Logistics in Real-Time: Inventory Routing Operations Under Stochastic Demand

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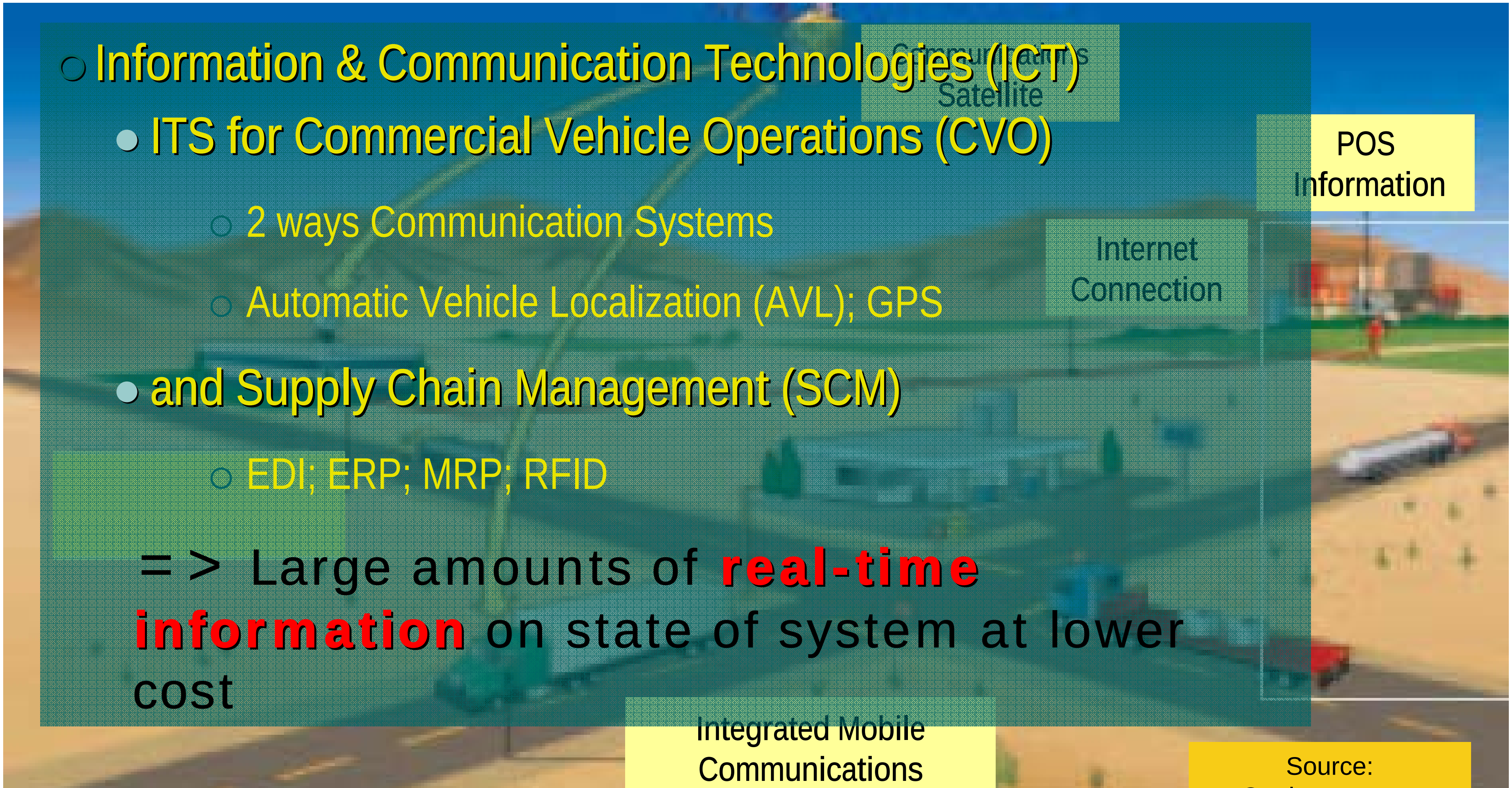
Outline

1. Introduction
2. The On-Line Inventory Routing Problem (OIRP)
3. Solution Approach
4. Real-Time Strategies
5. Simulation Experiments and Results
6. Conclusions

Motivation

- Information & Communication Technologies (ICT)
  - ITS for Commercial Vehicle Operations (CVO)
    - 2 ways Communication Systems
    - Automatic Vehicle Localization (AVL); GPS
  - and Supply Chain Management (SCM)
    - EDI; ERP; MRP; RFID

