



# A Short Course on Synchronous Machines and Synchronous Condensers

**G. Heydt   S. Kalsi   E. Kyriakides**

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American Superconductor**

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Session	Time	Topics	Instructor
Introductions	8:30 – 8:40		Bradshaw
1 Fundamentals of synchronous machines	8:40 – 9:50	<ul style="list-style-type: none"> <li>• Energy conversion</li> <li>• Synchronous machine construction</li> <li>• Energy transfer in a synchronous machine</li> <li>• Motor and generator action</li> <li>• Phasor diagram for synchronous machines</li> <li>• Losses</li> <li>• Superconducting designs</li> <li>• Power factor and torque angle</li> <li>• Example of calculations</li> <li>• Transients and damper windings</li> <li>• Saturation and the magnetization curve</li> </ul>	Heydt
B R E A K	9:50 – 10:00		

2 Synchronous condensers	10:00 10:30	–	<ul style="list-style-type: none"> <li>• What is a synchronous condenser?</li> <li>• Applications of synchronous condensers</li> <li>• Analysis</li> </ul>	Kalsi
3 Superconducting synchronous condensers	10:30 12:00	–	<ul style="list-style-type: none"> <li>• Superconductivity</li> <li>• The superconducting synchronous condenser (SSC)</li> <li>• Performance benefits of SSC in a grid</li> </ul>	Kalsi
LUNCH	12:00 – 1:30			

4 Synchronous machine models	1:30 – 2:30	<ul style="list-style-type: none"><li>• Park's transformation</li><li>• Transient and subtransient reactances, formulas for calculation</li><li>• Machine transients</li></ul>	Heydt
5 State estimation applied to synchronous generators	2:30 – 3:30	<ul style="list-style-type: none"><li>• Basics of state estimation</li><li>• application to synchronous generators</li><li>• demonstration of software to identify synchronous generator parameters</li></ul>	Kyriakides

B R E A K	3:30 – 3:40		
6 Machine instrumentation	3:40 – 4:30	<ul style="list-style-type: none"><li>• DFRs</li><li>• Calculation of torque angle</li><li>• Usual machine instrumentation</li></ul>	Heydt, Kyriakides, and Kalsi
Question and answer session	4:30 – 5:00		All participants

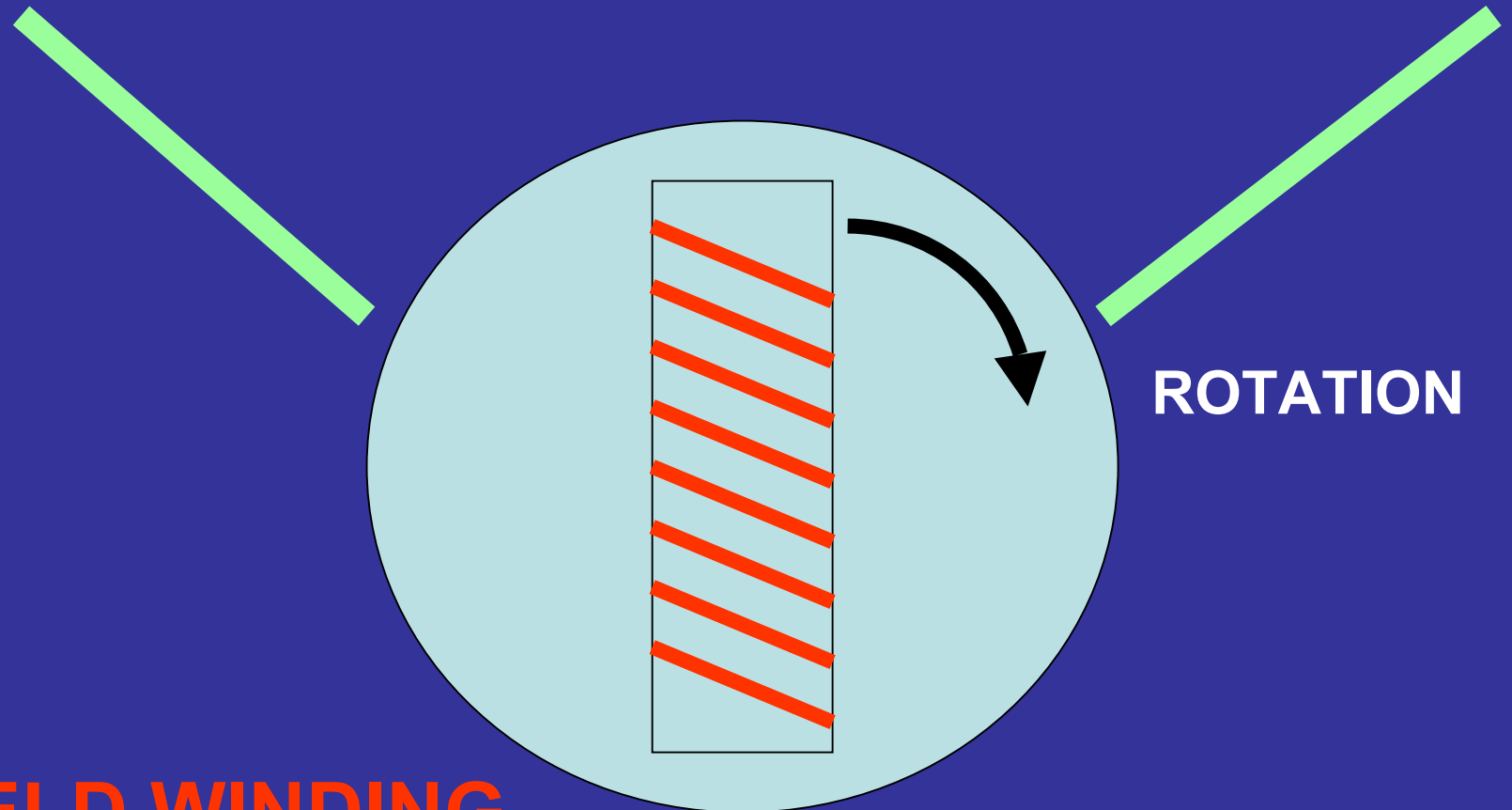
# SESSION 1

## Fundamentals of synchronous machines

# Synchronous Machines

- Example of a rotating electric machine
- DC field winding on the rotor, AC armature winding on the stator
- May function as a generator (**MECHANICAL** → **ELECTRICAL**) or a motor (**ELECTRICAL** → **MECHANICAL**)
- Origin of name: syn = equal, chronos = time

