

Smart Hydrogel Modelling

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*Dedicated first and foremost to my motherland,
and to Duer, Anne and my parents*

Preface

The science of mathematical modelling and numerical simulation is generally accepted as the third mode of scientific discovery (with the other two modes being experiment and analysis), making this field an integral component of cutting edge scientific and industrial research in most domains. This is especially so in advanced biomaterials such as polymeric hydrogels responsive to biostimuli for a wide range of potential BioMEMS applications, where multiphysics and multi-phase are common requirements. *These environmental stimuli-responsive hydrogels are often known as smart hydrogels.* In the published studies on the smart or stimuli-responsive hydrogels, the literature search clearly indicates that the vast majority are experimental based. In particular, although there are a few published books on the smart hydrogels, none is involved in the *modelling of smart hydrogels.*

For the few published journal papers that conducted mathematical modelling and numerical simulation, results were far from satisfactory, and showed significant discrepancies when compared with existing experimental data. This has resulted in ad hoc studies of these hydrogel materials mainly conducted by trial and error. This is a very time-consuming and inefficient process, and certain aspects of fundamental knowledge are often missed or overlooked, resulting in off-tangent research directions. Thus it is absolutely necessary to publish a book on the modelling and simulation of the smart hydrogels with real multidisciplinary requirement for establishment of a theoretical platform by developing the correct mathematical models and also the powerful numerical techniques required to solve these challenging highly nonlinear and coupled models.

Polymeric hydrogels are form of matters that possess both the properties of solid and liquid. Their structural framework chains are formed from networks of randomly crosslinked polymers that embody three different phases in general, namely the three-dimensional solid polymeric matrix network, interstitial fluid and ion species. Depending on the component characteristics and synthesis methods, the hydrogels can be designed or tailored to demonstrate the unique property of undergoing discrete or continuous volume transformation in response to infinitesimal changes of external environment stimuli, such as solution pH, electric field, temperature, solvent composition, glucose/carbohydrates, salt concentration/ionic strength, light/photon, pressure, coupled magnetic and electric fields. These magnificent features make the hydrogel better known as smart or stimuli-responsive hydrogels.

Due to their unique properties that include swelling/deswelling behaviour, sorption capacity, mechanical property, permeability and surface property, the hydrogels provide the instrumentation for creating functional materials for broad spectrum of applications as they can sense the environmental changes and eventually induce structural changes without a need for an external power source. Artificial muscle, microfluidic control, sensor/actuator, separation process and chromatographic packing are just few examples of the successful applications of hydrogels. Another exceptional promise of the hydrogels is their biocompatibility and biostability potentials, suggesting that the hydrogels are also an excellent substitution for the human body tissues or biomimetic applications. There are also extensive explorations of the hydrogels in the medical and pharmaceutical applications, such as drug delivery system, articular cartilage, biomaterial scaffold, corneal replacement and tissue engineering. As such, the multi-state characteristics of the smart polymer hydrogels and their wide-range multiphysics applications make the multi-disciplinary and multiphase the basic requirements for the mathematical models. For example, these models are required to be highly multi-disciplinary and at least to take into consideration the coupled chemo-electro-mechanical multi-fields and multiphase deformation of polymeric network solid matrices with flow of ions and interstitial fluid. This results in several mathematical challenges, in which usually the models consist of coupled nonlinear partial differential equations with requirements of moving boundary and localized high gradient.

A comprehensive study through modelling and simulation is thus warranted for theoretical understanding of the response behaviour of the smart hydrogels in BioMEMS devices subject to different environmental stimuli, due to the advantages of the material characteristics of the smart polymer hydrogels and their wide range of multiphysics applications. However, as mentioned above, there is a lack of open publications on modelling and simulation of the smart hydrogels, and this monograph is thus written to systematically document the response behaviour of the smart hydrogels to various environmental stimuli. A complete theoretical platform detailing of the fundamental theory for the smart polymer hydrogels is established. It is composed of several novel mathematical models which are already successfully developed. Response of the smart hydrogels to surrounding environment is examined in detail for the basic stimuli within common BioMEMS devices such as solution pH, externally applied electric voltage, temperature, solvent composition, glucose/carbohydrates and salt concentration/ionic strength. The effects of various material properties and environmental conditions on the responsive performance of the smart hydrogels, including Young's modulus, initially fixed charge density, effective crosslink density, ionic strength and valence of bath solution, initial volume fraction of polymeric network, initial geometry, are also investigated in various parametric studies. In addition, an analysis of drug delivery system for the controlled drug release from non-swellable micro-hydrogel particles is presented with consideration of drug dissolution and diffusion through the continuous matrices of spherical micro-hydrogel particles.

