PAPER

The Absolute Stability Analysis in Fuzzy Control Systems with Parametric Uncertainties and Reference Inputs

Bing-Fei WU[†a)], *Member*, **Li-Shan MA**[†*], *and* **Jau-Woei PERNG**[††], *Nonmembers*

SUMMARY This study analyzes the absolute stability in P and PD type fuzzy logic control systems with both certain and uncertain linear plants. Stability analysis includes the reference input, actuator gain and interval plant parameters. For certain linear plants, the stability (i.e. the stable equilibriums of error) in P and PD types is analyzed with the Popov or linearization methods under various reference inputs and actuator gains. The steady state errors of fuzzy control systems are also addressed in the parameter plane. The parametric robust Popov criterion for parametric absolute stability based on Lur'e systems is also applied to the stability analysis of P type fuzzy control systems with uncertain plants. The PD type fuzzy logic controller in our approach is a single-input fuzzy logic controller and is transformed into the P type for analysis. In our work, the absolute stability analysis of fuzzy control systems is given with respect to a non-zero reference input and an uncertain linear plant with the parametric robust Popov criterion unlike previous works. Moreover, a fuzzy current controlled RC circuit is designed with PSPICE models. Both numerical and PSPICE simulations are provided to verify the analytical results. Furthermore, the oscillation mechanism in fuzzy control systems is specified with various equilibrium points of view in the simulation example. Finally, the comparisons are also given to show the effectiveness of the analysis method.

key words: *steady state error, parametric absolute stability, fuzzy logic control system, Lur'e system, Popov criterion, robust*

1. Introduction

Fuzzy logic controller (FLC) has become a conventionally adopted control algorithm, and has been employed in various industrial applications [1], since Mamdani [2] proposed the first linguistic FLC based on expert experience to control a laboratory steam engine. The FLC design does not require an accurate mathematical model. Unlike traditional nonlinear controllers, FLC can work with imprecise inputs, and can deal with nonlinearity and uncertainty. Therefore, many studies are devoted to this field. Conversely, since the accurate mathematical model is not required to design FLC, the design procedure is still based on trial and error. Hence, the stability and performance of FLC cannot be guaranteed. Systematic analysis and synthesis schemes [3]–[26] have recently been developed to improve this issue.

Manuscript received November 28, 2008.

Manuscript revised February 18, 2009.

[†]The authors are with the Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu, 300, Taiwan.

[††]The author is with the Department of Mechanical and Electro-Mechanical Engineering, National Sun Yat-sen University, Kaohsiung, 804, Taiwan.

[*]The author is also with the Department of Electronic Engineering, Chienkuo Technology University, Changhua, 500, Taiwan.

a) E-mail: bwu@cc.nctu.edu.tw

DOI: 10.1587/transfun.E92.A.2017

Some methods [3]–[10] adopt the Takagi-Sugeno (T-S) fuzzy models to determine the stability of fuzzy control systems by the Lyapunov function or linear matrix inequality (LMI). The overall plant is first represented as a T-S fuzzy model by a fuzzy blending of each linear system model. The controller is then designed based on this T-S fuzzy model by Lyapunov function or LMI. However, an appropriate fuzzy model may be difficult to formulate for an arbitrary nonlinear dynamic system. Additionally, a common Lyapunov function for general cases, and an existing positive-definite matrix, are both difficult to obtain. Besides the T-S fuzzy model, Lyapunov functions are also adopted to design and analyze the robust PD fuzzy controller for bounded uncertainties or nonlinearities of the system, using the Popov-Lyapunov approach [11]. In addition, the stability on the T-S fuzzy model is analyzed by the Kharitonov theorem incorporated with the Schur and Hurwitz criterions [12]. Recently, the developments of fuzzy logic control designs almost focus on the T-S fuzzy models control. The stability analyzes all apply the time-domain LMI approach. The main research directions include model uncertainties [13]–[20] and time-delay [21]–[23] or both [24], [25]. The stability issues due to the reference input influence are not to be discussed in the T-S fuzzy models control.

Kickert and Mamdani [26] first applied the describing function approach (DF) to analyze the stability of fuzzy control systems by granting fuzzy control systems as a multilevel relay model. The describing function of FLC can, under reasonable assumptions, be obtained to predict the existence of a limit cycle in fuzzy logic control systems [27], [28]. DF provides an approximate approach to obtain the stability of unforced fuzzy control systems. DF may yield inaccurate or incorrect analysis results, because it is an aggressive and approximate approach. In other words, under some assumptions, DF can only be applied to analyze fuzzy system stability successfully. Additionally, the steady state error and transient response of fuzzy control systems with the sinusoidal and exponential input describing functions techniques are analyzed in [29] and [30], respectively.

The choice of parameters in fuzzy control systems with phase plane approach was proposed in [31]–[33]. Then, the phase plane analysis can be utilized to design fuzzy rules, or measure the performance and stability of a specific set of fuzzy rules. Phase plane analysis is a simple graphical approach, in which the system trajectories are inspected to provide information on system stability and performance. However, it is restricted to second order dynamic systems.

The extension of classical circle criteria is also applied to analyze the stability of linear systems with fuzzy logic controllers [34], [35]. The extended circle criteria can be employed to test the SISO and MIMO systems [34]. The extended circle criteria for MISO and MIMO are presented in [35] for testing the robust stability in PI, such as fuzzy control systems with uncertain plant gains. This algorithm limits the nonlinearity of fuzzy controller to the sector bound.

The Popov is a frequency domain stability criterion for closed loop nonlinear systems of Lur'e type. Fuzzy control systems can be regarded as Lur'e type systems. Kandel et al. [36] adopted the Popov criterion to analyze the stability of fuzzy control systems with controller as multi-level relay. Furutani et al. [37] utilized the shifted Popov criterion to manage the fuzzy controller with both time-variant and time-invariant parts. However, the Popov criteria applied to the stability analyzes on the fuzzy logic control do not consider the effect of reference input.

On the other hand, the latest research developments on the Lur'e systems stability analyzes concentrate on the systems with model uncertainties [38]–[41] and time-delay [42], [43] or both [44]–[46]. The main approaches include the time-domain LMI [38]–[44] and the classical frequency-domain [45], [46] methods. The stability issues due to the reference input influence are not even discussed except in [51]. By [51], we can predict that the stability of fuzzy control systems will crash due to reference input shift, so it is important to take the reference inputs as one of the parameters for stability analyzes of fuzzy control systems.

On short, the recent stability analysis developments on the Lur'e type systems almost always use the time-domain LMI approach. The concerned issues are on uncertainties and time-delay or both. However, the development directions don't concern the reference input influence on stability.

Other investigations on fuzzy logic control systems can be described as follows. Butkiewicz [47] investigated the steady error of a fuzzy control system with respect to different fuzzy reasoning processes [47]. Tao and Taur [48] designed a robust complexity-reduced PID-like fuzzy controller for a plant with fuzzy linear model in [48]. Malki et al. [49] derived a fuzzy PD controller from the conventional continuous-time linear PD controller [49], in which the proportional and derivative gains are a nonlinear function of the input signal. The stability of this new type fuzzy PD controller is ensured by the small gain theorem. Taur and Tao [50] analyzed and designed region-wise linear fuzzy controllers (RLFC) [50], and found that the RLFCs generally performed better than the PD controllers.

Our work analyzes the absolute stability in P and PD type fuzzy control systems with both certain and uncertain linear plants. The control functions in P and PD type fuzzy controllers are known to be piecewise linear, and can be described with mathematical equations. The equilibrium points of each piecewise linear surface in a P type fuzzy control system with a certain linear plant can be calculated by this description. The unique error equilibrium point of the overall system can be obtained by determining whether the error equilibrium point located in its own error region. Therefore, the error equilibrium points in the reference and actuator gain parameter space can be analyzed. Additionally, the absolute stability can be analyzed using the frequency and time domain approaches. Since a P type fuzzy control system is a Lur'e system, its stability can be tested by the Popov criteria in the frequency domain. In the time domain, the stability can be tested by linearizing the system with regard to the equilibrium point. Conversely, the stability of a P type fuzzy control system can be tested by the parametric robust Popov criterion [51] incorporated with the Kharitonov theorem for uncertain linear plant and interval parameters, including actuator gain, reference input and plant parameters. Notably, the actuator gain can be included in one of the plant parameters. For a PD type fuzzy control system, single-input fuzzy logic controller (SFLC) [52] is introduced into our analysis. In a certain linear plant situation, the equilibrium point of fuzzy control systems can be analyzed using the same P type fuzzy analysis concepts. A PD type fuzzy control system with an SFLC controller can be transformed into a P type system, so that its stability can be analyzed with the Popov and linearization methods. The parametric absolute stability of Lur'e systems can also be applied to a transformed PD type fuzzy control system when the plant is uncertain. For comparison with theoretical analysis, a fuzzy current controlled RC circuit is designed with a PSPICE model. Simulation results including both numerical and PSPICE confirm the theoretical analysis. Additionally, the mechanism of oscillations in fuzzy control systems is interpreted with a viewpoint of equilibrium points in a simulation example. Finally, the comparisons also are made to exhibit the effectiveness of the analysis method. The applied method parametric robust Popov criterion will be compared with the robust Lur'e test [54], the robust circle criterion [54], and the robust Popov criterion [54]. In compared methods, the stability of uncertain fuzzy control systems which are considered as stable by compared methods will crash under the effect of the reference inputs. On the other hand, by the applied analysis method, the stability can be guaranteed for the certain interval reference inputs. In brief, this study can provide a valuable reference in designing fuzzy control systems.

In conclusion, the stability analysis is extended to a non-zero reference input and an uncertain linear plant. This is in contrast to the approach employed by Kim et al. [27], in which DF is derived and applied to analyze the stability of fuzzy control systems for zero reference inputs and certain linear plants. The DF method may yield inaccurate or incorrect analysis results without restricted assumptions. By contrast, the Popov criterion based on the Kharitonov theory can guarantee an exact stability investigation. Moreover, SFLC [52] is applied in the analysis of a PD type fuzzy control system. SFLC is an efficient FLC, owing to its 1-D fuzzy rules only. By this feature, the SLFC can be implemented as an analog circuit and applied for high frequency control. This work first investigates the steady state error and robust stability analysis for linear plants using the proposed structure transformation. Additionally, an analog fuzzy control system is designed with a PSPICE model to verify the analysis results. Finally, the explanations for unstable oscillations in fuzzy control systems are presented with the equilibrium concept.

This paper is organized as follows. Section 2 describes the P and PD type fuzzy control systems. Section 3 analyzes the equilibrium points and stability in P type fuzzy control system. Section 4 then performs the same analyzes in a PD type fuzzy control system. Section 5 provides simulation results with Matlab and PSPICE simulators. In Sect. 6, the comparisons are made to show the superiority of the applied analysis method. Conclusions are finally drawn in Sect. 7.

2. The Fuzzy Logic Control System

Both P and PD type fuzzy logic control systems include a linear plant with time-invariant uncertainty, adjustable actuator gain and reference input. Moreover, the fuzzy logic controllers are the cores of systems. An FLC can be taken as multiple bends of piecewise linear functions, since it has singleton and specific membership functions. Hence, a fuzzy logic control system can be treated as a Lur'e type system.

2.1 Fuzzy Logic Controller

Consider the fuzzy logic control system in Fig. 1. The IF-THEN rules in single input fuzzy logic controller can be described as:

$$Rule_i : \text{If } e \text{ is } M_i, \text{ then } u_f \text{ is } u_i, \tag{1}$$

where e is the control error and M_i and u_i denote fuzzy sets. If a singleton is applied in a fuzzifier, then the product inference and center average are formulated in the inference engine and defuzzifier, respectively. The output of the fuzzy logic controller can be represented as

$$u_f = \sum_i \Omega_i(e) u_i, \tag{2}$$

where $\Omega_i(e) = \dfrac{M_i(e)}{\sum\limits_j M_j(e)}$.

For simplification, this study uses the fuzzy rules and membership functions listed in Table 1 [27] and Fig. 2 are adopted in this paper, respectively. Table 2 presents the fuzzy controller parameters. Figure 3 shows the control function of the fuzzy controller, which can be described as:

$$u_f = \sigma(e) = \begin{cases} segment\ 1 : k_2 e + c_2, & e \in [a_2, a_3] \\ segment\ 2 : k_1 e + c_1, & e \in [a_1, a_2] \\ segment\ 3 : k_0 e, & e \in [-a_1, a_1] \\ segment\ 4 : k_1 e - c_1, & e \in [-a_2, -a_1] \\ segment\ 5 : k_2 e - c_2, & e \in [-a_3, -a_2] \end{cases} \tag{3}$$

where
$0 < a_1 < a_2 < a_3, 0 < b_1 < b_2 < b_3, c_1 = b_2 - k_1 a_2,$
$c_2 = b_3 - k_2 a_3, k_0 = \dfrac{b_1}{a_1}, k_1 = \dfrac{b_2 - b_1}{a_2 - a_1}$, and $k_2 = \dfrac{b_3 - b_2}{a_3 - a_2}$.

Remark 1: The assumptions $0 < a_1 < a_2 < \cdots < a_n$ and $0 < b_1 < b_2 < \cdots < b_n$ are satisfied for n multiple bends of a control function. The control output of the static fuzzy system is given by:

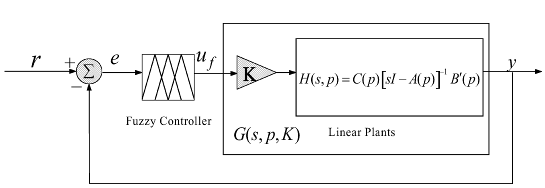


Fig. 1 The P type fuzzy control system.