=> Large amounts of **real-time information** on state of system at lower cost

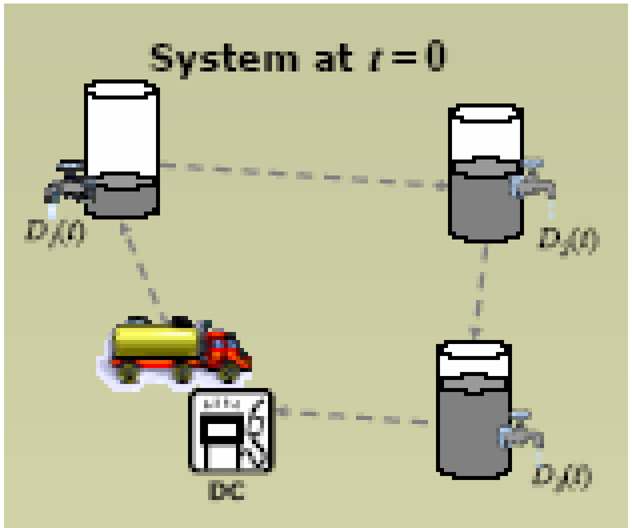


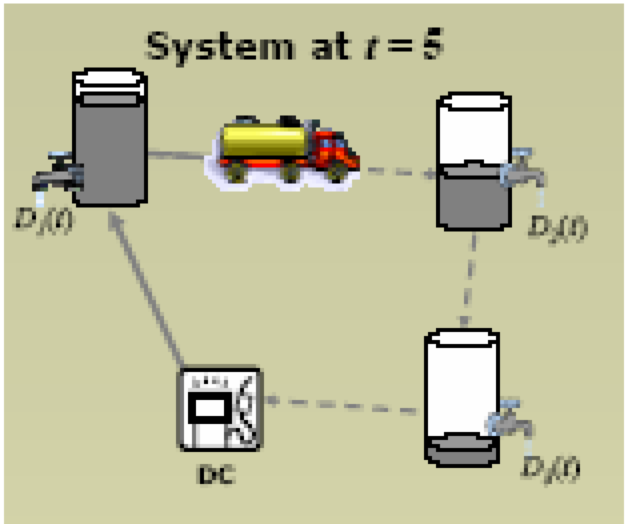
Source: Qualcomm.com

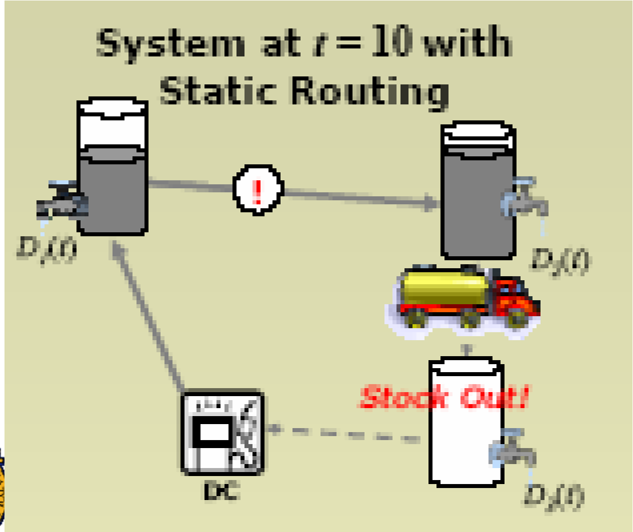
Motivation

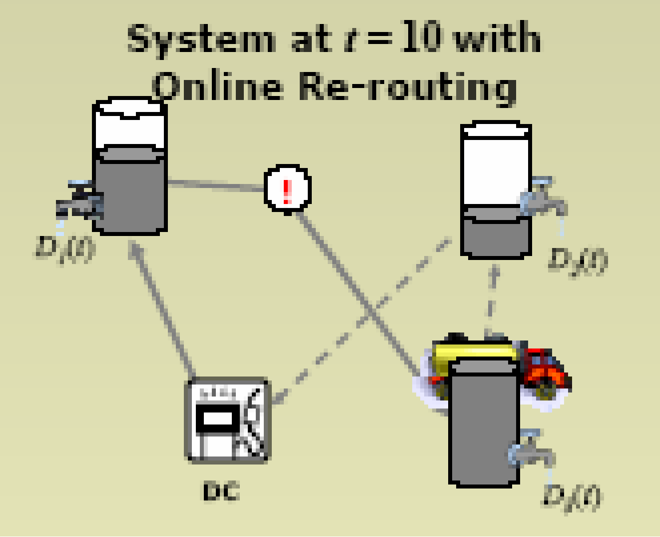
- VMI Agreements and Vertically Integrated Distribution Systems
- Extend Dynamic Vehicle Routing Problems to Incorporate Inventory Control
- In all previously studied **Inventory Routing Problems (IRP)** ***routes are not modified after leaving the depot*** (Baita et al, 1998; Campbell et al, 1998; Kleywegt et al, 2002)

OIRP Instance









Objectives

- Formulate the Online Inventory Routing Problem (OIRP) taking into account explicitly real-time information about state of the system
- Develop efficient operational strategies for coordinating ***Transportation*** and ***Inventory Control*** operations under **real-time information**
- Evaluate the benefits of the proposed strategies and the value of using real-time information

OIRP Definition

- System is composed of:
  - A ***Depot,*** *i=0,* with infinite capacity where the vehicle is refilled
  - ***Set of Customers*** , $i=1, 2, \ldots, N$
    - Independent and known stochastic demand processes
      - *Compound Poisson* (with parameters λ_i, θ_i)
      - independent across customers
    - Demand during stock-out is lost with a penalty p_i per unit
    - Stock accrue inventory holding cost h_i per unit per unit of time
    - Fixed Capacity, u_i , maximum inventory level.
  - A ***Vehicle***, that transports the product between facilities
    - Travel Times are deterministic
    - Moves at Constant Speed according to Euclidean metrics
    - Fixed capacity, Υ

