On Unary Fragments of *MTL* and *TPTL* over Timed Words.

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ICTAC,2014 Bucharest, Romania

September 19, 2014

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- In general satisfiability checking of full *MTL* and *TPTL* is undecidable.
- Unlike *LTL*, they are more expressive with *past* operators.
- Thus it becomes interesting to study satisfiability checking and expressiveness for different fragments of these logics.

Presentation Flow

- Preliminaries
- Satisfiability Checking
  - $MTL[\Diamond_I, \diamondminus_I]$
  - $2 - TPTL[\Diamond_I]$
  - $MTL[\Diamond_I]$
- Expressiveness
- Conclusion
- Future Work

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- A strictly monotonic timed word is a timed word where timestamps are strictly increasing that is $i_1 > i_2$ implies $t_{i_1} > t_{i_2}$.In general,$i_1 > i_2$ implies $t_{i_1} \geq t_{i_2}$

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where I is interval of the form $\langle x, y \rangle$, $x \in \mathcal{N} \cup \{0\}$, $y, x \in \mathcal{N} \cup \{0, \infty\}$ and $\langle ... \rangle \in \{[...], (...), [...), (...]\}$

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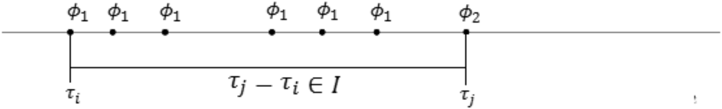
$$\phi_2 \ and \ \tau_j - \tau_i \in I \ and \ \forall \ i \leq k < j \ \rho, k \models \phi_1$$



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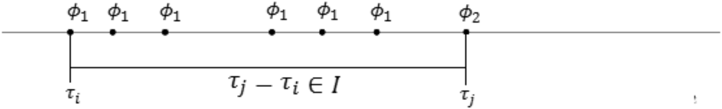

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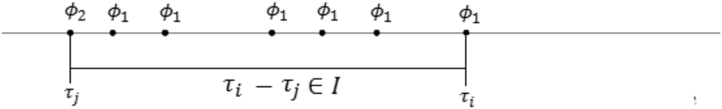

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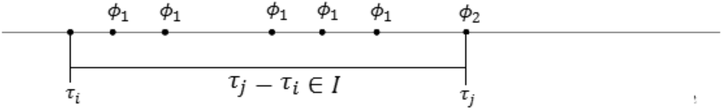

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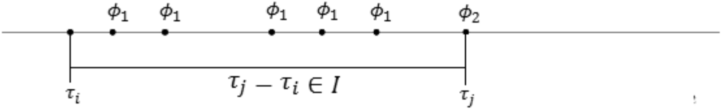

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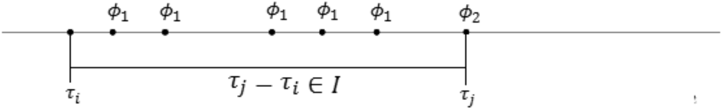

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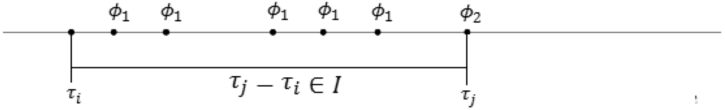

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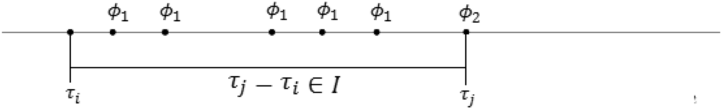

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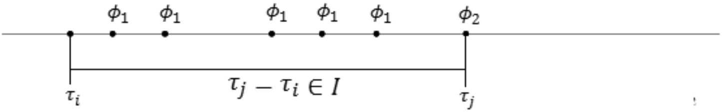

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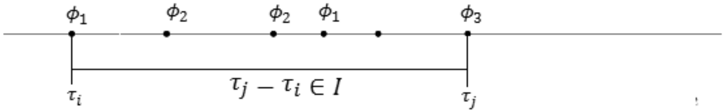
- By restricting set of allowed intervals. e.g. *MITL*.
- By restricting set of operators. e.g. *MTL*$[\mathsf{U}_I]$.

TPTL Syntax

$$\phi ::= x \mid \phi \wedge \phi \mid \phi \vee \phi \mid \neg \phi \mid \phi \,\mathsf{S}\, \phi \mid \phi \,\mathsf{U}\, \phi \mid \mathsf{O}\phi \mid \bar{\mathsf{O}}\phi \mid y.\varphi \mid y \in I$$

where y is a clock (freeze) variable. Note, all the strict and unary operators can be defined similarly as before.

- Note that the truth of the formula is defined at a point i in a timed word ρ with valuation of the freeze variables ν. Thus the model is ρ, i, ν.
- All the unary and strict modal operators can be defined similarly.
- Following is the model of the formula $x.\Diamond(\phi_1 \wedge \Diamond(\phi_2 \wedge \Diamond(\phi_3 \wedge x \in I)))$



Deterministic Two Counter Machine

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- A Two counter machine can be defined as 4 tuple $M = (P, C, D, I)$, where
  - P is a program counter whose value is bounded $value(P) \in 0, 1, \ldots, n$ where $n \in N$.
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- Whether a unique run of a DTCM is halting or not is undecidable.