Table 1 Rules of the fuzzy logic controller.

e	NBE	NME	NSE	ZRE	PSE	PME	PBE
u_f	NBU	NMU	NSU	ZRU	PSU	PMU	PBU

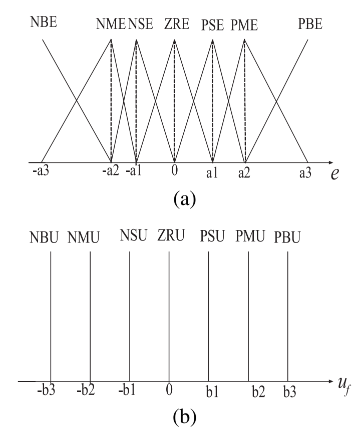


Fig. 2 The membership functions of the fuzzy logic controller.

Table 2 Parameters of the fuzzy logic controller.

	NBE	NME	NSE	ZRE	PSE	PME	PBE
e	$-a_3$	$-a_2$	$-a_1$	0	a_1	a_2	a_3
	NBU	NMU	NSU	ZRU	PSU	PMU	PBU
u_f	$-b_3$	$-b_2$	$-b_1$	0	b_1	b_2	b_3

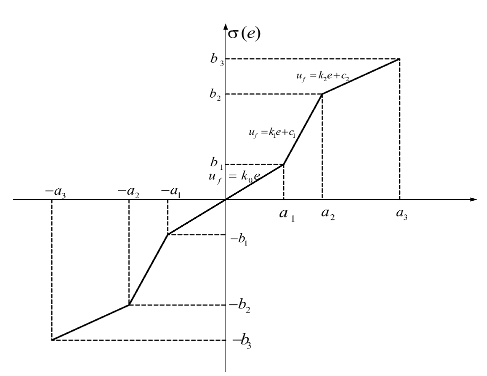


Fig. 3 The control function of the fuzzy logic controller.

$$u_f = \sigma(e) = \begin{cases} k_n e + c_n, & e \in [a_n, a_{n+1}], \\ k_0 e, & e \in [-a_1, a_1], \\ k_n e - c_n, & e \in [-a_{n+1}, -a_n], \end{cases} \tag{4}$$

where $c_n = b_{n+1} - k_n a_{n+1}, k_n = \dfrac{b_{n+1} - b_n}{a_{n+1} - a_n}$,
and $n = 1, 2, 3, \ldots, n$. The control function satisfies

$$0 \le \hat{e}[\sigma(e + \hat{e}) - \sigma(e)] \le k(e)\hat{e}^2, \ \forall e \in \mathbf{O}, \ \forall \hat{e} \in \Re, \tag{5}$$

where $\sigma(0) = 0, k > 0$ and $\mathbf{O}$ indicates some neighborhood of $e = 0$.

2.2 P Type Fuzzy Logic Control System

Figure 1 illustrates a P type fuzzy control system with a fuzzy logic controller, a parametric linear time-invariant system and adjustable parameters, which include actuator gain K and reference input r. The control function of the fuzzy controller is a piecewise linear function, and is depicted in Fig. 3.

The linear plant $H(s, p)$ shown in Fig. 1 can be presented as

$$H(s, p) = C(p)[sI - A(p)]^{-1} B'(p), \tag{6}$$

where $A(p) \in \Re^{n\times n}$ and $A(p)$ is a stable matrix; $B'(p) \in \Re^{n\times 1}$; $C(p) \in \Re^{1\times n}$, the parameter vector p exists in a compact and simple connected region $\mathbf{P} \subset \Re^l$. The transfer function $G(s, p, K)$ with amplifier gain $K \in \Re$ can be stated as

$$G(s, p, K) = C(p)[sI - A(p)]^{-1} B(p, K), \tag{7}$$

where $B(p, K) = KB'(p) \in \Re^{n\times 1}$, and $K \in \Re$. The overall static fuzzy logic control system in Fig. 1 can be described as:

$$\begin{aligned} \dot{x} &= A(p)x + B(p, K)u_f, \\ y &= C(p)x, \end{aligned} \tag{8}$$

where the control input $u_f = \sigma(e)$; the control error $e = r - y, x \in \Re^n, e \in \Re$ and $y \in \Re$; the reference input r is a constant value, and r is a constant value, and $r \in \Re$.

The closed loop system is given by

$$\dot{x} = A(p)x + B(p, K)\sigma[r - C(p)x]. \tag{9}$$

The error equilibrium points and relative stability under the influence of parameters including actuator gain K, reference input r and time invariant uncertainty in linear plants are addressed. The parameter vector is defined as (r, p, K).

2.3 PD Type Fuzzy Logic Control System

This subsection discusses the PD type SFLC depicted in Fig. 4. The SFLC's output u_f is proportional to a negative signed distance D_s. Additionally, the number of the fuzzy rules, as shown in Table 3 [52], is significantly reduced into 1-D space, as in Table 4, owing to the single input and skew-symmetric property. Due to the skew-symmetric property of the rule table, $(e, \dot{e})$ can be split into five regions. Figure 5 illustrates an example of this division of $(e, \dot{e})$. The reduced 1-D rules improve the efficiency of the controller by saving time cost for a look up rule table, although it also adds the calculation time of signed distance. Therefore, the SFLC is suitable for implementation in circuit control. The SFLC is introduced in this section for further equilibrium points and stability analysis in the following sections.

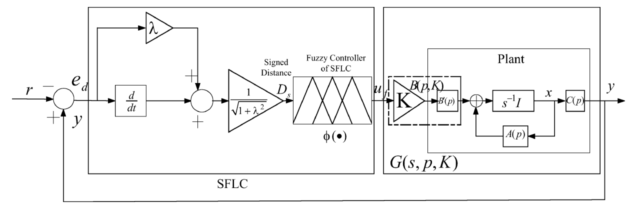


Fig. 4 The single-input fuzzy logic control system.

Table 3 Rules of conventional FLC with control error defined as e_d.

$\dot{e}_d$ \ e_d	NB	NS	ZR	PS	PB
PB	ZR	NS	NS	NB	NB
PS	PS	ZR	NS	NS	NB
ZR	PS	PS	ZR	NS	NS
NS	PB	PS	PS	ZR	NS
NB	PB	PB	PS	PS	ZR

Table 4 Rules of SFLC.

D_s	NBE	NME	NSE	ZRE	PSE	PME	PBE
u_f	PBU	PMU	PSU	ZRU	NSU	NMU	NBU

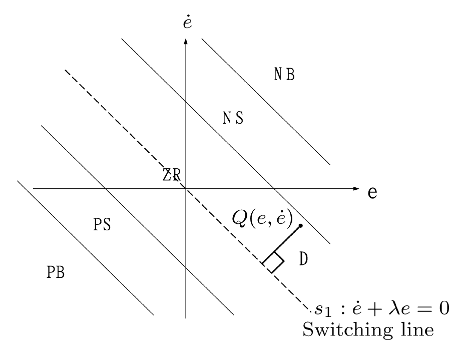


Fig. 5 The skew-symmetric property in $(e, \dot{e})$ and the calculation of signed distance.

(1) Calculation of signed distance

The control error in SFLC is defined as

$$e_d(t) = y - r. \tag{10}$$

The switching line s_l as shown in Fig. 5 is given by

$$s_l : \dot{e}_d + \lambda e_d = 0. \tag{11}$$

The signed perpendicular distance D_s of general point $Q(e_d, \dot{e}_d)$ to a switching line is calculated as follows:

$$D_s = \mathrm{sgn}(s_l)D = \frac{\dot{e}_d + \lambda e_d}{\sqrt{1 + \lambda^2}}, \tag{12}$$

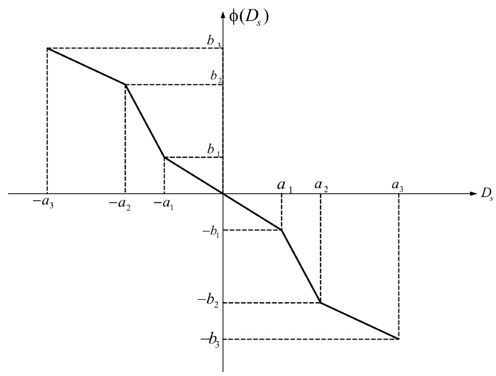


Fig. 6 The control function of the fuzzy logic controller in SFLC.

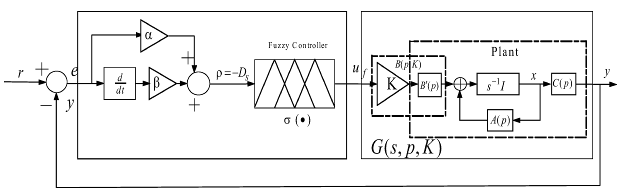


Fig. 7 The transition formation in the transformation.

where, $D = \dfrac{|\dot{e}_d + \lambda e_d|}{\sqrt{1+\lambda^2}}$ is shown in Fig. 5 and

$$\operatorname{sgn}(s_l) = \begin{cases} 1 & \text{for } s_l > 0 \\ -1 & \text{for } s_l < 0. \end{cases}$$

The control output $u_f = \phi(D_s)$ is defined according to the control rule in SFLC as given in Table 4 and Fig. 4.

(2) The presentation of the SFLC system

The SFLC system can be described as:

$$\begin{aligned} \dot{x} &= A(p)x + B(p,K)u_f, \\ y &= C(p)x, \end{aligned} \tag{13}$$

where the control input $u_f = \phi(D_s)$.

(3) The analytic representation of the SFLC system

If Tables 2, 4 and Fig. 2 are applied into the controller in SFLC, then the control function $\phi(\cdot)$ of the fuzzy controller is as displayed in Fig. 6. The surface of the fuzzy controller in SFLC is typically oddly symmetrical; therefore, the control force is given by

$$u_f = \phi(D_s) = \sigma(-D_s) = \sigma(\rho), \tag{14}$$

where $\rho = -D_s = \dfrac{\dot{e} + \lambda e}{\sqrt{1+\lambda^2}}$.

In the following analysis, this representation as illustrated in Fig. 7 is applied to PD type analysis. In Sect. 4, the SFLC system is reformatted as a special P type fuzzy control system, and is employed to analyze the equilibrium point and stability.

3. Equilibrium Points and Stability Analysis in P and PD Type Fuzzy Control Systems

3.1 Equilibrium Point Analysis for P Type Fuzzy Control Systems with Linear Plants

This section presents the analysis of error equilibrium points and stability in P type fuzzy control systems. The equilibrium point in fuzzy control systems can be derived when equilibrium points can be solved. Moreover, the stability of the equilibrium point can be judged with the linearizing system around the equilibrium or the Popov criterion in the following subsection. If the error equilibrium points of the overall system are stable, then the steady state error can be derived from this result. By (9), let $\dot{x} = 0$, then

$$Ax + B(K)\sigma[r - Cx] = 0. \tag{15}$$

If A^{-1} exists, then (16) is obtained.

$$x + A^{-1}B(K)\sigma(e) = 0, \tag{16}$$

where $e = r - Cx$.

Multiply the result by C in (16), and let $Cx = r - e$, then

$$e - r - CA^{-1}B(K)\sigma(e) = 0. \tag{17}$$

The state equilibrium points represented as x^e, and the error equilibrium points denoted as e^e, can be determined from (16) and (17), respectively.

Assumption 1: The unique solution exists in (17). In other words, an error equilibrium point uniquely exists. Under Assumption 1, the error equilibrium points can be solved from (18) by replacing (4) in each segment.

$$\begin{aligned} &e^e - r - CA^{-1}B(K)(k_n e^e + c_n) = 0 && \text{if } e \in [a_n, a_{n+1}], \\ &e^e - r - CA^{-1}B(K)(k_0 e^e) = 0 && \text{if } e \in [-a_1, a_1], \\ &e^e - r - CA^{-1}B(K)(k_n e^e - c_n) = 0 && \text{if } e \in [-a_{n+1}, -a_n]. \\ &\quad n = 1, 2, 3, \ldots. \end{aligned} \tag{18}$$

One of these error equilibrium points is the unique point of the overall system. The unique point is identified by checking whether e^e is located in its own error region.

3.2 Stability Analysis for P Type Fuzzy Control Systems with a Certain Linear Plant

In the certain linear plant case, the stability can be determined by the time or frequency domain approaches proposed in [51]. In the time domain approach, the eigenvalues of the linearizied system (8) can be applied to determine the stability. In the frequency domain, the Popov criterion is utilized to test stability.

(1) Frequency domain approach

Consider the error dynamic system for a given parameter vector (r, p, K).

$$\dot{\hat{x}} = A(p)\hat{x} + B(p)\hat{\sigma}(-C(p)\hat{x}), \tag{19}$$

where $\hat{x} = x - x^e(r, p, K)$, and

$$\hat{\sigma}(-C(p)\hat{x}) = \sigma[-C(p)\hat{x} + e^e(r, p, K)] - \sigma[e^e(r, p, K)].$$

The error equilibrium point of the P type fuzzy control system is given by

$$e^e(r, p, K) = r - C(p)x^e(r, p, K). \tag{20}$$

The error dynamic system is also of Lur'e type. The function $\hat{\sigma}$ satisfies the following sector condition if $e^e(r, p, K) \in \mathbf{O}$.

$$0 \le \hat{e}\hat{\sigma}(\hat{e}) \le k[e^e(r, p, K)]\hat{e}^2,\ \forall \hat{e} \in \Re, \tag{21}$$

where $\hat{e} = e - e^e(r, p, K)$ and $k > 0$. By the Popov criterion, (19) is absolutely stable for a given (r, p, K), if there exists a real number $\nu = \nu(r, p, K)$ satisfying

$$\mathrm{Re}[(1 + j\omega\nu)G(j\omega, p, K)] + \frac{1}{k[e^e(r, p, K)]} > 0\ \forall \omega \in \Re, \tag{22}$$

where $G(s, p, K) = C(p)[sI - A(p)]^{-1}B(p, K)$.