OIRP Definition

- System is operated by a Central decision maker who has:
  - **Real-time information about the state of the system**:
    - Inventory Levels (I_i) at each facility
    - Location and load remaining in the vehicle
  - and **can update truck plans at any time**
    - if a truck is traveling when a decision to update its plan is taken a ***time projection*** is used
- The **objective** is to **minimize the expected average operating costs** per unit of time
  - **Transportation Costs**, TC [$/distance]
  - **Inventory Holding Costs**, h_i [$/units-day]
  - **Stock-out Costs** or Lost Sale Penalties, p_i [$/units]

Main Decisions in the OIRP

- Truck Plans are specified by
  - **Sequence** of facilities to be visited
  - **Amounts to be delivered** to subsequent facilities in the route
  - **Departure time from the depot**
    (only facility where the truck can be idle)
- Main decisions (variables) at any plan update epoch are:
  - **Modify the set** or the sequence of subsequent customers to be visited
  - **Divert** a truck from its current destination to visit a different facility
  - **Adjust amounts** to be delivered to subsequent facilities in the route
  - Modify the amount of **time spent at the depot**

Solution Approach

- Main difficulty are the effects of **short-term decisions on long-run costs**
  - since deliveries can be combined on the same route, transportation costs are not fixed

  ⇒ **Optimal Policy** to serve each customer would **depend** not only on its inventory level, but also on the **complete state of the system**

- A Hierarchical Rolling Horizon Approach is Proposed:
  1. Optimal Refilling Levels for each facility (long-term impacts)
  2. Routing Problem to Update Plans (short-term impacts)

Solution Approach

- Rolling Horizon Strategies are specified by two main definitions:
  - **When** plans are updated
    - **Event** driven strategies (ex. at tour completions)
    - **Time** driven strategies (ex. every hour or period)
    - **Mixed** (event and time driven) strategies
  - **How** plans are updated
    - Fast Local Online Operations (Insertion Heuristics)
    - Optimization Formulations for Associated Off-Line Problems
    - Stochastic Optimization Formulations

Reorder Quantities in the Off-Line IRP

1. **Optimal Refilling Levels** (s,S) for each customer
   - Assumptions:
     - Truck visits only one customer per route (No pattern of visits)
     - Customers should wait for service, if the truck is serving other facilities
     - Quantities delivered to each customer could be updated upon arrival
   - Levels are used as target in (2) to refill in each visit
   - Levels are obtain minimizing the total expected cost of the system using a steepest decent numerical procedure

Reorder Quantities in the Off-Line IRP

$$\text{Min} \sum_i AC_i \approx \text{Min} \sum_i \left\{ AC_i \approx \frac{FTC_i + h_i\left(\frac{S_i - s_i}{\mu_i} + L_i\right)\left((s_i - L_i\mu_i) + \frac{1}{2}(S_i - s_i + L_i\mu_i)\right) + p_i \cdot \sigma_i\sqrt{L_i} \cdot G\left(\frac{s_i - L_i\mu_i}{\sigma_i\sqrt{L_i}}\right)}{\left(\frac{S_i - s_i}{\mu_i} + L_i\right)} \right\}$$

Inventory holding cost, per cycle

Expected stockout cost, per cycle

Expected Cycle Length

s.t. $L_i = T + d_{io} + W_i$

Total Lead Time

$$W_i = \sum_{j \neq i} \left\{ \underbrace{\Pr\{\text{cust. } j \text{ is in service}\}}_{\beta_j} \cdot \left(d_{oj}\right) + \underbrace{\Pr\{\text{cust. } j \text{ is waiting for service}\}}_{\gamma_j} \cdot \left(2d_{oj}\right) \right\}$$

Waiting time

$$\beta_j = \frac{2d_{0j}}{\left(\frac{S_j - s_j}{\mu_j}\right) + L_j} \quad \text{and} \quad \gamma_j = \frac{W_j}{\left(\frac{S_j - s_j}{\mu_j}\right) + L_j}$$

$$\rho = \sum_i \left(\mu_i / Q_i\right) \cdot (2d_{0i}) < 1$$

where: the total demand during L is approximated as $N(L\mu, L\sigma^2)$

$G(\ )$ is the loss function

Mathematical Formulation Off-Line IRP

2. Off-Line IRP used to Update Plans

- Variables: t_i : Arrival time at facility i , for $i \in \Im$

  q_i : Delivery size at facility i , for $i \in \Im$

- Problem:
  - Minimize the sum of:
    - **Total Transportation Costs**
      - Total **distance** required to visit all customers
      - **Incremental Transportation Costs (*ITC*) :** additional distance due to refills lower than S_i
    - **Expected Lost Sale Penalties (*LSP*)**
  - Subject to:
    - All customers should be visited once during the planning horizon
    - After a delivery, inventory levels at each facility should exceed s_i and not exceed S_i

Mathematical Formulation Off-Line IRP

Incremental Transportation Costs (*ITC*)

Additional distance associated with refills lower than S

$$ITC(q_i, t_i / \iota_i) = TC \cdot 2d_{0i} \cdot \left(1 - \frac{q_i}{S_i}\right)$$

where: $(s - \iota + \mu \cdot t) \leq q \leq \max\{S, (S - \iota + \mu \cdot t)\}$