Channel Machines

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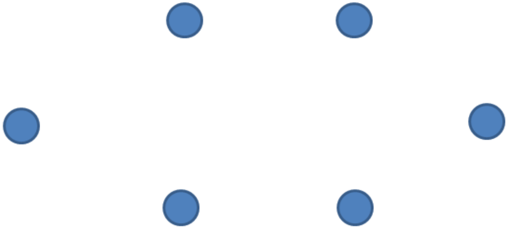
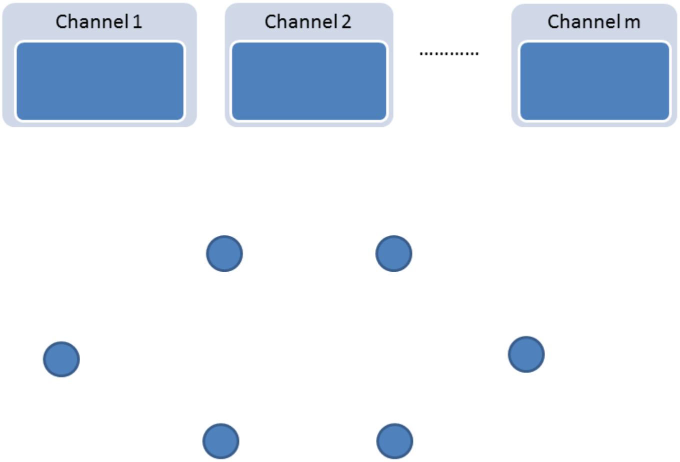


Channel Machines

5-Tuple $\{S, C$



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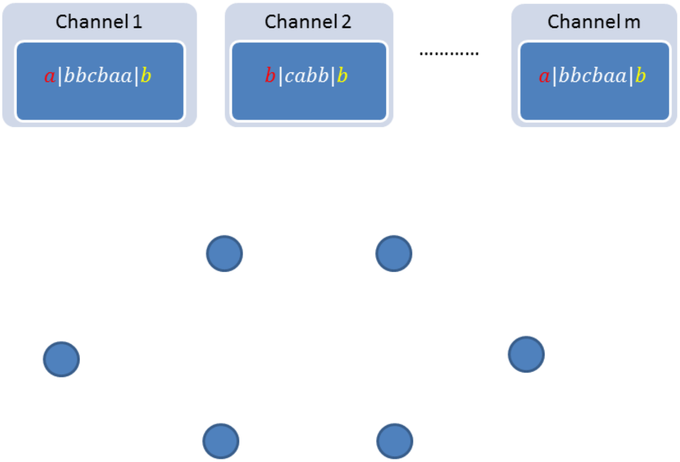
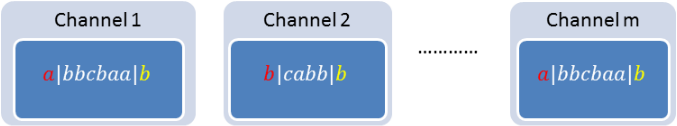
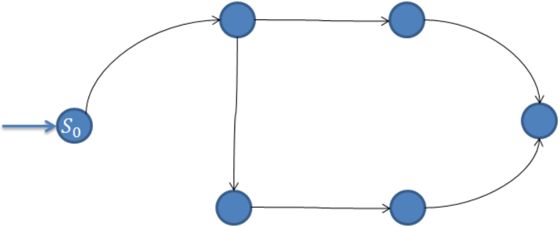


Channel Machines

5-Tuple $\{S, C, M, T, s_0\}$





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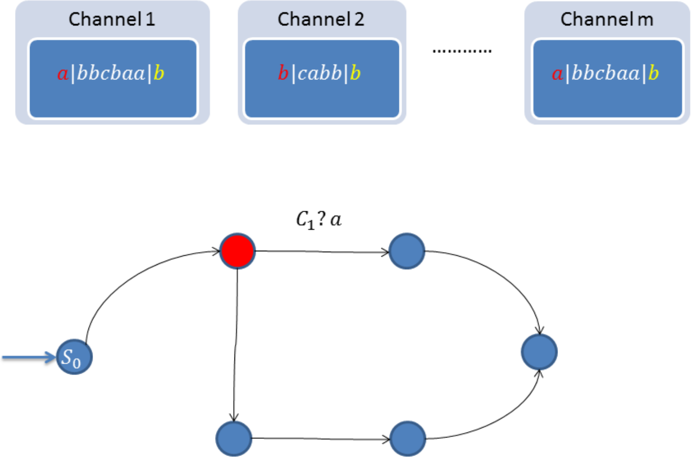

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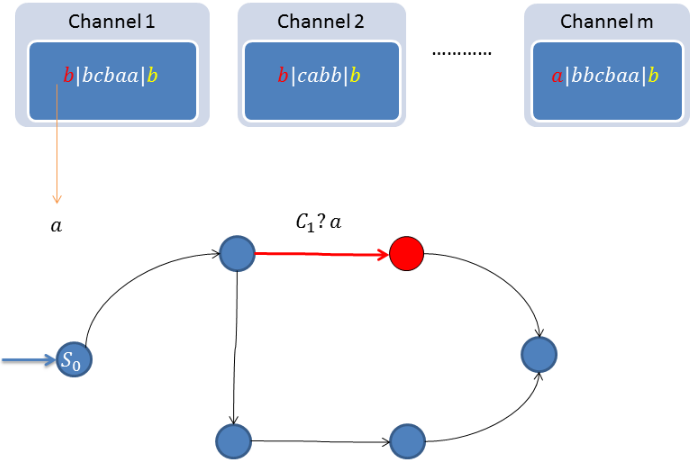