(2) Time domain approach

Under an arbitrary parameter vector (r, p, K), if an equilibrium state $x^e(r, p, K)$ of the system exists, then the stability can be determined from the linearization of (9) near the state equilibrium point.

Remark 2: If the unique state equilibrium is stable, then the steady state error in fuzzy control systems can be obtained from the state equilibrium by $e^e = r - Cx^e$.

3.3 Stability Analysis for P Type Fuzzy Control Systems with an Uncertain Linear Plant

In this subsection, the parametric absolute stability can be tested using the parametric robust Popov criterion incorporated with Kharitonov theorem, when the parameter vector $(r, p, K) \in R_{ref} \times \mathbf{P} \times \mathbf{K}$, where $R_{ref} = [\underline{r}, \overline{r}] \subset \Re$. The value of $e^e(r, p, K)$ is difficult to calculate from the results in the previous subsection, because fuzzy control function $\sigma(\cdot)$ is sometimes impossible to obtain mathematically, and parameters (r, p, K) vary in a range in real application. Therefore, the stability analysis by the parametric robust Popov criterion in [51] is adopted to handle this situation.

Theorem 1: Consider the uncertain P type fuzzy control system (9) satisfying the following conditions. Then, the P type fuzzy control system is parametric absolute stable.

(1) If the fuzzy controller is continuous, and for some neighborhood $\mathbf{O}$ of $e = 0$ satisfies

$$0 \le \hat{e}[\sigma(e + \hat{e}) - \sigma(e)] \le k(e)\hat{e}^2,\ \forall e \in \mathbf{O},\ \forall \hat{e} \in \Re, \text{ and } \sigma(0) = 0, \tag{23}$$

where $k(e)$ is a positive number depending on $e \in \mathbf{O}$.

(2) If

$$-C(p)A^{-1}(p)K(p, K) + \frac{1}{k(0)} > 0, \forall p \in \mathbf{P} \tag{24}$$

holds, for any $(r, p, K) \in R_{ref} \times \mathbf{P} \times \mathbf{K}$ and any σ satisfying the sector condition (23), there exists a solution $e = e^e(r, p, K)$ of (17) in $\mathbf{O}^e(r, p, K)$, where

$$\mathbf{O}^e(r, p, K) = \begin{cases} \left[\frac{r}{\zeta_0(p)}, r\right] \ (\text{when } r\{C(p)A^{-1}(p)B(p.K)\} \le 0), \\ \left[r, \frac{r}{\zeta_0(p)}\right] \ (\text{when } r\{C(p)A^{-1}(p)B(p.K)\} > 0). \end{cases} \tag{25}$$

and $\zeta_0(p) = 1 - C(p)A^{-1}(p)B(p, K)k(0)$. A more detail proof on (15) and (16) can be referred in the Lemma 1 of [51].

(3) If for a given region R_{ref} of r and for any $p \in \mathbf{P}$, the condition $\mathbf{O}_R^e(p) \subset \mathbf{O}$ is satisfied, and a real number $\nu_o = \nu_o(r, p, K)$ exists such that the following inequality holds

$$\mathrm{Re}[(1 + j\omega\nu_0)G(j\omega, p, K)] + \frac{1}{k_R(r, p, K)} > 0,\ \forall \omega \in \Re, \tag{26}$$

where

$$k_R(p) = \max\{k(e) : e \in \mathbf{O}_R^e(r, p, K)\}, \tag{27}$$

and $\mathbf{O}_R^e(r, p, K)$ represents the region containing $e^e(r, p, K)$ for all $r \in R_{ref}$.

Proof: Since the uncertain P type fuzzy control system (9) satisfies the conditions described in Theorem 1 of [51], the P type fuzzy control system with Lur'e type is parametric absolute stable.

Remark 3: $k_R(r, p, K)$ is hard to find, so Corollary 1 is derived for Theorem 1.

Corollary 1: Suppose that $R_{ref} \subset \mathbf{O}$. Moreover, assume that for any $p \in P, G(0, p, K) > 0$, $k_R^* = \max\{k(e) : e \in R_{ref}\}$, and there exists a real number $\nu_o = \nu_o(r, p, K)$ letting the inequality hold.

$$\mathrm{Re}[(1 + j\omega\nu_0)G(j\omega, p, K)] + \frac{1}{k_R^*} > 0,\ \forall \omega \in \Re. \tag{28}$$

The P type fuzzy control system is then parametric absolute stable.

Remark 4:

(1) This test can be extended to the general P type fuzzy control functions design.
(2) The assumption in Corollary 1 does not lose generality, since most systems have $G(0, p, K) > 0$.
(3) The effect of K can be combined into plant parameters p.

The existence of $\nu_o = \nu_o(p)$ for every $p \in \mathbf{P}$ should be guaranteed in Theorem 1 and Corollary 1. This is generally a difficult problem. Therefore, the parametric robust Popov criterion incorporated with Kharitonov [51], [53], [54] for interval Lur'e systems is introduced into a parametric absolute stable analysis.

Consider the following as a family of interval plants

$$G(s, p, K) = \frac{Q(s)}{P(s)}, \tag{29}$$

where $Q(s)$ and $P(s)$ belong to the families of real interval polynomials $\mathbf{Q}(s)$ and $\mathbf{P}(s)$, respectively.

$$\begin{aligned}\mathbf{Q}(s) = \{&Q(s) : Q(s) = q_0 + q_1 s + \cdots + q_\tau s^\tau, \\ &\text{and } q_i \in [q_i^-, q_i^+], \text{ for all } i = 0, \ldots, \tau\},\end{aligned}$$

and

$$\begin{aligned}\mathbf{P}(s) = \{&P(s) : P(s) = p_0 + p_1 s + \cdots + p_n s^n, \\ &\text{and } p_j \in [p_j^-, p_j^+], \text{ for all } j = 0, \ldots, n\}.\end{aligned} \tag{30}$$

$K_Q^i(s), i = 1, 2, 3, 4$ and $K_P^j(s), j = 1, 2, 3, 4$ represent the Kharitonov polynomials associated with $\mathbf{Q}(s)$ and $\mathbf{P}(s)$, respectively. The Kharitonov systems associated with $G(s, p, K)$ are defined as the 16 plants of the following set,

$$G_K(s) := \left\{ \frac{K_Q^i(s)}{K_P^j(s)} : i, j \in \{1, 2, 3, 4\} \right\}, \tag{31}$$

where

$$\begin{aligned}
K_Q^1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + q_4^- s^4 + q_5^- s^5 + q_6^+ s^6 + \cdots ;\\
K_Q^2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + q_4^+ s^4 + q_5^+ s^5 + q_6^- s^6 + \cdots ;\\
K_Q^3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + q_4^+ s^4 + q_5^- s^5 + q_6^- s^6 + \cdots ;\\
K_Q^4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + q_4^- s^4 + q_5^+ s^5 + q_6^+ s^6 + \cdots ;\\
K_P^1(s) &= p_0^- + p_1^- s + p_2^+ s^2 + p_3^+ s^3 + p_4^- s^4 + p_5^- s^5 + p_6^+ s^6 + \cdots ;\\
K_P^2(s) &= p_0^+ + p_1^+ s + p_2^- s^2 + p_3^- s^3 + p_4^+ s^4 + p_5^+ s^5 + p_6^- s^6 + \cdots ;\\
K_P^3(s) &= p_0^+ + p_1^- s + p_2^- s^2 + p_3^+ s^3 + p_4^+ s^4 + p_5^- s^5 + p_6^- s^6 + \cdots ;\\
K_P^4(s) &= p_0^- + p_1^+ s + p_2^+ s^2 + p_3^- s^3 + p_4^- s^4 + p_5^+ s^5 + p_6^+ s^6 + \cdots ;
\end{aligned}$$

Theorem 2: An P type fuzzy control system is absolutely stable in sector $[0, k]$ for all $G(s) \in G(s, p, K)$, if a real ν_o can be obtained by verifying the robust Popov condition for $G(s) \in G_K(s)$ to satisfy inequality (28).

Proof: By Theorem 1, the P type fuzzy control system of Lur'e type can be tested by the Popov criterion. Hence, the parametric robust Popov criterion incorporated with Kharitonov for interval Lur'e systems [51], [53], [54] can be considered here for parametric absolute stability analysis of P type fuzzy control systems.

Remark 5: Theorem 2 implies that only 16 Popov plots need to be drawn from family $G_K(s)$ to check that the P type fuzzy logic control system is stable when the robust Popov condition (28) holds for the whole family $G(s)$.

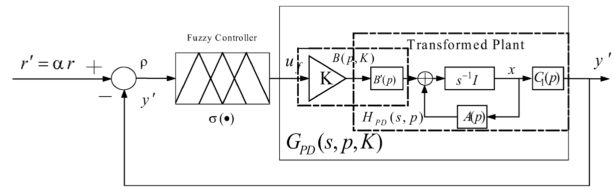


Fig. 8 The transformed SFLC with the special P type fuzzy control system formation.

3.4 Transformation SFLC from PD to P Type

In the following, the SFLC is transformed from PD to P type, so that the equilibrium point and stability can be analyzed by the transformed special P type fuzzy logic control system. From Fig. 4, the factor $\frac{1}{\sqrt{1+\lambda^2}}$ of SFLC is integrated into both the proportional and derivative factors. The α and β in Fig. 7 are then defined as

$$\alpha = \frac{\lambda}{\sqrt{1+\lambda^2}}, \text{ and } \beta = \frac{1}{\sqrt{1+\lambda^2}}. \tag{32}$$

Assumption 2: $CB = 0$.

According to Assumption 2 and Fig. 7, the following derivation can be obtained.

$$e = r - y = r - Cx. \tag{33}$$

By differentiating both sides, then

$$\dot{e} = -C\dot{x} = -C(AX + Bu_f) = -CAx. \tag{34}$$

From (33) and (34), then

$$\rho = \alpha e + \beta \dot{e} = \alpha(r - Cx) + \beta(-CAx) = r' - C_1 x, \tag{35}$$

where $C_1 = (\alpha C + \beta CA)$, and $r' = \alpha r$.

After transformation, the transformed plant in Fig. 8 can be obtained

$$G_{PD}(s, p, K) = C_1(p)[sI - A(p)]^{-1} B(p, K). \tag{36}$$

From Fig. 8, the special P type transformation from the SFLC system can be described as:

$$\begin{aligned}\dot{x} &= A(p)x + B(p, K)u_f,\\ y' &= C_1(p)x,\end{aligned} \tag{37}$$

where the control input $u_f = \sigma(\rho)$, and control error $\rho = r' - y'$.

The transfer function $H_{PD}(s, p)$ of the transformed plant in Fig. 8 can be described as

$$H_{PD}(s, p) = C_1(p)[sI - A(p)]^{-1} B'(p), \tag{38}$$

3.5 Equilibrium Point Analysis for PD Type Fuzzy Control Systems with Linear Plants

From Fig. 7, the equilibrium point can be analyzed

$$\dot{x} = A(p)x + B(p, K)\sigma(\rho). \tag{39}$$

Let $\dot{x} = 0$,

$$0 = A(p)x + B(p, K)\sigma(\rho). \tag{40}$$

If $A^{-1}(p)$ exists, then

$$x + A^{-1}(p)B(p, K)\sigma(\rho) = 0. \tag{41}$$

By multiplying the result of (40) by C and using (35), then

$$-C(p)x - C(p)A^{-1}(p)B(p, K)\sigma(\alpha e + \beta\dot{e}) = 0. \tag{42}$$

When $t \to \infty$, $\dot{x} = 0$ and $\dot{e} = 0$ are implied. By $\dot{e} = 0$,

$$e^e - r - C(p)A^{-1}(p)B(p, K)\sigma(\alpha e^e) = 0. \tag{43}$$

Remark 6: The error equilibrium point of the PD type fuzzy control system is

$$e^e = -e^e_d. \tag{44}$$

3.6 Stability Analysis for PD Type Fuzzy Control Systems with Linear Plants

The transformed P type of SFLC in Fig. 8 can be employed to analyze the stability of SFLC for a given (r, p, K).

(1) Frequency domain approach

Consider the error dynamic system in Fig. 8 for the given parameter vector (r, p, K).

$$\dot{\tilde{x}} = A(p)\tilde{x} + B(p, K)\tilde{\sigma}(-C_1(p)\tilde{x}), \tag{45}$$

where

$$\begin{aligned} &\tilde{x} = x - \breve{x}^e(r, p, K),\\ &\tilde{\sigma}(-C_1(p)\tilde{x}) = \sigma[-C_1(p)\tilde{x} + \breve{e}^e(r, p, K)] - \sigma[\breve{e}^e(r, p, K)],\\ &\text{and } \breve{e}^e(r, p, K) = r' - C_1(p)\breve{x}^e(r, p, K). \end{aligned} \tag{46}$$

The error dynamic system is also of Lur'e type. The function $\tilde{\sigma}$ satisfies the following sector condition, if $\breve{e}^e(r, p, K) \in \mathbf{O}$.

$$0 \le \tilde{e}\tilde{\sigma}(\tilde{e}) \le k[\breve{e}^e(r, p, K)]\tilde{e}^2,\ \forall\tilde{e} \in \Re, \tag{47}$$

where $\tilde{e} = e - \breve{e}^e(r, p, K)$, and $k > 0$.