# Synchronous Machines



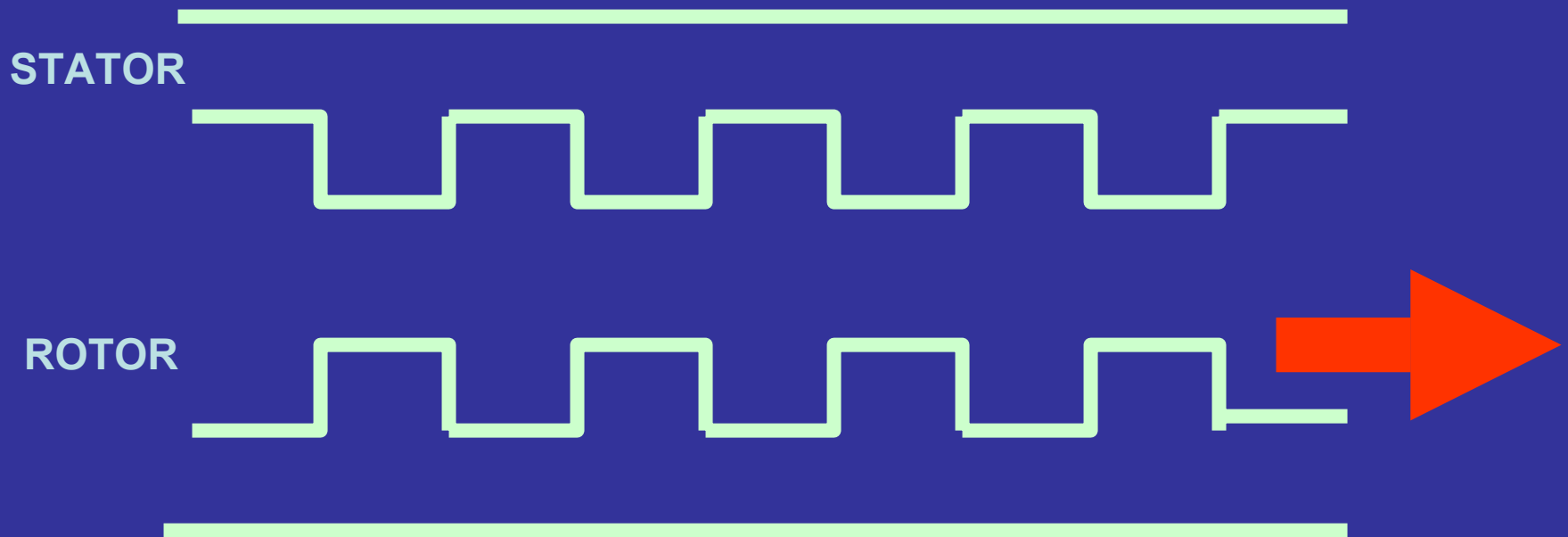
ROTATION

- **FIELD WINDING**
- **ARMATURE WINDING**



# Synchronous Machines

The concept of air gap flux



# Synchronous Machines

- The inductance of the stator winding depends on the rotor position
- Energy is stored in the inductance
- As the rotor moves, there is a change in the energy stored
- Either energy is extracted from the magnetic field (and becomes mechanical energy – that is, it is a motor)
- Or energy is stored in the magnetic field and eventually flows into the electrical circuit that powers the stator – this is a generator

# Synchronous Machines

The basic relationships are

$$\text{POWER} = (\text{TORQUE}) (\text{SPEED})$$

$$\text{ENERGY} = (1/2) (L I^2)$$

$$\text{POWER} = d(\text{ENERGY}) / d(\text{TIME})$$

# Synchronous Machines

Consider the case that the rotor (field) is energized by DC and the stator is energized by AC of frequency  $f$  hertz.

There will be average torque produced only when the machine rotates at the same speed as the rotating magnetic field produced by the stator.

$$\text{RPM} = ( 120 f ) / (\text{Poles})$$

Example:  $f = 60$  Hz, two poles, RPM = 3600 rev/min

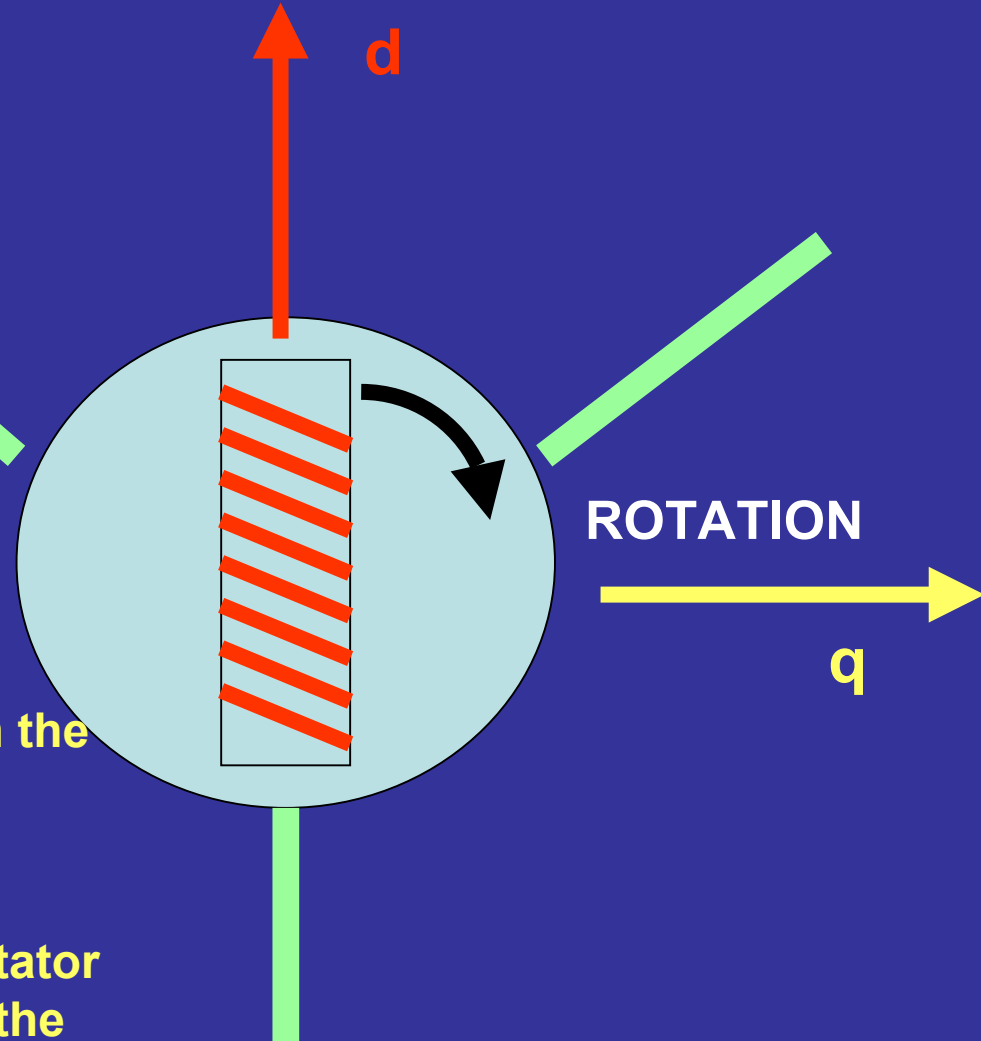
# Synchronous Machines

The axis of the field winding in the direction of the DC field is called the rotor direct axis or the d-axis. 90 degrees later than the d-axis is the quadrature axis (q-axis).