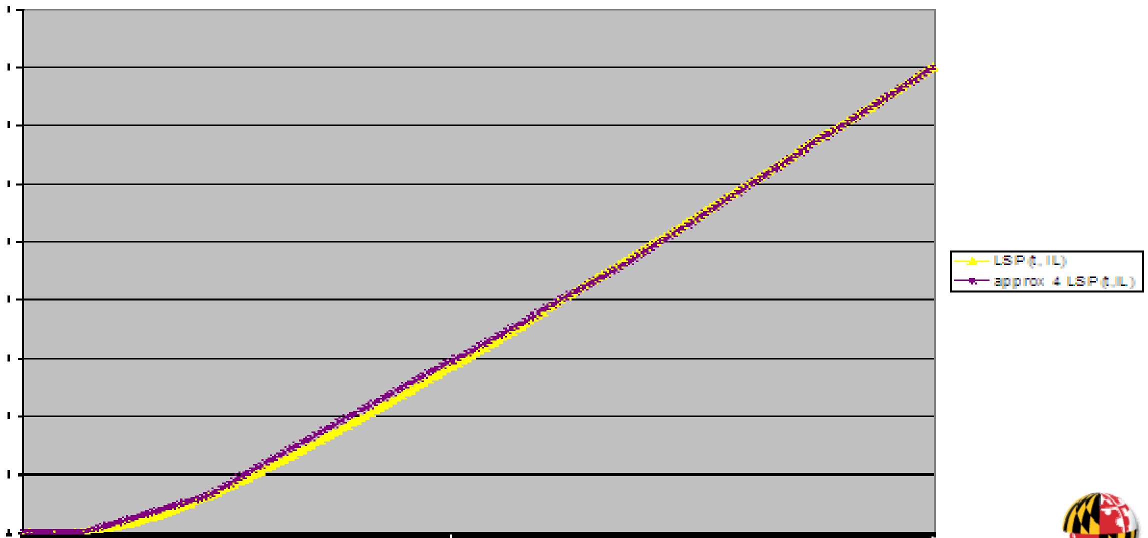
Mathematical Formulation Off-Line IRP

Expected Lost Sale Penalty (*LSP*)

$$LSP(t/\iota) = p \cdot \int_{\iota}^{\infty} (u-\iota) f_{D(t)}(u)\,du \qquad = p \cdot \sigma\sqrt{t} \cdot G\left(\frac{\iota - t\mu}{\sigma\sqrt{t}}\right)$$

, where: the total demand during t is approximated as $N(t\mu, t\sigma^2)$

$G(\ )$ is the loss function



Mathematical Formulation Off-Line IRP

Min. $$TC \cdot \sum_{i,j \neq i} d_{ij} \cdot \left(\sum_{r} x_{ij}^{r} \right) + \sum_{i=1}^{N} \sum_{r} ITC_i(q_i^r, t_i / \iota_i) \cdot y_i^r + \sum_{i=1}^{N} LSP_i(t_i / \iota_i)$$

s.t.

- each customer is visited
- continuity of flows
- Consistency between arrival times at each facility and travel times between them and the initial conditions
- truck capacity is not exceeded
- quantities delivered by current route cannot exceed the load remaining in the vehicle
- inventory level should be greater than R_i after refilling,
  - except at current facility if the load remaining in the truck is insufficient
- inventory levels should not exceed the order up to level S_i

