Low Crest Factor Modulation Techniques for Orthogonal Frequency Division Multiplexing (OFDM)

Ashraf A. Eltholth[*], Adel R. Mekhail[*], A. Elshirbini[*], M. I. Dessouki[†] and A. I. Abdelfattah [††]

[*] National Telecommunication Institute, Cairo, Egypt
[†] Faculty of electronic Engineering, Menouf, Egypt
[††] Faculty of Engineering, Mansoura, Egypt

ABSTRACT

OFDM suffer from a high peak-to-average ratio (PAR), caused by the addition of a large number of independently modulated sub-carriers in parallel at the transmitter. When subjected to a non-linear power amplifier, these signals may undergo significant spectral distortion, leading to both in-band and out-of-band interference, and an associated degradation in system performance. In this paper we compare between different modulations techniques in OFDM for the spectral efficiency, BER, and PAR reduction, which reduce the effect of nonlinear amplifier. Simulations of the proposed system models using Matlab are used to compare between three linear modulation families, namely M-QAM, M-PSK and MSK to be used for OFDM. Also we discuss the benefits of using MSK as the lowest crest factor modulation technique to be used for OFDM.

Keywords: OFDM, Crest Factor, MSK, HPA, PAPR.

1. INTRODUCTION

Besides its lot of advantages, OFDM suffers from some drawbacks that become apparent when used in the real world; a major obstacle is the very high peak to average power ratio (PAR), or equivalently, a large crest factor (CF) [1]. This problem arises because of the nature of the composite time OFDM signal as it is the sum of N-modulated sub-carriers, and carriers may add-up constructively forming a very high peak. This spurious high amplitude peaks in the composite time signal compared to the average signal power, the instantaneous power of these peaks is high, and consequently, so is the Peak-to-Average power Ratio (PAR). The occurrence of these peaks seriously hampers practical implementations specially the power amplifiers. The impacts of amplifier-induced nonlinear distortions are: the in band waveform distortion, resulting in a signal to noise ratio degradation and the out of band radiation resulting in adjacent channel interference. However, in most practical cases the out of band radiation is the limiting factor, which defines the amplifier back-off requirements [2]. Simply dimensioning the system components to be able to cope with the worst case signal peaks is practically impossible. That is why solutions have been proposed to counter act PAR problem. In this paper we consider the choice of low crest factor modulation techniques to be used in the OFDM system. We have compared among several families of modulation techniques. We propose to use a modulation technique with high spectral efficiency and less sensitive to nonlinear channel effects, which is the Minimum Shift Keying (MSK). Sections 2 will describe the statistical properties of the OFDM composite time signal, while in section 3 we will show the performance of OFDM system using the mentioned modulation techniques and in section 4 we will discuss the applicability of power amplifiers to OFDM systems.

2. STATISTICAL PROPERTIES OF OFDM SIGNAL

The OFDM transmitted base-band signal may be represented as [3, 4]:

$$x(t) = \sum_{k=0}^{N-1}(a_k + jb_k)\exp(-j2\pi w_k t)$$

(1)

Where N is the number of sub-carriers, while a_k and b_k are the real and imaginary components of the complex modulating symbols, respectively. For example, for 16-QAM modulation a_k and b_k may assume the equi-probable values of {-3, -1, 1, 3}. From the central limit theory it follows that for large values of N (N≥64) both the real and imaginary components of x(t) becomes normally

distributed variable having a mean of zero. And thus, the absolute value will have Rayleigh distribution.

The crest factor of the discrete time representation x(k) is defined as the ratio of the peak magnitude value and the square root of the average power of this signal. For the OFDM signal as we mentioned it have a zero mean and thus the square root of the average power will be equal to the standard deviation δ. Thus the crest factor can be written as:

$$CF = \frac{\max(x(k))}{\sqrt{E(x^2)}} = \frac{\max(x(k))}{\delta} \qquad (2)$$

Note that, the peak to average power ratio, widely used in literature, is simply the square of the crest factor. Both quantities coincide, if expressed in logarithmic scale (dB).