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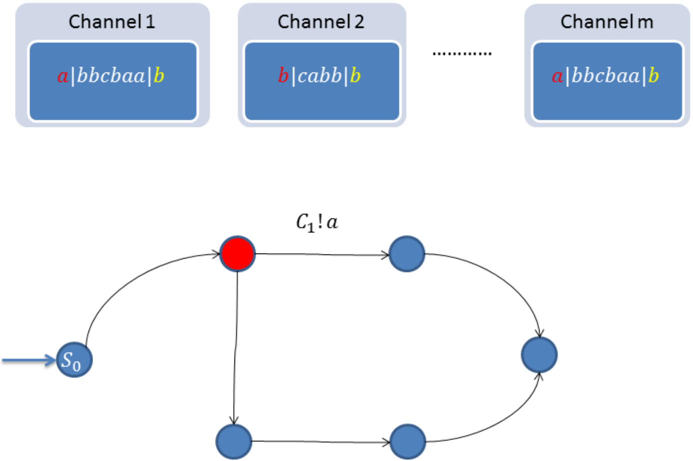

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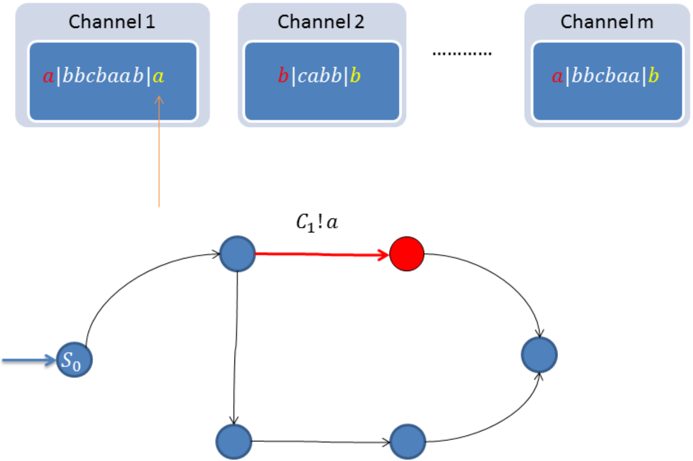

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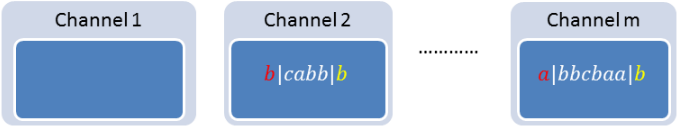

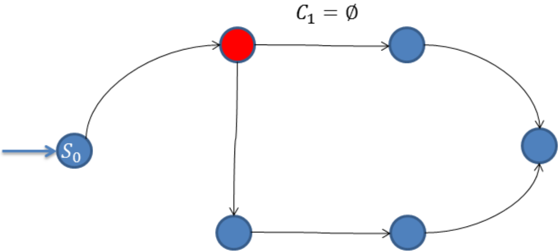

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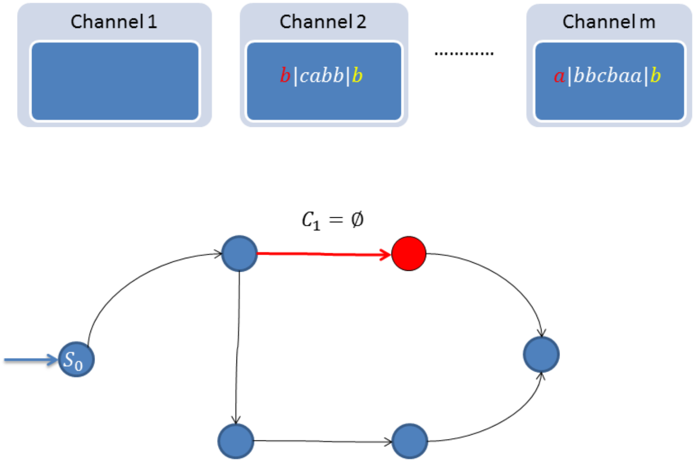

Channel Machines with Insertion Errors - *ICMET*.