From the Popov criterion, (39) is absolutely stable for a given (r, p, K), if a real number $\tilde{\nu}_0 = \tilde{\nu}_0(r, p, K)$ exists satisfying

$$\begin{aligned} &\mathrm{Re}[(1 + j\omega\tilde{\nu}_0)G_{PD}(j\omega, p, K)] + \frac{1}{k[e^e(r, p, K)]} > 0,\\ &\forall\omega \in \Re. \end{aligned} \tag{48}$$

(2) Time domain approach

Consider an arbitrary parameter vector (r, p, K) in SFLC. Suppose that an equilibrium state $x^e(r, p, K)$ of the system exists. The stability can be determined by the linearization of (37) near the error equilibrium point.

3.7 Stability Analysis for PD Type Fuzzy Control Systems with Uncertain Linear Plants

Since the transformed SFLC is a special P type fuzzy control system as shown in Fig. 8, the parametric Popov criterion [51] incorporated with Kharitonov theorem is adopted to analyze the stability of PD type fuzzy control systems with uncertainties.

4. Fuzzy Current Control RC Circuit System Design

The temperature control is an important issue in many industrial processes or medical applications. The temperature controls systems are analogous to RC electrical circuits and are governed by the following third-order equation (49) [55]. In our design, FLC is applied to control the RC electrical circuits to reach the specified output voltage. In other words, it is similar to regulate the temperature to desired set point This section specifies fuzzy current control RC circuit systems of P and PD types for verifying the theoretical analysis using PSPICE simulation.

In this section, the circuit structure is specified first. The fuzzy logic controller is then designed to construct the fuzzy control function, which is mapping I/O relation of the fuzzy controller. Finally, some components of the overall structure of the fuzzy logic control system are introduced.

4.1 The Block Diagram of the Fuzzy Current Control RC Circuit System

Figure 9 depicts the block diagram of a fuzzy current control RC circuit. The control objective of this system is to track a dc constant reference voltage r. To avoid the loading effect from the circuit of the next stage, the voltage buffer is utilized to feed the output voltage v_3 back into the controller to generate the control error voltage v_e. The core of this system is the fuzzy controller. Both P and PD type fuzzy controllers are designed in the circuit system. The control voltage v_{of} is transformed into the control current i_{ovc} with a voltage controlled current circuit. Finally, the amplified current $u(t)$ from the current amplifier is injected into circuit plant to let output voltage v_3 to track a reference voltage r.

4.2 Circuit Plant

The circuit plant in Fig. 10 [55] is composed of RC circuits and external current source control input $u(t)$. The output

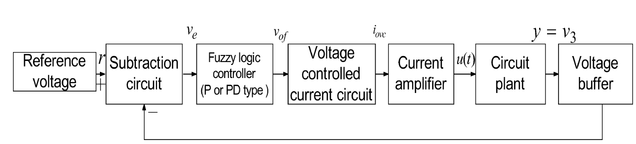


Fig. 9 The block diagram of a fuzzy current control RC circuit system.

voltage is v_3 Consider the transfer function of circuit plant

$$H(s) = \frac{Y(s)}{U(s)} = \frac{R_3C_1}{\Delta}, \tag{49}$$

where $\Delta = R_1R_2R_3C_1C_2C_3s^3 + C_1(R_1R_2C_1C_2 + R_2R_3C_2C_3 + R_2R_3C_1C_3 + R_1R_3C_1C_2 + R_1R_3C_1C_3)s^2 + C_1(R_2C_2 + R_2C_1 + R_3C_1 + R_1C_1 + R_3C_2 + R_3C_3)s + C_1$.

4.3 Fuzzy Logic Controller Circuit

The circuit of a fuzzy logic controller is shown in Fig. 11. This circuit is designed to construct the control function of the fuzzy controller. Figure 12 illustrates the relationship between the circuit parameters and the control function [56], [57].

4.4 Overall Design Circuit

Figure 11 shows the overall design circuit. For simplification, the voltage controlled current circuit, current amplifier and PD type signal generator are introduced in [58].

(1) Voltage controlled current circuit

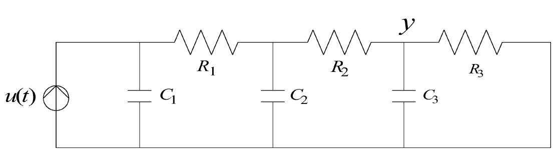


Fig. 10 The RC circuit plant [55].

Figure 11 displays the voltage controlled current circuit. If the following equalities (50) stand, then

$$\frac{R_{vc4}}{R_{vc3}} = \frac{R_{vc2}}{R_{vc1}}, \tag{50}$$

and

$$i_{ovv} = \frac{V_{of}}{R_{vc1}}. \tag{51}$$

(2) Current amplifier

The current amplifier is designed to normalize the signal from voltage controlled current circuit and amplifies it. The control input $u(t)$ from the current amplifier for the circuit plant is given by

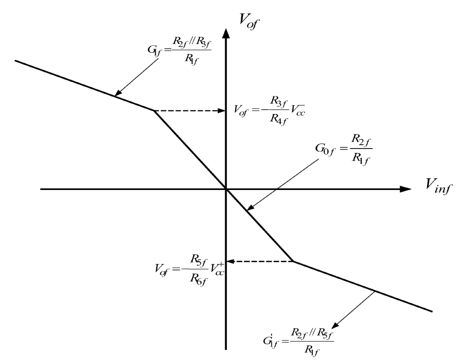


Fig. 12 The control function of a fuzzy controller with circuit design parameters.

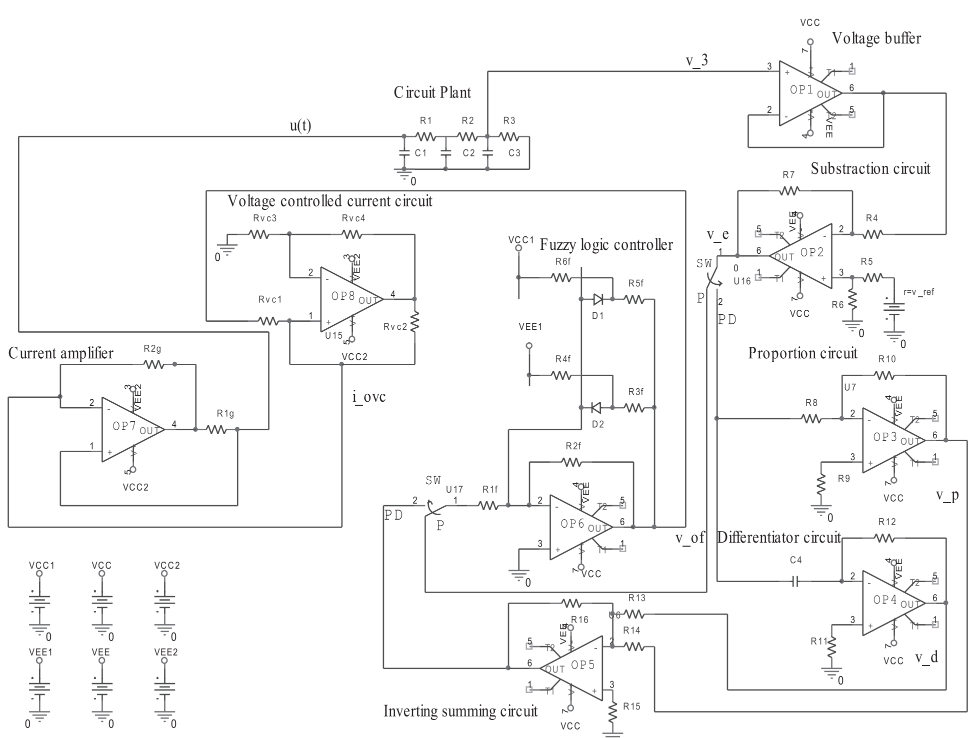


Fig. 11 The designed fuzzy current control RC circuit system.

$$u(t) = i_{og} = -\frac{R_{2g}}{R_{1g}} i_{ovc}. \tag{52}$$

(3) PD type signal generation

The derivative and proportional signals are generated by OP amplifier differentiator and OP inverting amplifier as illustrated in Fig. 11.

The OP amp differentiator is designed as

$$v_d = -R_{12}C_4 \frac{dv_e}{dt}. \tag{53}$$

The value $R_{12}C_4$ is chosen to meet β. Conversely, the OP inverting amplifier is given by

$$v_p = -\frac{R_{10}}{R_8} v_e, \tag{54}$$

where $\alpha = \dfrac{R_{10}}{R_8}$.

In Fig. 11, a P type fuzzy control system is chosen when two switches open at P positions. Conversely, a PD type fuzzy control system is selected when two switches close at PD.

5. Simulation Results

In this section, a fuzzy control RC circuit plant as shown in Fig. 10 is utilized to investigate the parametric equilibrium points and stability when the circuit plant is certain or uncertain with P and PD type fuzzy logic controllers, respectively. The varying parameters include reference input r, an adjustable parameter K and an interval circuit plant parameters p.

For the analysis of certain plants, the equilibrium points under the (r, K) parameter space with stable notation are given. The phase plane and time waveforms are given to verify the analytical results. The design circuit with PSPICE simulation is also provided to check theoretical analysis. On the other hand, the parametric robust Popov criterion is employed to test the stability of the parameter vector $(r, p, K) \in R_{ref} \times \mathbf{P} \times \mathbf{K}$. From this point of view, the effect of K can be combined into plant parameters by the previous introduction.

Let $R_1 = R_2 = R_3 = 1\,\Omega$, and $C_1 = C_2 = C_3 = 1F$ in (49), the third-order transfer with form

$$H(s) = \frac{q_0}{p_3 s^3 + p_2 s^2 + p_1 s + p_0}, \tag{55}$$

where $q_0 = 1, p_0 = 1, p_1 = 6, p_2 = 5$ and $p_3 = 1$. From Fig. 1, combining the adjustable parameter K, the transfer function is given by

$$G(s, K) = \frac{q_0 K}{p_3 s^3 + p_2 s^2 + p_1 s + p_0}. \tag{56}$$

The state space representation for $G(s, K)$ can be derived

$$A(p) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -p_0/p_3 & -p_1/p_3 & -p_2/p_3 \end{bmatrix},$$

$$B(K) = \begin{bmatrix} 0 \\ 0 \\ (q_0K)/p_3 \end{bmatrix},$$

$$\text{and } C(p) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{57}$$

The fuzzy rules are adapted in this simulation as follows:

$$\begin{aligned} &\mathit{Rule}\ \mathbf{1}: \text{If } e \text{ is } NBE, \text{ then } u_f \text{ is } NBU; \\ &\mathit{Rule}\ \mathbf{2}: \text{If } e \text{ is } NSE, \text{ then } u_f \text{ is } NSU; \\ &\mathit{Rule}\ \mathbf{3}: \text{If } e \text{ is } ZRE, \text{ then } u_f \text{ is } ZRU; \\ &\mathit{Rule}\ \mathbf{4}: \text{If } e \text{ is } PSE, \text{ then } u_f \text{ is } PSU; \\ &\mathit{Rule}\ \mathbf{5}: \text{If } e \text{ is } PBE, \text{ then } u_f \text{ is } PBU. \end{aligned} \tag{58}$$

Figure 13 illustrates the membership functions. Table 5 shows the fuzzy control system parameters. Figure 3 shows the control function, where $k_0 = 6, k_1 = 4/9$ and $c_1 = 5/9$.

Consider the following simulation with $K = 1 \sim 20$, $r = -1 \sim 1$ and the initial condition $x(0) = [0, 0, 0]'$. Table 6 lists the circuit components in Fig. 11. For practical considerations, the parameters of the fuzzy controller are selected as Table 6 in order to approach the ideal control function depicted in Fig. 14.

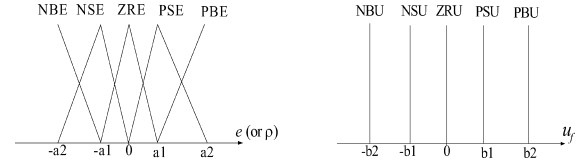
5.1 P Type Example Demonstrations

(1) Certain linear circuit plant

Under Assumptions 2, the equilibrium points of the fuzzy control systems in each segment can be calculated using (18).

$$e^e = \begin{cases} \text{segment 1}: \dfrac{rp_0 - q_0Kc_1}{p_0 + qKk_1}, & e^e \in [a_1, a_2], \\ \text{segment 2}: \dfrac{rp_0}{p_0 + q_0Kk_0}, & e^e \in [-a_1, a_1], \\ \text{segment 3}: \dfrac{rp_0 + q_0Kc_1}{p_0 + q_0Kk_1}, & e^e \in [-a_2, -a_1]. \end{cases} \tag{59}$$

The equilibrium point of one segment is e^e when $t \to \infty$ and



(a) (b)

Fig. 13 The membership functions of the fuzzy control system.

Table 5 Parameters of the fuzzy logic controller in simulations.

	NBE	NSE	ZRE	PSE	PBE
e(or ρ)	−1	−0.1	0	0.1	1
	NBU	NSU	ZRU	PSU	PBU
u_f	−1	−0.6	0	0.6	1

Table 6 Parameters of the fuzzy current control RC circuit system.