An OFDM system is simulated using Matlab with 512 sub-carriers; We have performed a measure of normality, i.e., wither the composite time signal approaches the normal distribution or not, as a verification of the applicability of central limit theory to OFDM, and we get the following result as shown in figure (1)

It can be noticed that the real and imaginary parts of the OFDM signal completely agree with the normal distribution.
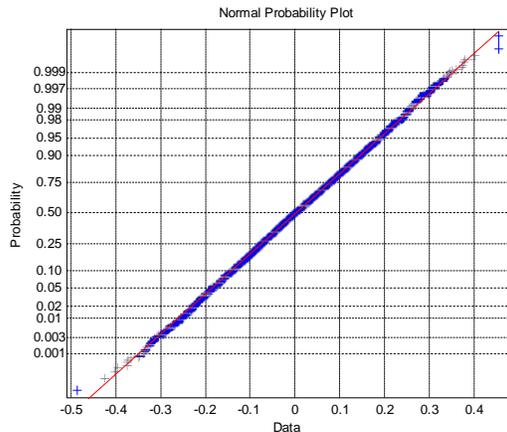

Fig.(1) Normal probability plot of OFDM signal

3. OFDM USING DIFFERENT MODULATORS

In this section we will simulate an OFDM system as shown in the block diagram in figure (2) using different modulation techniques.

We have simulated an OFDM system with different modulation techniques, namely, M-ary PSK, M-ary QAM (with M=4, 8, 16 and 32) and Minimum Shift Keying (MSK).

When designing a constellation diagram for a modulation technique, some considerations must be given to [5]:

a. The minimum Euclidean distance amongst phasors, which is characteristics of the noise immunity of the scheme

b. The minimum phase rotation amongst constellation points, determining the phase jitter immunity

c. The peak to average phasor power, which is a measure of robustness against non-linear distortion introduced by power amplifiers.
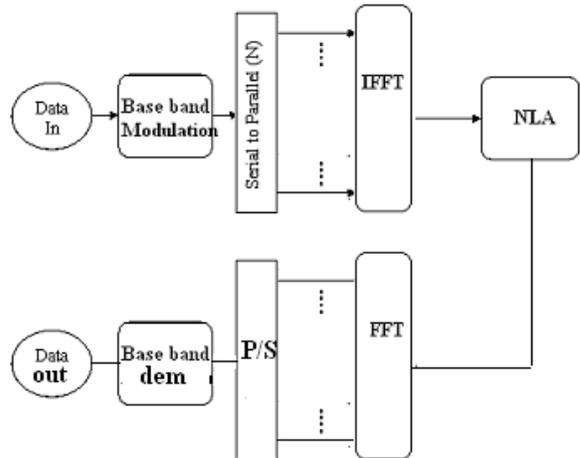

Fig.(2) Block diagram of OFDM system

The Bandwidth efficiency can be increased either by increasing the Number of signal phase levels, or by increasing the Number of signal amplitude levels [6],

1. Increasing the signal amplitude levels has the drawback that the signal envelope is not constant and therefore non-linear amplification may cause spreading of the signal spectrum and increase in BER.

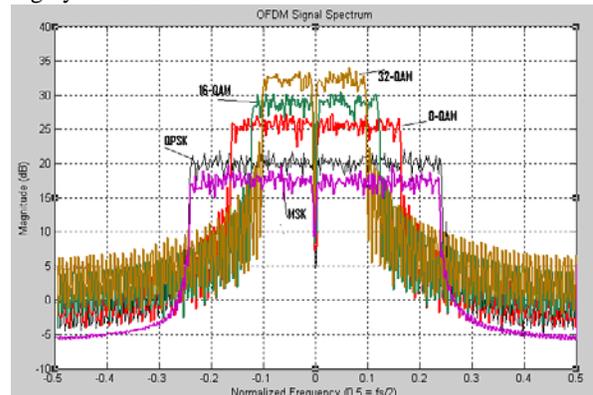2. Increasing the Number of phase levels will highly increase the BER.

Fig.(3)OFDM Spectrum with different modulation

We have noted that for M-PSK, as M increases no effect has been occurred to the dynamic range and the PAR remains nearly the same, while for M-QAM, as M increases the dynamic range increase and so the PAR, but both of them agree in the

spectral efficiency increase as M increase. This is shown in figure 3.

It is also noticeable from figure (3) that the Bandwidth of MSK is nearly the same as that of QPSK, but with lower power.