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Insertion Error Channel Machines with Emptiness Testing

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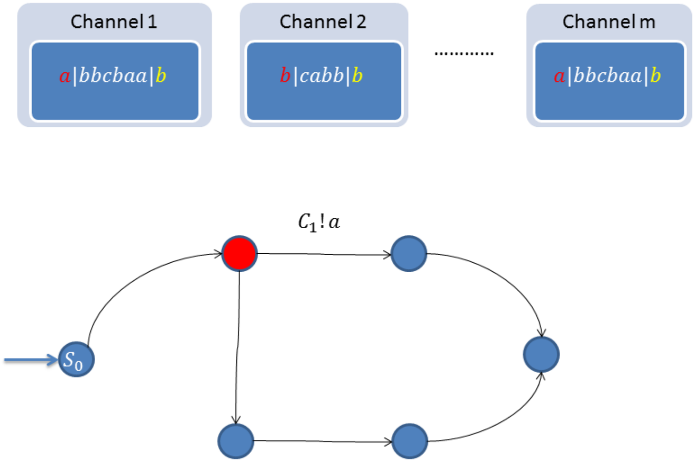

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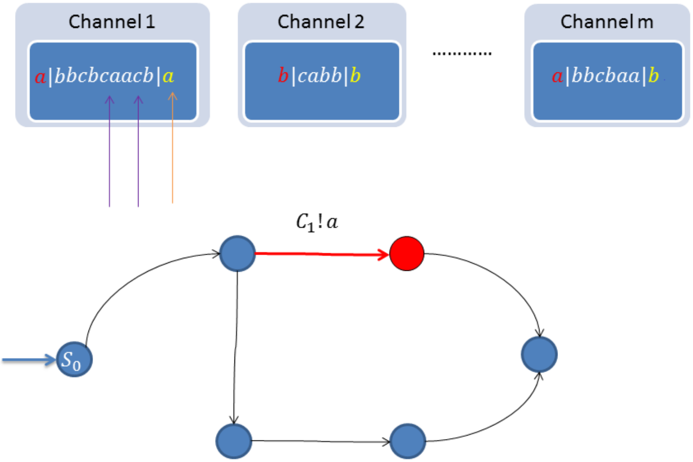
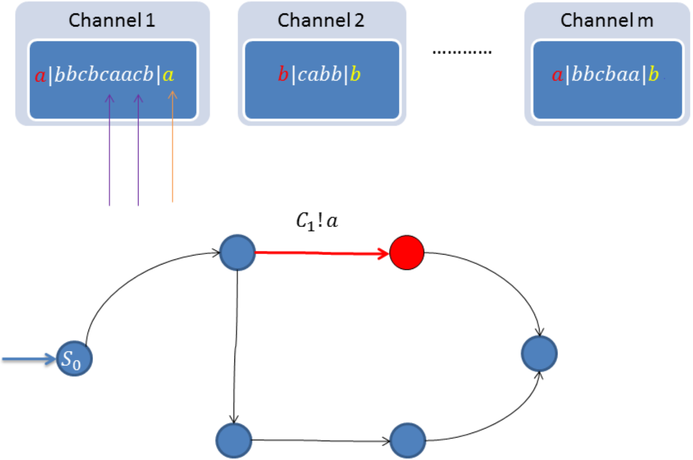


Insertion Error Channel Machines with Emptiness Testing

To verify whether a state is reachable from the initial state in ICMET is decidable with Non Primitive Recursive complexity.



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- Until Move is played in 2 parts:

EF Games - U Move

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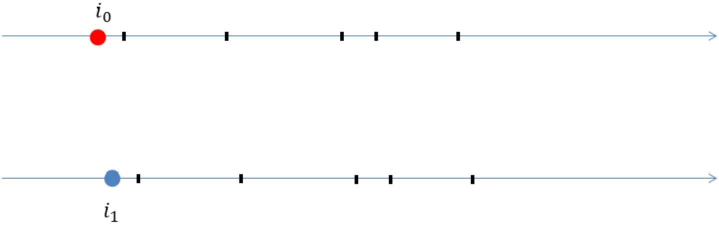

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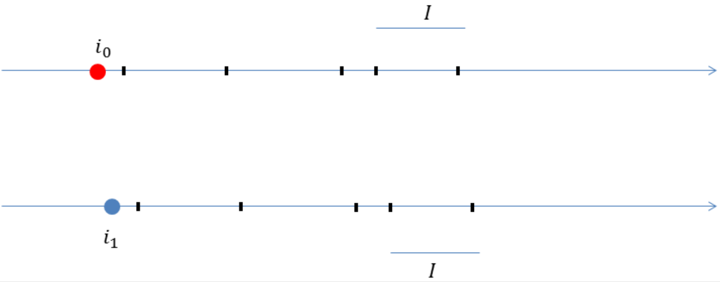

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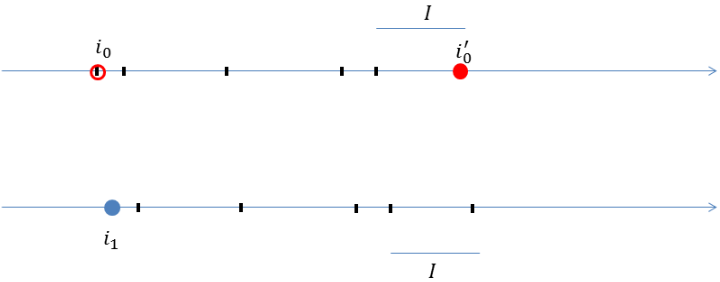