This is the first monograph of its kind, which primarily meets the needs of scientists and engineers in the broad areas of polymer materials science, biomaterials

engineering, biomedical engineering, sensor/actuator, micro-electro-mechanical system (MEMS) and BioMEMS, physics, chemistry, biophysics, biochemistry and bioengineering. It is especially useful for them as a reference source, and also if they wish to conduct further studies so as to extend their work to practical application. Other important primary readers are postgraduate students in the area of polymer materials science and biomedical engineering, especially those with involvement in the computational aspects such as the modelling and simulation of stimuli-responsive soft materials. Possible secondary readers include undergraduate students taking the advanced mechanical and electric engineering courses which involve sensor or actuator, MEMS and BioMEMS. The chapters on the fundamental theoretical development are especially useful to these students. Correspondingly, the course lecturers will also find this book a good reference source. This book provides both the casual and interested reader with insights into the special features and intricacies of smart polymer hydrogels when environmental stimuli are involved. It is also invaluable to design engineers in the polymer sensor/actuator, MEMS and BioMEMS industry and biomedical engineering, serving as a useful reference source with benchmark results to compare and verify their experimental data against.

The author would like to thank Professor Justin Hanes for his constant encouragement over the years, and especially for writing the Foreword to this book. Special thanks also go to Professors N.R. Aluru, Erik Bergersson, Khin-Yong Lam, Teng-Yong Ng and Zhigang Suo for their strong support and useful advice. Finally the author is very grateful to Drs J. Chen, J.Q. Cheng, J. Fu, R.M. Luo, Q.X. Wang, X.G. Wang, Z.J. Wang, S.N. Wu, G.P. Yan, Y.K. Yew and Z. Yuan for their invaluable contributions to this research.

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Author's Brief Biography



Dr. Hua Li received his BSc and MEng degrees in engineering mechanics from Wuhan University of Technology, P.R.C., in 1982 and 1987, respectively. He obtained his PhD degree in mechanical engineering from the National University of Singapore in 1999. From 2000 to 2001, Dr. Li was a postdoctoral associate at the Beckman Institute for Advanced Science and Technology, University of Illinois at Urbana-Champaign. At the end of 2005, he was a visiting scientist (on invitation) at the Department of Chemical and Biomolecular Engineering of Johns Hopkins University. From 2001 to 2006, he was a research scientist in the A*STAR Institute of High Performance Computing. Dr. Li is currently an assistant professor in School of Mechanical & Aerospace Engineering at Nanyang Technological University. His research interests include the modelling and simulation of MEMS focusing on the use of smart hydrogels in BioMEMS applications, the development of advanced numerical methodologies and the dynamics of high-speed rotating shell structures. He has co-authored a book on “*Rotating Shell Dynamics*” published by Elsevier and two book chapters, one on MEMS simulation and the other on hydrogel

drug delivery system modelling, and authored/co-authored over 90 articles published in top international peer-reviewed journals. He is also extensively funded by agencies and industries, including the principal investigator of a Computational BioMEMS project awarded under A*STAR's strategic research programme in MEMS.

Foreword

Since the author told me that he planned to write a book about the mathematical modelling of smart hydrogels, I have looked forward to its completion with great anticipation. I have known the author and have followed his research for years; this work is a consummation of his extraordinary contributions in the area of soft active materials (SAMs). To write this book, the author has exhausted his knowledge, experience and free time. I believe this book will be of great value to both experts and also those with a casual interest in hydrogels, soft materials, drug delivery, BioMEMS and related fields of active scientific investigation. Its author, professor Hua Li, is among the most highly respected scientists in the world in the area of BioMEMS hydrogel theory. It is an honour to write the Foreword for his book.

Smart hydrogels have wide-ranging applications in bioengineering, such as in the development of soft sensors/actuators, controlled drug delivery systems and stimuli-responsive BioMEMS devices. The majority of relevant studies in these areas are experiment based; considerably less attention has been paid to theoretical aspects. This monograph provides a comprehensive and systematic study for modelling smart polymer hydrogels in the BioMEMS environment. It covers development of the models characterized in chemo-electro-mechanical multi-energy coupled domains and expressed in form of nonlinear partial differential governing equations for smart hydrogels. It also documents benchmark results, namely simulating the performance and predicting the characteristics of smart hydrogels responding to solution pH, externally applied electric voltage, environmental temperature, glucose/carbohydrates and salt concentration/ionic strength, which are basic stimuli in common BioMEMS devices.

This book is written in a straightforward manner without losing depth, such that it makes informative reading for a graduate student working or intending to work in this area. It will also undoubtedly serve as a rich reference source for experts in the field. I congratulate professor Hua Li for a tremendous achievement, and I hope and expect that the reader will benefit greatly from it.

