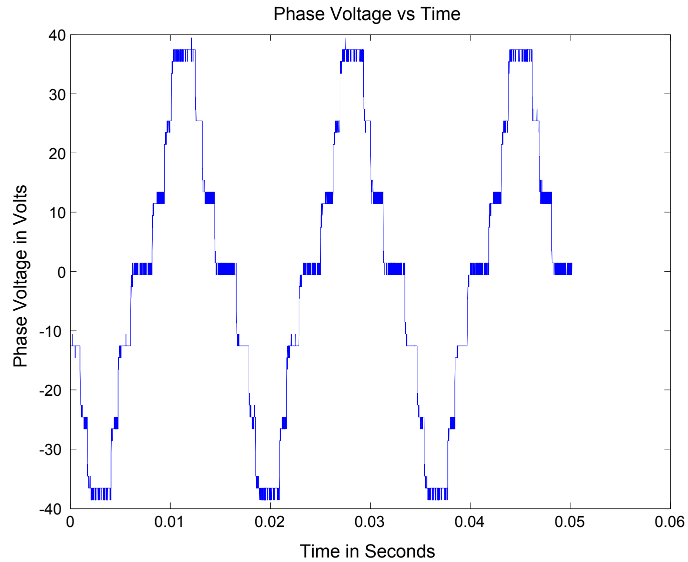

Fig. 6. Phase voltage when $m = 2$

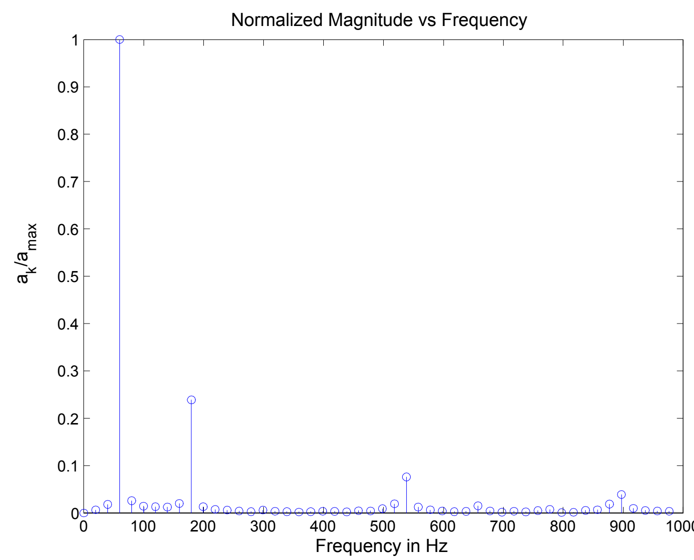

Fig. 7. Normalized FFT $a_k/a_{\max}$ vs frequency for $m = 2$

V. Elimination of Harmonics in a Multilevel Converter with Non Equal DC sources

The above methodology can be extended to the case where the separate DC sources do not have equal voltage levels [3]. In this case, the Fourier series expansion of the (stepped) output voltage waveform of the multilevel inverter is given by

$$V(\omega t) = \frac{4V_{dc}}{\pi} \times \sum_{n=1,3,5,...}^{\infty} \frac{1}{n}\left(V_1\cos(n\theta_1) + \cdots + V_s\cos(n\theta_s)\right)\sin(n\omega t) \tag{5}$$