EF Games - U Move

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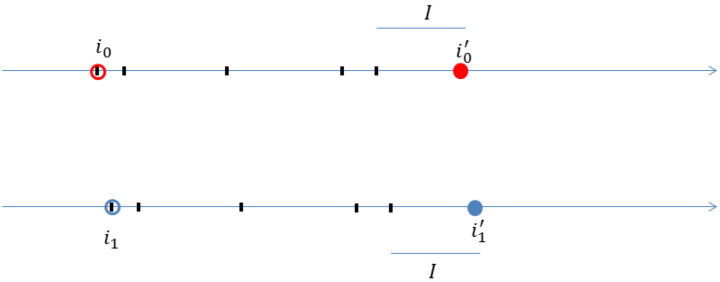
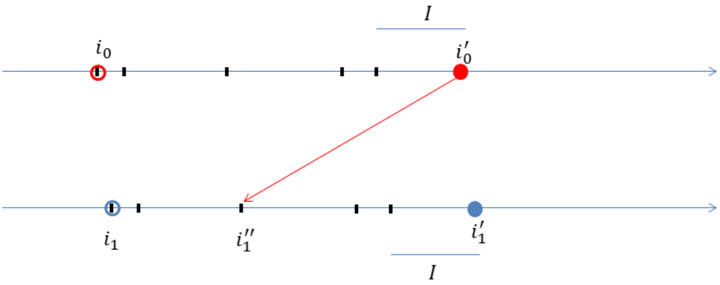


U $-$ *part*



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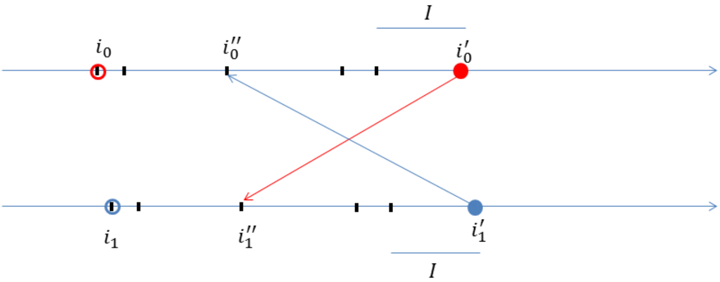

EF Games

- **Game equivalence:** $(\rho_0, i_0) \approx_k (\rho_1, i_1)$ iff for every k-round $MTL[U_I, S_I]$ EF-game over the words ρ_0, ρ_1 starting from the configuration (i_0, i_1), the Duplicator always has a winning strategy.

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- **EF Theorem of *MTL***: Game equivalence $\equiv$ Formula equivalence.

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- Let spoiler choose a pair of words ρ_0, ρ_1 such that $\rho_0 \models \varphi$ and $\rho_1 \models \neg\varphi$.
- Play n round L_2 EF Game. If the duplicator wins then L_2 doesn't have an equivalent formula φ.

Related Work

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- For $MITL[\Diamond_\infty, \diamondminus_\infty]$ is *NP*-complete.[Pandya *et al.*]

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- For $MTL[\Diamond_I]$ is non primitive recursive.
- For 2 clock $TPTL[F]$ is undecidable.
- Expressiveness Picture of Unary MTL.

Presentation Flow

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Segment of Timed Word Showing Encoding of Increment Operation

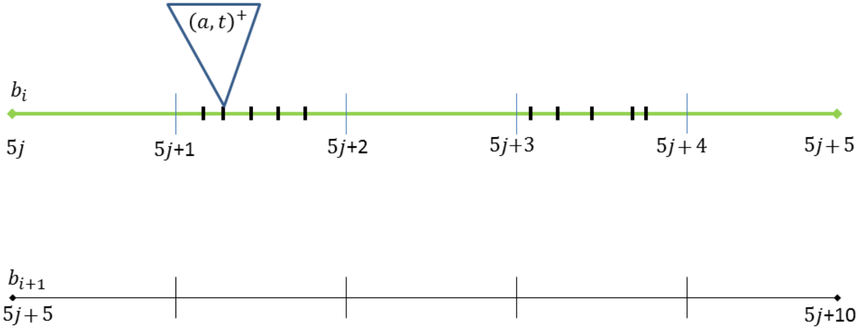

Figure: Run showing increment d

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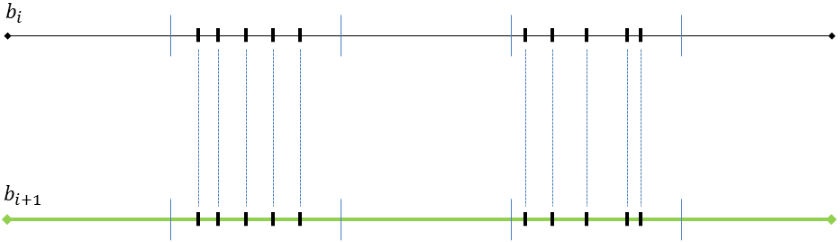

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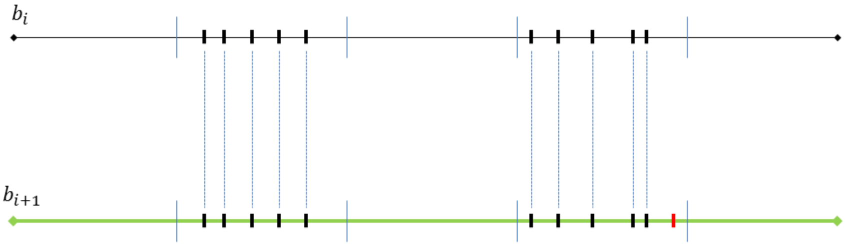

