The story of *c*

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The letter c is the standard symbol for the speed of light, but that was not always the case. I describe how c was first introduced into the theory of electromagnetism and the stages by which it came to be used to denote the speed of light. © *2006 American Association of Physics Teachers.*
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I. INTRODUCTION

One of the best known symbols in all of science is c, the speed of light. Even people who know nothing else about science have seen the equation

$$E = mc^2 \tag{1}$$

and know that c in Eq. (1) is the speed of light. How c came to denote the speed of light is an interesting story in the history of physics.

II. THE RATIO OF ELECTRICAL UNITS

The symbol c was originally introduced by Wilhelm Weber as a ratio of units of electric charge. These units were defined in terms of either the electrostatic force between two charges or the electromagnetic force between two current elements. The electrostatic force between charges is given by Coulomb's law which was developed by Coulomb and others at the end of the 18th century.[1] Coulomb's law can be written as

$$\mathbf{F}_{12} = kq_1q_2\frac{\mathbf{r}}{r^3}, \tag{2}$$

where $\mathbf{F}_{12}$ is the force exerted by charge q_2 at position $\mathbf{r}_2$ on charge q_1 at position $\mathbf{r}_1$, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, and k is a constant determined by the units. Equation (2) and the following equations are written in vector notation rather than in the notation of the 19th century.

Contemporary treatments of electromagnetism[2] are typically based on a formulation developed by Biot and Savart.[3,4] However, Weber's discussion is based on a formulation developed by Ampère.[6] Let $d\mathbf{s}_1$ ($d\mathbf{s}_2$) be an element of length of wire located at position $\mathbf{r}_1$ ($\mathbf{r}_2$) and carrying current I_1 (I_2). Then Ampère showed that the current in $d\mathbf{s}_2$ exerts on the current in $d\mathbf{s}_1$ a magnetic force

$$d^2\mathbf{F}_{12} = -2k'I_1I_2\frac{\mathbf{r}}{r^3}\left[(d\mathbf{s}_1\cdot d\mathbf{s}_2) - \frac{3}{2}\frac{(d\mathbf{s}_1\cdot\mathbf{r})(d\mathbf{s}_2\cdot\mathbf{r})}{r^2}\right], \tag{3}$$

where k' is another constant determined by the units. The Biot-Savart and Ampére expressions give different values for the force between two current elements, but give the same value for the force of a complete circuit on a current element.

We can define a unit of charge by choosing either k or k'. Once one of these constants is chosen, the other must be determined experimentally. We know that the quantity $\sqrt{k/k'}$ is equal to the speed of light, but this relation was not known in the early 19th century.

Several choices have been made for k and k'. In the International System (SI) units we write $k=1/4\pi\epsilon_0$, $k'=\mu_0/4\pi$, and choose $\mu_0\equiv 4\pi\times 10^{-7}$ N/A^2. This choice was introduced by Giorgi in 1901.[7] Three other systems of units that were defined in the early 19th century are important for this discussion. The electrostatic system of units (esu) is defined by choosing $k\equiv 1$. The electromagnetic system of units (emu) is defined by choosing $k'\equiv 1$. And the electrodynamic system of units (edu) is defined by choosing $2k'\equiv 1$. Note that $q_{\text{emu}}/q_{\text{esu}}=\sqrt{k/k'}$ and $q_{\text{edu}}/q_{\text{esu}}=\sqrt{2k/k'}$.

In 1846 Weber[8] expressed the force between two moving charges as (in our notation)

$$\mathbf{F}_{12} = kq_1q_2\frac{\mathbf{r}}{r^3}\left\{1 - \frac{k'}{2k}\left[\left(\frac{dr}{dt}\right)^2 - 2r\frac{d^2r}{dt^2}\right]\right\}. \tag{4}$$

Equation (4) was derived from Eqs. (2) and (3) and a model of electric current introduced by Fechner.[10] In Fechner's model, one unit of current flows past a point when one unit of positive charge and one unit of negative charge (in the opposite direction) flow past the point in one unit of time. Weber used electrostatic units for which $k=1$ and wrote $a^2/16$ in place of $k'/2k$. Thus, $4/a=q_{\text{edu}}/q_{\text{esu}}$. In 1851,[11] Weber replaced $4/a$ by a and in 1852 (Ref. 12) by c. This c is not equal to the speed of light but is larger by a factor of $\sqrt{2}$. Nevertheless, it is from this usage that the letter c eventually came to denote the speed of light.

Why did Weber choose c to denote $q_{\text{edu}}/q_{\text{esu}}$? Did this letter have some significance or was it a purely arbitrary choice? Asimov[14] states that c stands for the Latin *celeritas* meaning speed, but he gives no historical evidence to support this statement. It is likely that Asimov chose *celeritas* himself with no historical precedent.

Gibbs[15] suggests that c stands for constant. This suggestion requires a little explanation. The contemporary German word is *Konstante* which starts with K. Reprints of Refs. 11 and 22 in *Wilhelm Weber's Werke* and in *Ostwalds Klassiker der Exakten Wissenschaften* use the word *Konstante*. However, in the original papers in the *Annalen der Physik* (and in Ref. 23) the word is spelled *Constante*.[16] Thus Gibbs's suggestion is plausible.