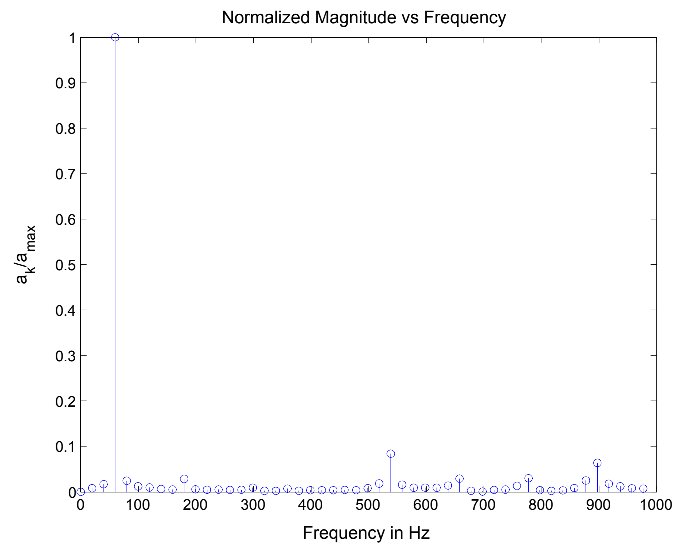

Fig. 8. Normalized FFT $a_k/a_{\max}$ vs frequency for $m = 1.82$ and $\theta_1, \theta_2, \theta_3$ chosen to give smallest distortion due to the 11^{th} and 13^{th} harmonics.

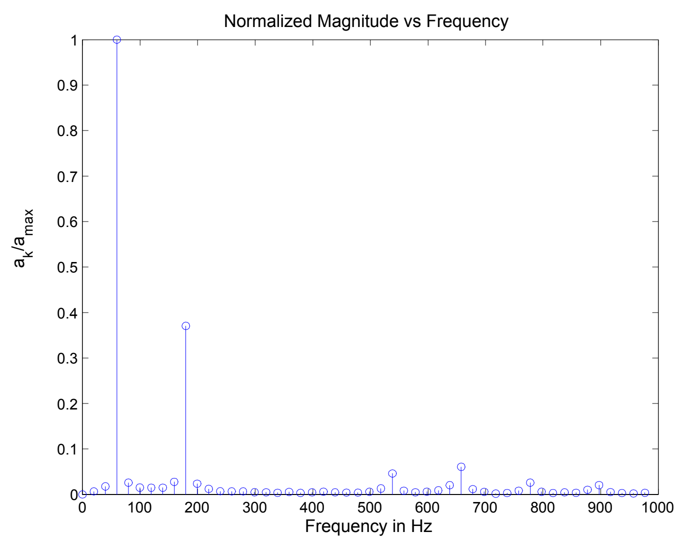

Fig. 9. Normalized FFT $a_k/a_{\max}$ vs frequency for $m = 1.82$ and $\theta_1, \theta_2, \theta_3$ chosen to which give a larger distortion generated by the 11^{th} and 13^{th} harmonics.

where s is the number of DC sources and the *product* V_iV_{dc} is the value of the i^{th} DC source (if all the DC sources have the same value V_{dc}, then $V_1 = V_2 = \cdots = V_s = 1$). With three DC sources, the switching angles are chosen so as to not generate the $5^{th}, 7^{th}$ order harmonics while achieving the desired fundamental voltage. The mathematical statement of these conditions is then

$$\begin{aligned} V_1\cos(\theta_1) + V_2\cos(\theta_2) + \cdots + V_s\cos(\theta_s) &= \frac{\pi V_f}{4V_{dc}} \\ V_1\cos(5\theta_1) + V_2\cos(5\theta_2) + \cdots + V_s\cos(5\theta_s) &= 0 \\ V_1\cos(7\theta_1) + V_2\cos(7\theta_2) + \cdots + V_s\cos(7\theta_s) &= 0. \end{aligned} \quad (6)$$

This is a system of 3 transcendental equations in the unknowns $\theta_1, \theta_2, \theta_3$. As before, equations (5) are first converted to a polynomial system by setting $x_1 = \cos(\theta_1), x_2 = \cos(\theta_2), x_3 = \cos(\theta_3)$ and using trigonometric identities to transform (5) into the equivalent conditions

$$\begin{aligned} p_1(x) &\triangleq V_1x_1 + V_2x_2 + V_3x_3 - m = 0 \\ p_5(x) &\triangleq \sum_{i=1}^{3} V_i\left(5x_i - 20x_i^3 + 16x_i^5\right) = 0 \\ p_7(x) &\triangleq \sum_{i=1}^{3} V_i\left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0 \end{aligned} \quad (7)$$