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6. Extensions

Operational Strategies with Real-Time Information

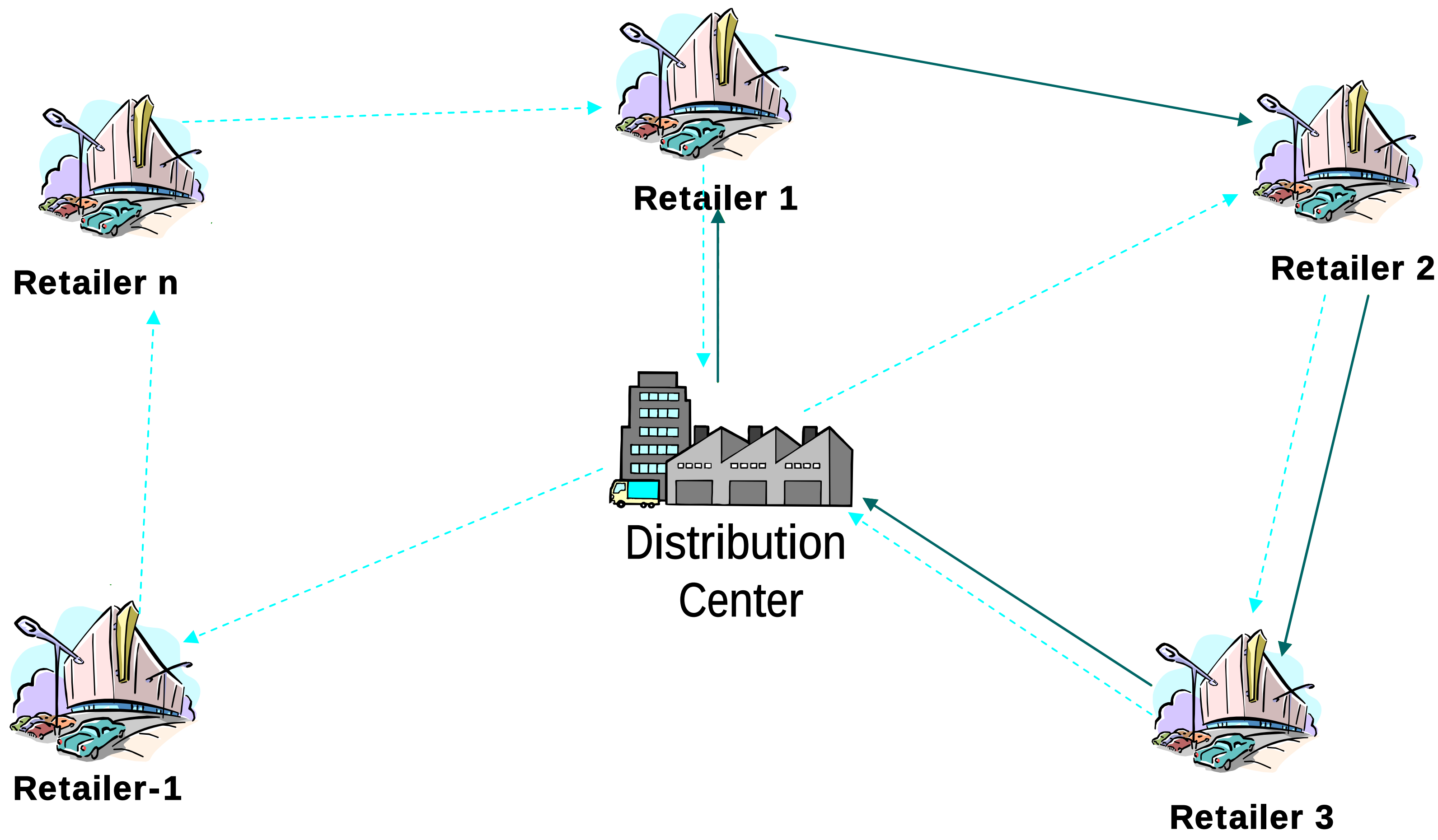
1) BENCH: Benchmark

- each retailer manages his own inventory, placing orders to a central supplier that schedules deliveries for previous day orders once a day
  - Each Customer follows an optimal (*s, S*) policy to control his inventory
  - the supplier creates routes solving a vehicle routing problem (VRP) based on the orders received at the end of each day

2) <u>RTC</u>: Re-plan at Tour Completions

- off-line IRP is solved each time the truck returns to the depot
- Implement only the 1st tour of the current solution of the off-line IRP

RTC: Re-plan at Tour Completions



Operational Strategies with Real-Time Information

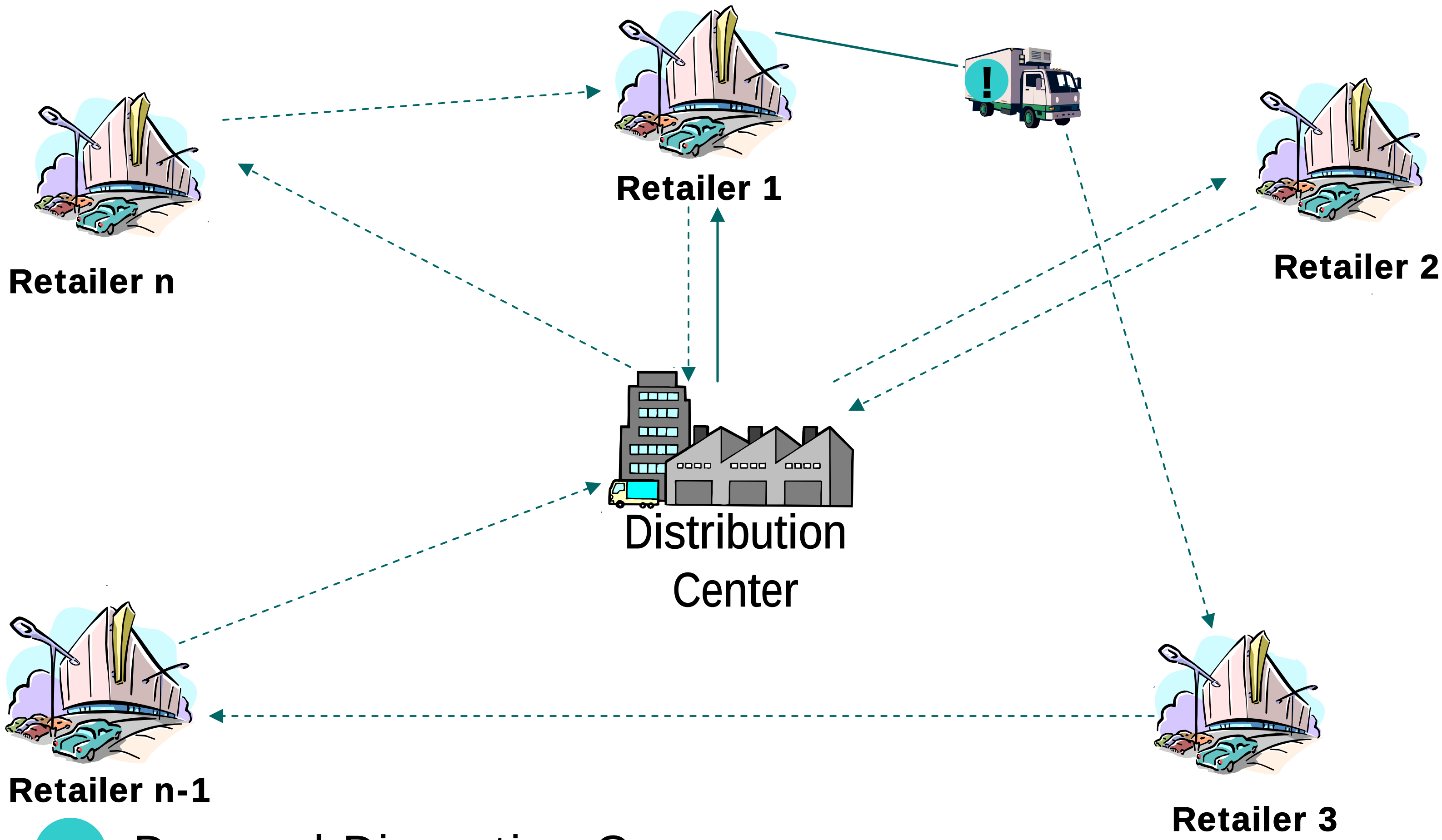
3) <u>RDE</u>: Re-plan at Delivery Epochs