Circuit Blocks	Circuit components
Circuit plant	$R_1 = R_2 = R_3 = 1\,\Omega$ and $C_1 = C_2 = C_3 = 1F$.
Subtraction circuit	$R_4 = R_5 = R_6 = R_7 = 25\,\mathrm{k}\Omega$ and $v_{ref} = 0.2\,\mathrm{V}$.
Proportion circuit	$R_8 = 1\,\mathrm{k}\Omega, R_9 = 10\,\mathrm{k}\Omega$ and $R_{10} = 1\,\mathrm{k}\Omega$.
Differentiator circuit	$R_{11} = 10\,\mathrm{k}\Omega, R_{12} = 0.9\,\mathrm{k}\Omega$ and $C_4 = 100\,\mu\mathrm{F}$.
Inverting summing circuit	$R_{13} = R_{14} = R_{15} = R_{16} = 10\,\mathrm{k}\Omega$.
Fuzzy controller	$R_{1f} = 2\,\mathrm{k}\Omega$, $R_{2f} = 12\,\mathrm{k}\Omega$, $R_{3f} = R_{5f} = 400\,\Omega$ $R_{4f} = R_{6f} = 13\,\mathrm{k}\Omega$, and D_1 and D_2 : $1N4148$.
Voltage controlled current circuit	$R_{vc1} = R_{vc2} = R_{vc3} = R_{vc4} = 10\,\mathrm{k}\Omega$.
Current amplifier	R_{1g} and R_{2g} are chosen to meet the selected K with voltage controlled current circuit design. P type design: Stable $R_{1g} = 1\,\Omega$ and $R_{2g} = 50\,\mathrm{k}\Omega$. Unstable $R_{1g} = 1\,\Omega$ and $R_{2g} = 40\,\mathrm{k}\Omega$. PD type design: Stable $R_{1g} = 1\,\Omega$ and $R_{2g} = 90\,\mathrm{k}\Omega$. Unstable $R_{1g} = 1\,\Omega$ and $R_{2g} = 100\,\mathrm{k}\Omega$.
Power source	VCC = 15 V, VEE = −15 V, VCC1 = 8 V, VEE = −8 V, VCC2 = 30 V, and VEE2 = −30 V.
Operational amplifiers in design	P type design: OP amps 1-6 with OPA602, and OP amps 7-8 with LM675 (Power op amp). PD type design: OP amps 1-6 with OPA602, OP amps 7 with OPA501 (Power op amp) and OP amps 8 with LM675.

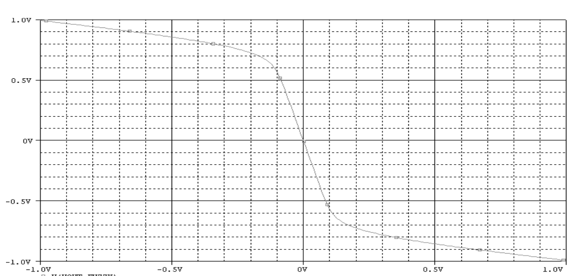


Fig. 14 The fuzzy control function with PSPICE simulation by Table 6 parameters.

$e \to e^e$. Equation (60) can be solved by linearizing (9) and using (57)

$$\tilde{A}(r, p) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(1 + K\zeta(r, p, K)) & -6 & -5 \end{bmatrix}. \qquad (60)$$

The stability can be determined by $\hat{A}$. $\zeta \in \{k_1, k_1\}$ denotes the slope of e^e in the control function, and ζ is determined by e^e from (18). In (18), the reference r and actuator gain K affect e^e. Figure 15 depicts the analysis of the stability of equilibrium points. The reason for the formation of unstable oscillations is discussed in Sect. 2). Figures 16 and 17 display the verification of the analysis in Fig. 15, with respect to P1 (unstable) and P2 (stable point).

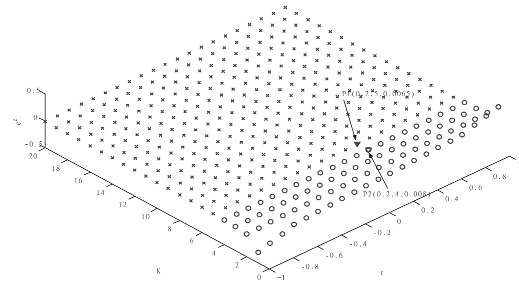
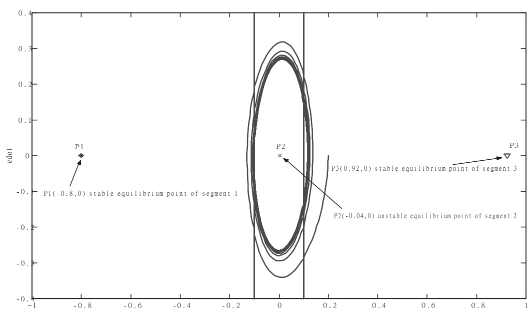
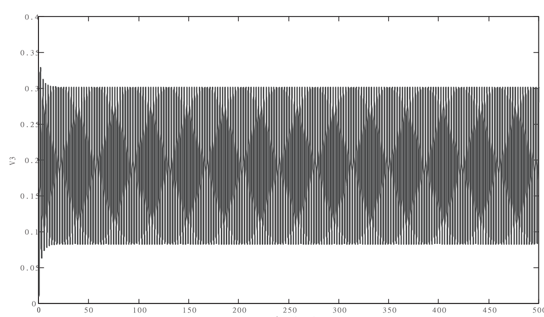


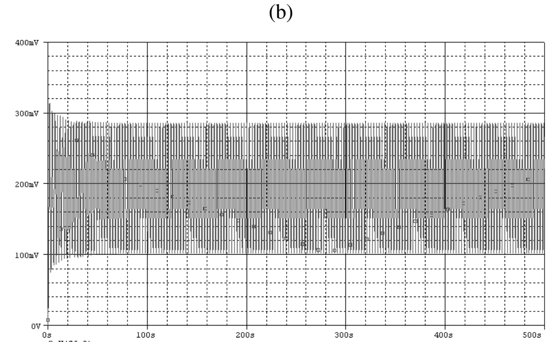
Fig. 15 The equilibrium stability of the P type fuzzy control system by Table 5 for (r, K), where o indicates a stable equilibrium, and $\times$ denotes an unstable equilibrium.



(a)



(b)

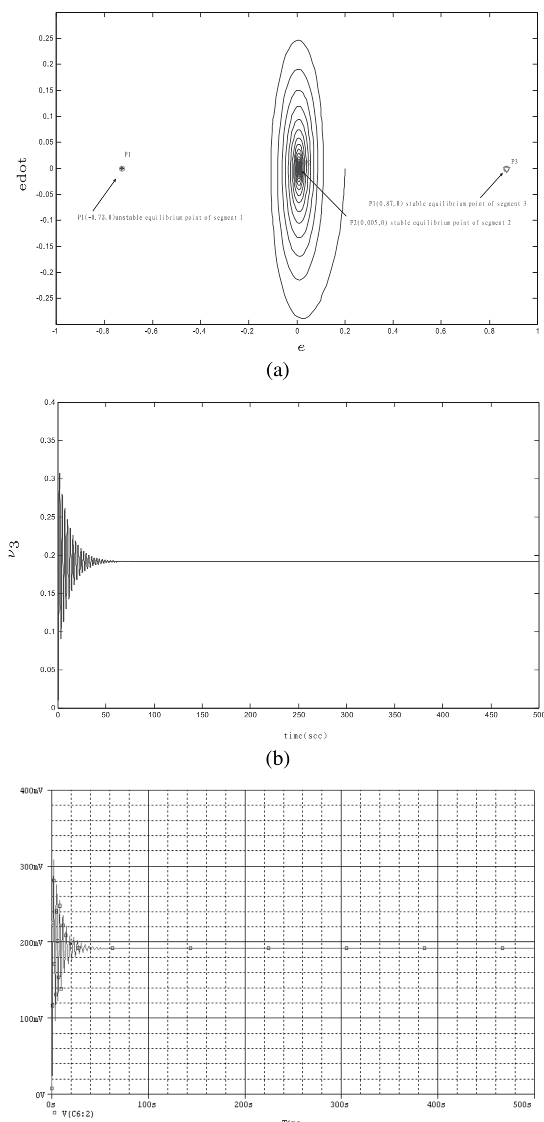


(c)

Fig. 16 (a) The phase plane of $(e, \dot{e})$ when $(r, K) = (0.2, 5)$; (b) The time waveform when $(r, K) = (0.2, 5)$; (c) PSPICE waveform when $(r, K) = (0.2, 5)$.

(2) Mechanism of oscillations in the fuzzy control system

In this example, the P type fuzzy control system is a piecewise-linear system with three segments. An equilibrium $(e, \dot{e}^e = 0)$ exists in every segment for a specific (r, K)



(a)

(b)

(c)

Fig. 17 (a) The phase plane of $(e, \dot{e})$ when $(r, K) = (0.2, 4)$; (b) The time waveform when $(r, K) = (0.2, 4)$; (c) PSPICE waveform when $(r, K) = (0.2, 4)$.

pair. Figure 16(a) shows the three error equilibriums of every piecewise segment in the phase plane of $(e, \dot{e})$ when $(r, K) = (0.2, 5)$. Three equilibrium points are represented as $*$ (stable equilibrium point for segment 1), x (unstable equilibrium point for segment 2) and ∇ (stable equilibrium point for segment 3), for segments 1-3, respectively. Assume that $(e, \dot{e})$ locates in segment 1 initially. $(e, \dot{e})$ is pulled into the equilibrium point '$*$' of segment 1 located in segment 3. When $(e, \dot{e})$ enters segment 2, $(e, \dot{e})$ is pushed away from equilibrium point x of segment 2. After $(e, \dot{e})$ is pushed away from segment 2 and enters segment 3, $(e, \dot{e})$ is pulled back to the equilibrium point of segment 3 ∇. The limit cycle is formulated by pushing and pulling.

Conversely $(e, \dot{e})$ crosses the segments 1, 2, and 3, is all pulled into equilibrium points and finally $(e, \dot{e})$ achieves the global equilibrium point of segment 2. The authors discuss in detail the stability under different design parameters [59].

Table 7 Alternative parameters of the fuzzy logic controller.

	NBE	NSE	ZRE	PSE	PBE
e(or ρ)	−1	−0.01	0	0.01	1
	NBE	NSE	ZRE	PSE	PBE
u_f	−1	−0.1	0	0.1	1

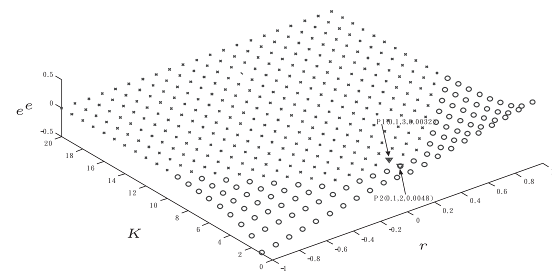


Fig. 18 The equilibrium with the stability of the alternative fuzzy controller by Table 7 for (r, K), where o denotes a stable equilibrium, and $\times$ indicates an unstable equilibrium.

(3) Alternative Control Function

In Fig. 15, the effect of reference for stability is not obvious. Therefore, the different fuzzy controllers in Table 7 are designed with different control functions. The results in Fig. 18 specify how the different controllers will influence the equilibrium points and stability besides r and K.

(4) Uncertain linear circuit plant

In this part, the stability of the fuzzy control system with interval plant is checked by Theorem 2. In the following simulations, $r \in [-1, 1]$, $K = 2$, $R_1 \sim R_3$ and $C_1 \sim C_3$ in circuit plant listed in Table 6 with tolerance ±5% and $k_R^* = 6$ in (28) are selected. The plant (56) for P type fuzzy control system can be rewritten as

$$G(s, K) = \frac{[q_0^-, q_0^+]K}{[p_3^-, p_3^+]s^3 + [p_2^-, p_2^+]s^2 + [p_1^-, p_1^+]s + [p_0^-, p_0^+]}, \tag{61}$$

where $[q_0^-, q_0^+] = [0.9, 1.1]$, $[p_3^-, p_3^+] = [0.74, 1.34]$, $[p_2^-, p_2^+] = [3.87, 6.14]$, $[p_1^-, p_1^+] = [5.14, 6.95]$, and $[p_0^-, p_0^+] = [0.95, 1.05]$.

It should be noted that the effect of interval actuator gain can be considered into $[q_0^-, q_0^+]$, so we just choose $K = 2$ in this example.

By Theorem 2, the absolute stability can be tested as shown in Fig. 19. Because the parameter in numerator is just one, only eight Popov curves are plotted enough to indicate the stability in such a case.

5.2 PD Type Example Demonstrations

In the following simulation, $\lambda = 10$ is selected in PD type fuzzy control system.

(1) Certain linear circuit plant

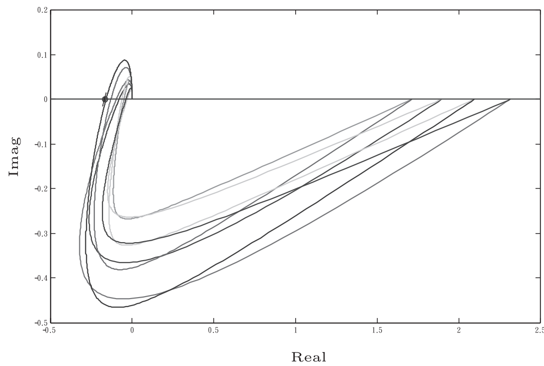


Fig. 19 The Popov plots for the P type fuzzy control system with uncertain circuit plant.

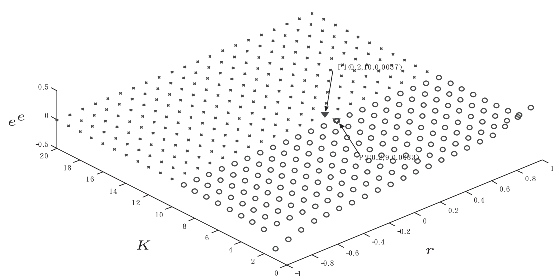


Fig. 20 The equilibriums with the stability of the PD type fuzzy control system by Table 5 for (r, K), where o indicates a stable equilibrium, and $\times$ denotes an unstable equilibrium.