When analyzing the statistical properties of the OFDM composite time signal we deduced the following: As shown in figure (4); The probability density function of the absolute OFDM signal agrees with the Rayleigh distribution for all used modulation techniques.
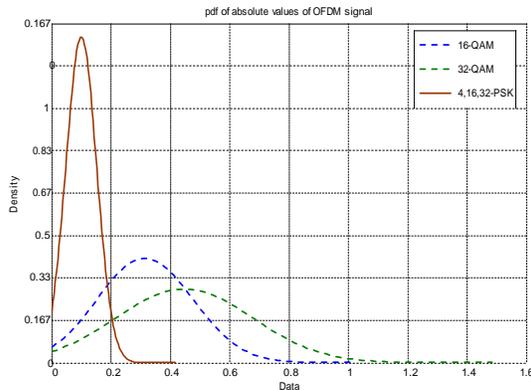

Fig.(4) PDF of OFDM with different modulation

And in agreement with figure (3), as M-increases in M-QAM the dynamic range increases, and in the case of MSK the dynamic range is less than that of QPSK.

Table I. Shows PAR, standard deviation δ and the dynamic range of the OFDM signal with the above mentioned modulation techniques

TABLE I.
PAR AND STD OF OFDM SIGNAL

MOD	PAR (dB)	STD (δ)	Absolute signal range
MSK	8.3902	0.0427	0.0642
QPSK	8.7001	0.0562	0.0912 ≡ MSK + 3 dB
8QAM	9.0063	0.0965	0.1470 ≡ QPSK +4 dB
16QAM	9.2989	0.1247	0.2213 ≡ 8QAM +3.5 dB
32QAM	10.082	0.1777	0.2780 ≡ 16QAM +2 dB

It is again in agreement with the above results. It is clear that although the PAR reduction due to the use of MSK instead of QPSK is slightly small, the true gain is the reduction in the dynamic range by **3 dB**, which enables us to use a low linearity and high efficiency power amplifiers. In addition a new indicator arises in the table which is the standard deviation δ, it is obvious from the table that MSK has the lowest δ while for M-QAM, as M increase δ increases also, since we can deal with OFDM signal as a narrow band Gaussian noise with a mean of zero and variance of δ^2 , then 68% of amplitude values ranges in [-δ,δ] and 99.994% of amplitude values

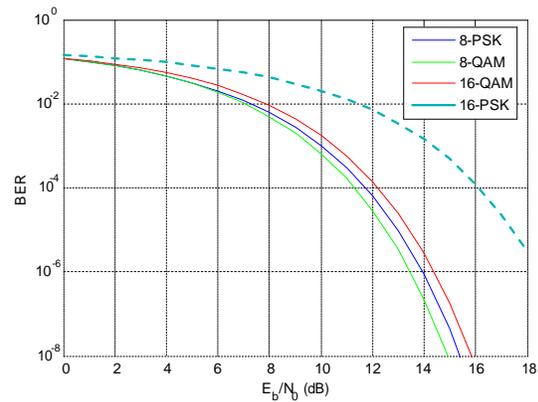ranges in [-4δ, 4δ], this can be a good indicator for clipping efficiency.
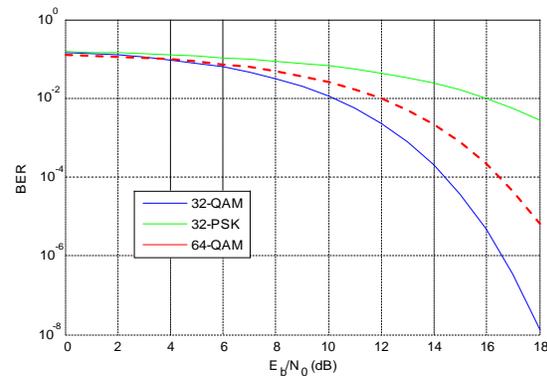

Fig.(5-a)BER of OFDM using 8,16 -PSK&QAM

Fig.(5-b) BER of OFDM using 32,64 -PSK&QAM

Regarding to the BER performance of the used modulation techniques, it is clear from figure (5-a) and (5-b) that, for M-PSK and M-QAM, the BER increases ad M increase while for the same M the BER of M-PSK is larger than that of M-QAM for the same E_b/N_o.

4. THE EFFECT OF POWER AMPLIFIERS

Due to the nature of OFDM signal generation, as we mentioned above, signals have large crest factor CF, which set a demand for linearity. PAs are divided into classes according to the biasing used, the most common used amplifier classes are A, B and C.