The basic expression for the voltage in the stator (armature) is

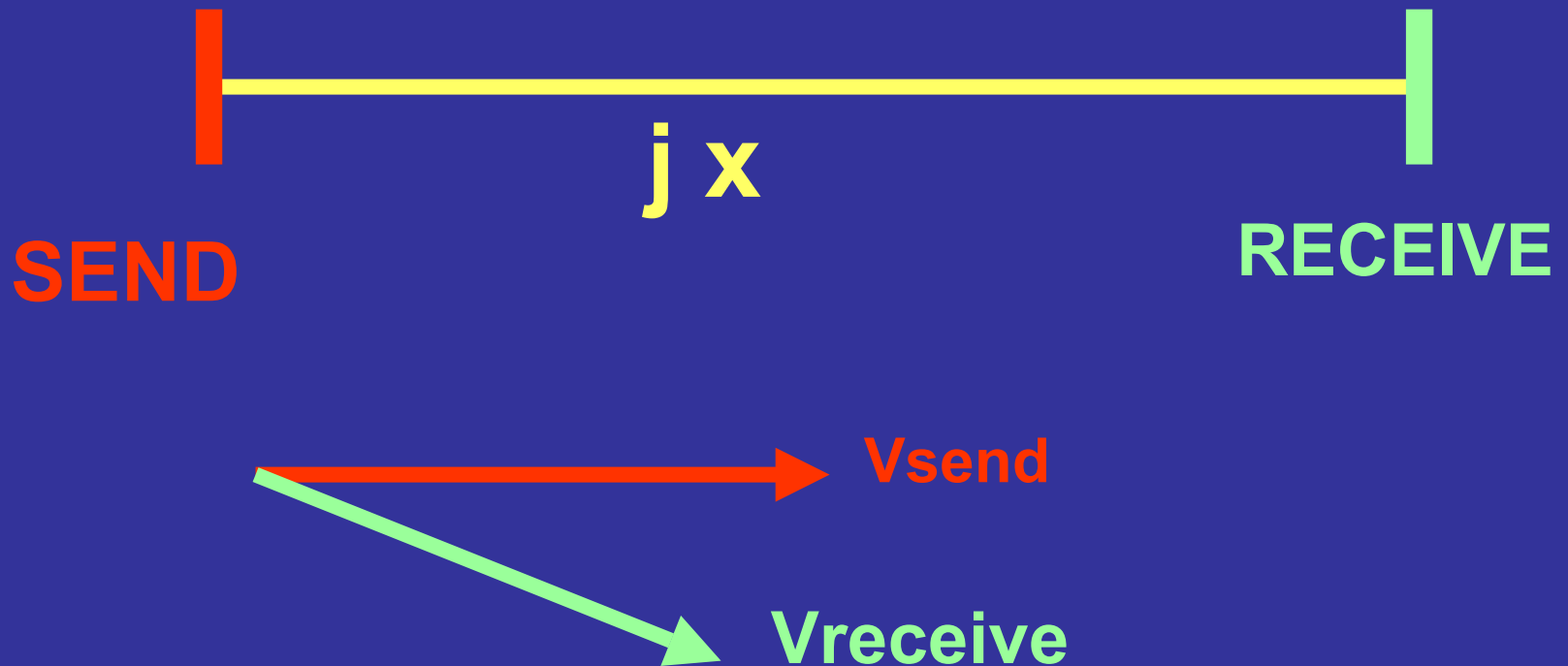
$$v = r i + d\lambda/dt$$

Where  $v$  is the stator voltage,  $r$  is the stator resistance, and  $\lambda$  is the flux linkage to the field produced by the field winding

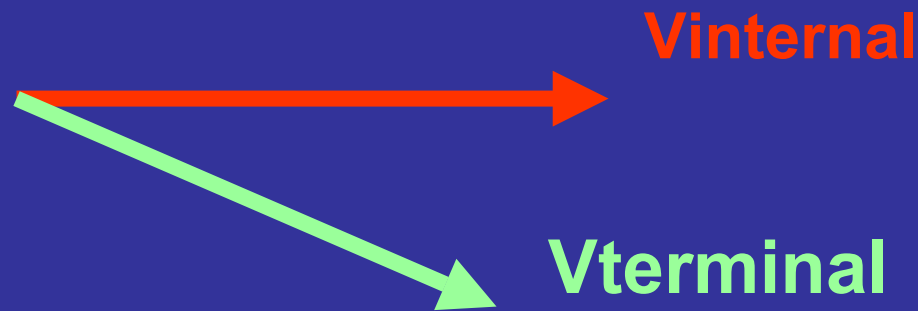


# Synchronous Machines

## Basic AC power flow



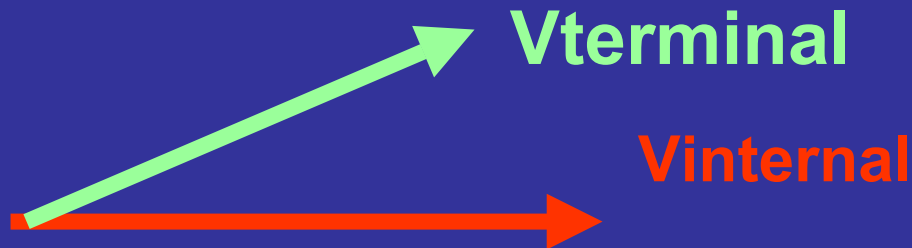
# Synchronous Machines



The internal voltage, often labeled  $E$ , is produced by the field interacting with the stator winding, and this is the open circuit voltage

**GENERATOR ACTION –  
POWER FLOWS FROM  
MACHINE TO EXTERNAL  
CIRCUIT,  $E$  LEADS  $V_t$**

# Synchronous Machines

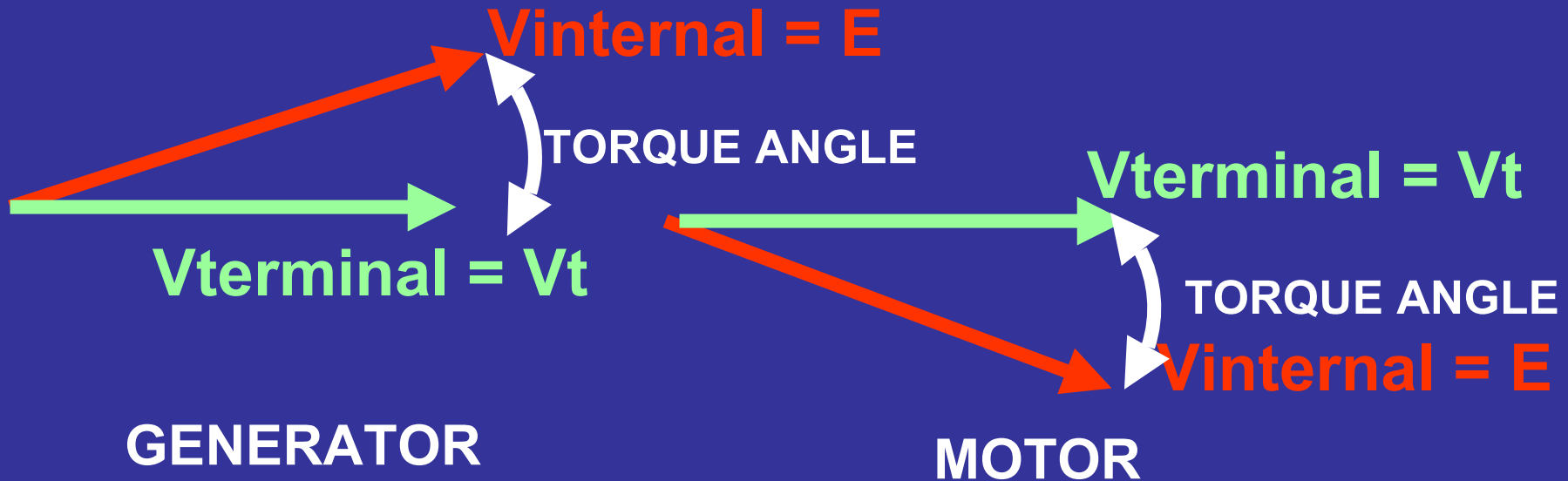


The internal voltage, often labeled  $E$ , is produced by the field interacting with the stator winding, and this is the open circuit voltage

**MOTOR ACTION – POWER FLOWS FROM EXTERNAL CIRCUIT INTO THE MACHINE,  $E$  LAGS  $V_t$**



# Synchronous Machines



The internal voltage, often labeled  $E$ , is produced by the field interacting with the stator winding, and this is the open circuit voltage

# Synchronous Machines

Active power will flow when there is a phase difference between  $V_{send}$  and  $V_{receive}$ .

This is because when there is a phase difference, there will be a voltage difference across the reactance  $jx$ , and therefore there will be a current flowing in  $jx$ . After some arithmetic

$$P_{sent} = [|V_{send}|] [|V_{receive}|] \sin(\text{torque angle}) / x$$

# Synchronous Machines

## Example

A synchronous generator stator reactance is 190 ohms, and the internal voltage (open circuit) generated is 35 kV line to line. The machine is connected to a three phase bus whose voltage magnitude is 35 kV line-line. Find the maximum possible output power of this synchronous generator