- Solve off-line IRP when truck arrives to a facility, and
- the solution implemented until the next delivery epoch
  - the amount specified by the solution is delivered at the current facility
  - truck is sent to the next facility specified by the solution

4) <u>RDE+div</u>: RDE + Plans are updated when demand disruptions occur. In this strategy, the truck could be diverted

- RDE+
- inventory levels are continuously monitored while the vehicle is traveling
- ***plans are updated*** based on projected state of the system whenever a ***disruption occurs***

div: En-Route Diversion



! Demand Disruption Occurs

Plan is updated based on projected state of the system

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Simulations

Set of **parameters fixed** used in the simulations:

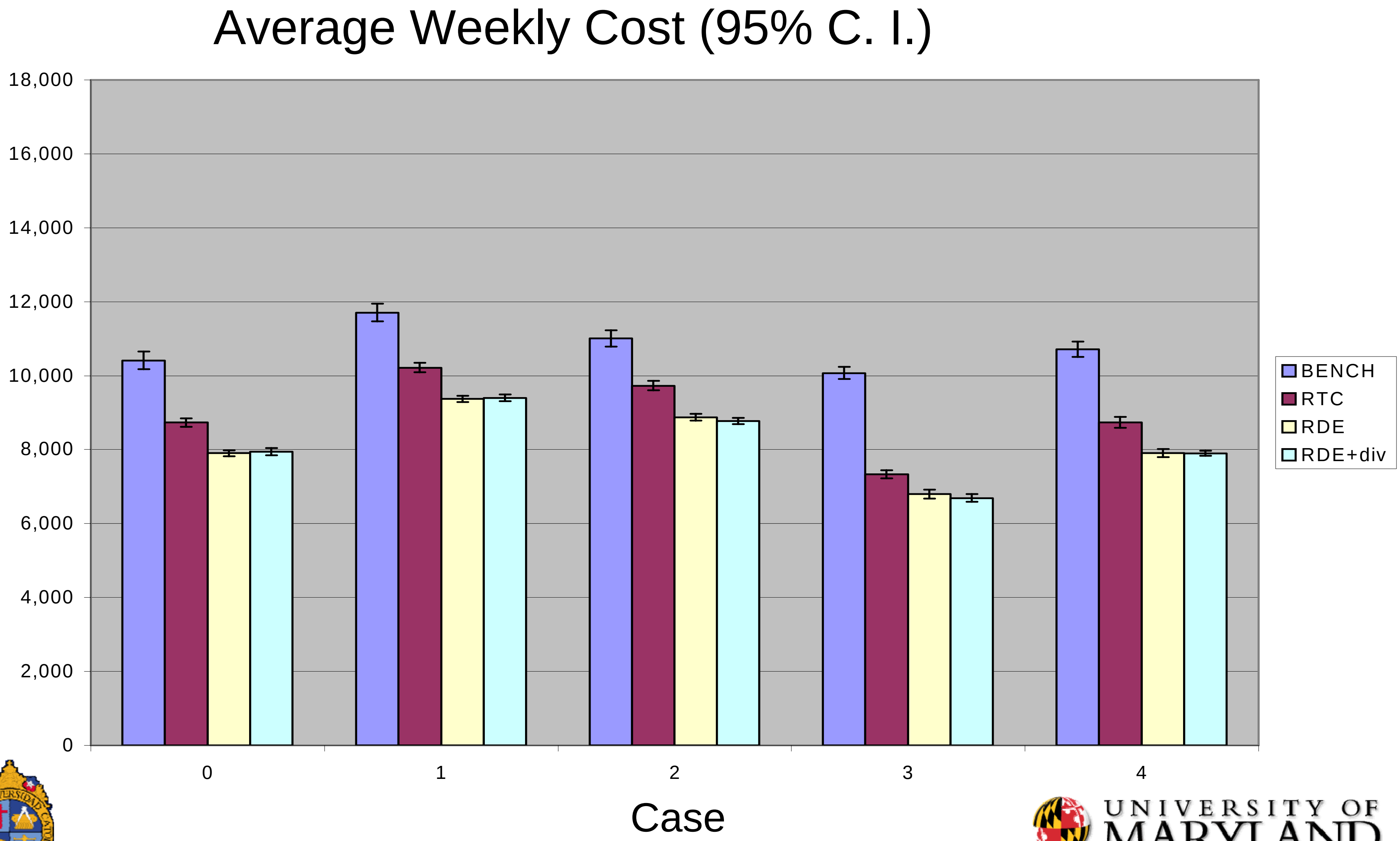
- N = 5 (number of customers considered)
- *Vehicle Capacity* = 400 [units]
- $\bar{t}$ = 100 [hrs] = 5 [days] length of the planning horizon (off-line problem)
- h = 5 [$/unit-day] customers
- p = 50 [$/unit] customers