Figure: Run showing increment *d*

$MTL[\Diamond_I, \Diamond\!\!\!-_I]$ Satisfiability

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- $\phi^{inc_c} \;=\; \Box[b_i \Rightarrow (COPY_C \wedge \Diamond_{[5,5]}(b_{i+1}) \wedge INC_D)]$

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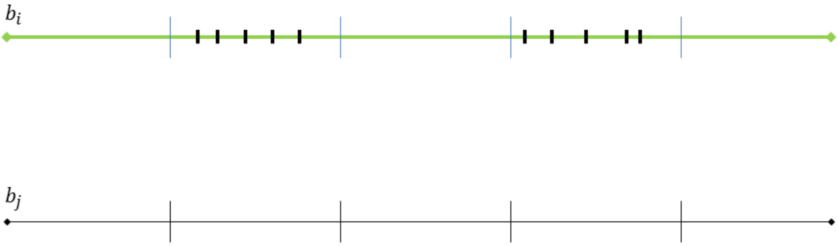
Correctness Idea



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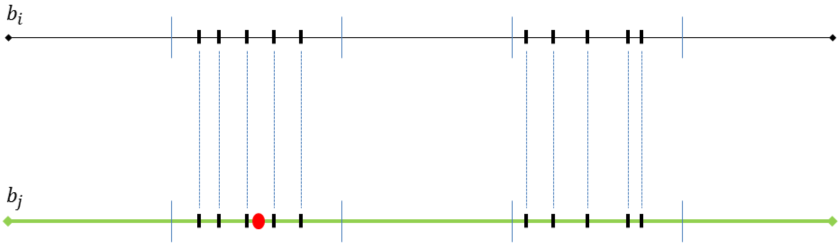

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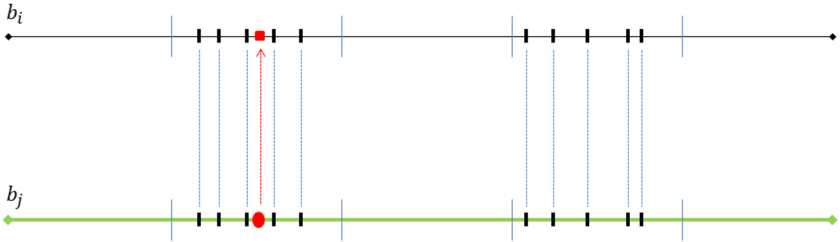

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1. In the previous logic past was helping in ensuring the precise copying of all the non-last a's. Here we try to specify similar restriction using two clocks.

$$COPY_1 = \Box x.[(a \wedge \Diamond(a \wedge x \in (0,1))) \Rightarrow \Diamond(a \wedge x \in [5,5])].$$

$$COPY_2 = \Box x.[(a \wedge \Diamond(a \wedge x \in (0,1))) \Rightarrow (\Diamond y.(a \wedge x \in (0,1) \wedge \neg\Diamond(a \wedge x \in (5,\infty) \wedge y \in (0,5))))].$$

This is the most important constraint in the encoding which results in undecidability.

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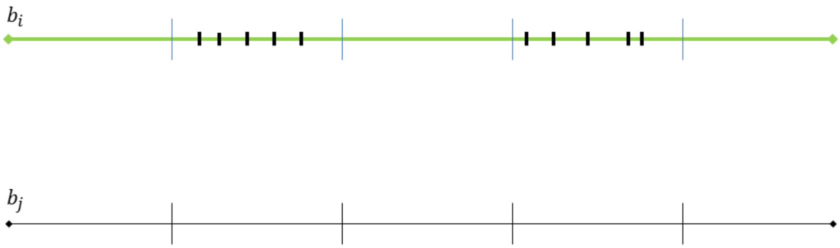

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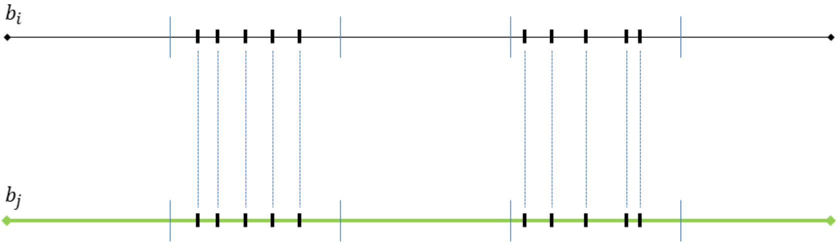

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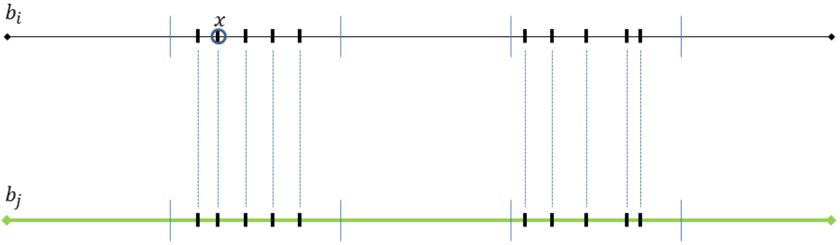