Class A amplifiers are the most linear among all classes, but with the lowest efficiency (50% at maximum). This poor efficiency causes high power consumption, which leads to worming in physical devices, and short battery life for mobile users. To achieve a better efficiency class B or C amplifiers are used but this costs a worse linearity. The high demands on linearity make class B or C unsuitable for OFDM systems. In practical applications of OFDM using QAM modulators, the amplifier is a compromise between class A and B, it is called class AB amplifier. NLA Model used for OFDM system is Rapp's SSPA with characteristic [7]:

$$v_{out} = \frac{v_{in}}{(1 + (|v_{in}|/v_{sat})^{2p})^{\frac{1}{2p}}}$$

Where v_{out} and v_{in} are complex i/p & o/p of the HPA , v_{sat} is the output saturation level (we use $v_{sat} = 0.2, 0.5, 0.8$ and 1) and P is "knee factor" that controls the smoothness of characteristic curve (we use P=2)
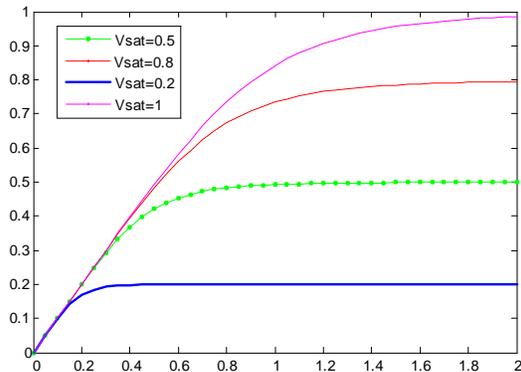

Figure (6) NLA input/output characteristic

Figure (6) shows the Input output characteristics of the amplifier with the above specified parameters.

When applying this amplifier using $v_{sat} = 0.1$ on the OFDM signal we have noticed the advantage of MSK over QPSK, as shown below in figure (7-a).

Also we have applied the amplifier on the OFDM signal using M-QAM with M=4, 8 and 16. Using $v_{sat} = 0.2$, we noticed that as M increases the distortion due to NLA increases and so the BER as shown in figure (7-b).
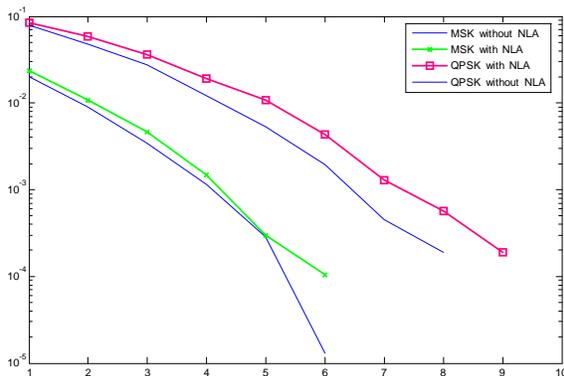

Figure (7-a) effect of NLA on MSK & QPSK

As regarding to our previous results, it can be noticed that, the MSK modulation gives us the lowest PAR when used in OFDM besides its main advantage, that it ignores any fading introduced amplitude fluctuation present in the received signal,

and hence facilitates the utilization of power efficient class C amplifier [6].
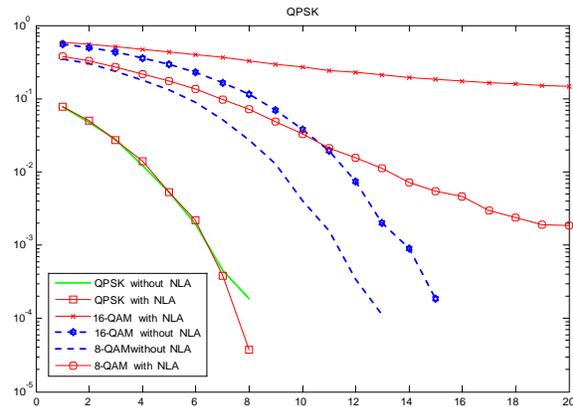

Figure (7-b) effect of NLA on M-QAM

5. CONCLUSIONS

For M-PSK, as M increases no effect has been occurred to the dynamic range and the PAR remains nearly the same, while for M-QAM, as M increases the dynamic range increase and so the PAR, but both of them agree in the spectral efficiency increase as M increase.

For M-PSK and M-QAM, the BER increases as M increase while for the same M the BER of M-PSK is larger than that of M-QAM for the same E_b/N_o.

Although the PAR reduction due to the use of MSK instead of QPSK is slightly small, the true gain is the reduction in the dynamic range by **3 dB**, which enables us to use a low linearity and high efficiency power amplifiers like class B or C.

MSK modulation gives us the lowest Crest Factor when used in OFDM besides its main advantage, that it ignores any fading introduced amplitude fluctuation present in the received signal, and hence facilitates the utilization of power efficient class C amplifier.

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