where $x = (x_1, x_2, x_3)$ and $m \triangleq V_f/(4V_{dc}/\pi)$. This is now a set of three *polynomial* equations in the three unknowns x_1, x_2, x_3. Further, the solutions must satisfy $0 \leq x_3 < x_2 < x_1 \leq 1$. One substitutes $x_3 = \left(m - (V_1x_1 + V_2x_2)\right)/V_3$ into p_5, p_7 to get the two polynomials equations

$$\begin{aligned} p_5\left(x_1, x_2, \frac{m - (V_1x_1 + V_2x_2)}{V_3}\right) &= 0 \\ p_5\left(x_1, x_2, \frac{m - (V_1x_1 + V_2x_2)}{V_3}\right) &= 0. \end{aligned}$$

Using a similar procedure as before, these are then solved using the method of resultants. The results are plotted on the left side of Figure 10.

This figure shows the switching angles $\theta_1, \theta_2, \theta_3$ vs. m for those values of m in which the system (2) has at least one solution set. The parameter m was incremented in steps of 0.01. Note that for m in the range from approximately 1.1 to 2.4, there are at least two different sets of solutions and sometimes three sets. Interestingly, there are also isolated values of m at $m \approx 0.7$ and $m \approx 0.8$ which are solutions of (6). The right side of Figure 10 is a plot of the set of switching angles chosen to give the smallest distortion generated by the 11^{th} and 13^{th} harmonics, i.e., the smallest value of $\sqrt{(p_{11}/11)^2 + (p_{13}/13)^2}$.

A. Experimental Work

The same prototype system describe previously was used. In the experimental study here with varying DC sources, the prototype system was configured to be a 7-level (3 SDCSs per phase) converter with each level being nominally 12 Volts ($V_{dc} = 12$ V). It turned out that $V_1V_{dc} = 12.56$ Volts, $V_2V_{dc} = 10.19$ Volts and $V_3V_{dc} = 12.01$ Volts (i.e., $V_1 = 12.56/12 = 1.05, V_2 = 0.85, V_3 = 1.01$). Due to space limitations, experimental results are only presented for the case with $m = 1.3$. The left side of Figure 11 is the plot of the phase voltage for $m = 1.3$. (The spikes on the plot are due to the low bit resolution of the sampling scope and are not present on the actual scope display). The corresponding FFT of this signal is given on the right side of Figure 11. This shows the normalized magnitude of the 5^{th} and 7^{th} harmonics are both about 0.01 which corresponds well with the predicted value of zero.