- **5 Cases of Facility Locations**
  - Distribution Center (DC) is located at the center for all cases
  - Case 0: Symmetric with all customers at 1.5 hrs. from the DC
  - Cases 1-4: Customers randomly located in a square region of 5X5 hrs

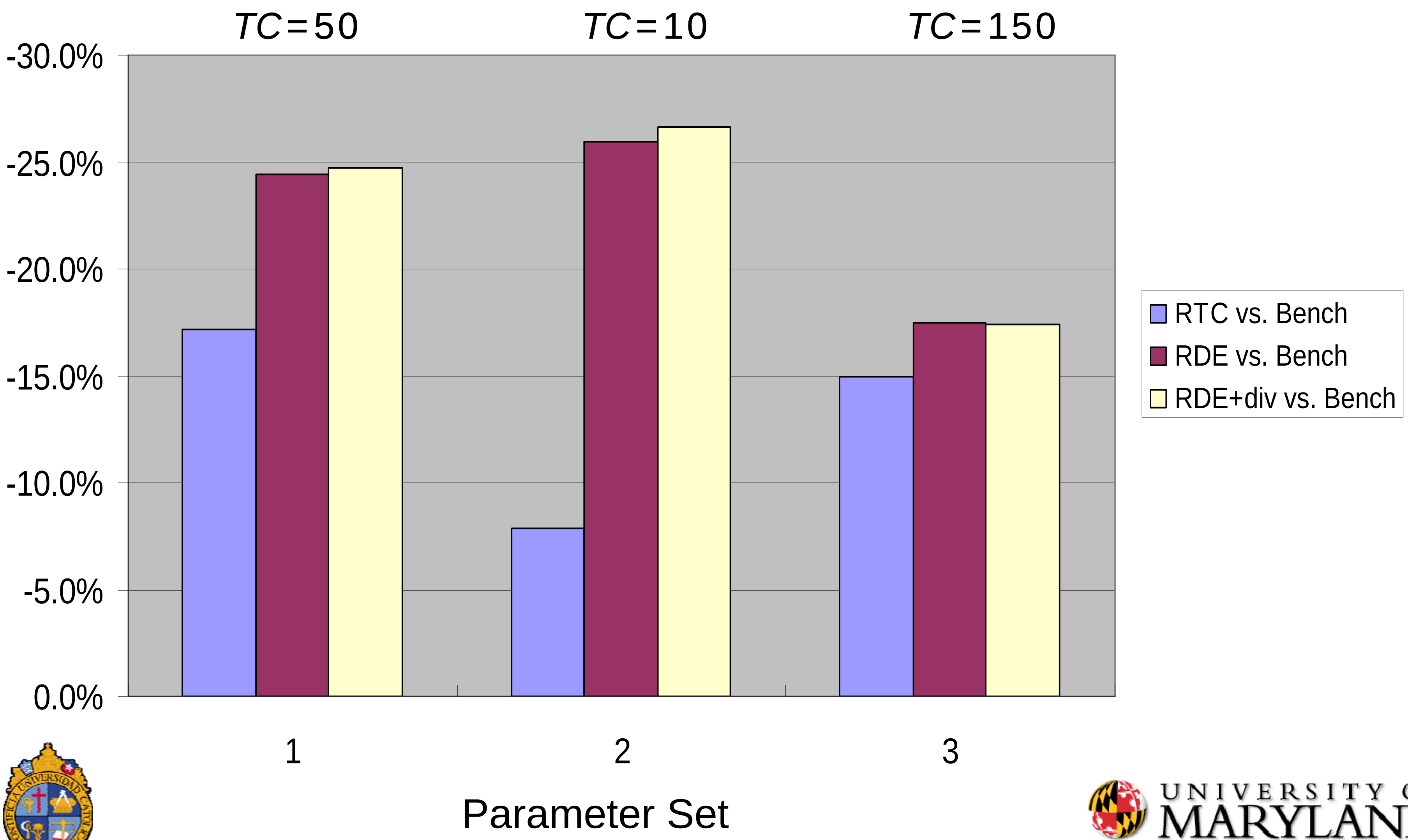
Simulations

Parameter Set	TC [$/ hr]	λ [arrivals/day]	Θ [units]	Approx. $N(\mu, \sigma^2)$
1	50	50	1	$N(50,10^2)$
2	10	50	1	$N(50,10^2)$
3	150	50	1	$N(50,10^2)$
4	50	10.5	4.8	$N(50,17^2)$
5	50	4.35	11.5	$N(50,25^2)$
6	50	2.4	20.8	$N(50,33^2)$