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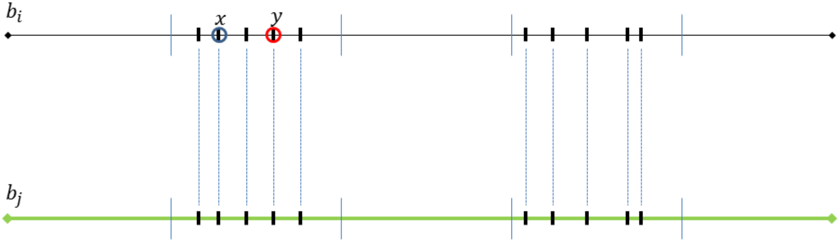

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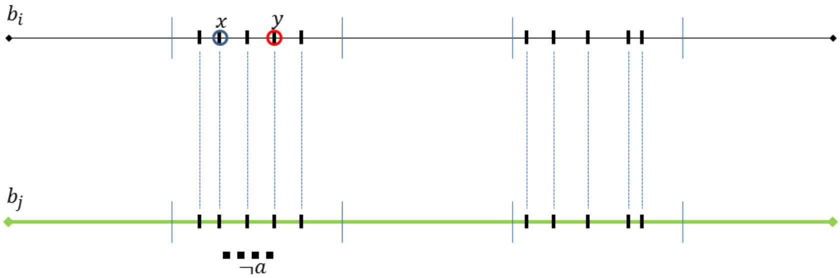

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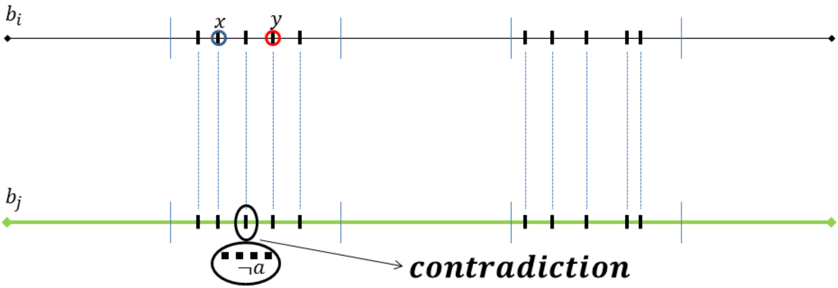

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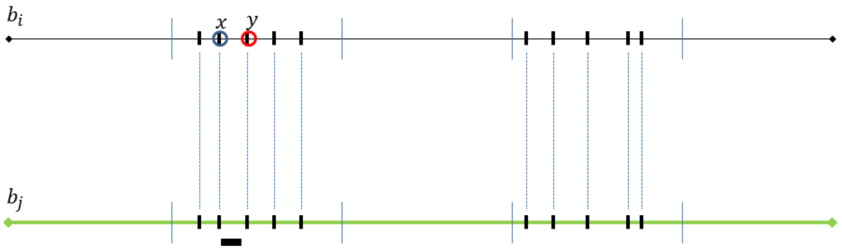

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Encoding Configuration

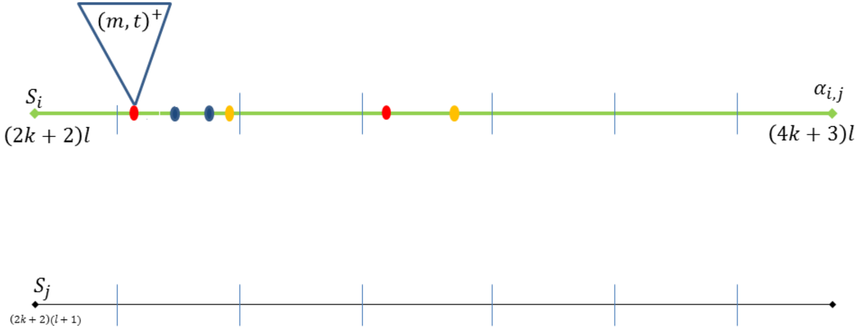

Figure: Encoding Configuration

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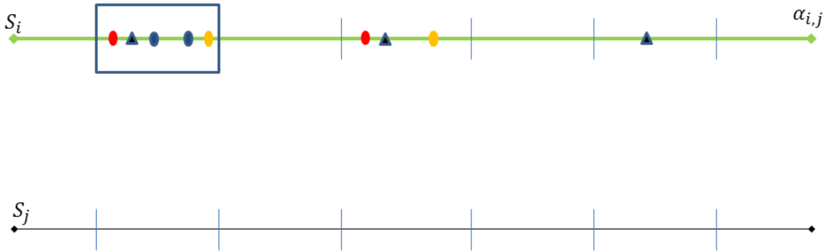
- We discuss Read($c?m$). The challenge is to assert that a particular point resembles head of the channel.
- For this we introduce a special symbol b which acts as a separator between head of the channel(first symbol in the unit integral interval) and the rest of the contents of channel.

Figure: Run showing $C_1?m$

Segment of Timed Word showing Read($C_1?m$)



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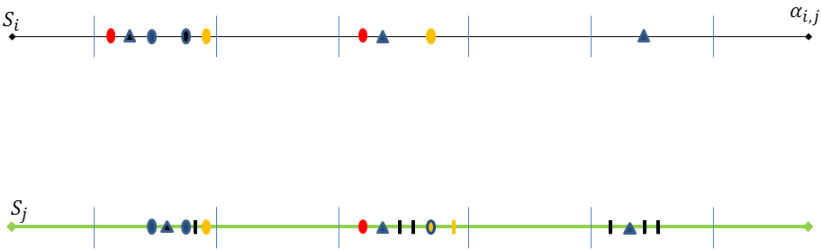

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Contd.