In this subsection, Fig. 7 demonstrates the PD type fuzzy control system. Under the Assumptions 1, and 2, the error equilibrium points of the fuzzy control systems in every segment can be obtained by (43).

$$
\begin{aligned}
e^e &= -e_d^e \\
&= \begin{cases}
\text{segment 1}: \dfrac{(rp_0 - qKc_1)\sqrt{1+\lambda^2}}{\lambda qKk_1 + p_0\sqrt{1+\lambda^2}}, & e^e \in [a_1, a_2], \\[2ex]
\text{segment 2}: \dfrac{rp_0\sqrt{1+\lambda^2}}{\lambda qKk_0 + p_0\sqrt{1+\lambda^2}}, & e^e \in [-a_1, a_1], \\[2ex]
\text{segment 3}: \dfrac{(rp_0 + qKc_1)\sqrt{1+\lambda^2}}{\lambda qKk_1 + p_0\sqrt{1+\lambda^2}}, & e^e \in [-a_2, -a_1].
\end{cases}
\end{aligned} \tag{62}
$$

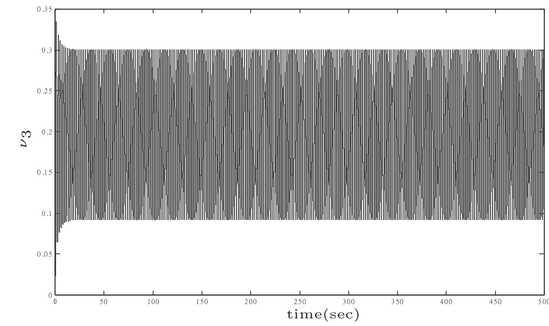
By linearizing (39) and using (57), (63) can be carried out, and Fig. 20 can be obtained.

$$
\begin{aligned}
\tilde{A}(r, p, K) &= A - \chi(r, p, K)B(K)C_1 \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(1 + K\chi(r, p, K)) & -6 & -5 \end{bmatrix}.
\end{aligned} \tag{63}
$$

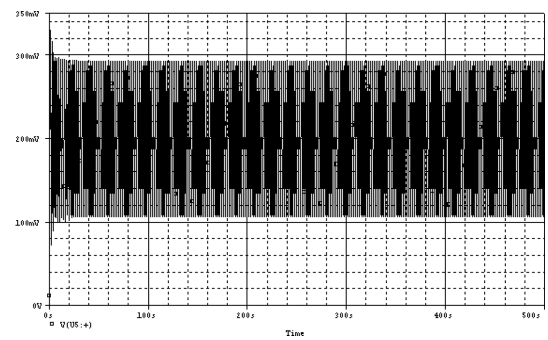
In the following, Figs. 21 and 22 verify the analysis in Fig. 20 with respect to P1 (unstable) and P2 (stable point).

(2) Alternative control function

The alternative controller in Table 7 obviously influences the equilibrium point and stability, when the reference is varying. Figure 23 shows the analytical results.

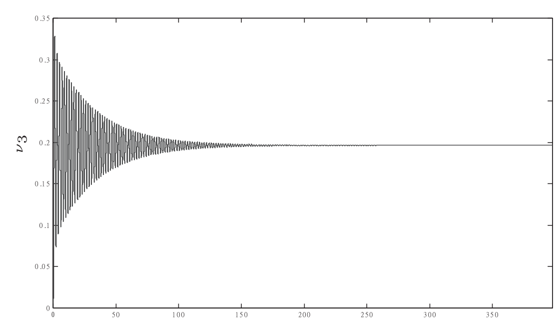


(a)

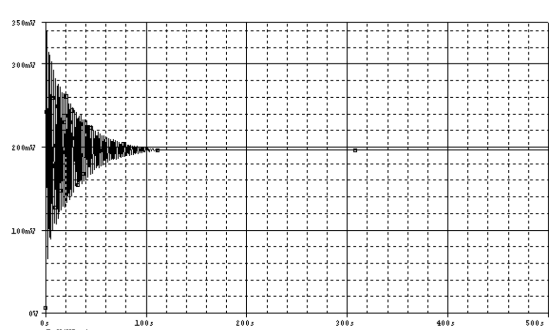


(b)

Fig. 21 (a) The time waveform when $(r, K) = (0.2, 10)$ (b) PSPICE waveform when $(r, K) = (0.2, 10)$.



(a)



(b)

Fig. 22 (a) The time waveform when $(r, K) = (0.2, 9)$; (b) PSPICE waveform when $(r, K) = (0.2, 9)$.

(3) Uncertain linear circuit plant

In this subsection, Fig. 8 is adopted to demonstrate the parametric stability of the PD type fuzzy control system. Fol-

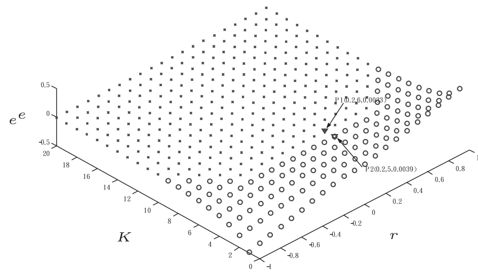


Fig. 23 Equilibrium with the stability of the PD type fuzzy control systems in Table 7 for (r, K), where o denotes a stable equilibrium, and $\times$ indicates an unstable equilibrium.

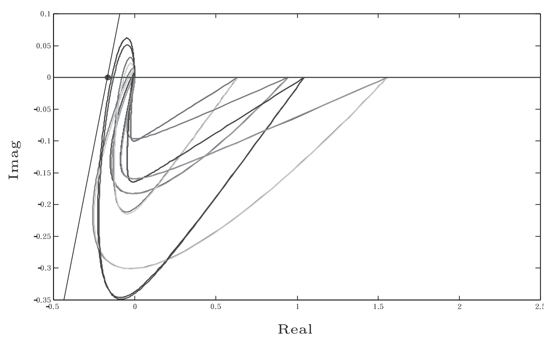


Fig. 24 The Popov plots for the PD type fuzzy control systems with the uncertain circuit plant.

lowing transformation, the analytic new plant for PD type fuzzy systems is given by (38):

$$H_{PD}(s)=\frac{R_2R_3^2C_1C_3(s+\lambda)}{\Upsilon}$$
$$\Upsilon=\sqrt{1+\lambda^2}[R_1R_2^2R_3^2C_1C_2C_3^2s^3+R_2R_3C_1C_3(R_1R_2C_1C_2 +R_2R_3C_2C_3+C_1C_3R_2R_3+R_1R_3C_1C_2+R_1R_3C_1C_3)s^2 +R_2R_3C_1C_3(R_2C_2+R_2C_1+R_3C_1+R_1C_1+R_3C_2+R_3C_3)s +R_2R_3C_3C_1]. \tag{64}$$

In the following simulation, $r \in [-1, 1]$, $K = 1$, $R_1 \sim R_3$ and $C_1 \sim C_3$ in circuit plant, as listed in Table 6 with tolerance $\pm 5\%$ and $k_R^* = 6$ in (28), are specified to evaluate the stability of a PD type fuzzy control system. From (36), the analytic new plant for PD type fuzzy control system can be recast as

$$G_{PD}(s,K)=\frac{K([\tilde{q}_1^-,\tilde{q}_1^+]s+[\tilde{q}_0^-,\tilde{q}_0^+])}{[\tilde{p}_3^-,\tilde{p}_3^+]s^3+[\tilde{p}_2^-,\tilde{p}_2^+]s^2+[\tilde{p}_1^-,\tilde{p}_1^+]s+[\tilde{p}_0^-,\tilde{p}_0^+]}, \tag{65}$$

where $[\tilde{q}_1^-,\tilde{q}_1^+] = [0.77, 1.28]$, $[\tilde{q}_0^-,\tilde{q}_0^+] = [7.74, 12.76]$, $[\tilde{p}_3^-,\tilde{p}_3^+] = [6.02, 16.37]$, $[\tilde{p}_2^-,\tilde{p}_2^+] = [33.34, 74.24]$, $[\tilde{p}_1^-,\tilde{p}_1^+] = [44.33, 80.81]$ and $[\tilde{p}_0^-,\tilde{p}_0^+] = [8.19, 12.22]$. The total of sixteen Popov curves illustrated in Fig. 24 are plotted to verify that the PD type fuzzy control system is stable according to Theorem 2.

Table 8 Parameters of fuzzy logic controller for the robust Lur's test.

	nbe	nme	nse	zre	pse	pme	pbe
e	−2000	−1025	−1000	0	1000	1025	2000
	nbu	nmu	nsu	zru	psu	pmu	pbu
u_f	−740	−350	100	0	100	350	740

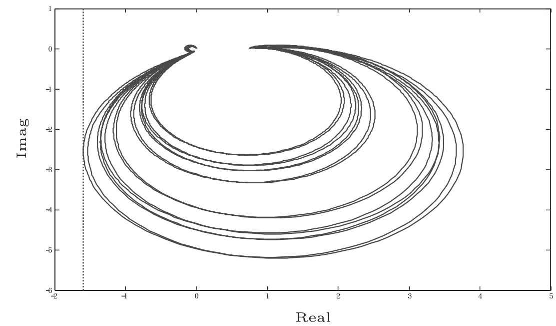


Fig. 25 The robust Lur'e test.

6. Comparisons with Other Approaches

In this section, we will illustrate the stability of uncertain fuzzy control systems which are considered as stable by compared methods will crash under the effect of the reference inputs. On the other hand, the stability can be tested with our applied method and guaranteed under the effect of the reference inputs. It should be noted that the applied parametric robust Popov criterion will be comprised with the robust Lur'e test [54], the robust Circle criterion [54], and the robust Popov criterion [54]. In the following, we consider the P type fuzzy control system in Fig. 1 to demonstrate the comparisons. Because the PD type fuzzy control systems can be transformed into P type ones, we will not exhibit the PD cases additionally.

6.1 Robust Lur'e Test

Consider the stable interval plant [54] in Fig. 1:

$$G(s,K)=\frac{K([q_1^-,q_1^+]s+[q_0^-,q_0^+])}{s^4+[p_3^-,p_3^+]s^3+[p_2^-,p_2^+]s^2+[p_1^-,p_1^+]s+[p_0^-,p_0^+]}, \tag{66}$$

where $[q_0^-,q_0^+] = [3, 3.3]$, $[q_1^-,q_1^+] = [3, 3.2]$, $[p_0^-,p_0^+] = [3, 4]$, $[p_1^-,p_1^+] = [2, 3]$, $[p_2^-,p_2^+] = [24, 25]$, and $[p_3^-,p_3^+] = [1, 1.2]$. For the following stability test demonstrations, the default values in the parameters are chosen: $q_0 = 3.2, q_1 = 3.1, p_0 = 3.5, p_1 = 2.5, p_2 = 24.5$, and $p_3 = 1.1$. The parameters in membership functions of the fuzzy logic controller can be chosen such Table 8. The actuator gain $K = 1$. The total sixteen robust Lur'e curves will be illustrated to test the stability of the fuzzy control systems.

From Fig. 25, $-1/k_L \approx -1.60$ is obtained. Therefore, the control surface $\sigma(\cdot)$ of fuzzy logic controller should belong to sector bound $[0, k_L \approx 0.63]$ as shown in Fig. 26, and the fuzzy logic control system is robust absolutely stable.

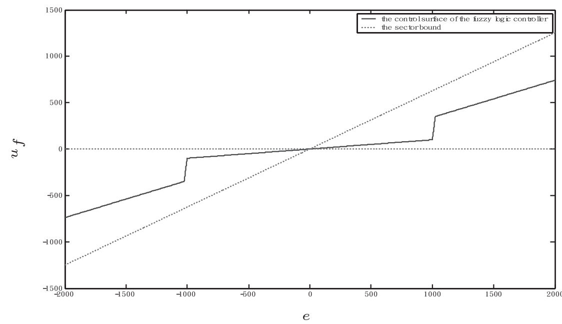


Fig. 26 The sector bound from the robust Lur'e test and the control surface of the fuzzy logic controller.

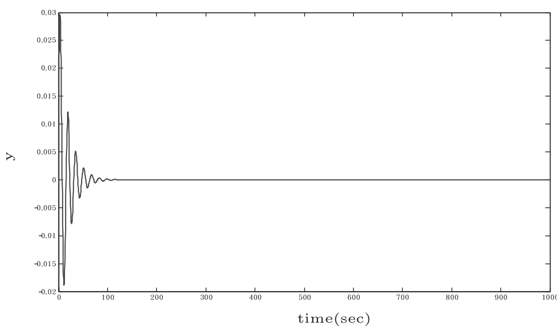


Fig. 27 The time waveform of the stable test case respect to the robust Lur'e test.

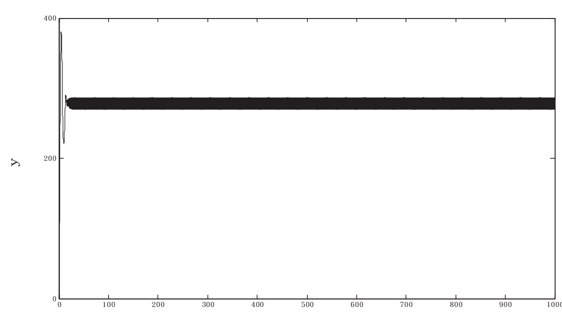


Fig. 28 The time waveform of the unstable test case respect to the robust Lur'e test.

If the parameters in membership functions of the fuzzy logic controller are chosen such Table 8, then the fuzzy control system is stable. The stable and unstable test cases respect to the robust Lur'e test are with a pulse reference input for testing $r = 0$ and a constant input $r = 1300$, respectively. The stable and unstable output waveforms are shown in Figs. 27 and 28, respectively. In this case, we can find that if the reference input is increased, the stability of the fuzzy control system which is considered as stable will crash.