# Synchronous Machines

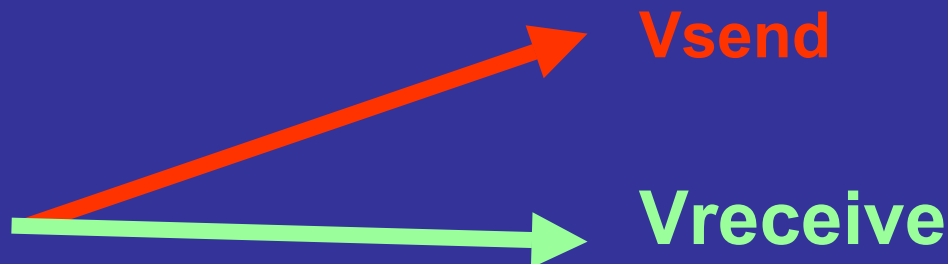
## Example

Work on a per phase basis

35 kV line-line = 20.2 kV l-n

Max P occurs when torque angle is 90 degrees

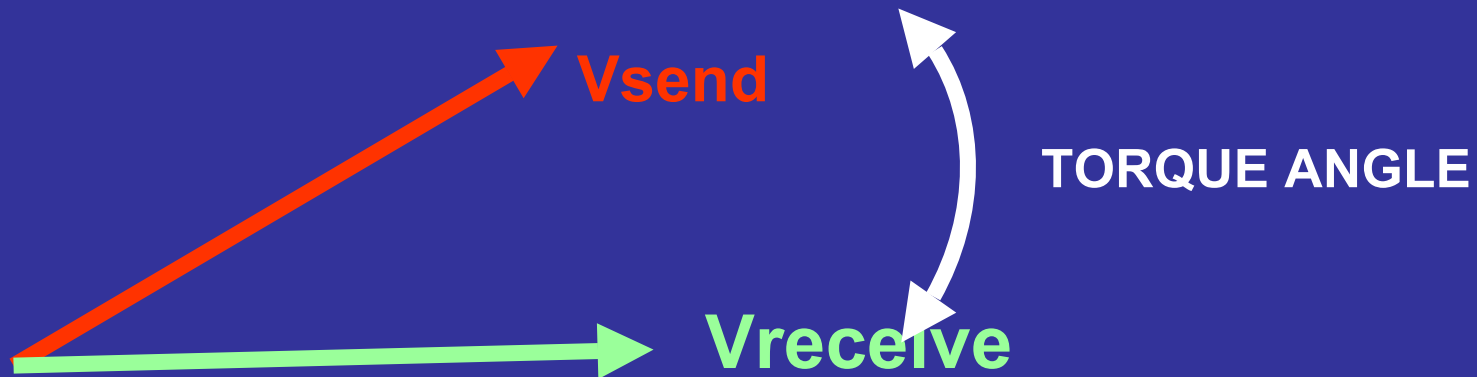
$$P = (20.2\text{K})(20.2\text{K})(\sin(90))/190$$
$$= 2.1 \text{ MW per phase} = 6.3 \text{ MW three phase}$$



# Synchronous Machines

## Example

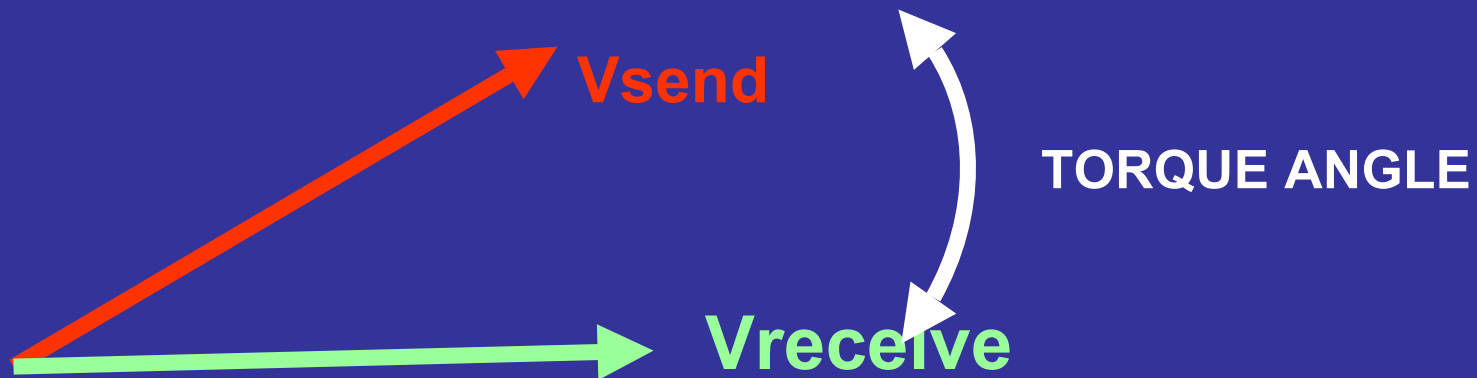
If the phase angle is limited to 45 degrees, find the generator power output



# Synchronous Machines

## Example

$$P = 6.3 \sin(45) = 4.6 \text{ MW}$$



# Synchronous Machines

## Losses

**Rotor:** resistance; iron parts moving in a magnetic field causing currents to be generated in the rotor body; resistance of connections to the rotor (slip rings)

**Stator:** resistance; magnetic losses (e.g., hysteresis)

**Mechanical:** windage; friction at bearings, friction at slip rings

**Stray load losses:** due to nonuniform current distribution

$$\begin{aligned}\text{EFFICIENCY} &= \text{OUTPUT} / \text{INPUT} \\ &= 1 - (\text{LOSSES}) / \text{INPUT}\end{aligned}$$

# Synchronous Machines

## Losses

Generally, larger machines have the higher efficiencies because some losses do not increase with machine size. For example, many generators in the 5 MW class and above have efficiencies greater than 97%

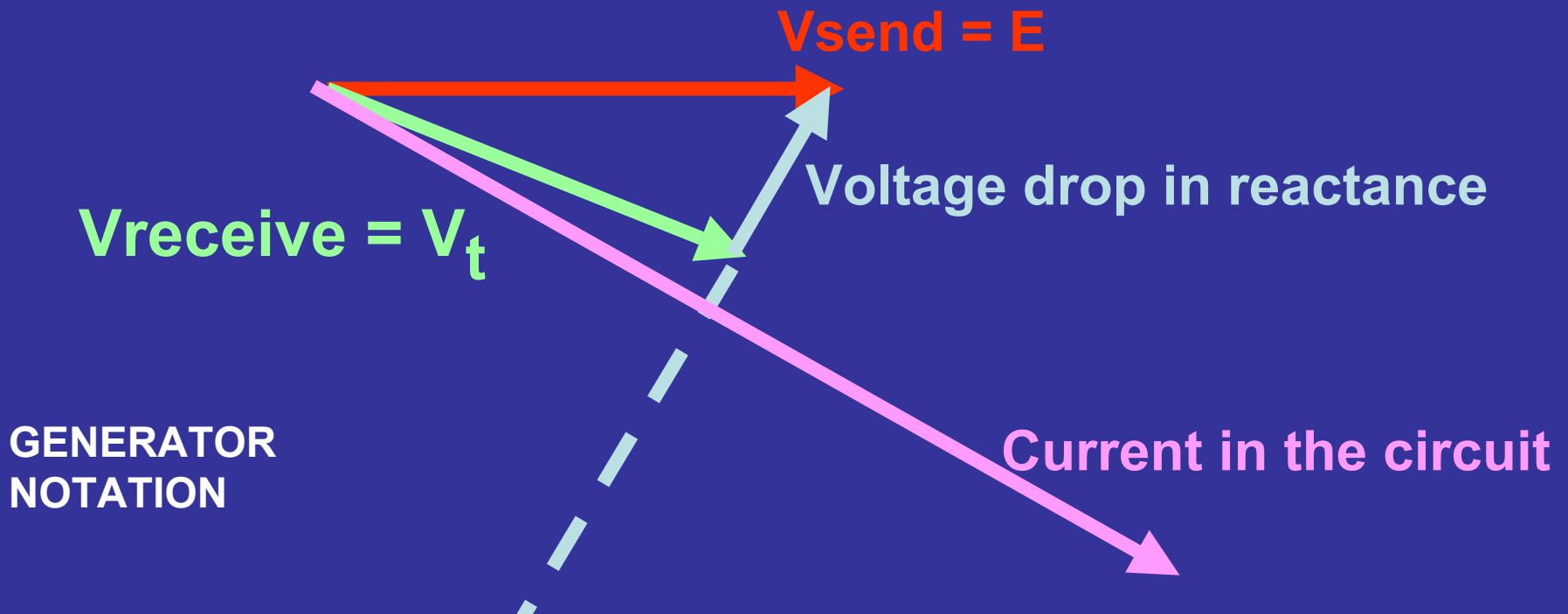
But 3% of 5 MW is still 150 kW – and for large units – e.g. 600 MW, 3% of 600 MW is 18 MW!