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Contents

1	Introduction	1
1.1	Definition and Application of Hydrogel	1
1.2	Historical Development of Modelling Hydrogel	3
1.2.1	Steady-State Modelling for Equilibrium of Smart Hydrogels	4
1.2.2	Transient Modelling for Kinetics of Smart Hydrogels	29
1.2.3	A Theoretical Formalism for Diffusion Coupled with Large Deformation of Hydrogel	43
1.2.4	Remarks	45
1.3	About This Monograph	45
	References	47
2	Multi-Effect-Coupling pH-Stimulus (MECpH) Model for pH-Sensitive Hydrogel	57
2.1	Introduction	57
2.2	Development of the MECpH Model	57
2.2.1	Electrochemical Formulation	58
2.2.2	Mechanical Formulation	67
2.3	Computational Domain, Boundary Condition and Numerical Implementation	72
2.4	Model Validation with Experiment	76
2.5	Parameter Studies by Steady-State Simulation for Equilibrium of Hydrogel	78
2.5.1	Influence of Initially Fixed Charge Density of Hydrogel	81
2.5.2	Influence of Young's Modulus of Hydrogel	85
2.5.3	Influence of Initial Geometry of Hydrogel	91
2.5.4	Influence of Ionic Strength of Bath Solution	95
2.5.5	Influence of Multivalent Ionic Composition of Bath Solution	102
2.6	Remarks	108
	References	111

3 Multi-Effect-Coupling Electric-Stimulus (MECe) Model for Electric-Sensitive Hydrogel	115
3.1 Introduction	115
3.2 Development of the MECe Model	115
3.2.1 Formulation of the MECe Governing Equations	116
3.2.2 Boundary and Initial Conditions	126
3.3 Steady-State Simulation for Equilibrium of Hydrogel	127
3.3.1 Numerical Implementation	127
3.3.2 Model Validation with Experiment	131
3.3.3 Parameter Studies	132
3.4 Transient Simulation for Kinetics of Hydrogel	147
3.4.1 Numerical Implementation	147
3.4.2 Model Validation with Experiment	150
3.4.3 Parameter Studies	151
3.5 Remarks	170
References	171
4 Multi-Effect-Coupling pH-Electric-Stimuli (MECpHe) Model for Smart Hydrogel Responsive to pH-Electric Coupled Stimuli	173
4.1 Introduction	173
4.2 Development of the MECpHe Model	174
4.3 Numerical Implementation	180
4.4 Model Validation with Experiment	182
4.5 Parameter Studies by Steady-State Simulation for Equilibrium of Hydrogel	184
4.5.1 Influence of Solution pH Coupled with External Electric Voltage	184
4.5.2 Influence of Initially Fixed Charge Density of Hydrogel	191
4.5.3 Influence of Ionic Strength	199
4.5.4 Influence of Ionic Valence	209
4.6 Remarks	214
References	217
5 Multi-Effect-Coupling Thermal-Stimulus (MECtherm) Model for Temperature-Sensitive Hydrogel	219
5.1 Introduction	219
5.2 Development of the MECtherm Model	220
5.2.1 Free Energy	220
5.2.2 Poisson–Nernst–Planck Formulation	223
5.3 Numerical Implementation	223
5.4 Model Validation with Experiment	228
5.5 Parameter Studies by Steady-State Simulation for Thermo-Sensitive Ionized Hydrogel	229
5.5.1 Influence of Initially Fixed Charge Density	230
5.5.2 Influence of Bath Solution Concentration	233

5.5.3	Influence of Effective Crosslink Density	237
5.5.4	Influence of Initial Volume Fraction of Polymeric Network	240
5.6	Transient Modelling of Temperature-Sensitive Neutral Hydrogel	243
5.6.1	Model Formulation in Eulerian Frame	246
5.6.2	Analysis	261
5.6.3	Model Formulation in Lagrangian Frame and Boundary and Initial Conditions	268
5.6.4	Numerical Implementation	270
5.6.5	Simulations and Discussions	272
5.7	Remarks	286
	References	288
6	Novel Models for Smart Hydrogel Responsive to Other Stimuli: Glucose Concentration and Ionic Strength	295
6.1	Introduction	295
6.2	Multi-Effect-Coupling Glucose-Stimulus (MECglu) Model for Glucose-Sensitive Hydrogel	296
6.2.1	Development of the MECglu Model	298
6.2.2	Model Validation with Experiment	306
6.3	Multi-Effect-Coupling Ionic-Strength-Stimulus (MECis) Model for Ionic Strength-Sensitive Hydrogel	310
6.3.1	Development of the MECis Model	312
6.4	Remarks	328
	References	329
7	Simulation of Controlled Drug Release from Non-Swellable Micro-Hydrogel Particle	335
7.1	Introduction	335
7.2	Formulation of Model	335
7.3	Numerical Implementation	338
7.4	Comparison with Experiment	339
7.5	Parameter Studies by Transient Simulation	341
7.5.1	Identification of Physical Parameters	341
7.5.2	Influence of Mean Radius of Micro-Hydrogel Particle	344
7.5.3	Influence of Equivalent Drug Saturation Concentration	344
7.5.4	Influence of the First-Order Drug Dissolution Rate	344
7.5.5	Influence of Drug Diffusion Coefficient	345
7.6	Remarks	345
	References	345
	References	347
	Acknowledgements	349
	Index	351