Weber did not say why he chose c. Weber first used a. Perhaps he decided that he didn't like a or b and simply moved on to the next letter in the alphabet. There is a Sidney Harris cartoon[17] showing Einstein looking at a blackboard on which he has written and crossed out $E=ma^2$ and $E=mb^2$. Perhaps something like that cartoon actually happened to Weber. In the absence of evidence we cannot draw any conclusions.

To avoid confusing Weber's use of c with current usage, I will follow Rosenfeld[18] and Assis[19] in the remainder of this paper and denote Weber's c by c_W, reserving c for the current usage. Thus, $c_W = q_{\text{edu}}/q_{\text{esu}}$, $c = q_{\text{emu}}/q_{\text{esu}}$ = the speed of light, and $c_W = \sqrt{2}c$.

III. THE RATIO OF UNITS AND THE SPEED OF LIGHT

During the 1850s and 1860s Weber, Kirchhoff, and Ludwig Lorenz used the constant c_W and found situations in which electromagnetic effects propagate with the speed $c_W/\sqrt{2}$. In 1857, Kirchhoff studied electric currents in thin wires[20] and extended media.[21] He found that in media with vanishing resistance, electric currents propagate with speed $c_W/\sqrt{2}$. From the value of c_W, determined experimentally by Weber and Kohlrausch[22] in 1856, Kirchhoff recognized that $c_W/\sqrt{2}$ is equal to the speed of light. Weber[24] did similar studies and drew the same conclusion at about the same time but independently of Kirchhoff. However, his work was not published until 1864. Kirchhoff and Weber were not studying electromagnetic waves but electric currents in conductors. But it is interesting that, several years before Maxwell, they found electromagnetic effects that propagate with the speed of light.

In 1867, Lorenz[25] extended Kirchhoff's treatment to free space and predicted the existence of electromagnetic waves traveling with speed $c_W/\sqrt{2}$. Although published after Maxwell's work, Lorenz's paper appears to be independent.

James Clerk Maxwell developed his theory of the electromagnetic field in three papers[26–28] published between 1856 and 1865. In the second of these papers[27] Maxwell first introduced the displacement current and predicted the existence of electromagnetic waves with a speed in vacuum equal to $q_{\text{emu}}/q_{\text{esu}}$. This quantity could be calculated from the value of c_W determined by Weber and Kohlrausch.[22] Maxwell noted that $q_{\text{emu}}/q_{\text{esu}}$ is equal to the speed of light measured in 1849 by Fizeau[30] and concluded that light is an electromagnetic wave. In this discussion Maxwell used E for $q_{\text{emu}}/q_{\text{esu}}$ and V for the speed of light. In his third paper[28] Maxwell again used V for the speed of light, but replaced E by v for $q_{\text{emu}}/q_{\text{esu}}$.

Maxwell continued his use of the notation V for the speed of light and v for $q_{\text{emu}}/q_{\text{esu}}$ in his *Treatise on Electricity and Magnetism*[31] first published in 1873. In the last chapter of the treatise[32] Maxwell wrote Weber's expression for the force between two charges as

$$\frac{ee'}{r^2}\left\{1+\frac{1}{c^2}\left[r\frac{d^2r}{dt^2}-\frac{1}{2}\left(\frac{dr}{dt}\right)^2\right]\right\}. \qquad (5)$$

Here, without comment, he has used c rather than c_W, moving the factor 1/2 in Eq. (4) into the final bracket. This appears to be the first use of c to represent a quantity having a value equal to the speed of light.

After 1870, the electrodynamic system of units and c_W were not used. But c did not immediately come to be used for the speed of light. Several scientists, including Lodge,[33] Drude,[34,35] Hendrik Lorentz,[36] and Michelson,[39] in the late 19th and early 20th centuries, followed Maxwell and used V for the speed of light. A few other symbols were also used. Hertz, for example, used A for the reciprocal of the speed of light.[40] During this time writers distinguished between the speed of light and $q_{\text{emu}}/q_{\text{esu}}$, the speed of electromagnetic waves. For example, Drude used V for the speed of light and c for the speed of electromagnetic waves.[34,41] He did mention that these two quantities agree with one another. Heaviside[42] used v for the speed of electromagnetic waves. Curiously he used c for permittivity and with μ for permeability, wrote $v = 1/\sqrt{\mu c}$.

Einstein used V for the speed of light in his early papers on relativity.[43–45] Although he introduced the relation between mass and energy in Ref. 44, Einstein did not write Eq. (1) as $E = mV^2$ in these early papers. The closest he came appears to be the statement in Ref. 45: "… to an increase in the body's energy ΔE must always correspond an increase in the mass $\Delta E/V^2$, where V denotes the velocity of light." Reference 45 was published in 1907 and that same year Einstein began using c for the speed of light.[48]

By the final decades of the 19th century c was in common use to denote $q_{\text{emu}}/q_{\text{esu}}$, the speed of electromagnetic waves. However other symbols, most commonly V, were used when the discussion dealt specifically with the speed of light. The earliest use of c specifically for the speed of light that I have found is in a 1903 paper by Abraham.[49] Still, it is not possible to definitely rule out an earlier use. In any case Abraham's notation would have had particular influence on later generations of physicists because his electromagnetism text, first published in 1904, became widely used for graduate study. Its English translation was a standard graduate text in the United States, at least until the 1950s.

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