6.2 Robust Circle Criterion

Suppose the stable interval plant such the previous test and the parameters in membership functions of the fuzzy logic controller are chosen such Table 9. The total sixteen ro-

Table 9 Parameters of fuzzy logic controller for the robust Circle criterion.

	nbe	nme	nse	zre	pse	pme	pbe
e	−2000	−1020	−1000	0	1000	1020	2000
	nbu	nmu	nsu	zru	psu	pmu	pbu
u_f	−4158.4	−1630	−630	0	630	1630	4158.4

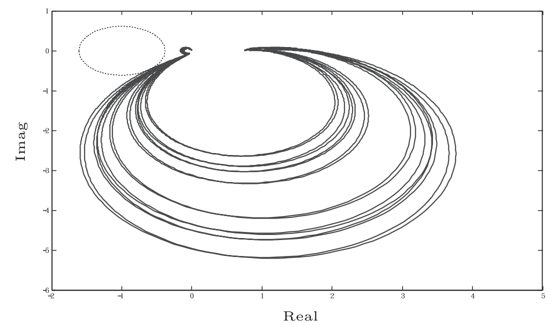


Fig. 29 Robust circle criterion.

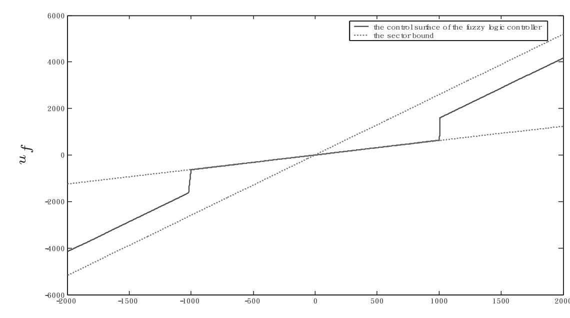


Fig. 30 The sector bound from the robust circle criterion and control surface of fuzzy logic controller.

bust circle curves will be illustrated to test the stability too. From Fig. 29, the circle center located on $(-1, 0)$, and radian is 0.6138. The circle cut the negative real axis at two points $-1/k_{C1} \approx -1.61$ and $-1/k_{C_2} \approx -0.39$. Therefore, the control surface $\sigma(\cdot)$ of the fuzzy logic controller should belong to the sector bound $[k_{C1} \approx 0.62, k_{C_2} \approx 2.59]$ as shown in Fig. 30, and the fuzzy logic control system is robust absolutely stable.

If the parameters in membership functions of the fuzzy logic controller are chosen such Table 9, then the fuzzy control system is stable. The stable and unstable test cases respect to the robust circle criterion are with a pulse reference input and a constant input $r = 2000$, respectively. The stable and unstable output waveforms are shown in Figs. 31 and 32, respectively. In this case, we can find that if the reference input is increased the stability of the fuzzy control system which is considered as stable will crash, too.

6.3 Robust Popov Criterion

Let's consider the stable interval plant such the previous test and the parameters in membership functions of the fuzzy logic controller are chosen such Table 8. The total sixteen

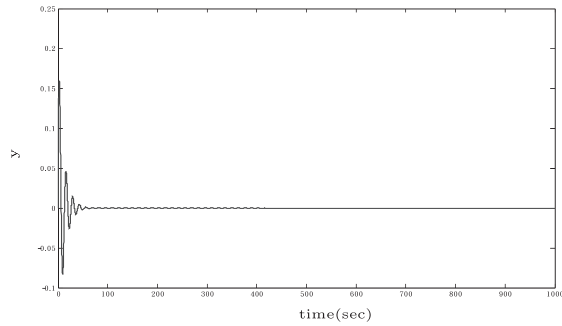


Fig. 31 The time waveform of the stable test case respect to the robust circle criterion.

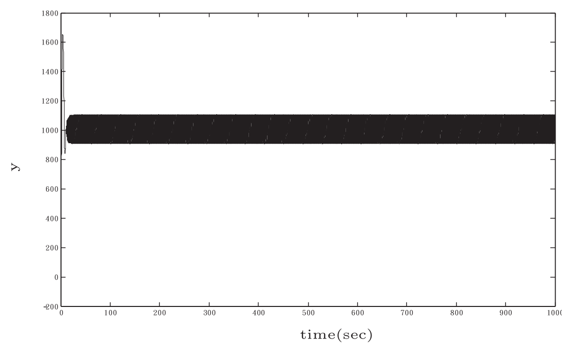


Fig. 32 The time waveform of the unstable test case respect to the robust circle criterion.

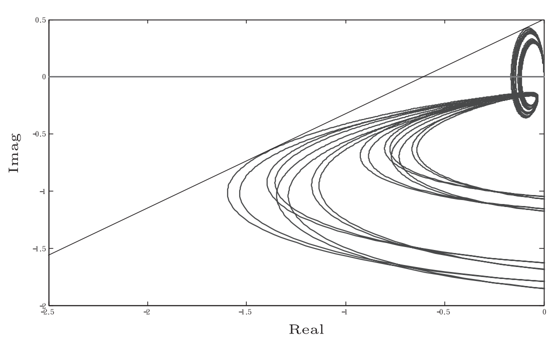


Fig. 33 Robust Popov criterion.

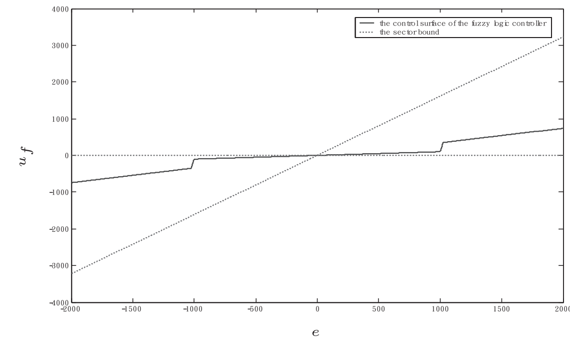


Fig. 34 The sector bound from the robust Popov criterion and control surface of fuzzy logic controller.

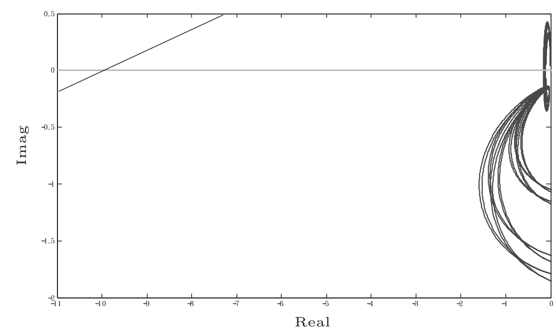


Fig. 35 Parametric robust Popov criterion for the reference inputs.

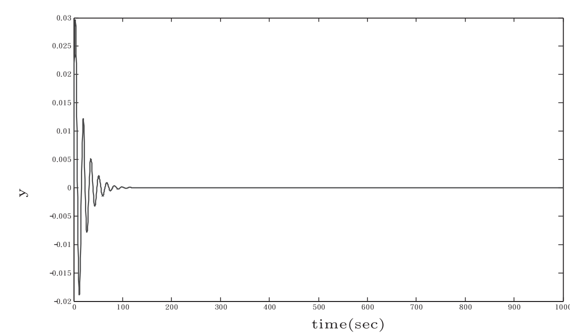


Fig. 36 The time waveform of the stable test case respect to the parametric robust Popov criterion with a bounded pulse reference.

robust Popov plots will be plotted to test the stability too. From Fig. 33, the Popov line cut the negative real axis at $-1/k_p \approx -0.62$ point. Therefore, the control surface $\sigma(\cdot)$ of the fuzzy logic controller should belong to the sector bound $[0, k_p \approx 1.61]$ as shown in Fig. 34, and the fuzzy logic control system is robust absolutely stable.

If the parameters in membership functions of the fuzzy logic controller are chosen such Table 8, then the fuzzy control system is stable. The stable and unstable test cases respect to the robust Popov criterion are with a pulse reference input and a constant input $r = 1300$, respectively. The stable and unstable output waveforms are identical the results as shown in Figs. 27 and 28, respectively. In this case, we also find that if the reference input is increased, the stability of the fuzzy control system which is considered as stable will crash.

6.4 Parametric Robust Popov Criterion

Let's suppose the stable interval plant such the previous test and the parameters in membership functions of the fuzzy logic controller are chosen such Table 8. If we consider the reference inputs $r = [-990, 990]$, Theorem 1 is applied to test the absolute stability of this fuzzy logic control system. By Theorem 1, $-1/k_R^* = -1/0.1 = -10$ is chosen. The total sixteen parametric robust Popov curve will be illustrated to test the robust stability with the reference input in Fig. 35. From Fig. 35, the fuzzy control system is robust absolutely stable. Figures 36–38 show the output waveforms for different reference inputs: a bounded pulse reference, $r = 990$ and

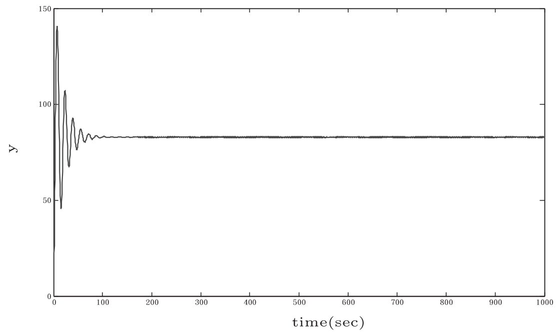


Fig. 37 The time waveform of the stable test case respect to the parametric robust Popov criterion with the reference input $r = 990$.

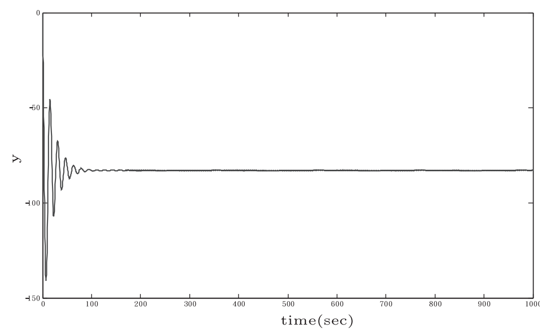


Fig. 38 The time waveform of the stable test case respect to the parametric robust Popov criterion with the reference input $r = -990$.

Table 10 The validity of the different robust stability tests.

	Parametric robust Popov criterion	Robust Lur'e test	Robust circle criterion	Robust Popov criterion
Zero reference inputs	Yes	Yes	Yes	Yes
Constant reference inputs	Yes	No	No	No

$r = -990$, respectively. These time waveforms show that the applied parametric robust Popov criterion is valid. In other words, by the applied parametric robust Popov criterion, the stability of the fuzzy control systems with uncertain interval plants can be guaranteed under the reference inputs in certain interval range.

6.5 A Brief Summary on the Comparisons

The following Table 10 is made for the comparisons with other robust criterions. It shows the applied parametric robust Popov criterion can deal with fuzzy logic control systems with the uncertain interval plants and the constant reference inputs cases. The other three approaches: the robust Lur'e test, the robust circle criterion and the robust Popov criterion just can deal with the uncertain interval plants and the zero reference inputs cases. In previous demonstrated examples, the stability will crash due to reference input shift. On the other hand, the stability of the fuzzy control systems with uncertain interval plants can be assured under the interval range reference inputs by the applied parametric robust Popov criterion.

7. Conclusions

This work analyzes the parametric absolute stability in P and PD type fuzzy logic control systems with both certain and uncertain linear plants with parameters such as the reference input, actuator gain and interval plan. For certain linear plants, the Popov and linearization methods are applied to analyze the stability in both P and PD type fuzzy control systems under different reference inputs and actuator gains. The steady state errors of the fuzzy control systems are also analyzed. For uncertain plants, the parametric robust Popov criterion based on the Lur'e system is applied to the stability analysis of P and PD type fuzzy control systems. Moreover, a fuzzy current controlled RC circuit is designed to compare theoretical analyzes with PSPICE simulation results. Finally, the oscillation phenomena in fuzzy control systems are interpreted from the point of view of the equilibriums in this simulation example. Compared with the other approaches, the absolute stability analysis of the fuzzy control systems with respect to a non-zero reference input and an uncertain linear plant is addressed with the parametric robust Popov criterion.

Acknowledgments

The work was supported by National Science Council under Grant no. NSC 97-2752-E-009-012-PAE. In addition, the authors also sincerely appreciate two anonymous Reviewers' valuable and constructive suggestions to let this paper demonstrate rigorously and completely.

References

[1] C.C. Lee, "Fuzzy logic in control systems: Fuzzy logic control-Parts I and II," IEEE Trans. Syst. Man Cybern., vol.20, no.2, pp.404–435, 1990.
[2] E.H. Mamdani, "Applications of fuzzy algorithms for control of simple dynamic plant," Proc. Inst. Elect. Eng., vol.121, pp.1585–1588, 1974.
[3] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," Fuzzy Sets Syst., vol.45, pp.135–156, 1992.
[4] S.G. Cao, N.W. Rees, and G. Feng, "Stability analysis of fuzzy control systems," IEEE Trans. Syst. Man Cybern. B, vol.26, no.1, pp.201–204, Feb. 1996.
[5] J. Joh, Y.H. Chen, and R. Langari, "On the stability issues of linear Takagi-Sugeno fuzzy models," IEEE Trans. Fuzzy Syst., vol.6, no.3, pp.402–410, Aug. 1998.
[6] S.G. Cao, N.W. Rees, and G. Feng, "Analysis and design of fuzzy control systems using fuzzy-state space models," IEEE Trans. Fuzzy Syst., vol.7, no.2, pp.192–200, April 1999.
[7] M. Sugeno, "On stability of fuzzy systems expressed by fuzzy rules with singleton consequents," IEEE Trans. Fuzzy Syst., vol.7, no.2, pp.201–224, April 1999.
[8] G. Feng, "Approaches to quadratic stabilization of uncertain fuzzy dynamic systems," IEEE Trans. Circuits Syst. I, vol.48, no.6, pp.760–769, June 2001.
[9] G. Feng, "Stability analysis of discrete-time fuzzy dynamic systems

based on piecewise Lyapunov functions," IEEE Trans. Fuzzy Syst., vol.12, no.1, pp.22–28, Feb. 2004.