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  - There is one and only one m before b (if c non-empty):

    $$\phi_{b_3} = \Box[\neg\{M \wedge \Diamond_{(0,1)}(M \wedge \Diamond_{(0,1)}(b)\}]$$

    $$\phi_{b_4} = \Box[S \Rightarrow \{\bigwedge_{j=1}^{k}(\Diamond_{(2j-1,2j)}(M) \Rightarrow \Diamond_{(2j-1,2j)}(M \wedge \Diamond_{(0,1)} b))\}]$$

- If transition is of the form $c_i = \emptyset$

$$\phi_{c_i=\emptyset} = S \wedge \Box_{(2i-1,2i)}(\neg action) \wedge \Box_{(0,2k+2)}(\bigwedge_{m \in M}(m \Rightarrow \Diamond_{[2k+2,2k+2]}(m))$$

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- If transition is of the form $c_i?m_x$ where $m_x \in M$

$$\phi_{c_i?m_x} = S \wedge \bigwedge_{j \neq i, j=1}^{k} \Box_{[2j-1,2j]}\{\bigwedge_{m \in M} m \Rightarrow \Diamond_{[2k+2,2k+2]}(m)\} \wedge \Diamond_{(2i-1,2i)}\{m_x \wedge \Diamond_{(0,1)}(b)\} \wedge \Box_{[2i-1,2i]}\{\bigwedge_{m \in M}(m \wedge \neg\Diamond_{(0,1)} b) \Rightarrow \Diamond_{[2k+2,2k+2]}(m)\}$$

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Expressiveness

Theorem: $MTL[\Diamond_I, \mathrm{O}]$ is incomparable to $MTL[\mathrm{U}_I]$ $\varphi \equiv a \,\mathrm{U}\, b$

$$A(0)A'(\delta)A'(2\delta)\ldots A'(1)$$

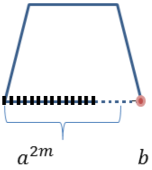

Figure: $\rho_0 \vdash \varphi$

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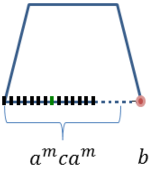

Figure: $\rho_1 \nvdash \varphi$

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Conclusion

- Not much is gained in terms of satisfiability checking by restricting to unary operators and non strict operators.
- Unlike binary, strict and non strict unary operators collide with strict monotonic restriction.

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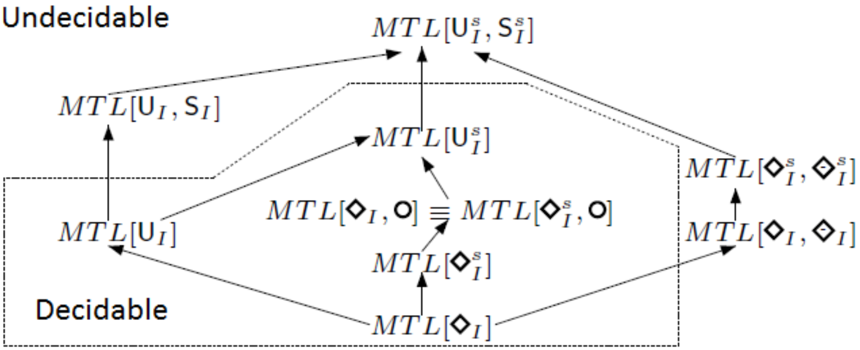

Figure: Expressiveness Hierarchy. Classes within Polygon have decidable satisfiability checking

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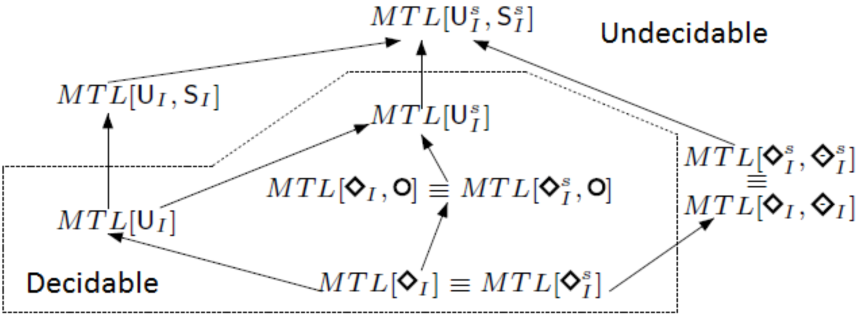

Figure: Expressiveness Hierarchy with Strict Monotonicity. Classes within Polygon have decidable satisfiability checking

Future Work

- Explore membership checking algorithms for these fragments.
- There is not much known about decidable fragments of *TPTL* with more than one clocks other than positive fragment.It would be interesting to explore those fragments too.
- To check how these unary fragments behave with different semantic restriction.
- And to come up with more restrictions which gives us low complexity satisfiability checking.
- Explore satisfiability checking on infinite words too.

Thank You