Lesson2
Lambda Calculus Basics

1/10/02

Chapter 5.1, 5.2

Outline

- Syntax of the lambda calculus
  - abstraction over variables
- Operational semantics
  - beta reduction
  - substitution
- Programming in the lambda calculus
  - representation tricks

Basic ideas

- introduce variables ranging over values
- define functions by (lambda-) abstracting over variables
- apply functions to values

$x + 1$

$\lambda x.\ x + 1$

$(\lambda x.\ x + 1)\ 2$

Abstract syntax

Pure lambda calculus: start with *nothing but variables.*

Lambda terms

t ::=

x	variable
λx . t	abstraction
t t	application

Scope, free and bound occurences

λx . t — body

binder

Occurences of x in the body t are bound.
Nonbound variable occurrences are called free.

(λx . λy. zx(yx))x

Beta reduction

Computation in the lambda calculus takes the form of beta-reduction:

(λx. t1) t2 → [x ⇨ t2]t1

where [x ⇨ t2]t1 denotes the result of substituting t2 for all free occurrences of x in t1.

A term of the form (λx. t1) t2 is called a beta-redex (or β-redex).

A (beta) normal form is a term containing no beta-redexes.

Beta reduction: Examples

(λx.λy.y x)(λz.u) → λy.y(λz.u)

(λx. x x)(λz.u) → (λz.u) (λz.u)

(λy.y a)((λx. x)(λz.(λu.u) z)) → (λy.y a)(λz.(λu.u) z)

(λy.y a)((λx. x)(λz.(λu.u) z)) → (λy.y a)((λx. x)(λz. z))

(λy.y a)((λx. x)(λz.(λu.u) z)) → ((λx. x)(λz.(λu.u) z)) a

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Evaluation strategies

- Full beta-reduction
  - any beta-redex can be reduced
- Normal order
  - reduce the leftmost-outermost redex
- Call by name
  - reduce the leftmost-outermost redex, but not inside abstractions
  - abstractions are normal forms
- Call by value
  - reduce leftmost-outermost redex where argument is a value
  - no reduction inside abstractions (abstractions are values)

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Programming in the lambda calculus

- multiple parameters through currying
- booleans
- pairs
- Church numerals and arithmetic
- lists
- recursion
  - call by name and call by value versions

Computation in the lambda calculus takes the form of beta-reduction:

$$(\lambda x.\ t1)\ t2 \to [x \mapsto t2]t1$$

where $[x \mapsto t2]t1$ denotes the result of substituting t2 for all free occurrences of x in t1.

A term of the form $(\lambda x.\ t1)\ t2$ is called a beta-redex (or β-redex).

A (beta) normal form is a term containing no beta-redexes.

Symbols

Symbols □□ □□ ⇨ ➔ ➙ ➝