[10] F.H. Hsiao, C.W. Chen, Y.W. Liang, S.D. Xu, and W.L. Chiang, "T-S fuzzy controller for nonlinear interconnected systems with multiple time delays," IEEE Trans. Circuits Syst. I, vol.52, no.9, pp.1883–1893, Sept. 2005.

[11] C.C. Fuh and P.C. Tung, "Robust stability analysis of fuzzy control systems," Fuzzy Sets Syst., vol.88, no.3, pp.289–298, 1997.

[12] J.C. Lo and Y.M. Chen, "Stability issues on Takagi-Sugeno fuzzy model-parametric approach," Fuzzy Sets Syst., vol.7, no.5, pp.597–607, 1999.

[13] H.K. Lam and Lakmal D. Seneviratne, "Stability analysis of interval type-2 fuzzy-model-based control systems," IEEE Trans. Syst. Man Cybern. B, vol.38, no.3, pp.617–628, June 2008.

[14] H.H. Choi, "Robust stabilization of uncertain fuzzy systems using variable structure system approach," IEEE Trans. Fuzzy Syst., vol.16, no.3, pp.715–724, June 2008.

[15] T.H.S. Li, S.H. Tsia, J.Z. Lee, M.Y. Hsiao, and C.H. Chao, "Robust H_∞ fuzzy control for a class of uncertain discrete fuzzy bilinear systems," IEEE Trans. Syst. Man Cybern. B, vol.38, no.2, pp.510–527, April 2008.

[16] C.T. Pang and Y.Y. Lur, "On the stability of Takagi-Sugeno fuzzy systems with time varying uncertainties," IEEE Trans. Fuzzy Syst., vol.16, no.1, pp.162–170, Feb. 2008.

[17] S. Xu, B. Song, J. Lu, and J. Lam, "Robust stability of uncertain discrete-time singular fuzzy systems," Fuzzy Sets Syst., vol.158, no.20, pp.2306–2316, 2007.

[18] H.K. Lam and L.D. Seneviratne, "BMI-based stability and performance design for fuzzy-model-based control systems subject to parameter uncertainties," IEEE Trans. Syst. Man Cybern. B, vol.37, no.3, pp.502–514, June 2007.

[19] L. Zhou, Q. Zhang, and C. Yang, "Practical stability analysis and synthesis of a classs of uncertain T-S fuzzy systems," Lecture Notes in Computer Science, vol.4223, pp.11–20, Sept. 2006.

[20] C.W. Park and H.J. Kang, "Graphical and numerical approach to robust stability analysis of fuzzy modeled systems with parametric uncertainty and disturbance," Fuzzy Sets Syst., vol.151, no.1, pp.99–117, 2005.

[21] H.K. Lam and F.H.F. Leung, "Stability analysis of discrete-time fuzzy-model-based control systems with time delay: Time delay-independent approach," Fuzzy Sets Syst., vol.159, no.8, pp.990–1000, 2008.

[22] B. Chen, X. Liu, and S. Tong, "Delay-dependent stability analysis and control synthesis of fuzzy dynamic systems with time delay," Fuzzy Sets Syst., vol.157, no.16, pp.2224–2240, 2006.

[23] H.N. Wu, "Delay-dependent stability analysis and stabilization for discrete-time fuzzy systems with state delay: A fuzzy Lyapunov-Krasovskii functional approach," IEEE Trans. Syst. Man Cybern. B, vol.36, no.4, pp.954–962, Aug. 2006.

[24] Z. Zuo and Y. Wang, "Robust stability and stabilization for nonlinear uncertain time-delay system via fuzzy control approach," IET Control Theory Appl, vol.1, no.1, pp.422–429, Jan. 2007.

[25] E. Tian and C. Peng, "Delay-dependent stability analysis and synthesis of uncertain T-S fuzzy systems with time varying delay," Fuzzy Sets Syst., vol.157, no.4, pp.544–559, 2006.

[26] W.J.M. Kickert and E.H. Mamdani, "Analysis of a fuzzy logic controller," Fuzzy Sets Syst., vol.1, no.1, pp.29–44, 1978.

[27] E. Kim, H. Lee, and M. Park, "Limit-cycle prediction of fuzzy control system based on describing function method," IEEE Trans. Fuzzy Syst., vol.8, no.1, pp.11–22, April 2000.

[28] J. Aracil and F. Gordillo, "Describing function method for stability analysis of PD and PI fuzzy controllers," Fuzzy Sets Syst., vol.143, no.2, pp.233–249, 2004.

[29] G. Abdelnour, J.Y. Cheung, C-.H. Chang, and G. Tinetti, "Steady-state analysis of a three-term fuzzy controller," IEEE Trans. Syst. Man Cybern., vol.23, no.2, pp.607–610, March 1993.

[30] G. Abdelnour, J.Y. Cheung, C-.H. Chang, and G. Tinetti, "Application of describing functions in the transient response analysis of a three-term fuzzy controller," IEEE Trans. Syst. Man Cybern., vol.23, no.2, pp.603–606, March 1993.

[31] M. Braae and D.A. Rutherford, "Selection of parameters for a fuzzy logic controller," Fuzzy Sets Syst., vol.2, no.3, pp.185–199, 1979.

[32] C.J. Harris and C.G. Moore, "Phase plane analysis tools for a class of fuzzy control systems" Proc. IEEE International Conference on Fuzzy Systems, pp.511–518, San Diego, CA, USA, 1992.

[33] H.X. Li and H.B. Gatland, "A new methodology for designing a fuzzy logic controller," IEEE Trans. Syst. Man Cybern., vol.25, no.3, pp.505–512, March 1995.

[34] K.S. Ray and D.D. Majumder, "Application of circle criteria for stability analysis of linear SISO and MIMO systems associated with fuzzy logic controller," IEEE Trans. Syst. Man Cybern., vol.14, no.2, pp.345–349, March/April 1984.

[35] R.E. Haber, G. Schmitt-Braess, R.H. Haber, A. Alique, and J.R. Alique, "Using circle criteria for verifying asymptotic stability in PI-like fuzzy control systems: Application to the milling process," IEE Proc., Control Theory Appl., vol.150, no.6, pp.619–627, Nov. 2003.

[36] A. Kandel, Y. Luo, and Y.-Q. Zhang, "Stability analysis of fuzzy control systems," Fuzzy Sets Syst., vol.105, no.1, pp.33–48, 1999.

[37] E. Furutani, M. Saeki, and M. Araki, "Shifted Popov criterion and stability analysis of fuzzy control systems," Proc. 31st Conf. on Decision and Control, pp.2790–2795, Tucson, AZ, USA, 1992.

[38] H. Wang, A. Xue, R. Lu, and J. Wang, "Reliable robust H_∞ tracking control for Lur'e singular systems with parameter uncertainties," Proc. American Control Conference, pp.4312–4317, Seattle, Washington, USA, June 2008.

[39] G. Arslan and K.Y. Lee, "Stability analysis of a fuzzy logic controller," Proc. American Control Conference, pp.3301–3305, Albuqueraue, New Mexico, USA, June 1997.

[40] T. Mori, T. Nishimura, and Y. Kuroe, "Stability of Lur'e systems with interval plants and general sector-type nonlinearities," Proc. American Control Conference, pp.1013–1017, Baltimore, Maryland, USA, June 1994.

[41] T. Mori, T. Nishimura, and Y. Kuroe, "Off-axis circle criteria for Lur'e systems with interval plants," Proc. Conference on Decision and Control, pp.585–586, Lake Buena Vista, Florida, USA, Dec. 1994.

[42] Y. Chen, Y. Zhang, and Q. Li, "Delay-dependent absolute stability of Lur'e systems with interval-varying delay," Proc. IEEE International Conference on Networking, Sensing and Control, pp.1696–1699, Sanya, Hainan, China, April 2008.

[43] Q.L. Han and D. Yue, "Absolute stability of Lur'e systems with time varying delay," IET Control Theory Appl, vol.1, no.3, pp.854–859, May 2007.

[44] Q.L. Han, "Absolute stability of time-delay systems with sector-bounded nonlinearity," Automatica, vol.41, no.12, pp.2171–2176, Dec. 2005.

[45] Y. Wang, X. Yan, Z. Zuo, and H. Zhao, "Further improvements on delay-dependent absolute stability of delayed systems with sector-bounded nonlinearity," Proc. IEEE International Symposium on Intelligent Control, pp.899–904, San Antonio, Texas, USA, Sept. 2008.

[46] R.Q. Lu, H.Y. Su, and J. Chu, "Robust H_∞ control for a class of uncertain Lur'e singular systems with time-delay," Proc. Conference on Decision and Control, pp.5585–5590, Maui, Hawaii, USA, Dec. 2003.

[47] B. Butkiewicz, "Steady-state error of a system with fuzzy controller," IEEE Trans. Syst. Man Cybern. B, vol.28, no.6, pp.855–860, Dec. 1998.

[48] C.W. Tao and J.S. Taur, "Robust fuzzy control for a plant with fuzzy linear model," IEEE Trans. Syst. Man Cybern. B, vol.13, no.1, pp.30–41, Feb. 2005.

[49] H.A. Malki, H. Li, and G. Chen, "New design and stability analysis of fuzzy proportional-derivative control systems," IEEE Trans.

Fuzzy Syst., vol.2, no.4, pp.245–254, Nov. 1994.
[50] J.S. Taur and C.W. Tao, "Design and analysis of region-wise linear fuzzy controllers," IEEE Trans. Syst. Man Cybern. B, vol.27, no.3, pp.526–532, June 1997.
[51] T. Wada, M. Ikeda, Y. Ohta, and D.D. Šiljak, "Parametric absolute stability of Lur'e systems," IEEE Trans. Autom. Control, vol.43, no.11, pp.1649–1653, Nov. 1998.
[52] B.J. Choi, S.W. Kwak, and B.K. Kim, "Design and stability analysis of single-input fuzzy logic controller," IEEE Trans. Syst. Man Cybern. B, vol.30, no.2, pp.303–309, April 2000.
[53] M. Dahleh, A. Tesi, and A. Vicino, "On the robust Popov criterion for interval Lur'e system," IEEE Trans. Autom. Control, vol.38, no.9, pp.1400–1405, Sept. 1993.
[54] S.P. Bhattacharyya, H. Chapellat, and L.H. Keel, Robust Control: The Parametric Approach, Prentice-Hall, Upper Saddle River, NJ, 1995.
[55] B. Friedland, Advanced Control System Design, Prentice-Hall, Englewood Cliffs, NJ, 1996.
[56] K. Viswanathan, R. Oruganti, and D. Srinivasan, "Nonlinear function controller: A simple alterative to fuzzy logic controller for a power electronic converter," IEEE Trans. Ind. Electron., vol.52, no.5, pp.1439–1448, Oct. 2005.
[57] P. Horowitz and W. Hill, The Art of Electronics, Cambridge University Press, Cambridge, UK, 2000.
[58] S. Franco, Design with Operational Amplifiers and Analog Integrated Circuits, 3rd ed., McGraw-Hill, NY, 2002.
[59] B.F. Wu, L.S. Ma, J.W. Perng, H.I. Chin, and T.T. Lee, "Stability analysis of equilibrium points in static fuzzy control system wit reference inputs and adjustable parameters," Proc. IEEE Congress on Computational Intelligence, pp.10255–10261, Vancouver, BC, Canada, July 2006.

Bing-Fei Wu was born in Taipei, Taiwan in 1959. He received the B.S. and M.S. degrees in control engineering from National Chiao Tung University (NCTU), Hsinchu, Taiwan, in 1981 and 1983, respectively, and the Ph.D. degree in electrical engineering from the University of Southern California, Los Angeles, in 1992. Since 1992, he has been with the Department of Electrical Engineering and Control Engineering, where he is currently a Professor. His research interests include fuzzy systems, nonlinear control, and intelligent control.

Li-Shan Ma was born in Changhua, Taiwan in 1968. He received the B.S. and M.S. degrees in electrical engineering from Chung Yuan Christian University, Chungli, Taiwan, in 1995 and 1997, respectively. Since 1999, he has been with the Department of Electronic Engineering, Chienkuo Technology University, Changhua, Taiwan, where he is currently a lecturer. Now, he is pursuing a Ph.D. degree in the Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu, Taiwan from 2001. His research interests include fuzzy systems, nonlinear control, and intelligent control.

Jau-Woei Perng was born in Hsinchu, Taiwan in 1973. He received the B.S. and M.S. degrees in electrical engineering from the Yuan Ze University, Chungli, Taiwan, in 1995 and 1997, respectively and the Ph.D. degree in electrical and control engineering from the National Chiao Tung University (NCTU), Hsinchu, Taiwan, in 2003. From 2004 to 2008, he was a Research Assistant Professor with the Department of Electrical and Control Engineering at NCTU. He is currently an Assistant Professor with the Department of Mechanical and Electro-Mechanical Engineering, National Sun Yat-sen University. His research interests include robust control, nonlinear control, fuzzy logic control, neural networks, systems engineering, and intelligent vehicle control.