- Cooling
- Damping

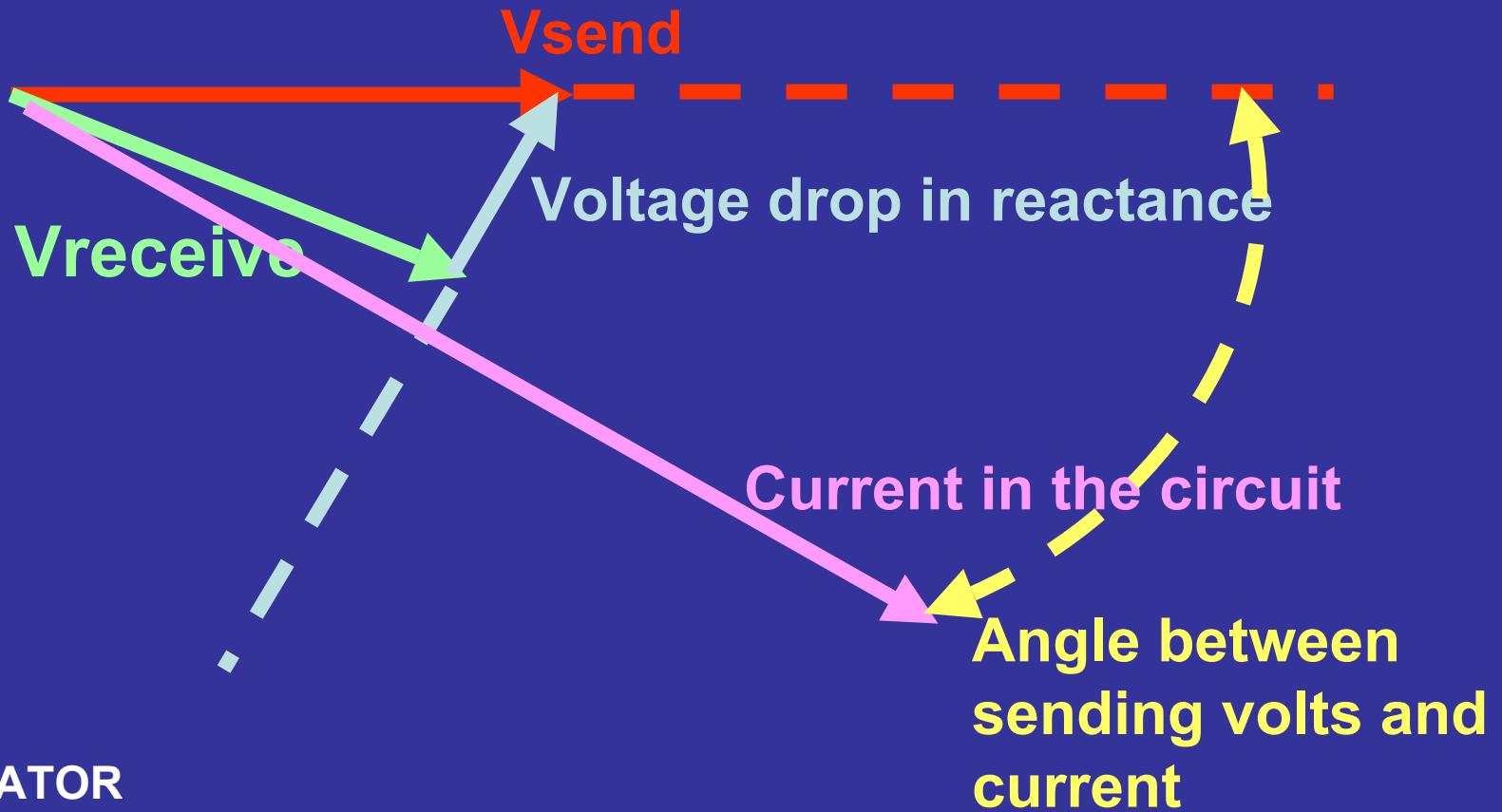


# Power factor

Power factor is the cosine between voltage and current in a sinusoidal AC circuit.