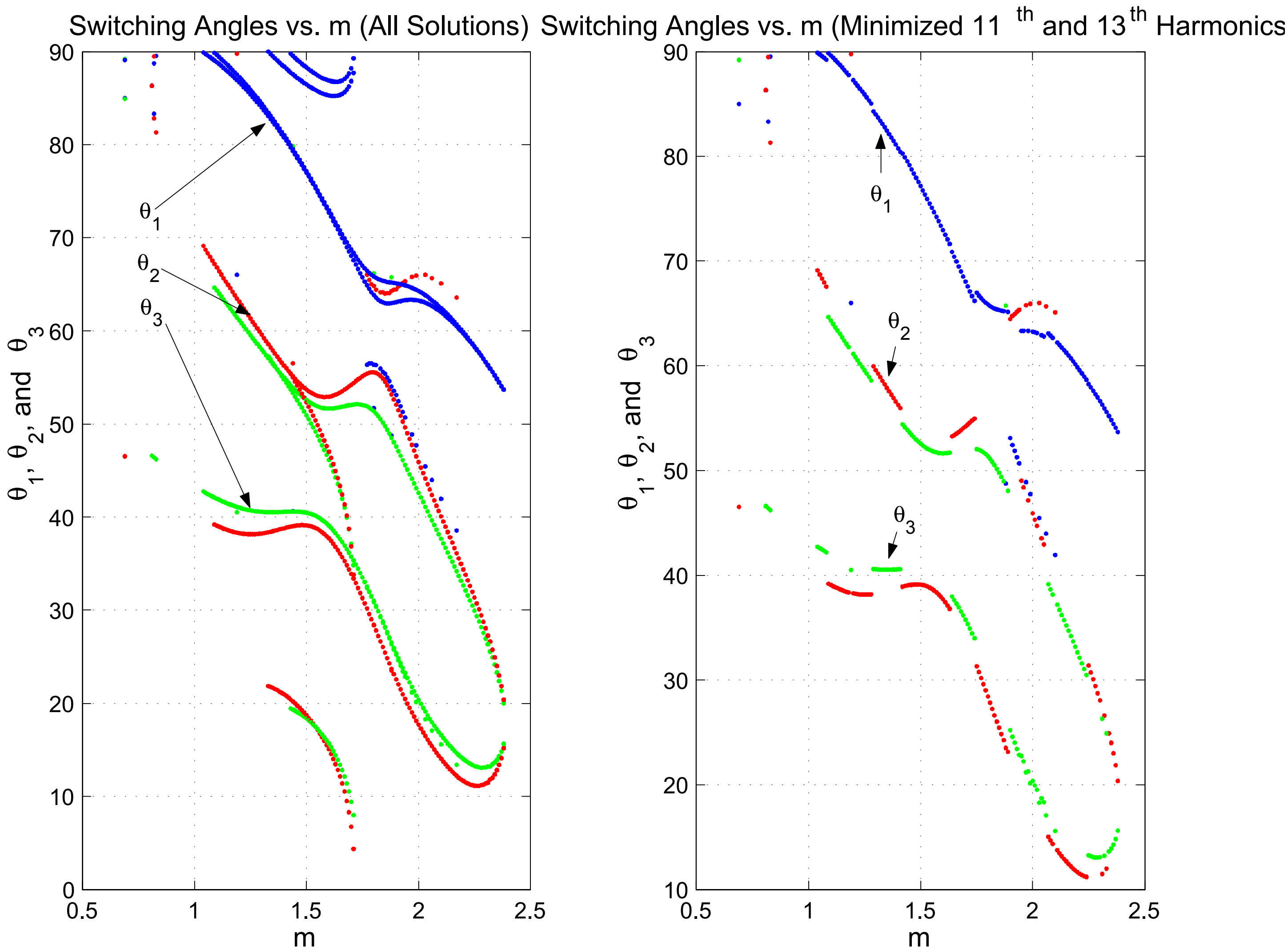

Fig. 10. Left: All sets of solutions $\theta_1, \theta_2, \theta_3$ vs. m (Only one set is annotated). Right: The solution set $\theta_1, \theta_2, \theta_3$ that generates the smallest distortion due to the 11^{th} and 13^{th} harmonics.

VI. Conclusions and Further Work

A full solution to the problem eliminating the 5^{th} and 7^{th} harmonics in a seven level multilevel inverter has been given. Specifically, resultant theory was used to completely characterize for each m when a solution existed and when it did not (in contrast to numerical techniques such as Newton-Raphson). Further, it was shown that for a range of values of m, there were two sets of solutions and these values were also completely characterized. The solution set minimized the 11^{th} and 13^{th} harmonics was chosen. Experimental results were also presented and corresponded well to the theoretically predicted results. The method also was extended to the case where the DC sources are not equal.

VII. Acknowledgements

Dr. Tolbert would like to thank the National Science Foundation for partially supporting this work through contract NSF ECS-0093884. Drs. Chiasson and Tolbert would like to thank Oak Ridge National Laboratory (especially Don Adams) for partially supporting this work through the UT/Battelle contract number 4000007596. The University of Tennessee is gratefully acknowledged for providing funding for the equipment in this project through its SARIF program. Finally, the authors would like to also thank Opal-RT Technologies for their courteous and professional help in support of this project.