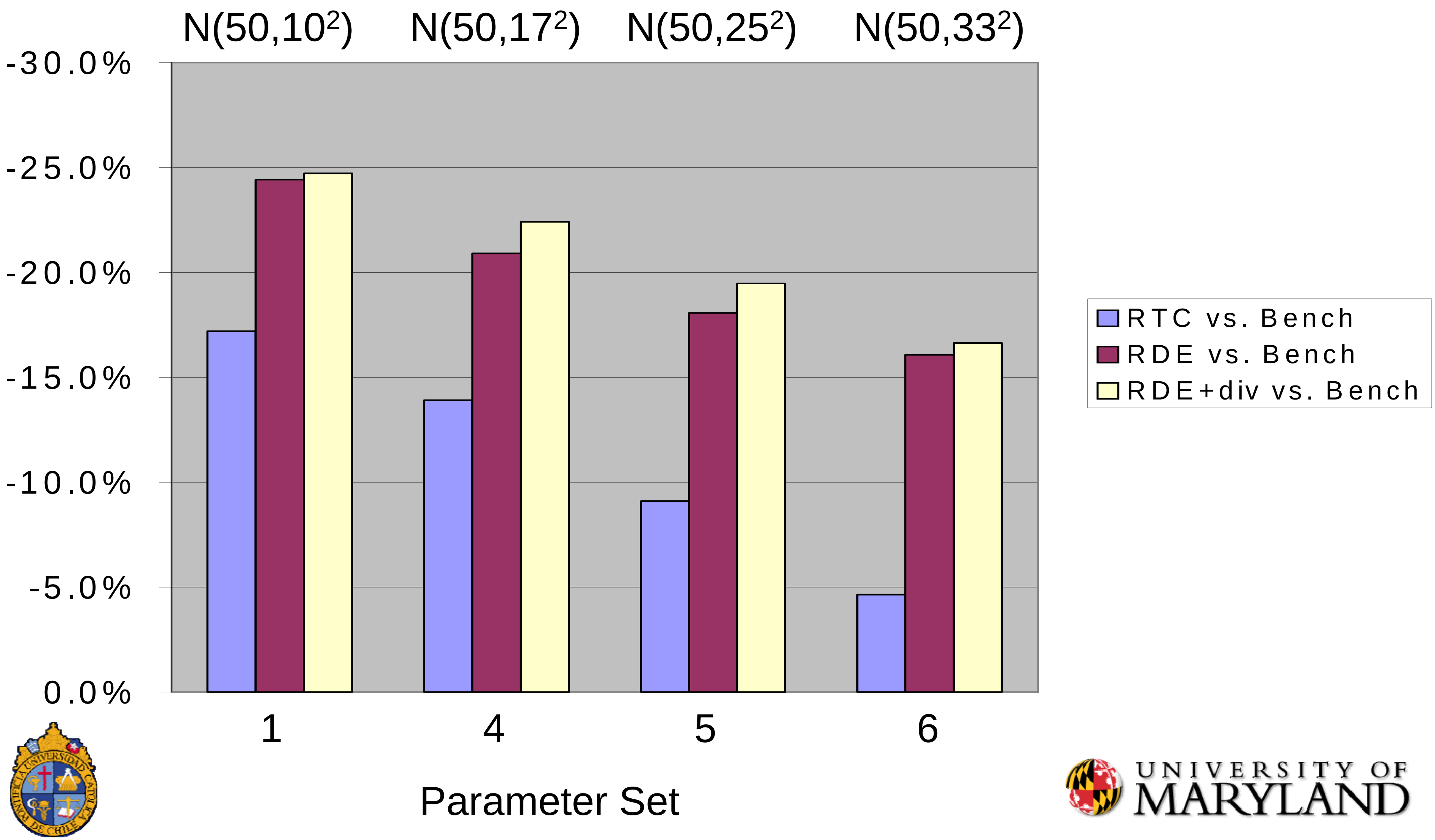
Results: Parameters Set 1 (*TC*=50) *D*~ N(50,10^2)



Results: Cost Reductions (%)



Results: Cost Reductions (%)



Results Summary (for scenarios studied)

- On-Line Strategies RDE and RDE+div :
  - **Decrease** Average Total Cost **by 21%** vs. BENCH
  - **Decrease Variability** in Average Cost
  - Benefits tend to be higher when clusters of customers are close to each other and/or near to the depot (case 3)
- Benefits of dynamic strategies tend to increase :
  - with higher inventory costs
  - slightly with greater demand variability
- The possibility of *diversion* does not show significant benefits in these experiments; However lost sales tend to be smaller

Extensions

- Design of A Priori Strategies
- Incorporate uncertainty **in travel times**
  - City Logistics, travel times between 2 points varies significantly depending on time of the day affecting the system performance
  - Incidents such as network congestion and accidents
- Design **strategies for larger size problems**
  - Larger number of customers
  - Multiple Vehicles
  - Multiple Distribution Center

    => Design of simple heuristics that achieve competitive results to the re-planning strategies presented

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Thank You

Q&A