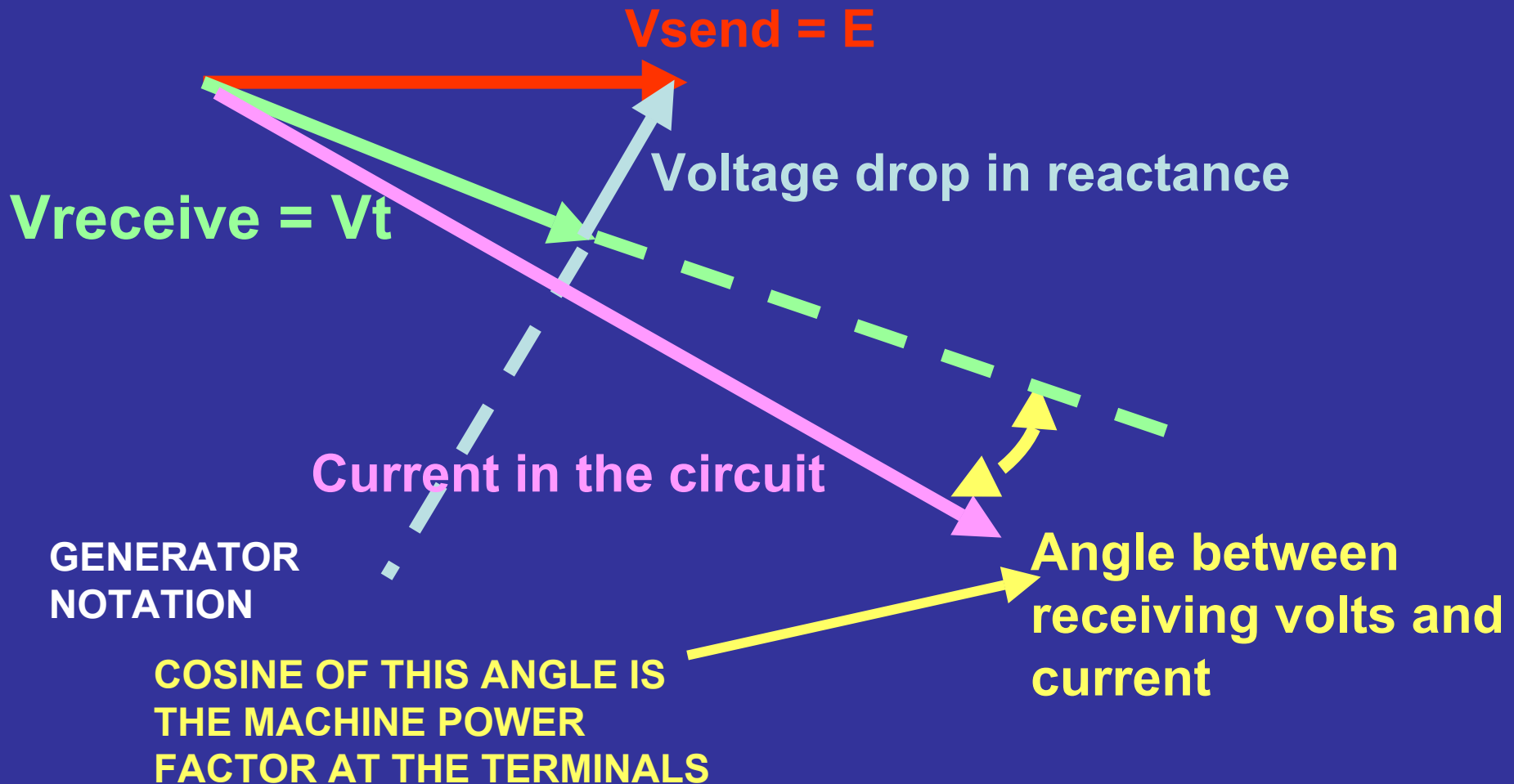


# Power factor

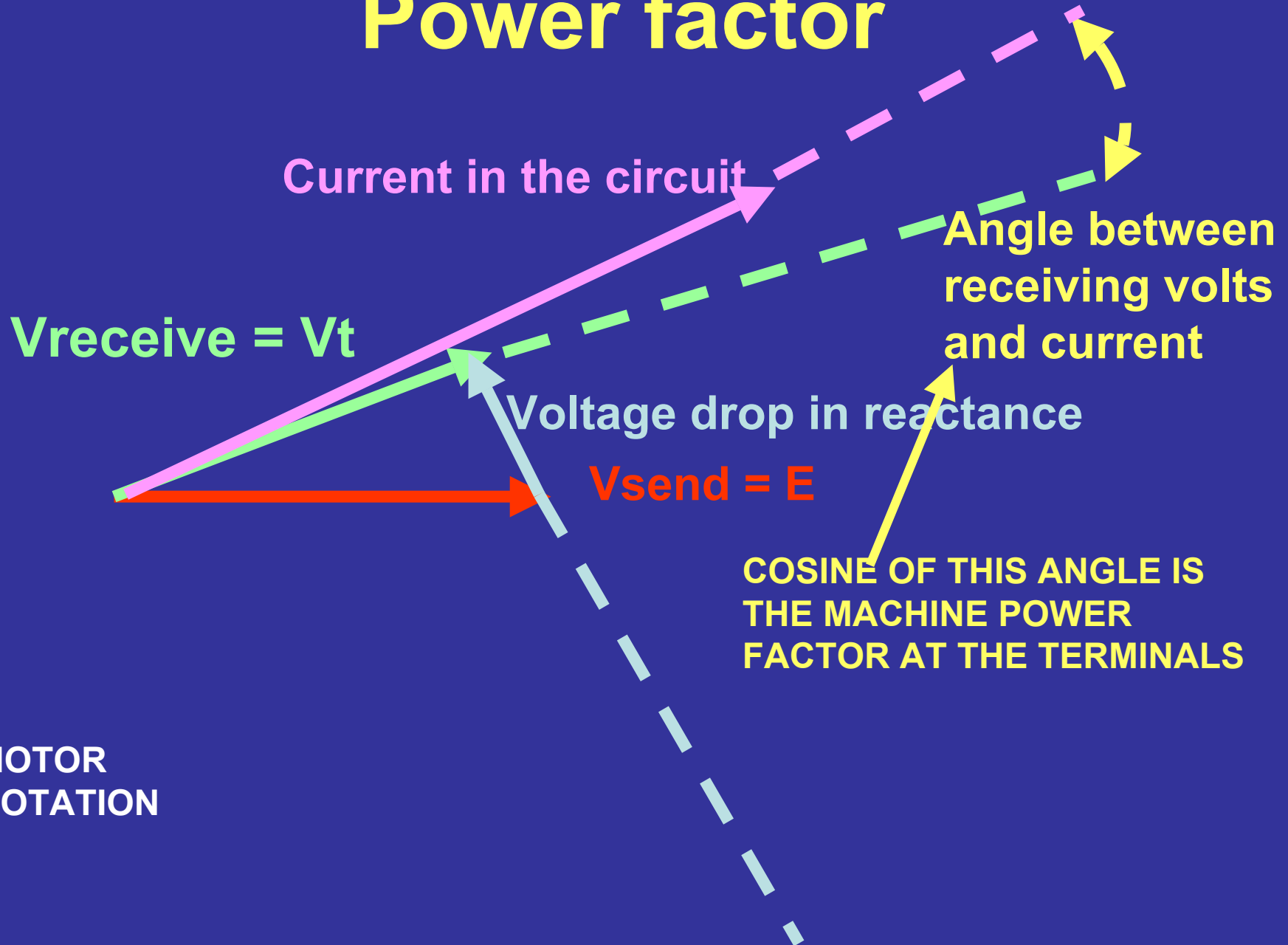


GENERATOR  
NOTATION

# Power factor



# Power factor



# Power factor

**Note that the power factor angle is controllable by the generated voltage  $E$  and hence by the DC field excitation.**

# Basic expressions

MOTOR  $V_t = E + jI_a X$

GENERATOR  $V_t = E - jI_a X$

# Power factor

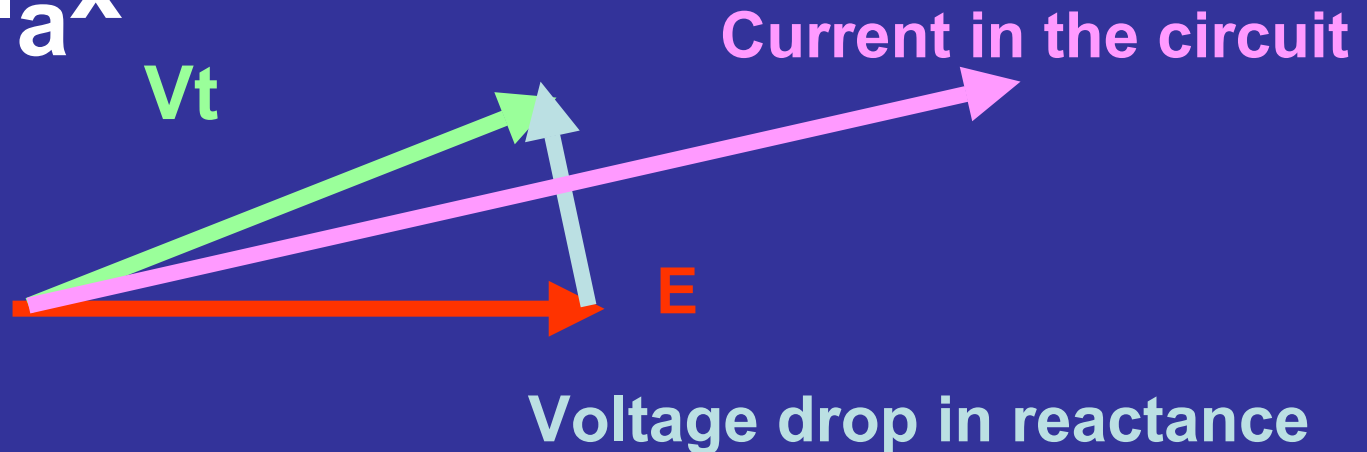
**Consider now a machine that:**

- 1. Is operated at successively smaller and smaller torque angle**
- 2. Greater and greater field excitation**

# Successively smaller and smaller torque angle

The machine torque angle is made smaller and smaller by reducing the electrical load (P)

$$V_t = E + jI_a X$$



MOTOR  
NOTATION

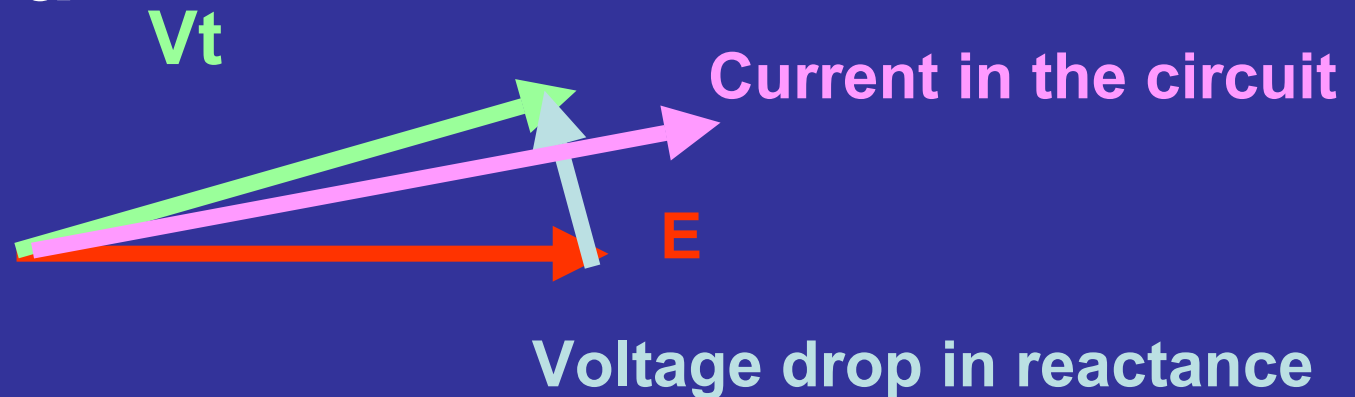
Voltage drop in reactance



# Successively smaller and smaller torque angle

The machine torque angle is made smaller and smaller by reducing the electrical load (P)