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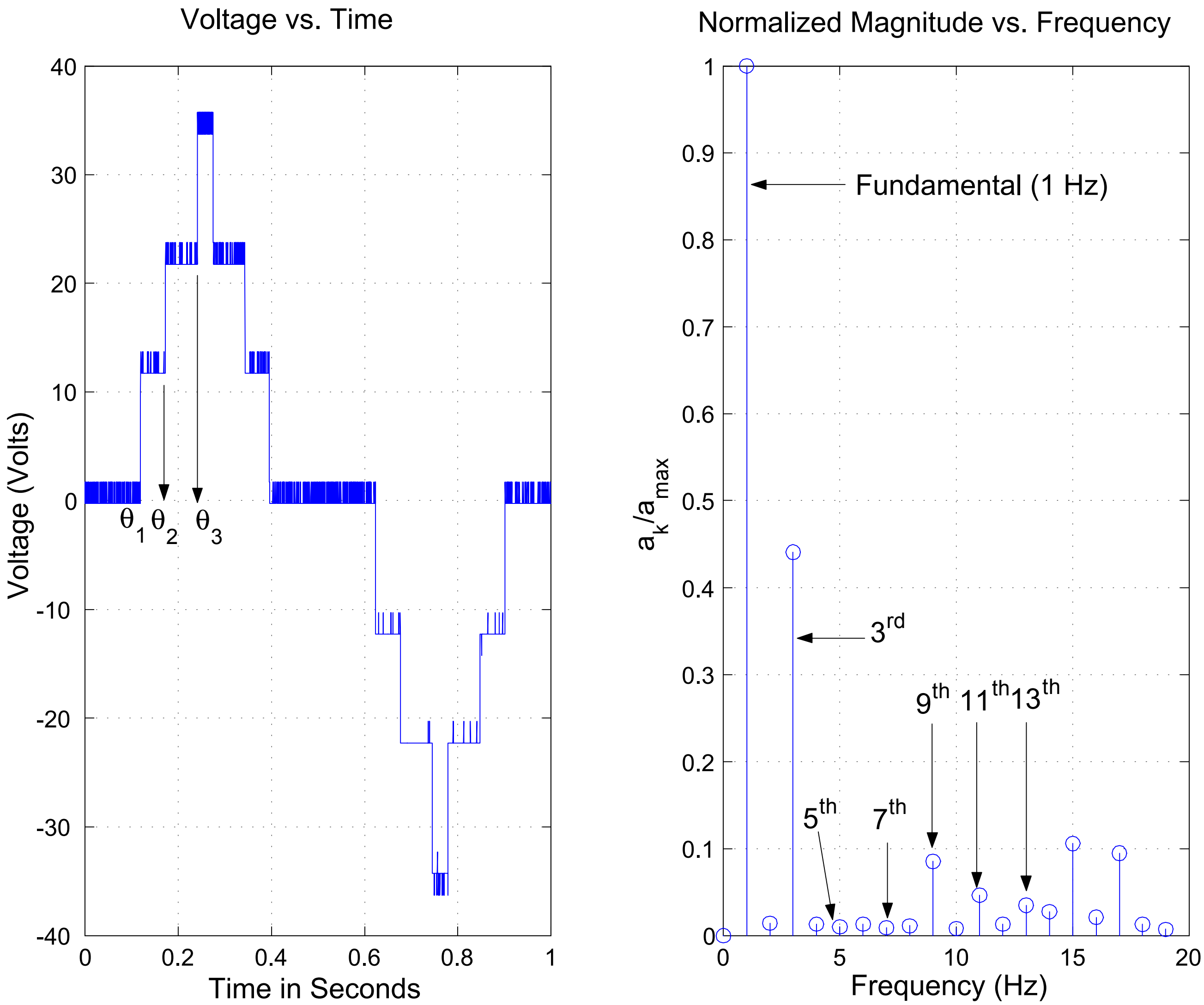

Fig. 11. Left: Sampled output voltage with $m = 1.3$. Right: FFT of the sampled output voltage.

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