$$V_t = E + jI_a X$$

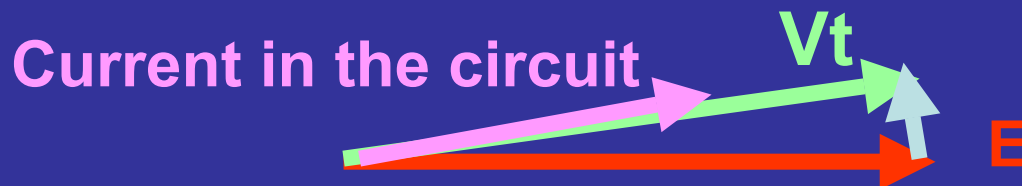


MOTOR  
NOTATION

# Successively smaller and smaller torque angle

The machine torque angle is made smaller and smaller by reducing the electrical load (P)

$$V_t = E + jI_a X$$



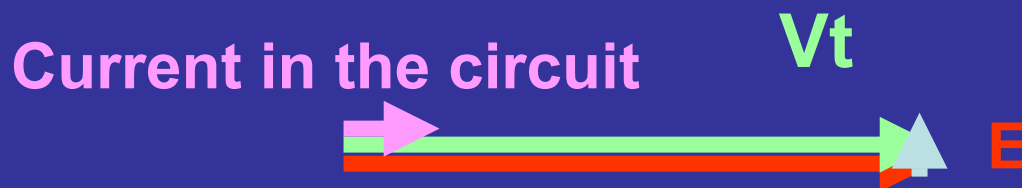
MOTOR  
NOTATION

Voltage drop in reactance

# Successively smaller and smaller torque angle

The machine torque angle is made smaller and smaller by reducing the electrical load (P)

$$V_t = E + jI_a X$$



MOTOR  
NOTATION

Voltage drop in reactance

# Successively greater field excitation

Increasing the field excitation causes  $E$  to increase

$$V_t = E + jI_a X$$

Current  
in the  
circuit



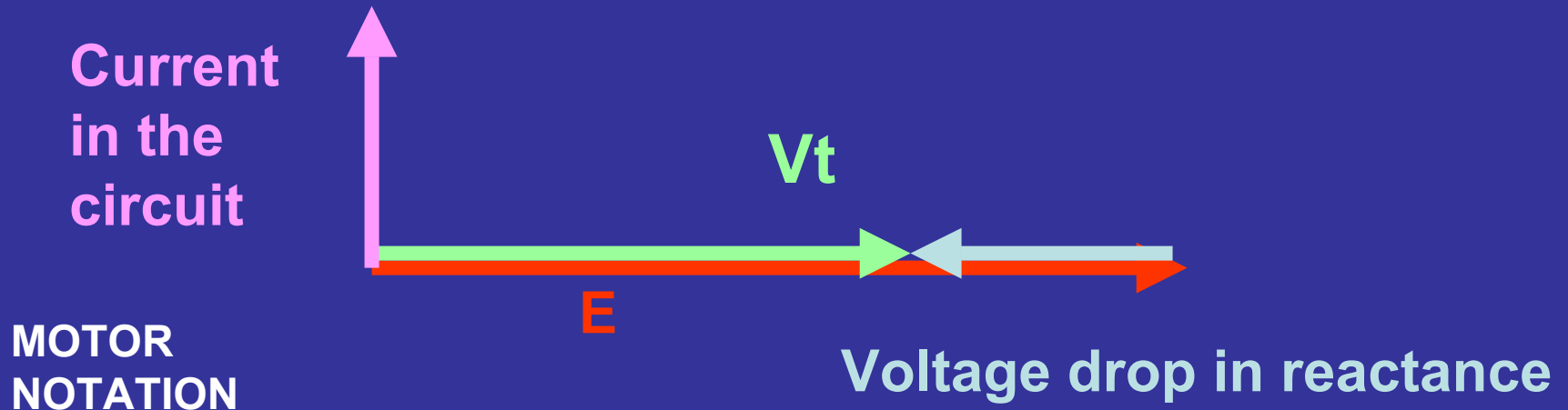
MOTOR  
NOTATION

Voltage drop in reactance

# Successively greater field excitation

Increasing the field excitation causes  $E$  to increase

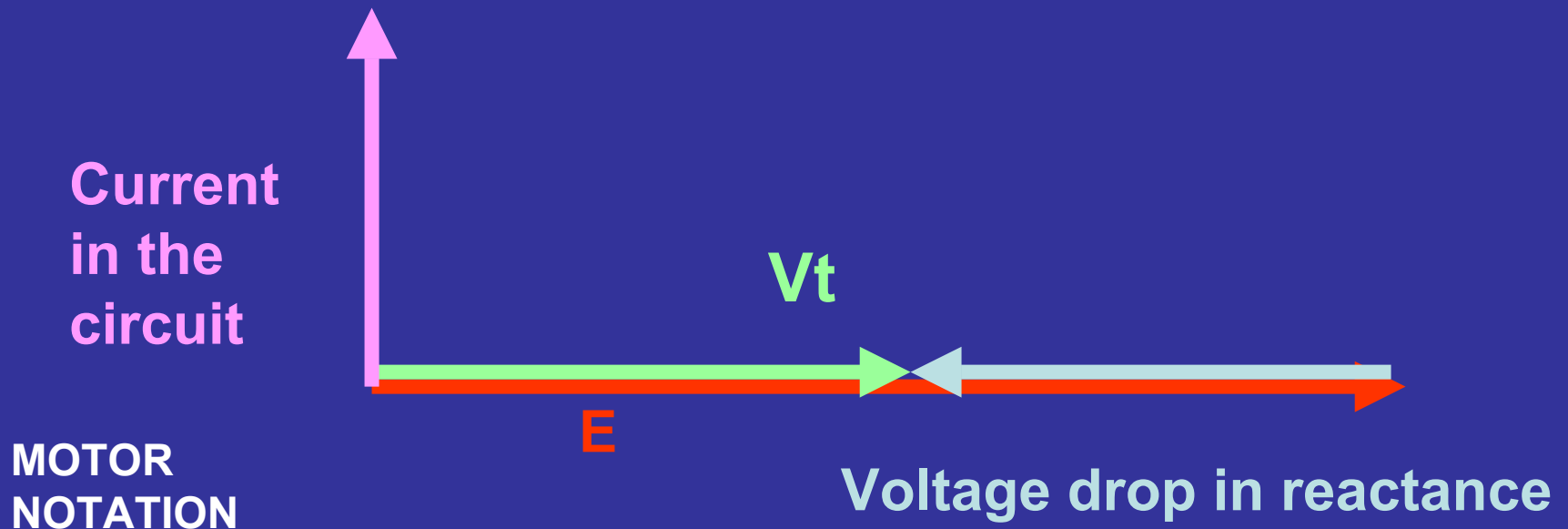
$$V_t = E + jI_a X$$



# Successively greater field excitation

Increasing the field excitation causes E to increase

$$V_t = E + jI_a X$$

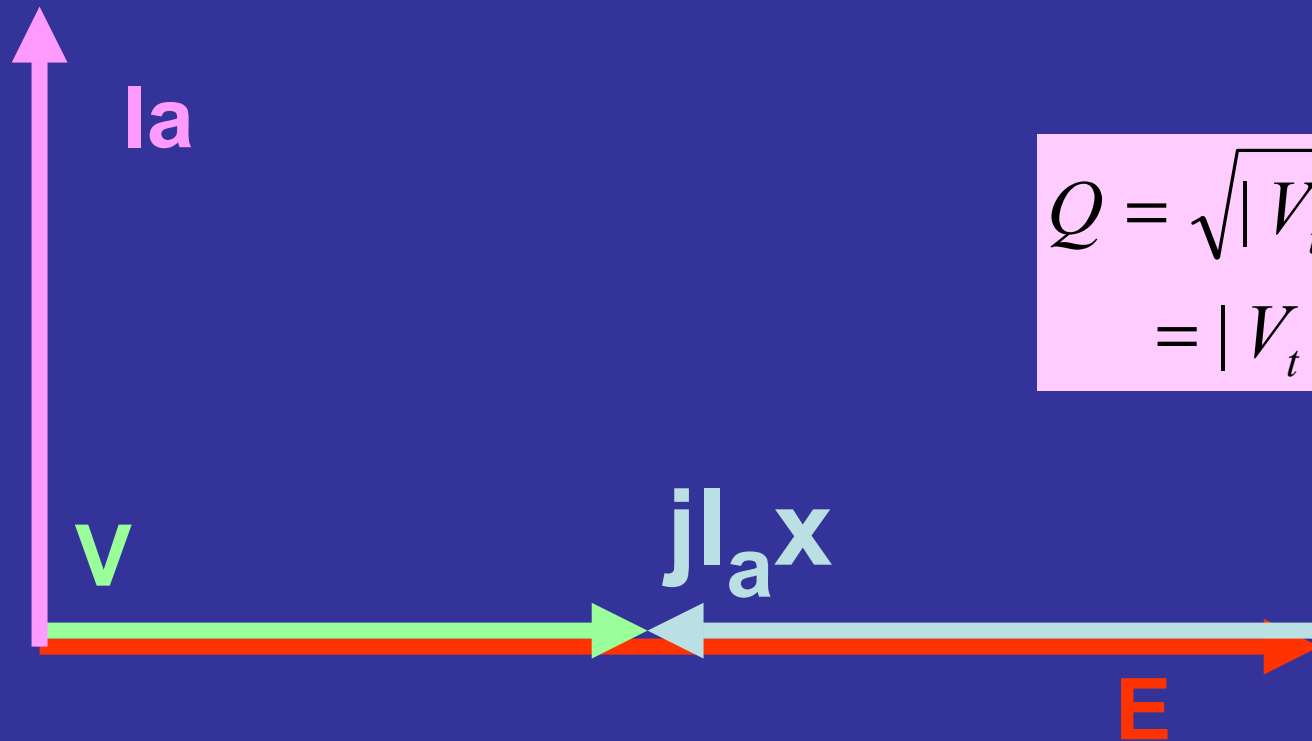


The foregoing indicates that as the machine (1) approaches zero power operation – the borderline between generator and motor operation, the active power to/from the machine goes to zero and (2) as the machine becomes overexcited, the power factor becomes  $\cos(90) = 0$ .

As the field excitation increases,  $|E|$  increases, and the machine current becomes higher – but the power factor is still zero. And  $I$  leads  $V_t$ . In theory, there is no active power transferred, but a high and controllable level of  $Q$ .

This mode of operation is called a synchronous condenser

# Synchronous condenser operation



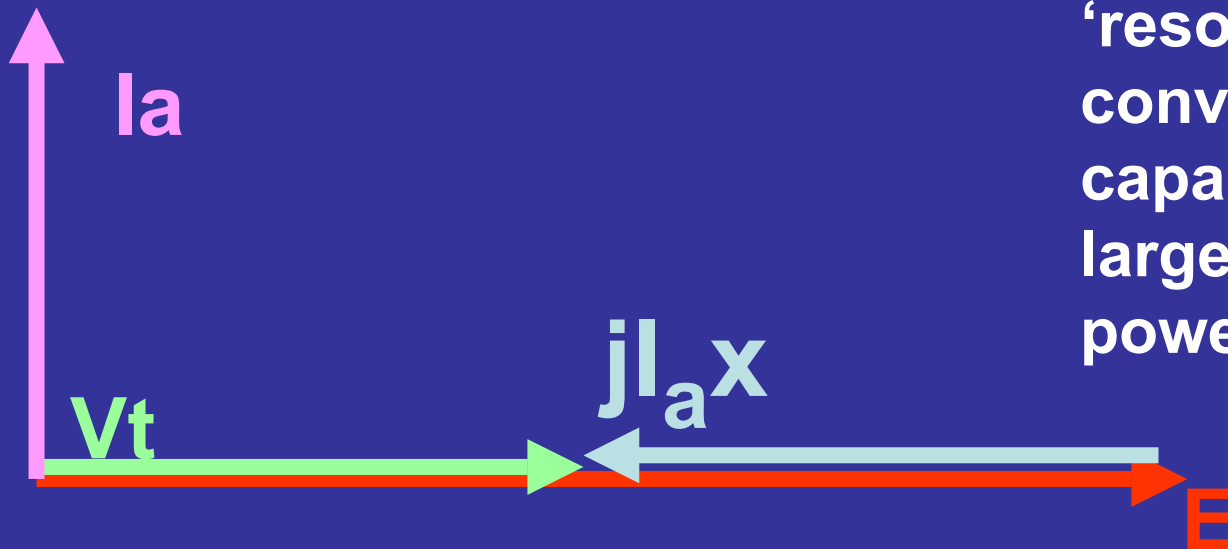
$$Q = \sqrt{|V_t|^2 |I_a|^2 - P^2}$$
$$= |V_t| |I_a|$$



# Synchronous condenser operation

Nearly zero active power flow, nearly zero power factor, nearly perpendicular  $I_a$  and  $V_t$ , current leads terminal voltage acting as a motor, it acts as a capacitor

Power factor correction, reactive power support, voltage support, reactive power can be varied by varying excitation, low loss, no 'resonance problems' of conventional fixed capacitors, potentially a large source of reactive power



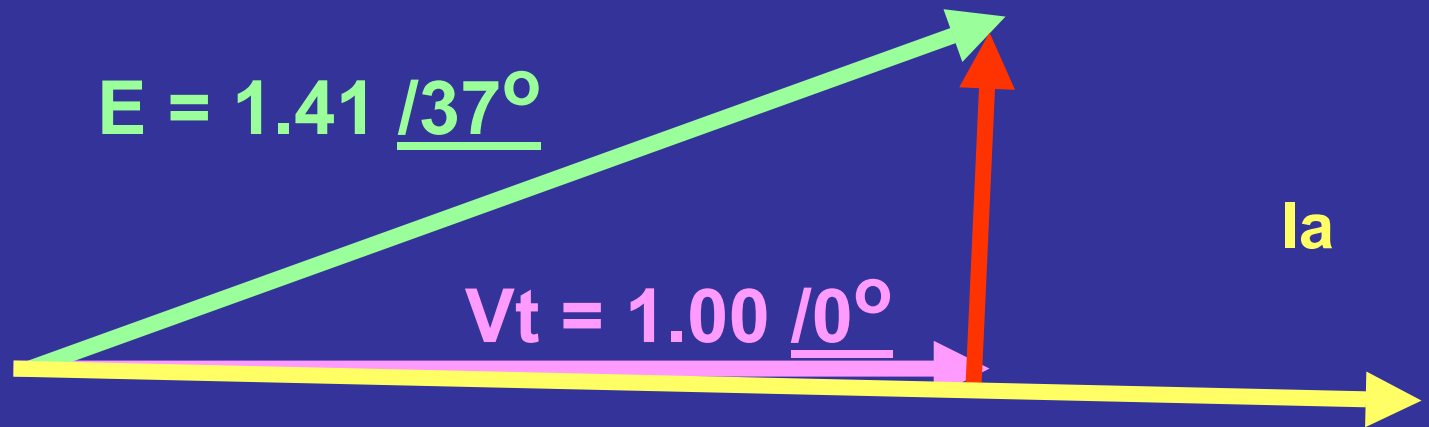
# Examples

**A synchronous generator is rated 100 MVA. The machine is intended to be operated at rated power at torque angle = 37 degrees. The armature resistance is 0.1%, and the reactance is 85%. The terminal voltage is rated 34.5 kV. Find the machine internal percent excitation and terminal pf when the machine operates at 100 MW. Estimate the armature  $I^2R$  losses.**

# Examples

$$\begin{aligned} P &= \frac{|V_t| |E| \sin(\delta)}{x} \\ &= \frac{(1)(|E| \sin(37^\circ))}{0.85} \\ &= 1.41 \end{aligned}$$

# Examples



$$1.00 \angle 0^\circ + jI_a(0.85) = 1.42 \angle 37^\circ$$

$$0.85 |I_a| \angle \phi + 90^\circ = 1.42 \angle 37^\circ - 1.00 \angle 0^\circ$$

$$|I_a| = 1.02 \quad \phi = -8.9^\circ$$

**POWER FACTOR**

$$\cos(-8.9^\circ) = 98.8\% \quad \text{LAGGING}$$

# Examples

**A six pole synchronous generator operates at 60 Hz. Find the speed of operation**

# Examples

$$\text{RPM} = ( 120 f ) / (\text{Poles})$$

$$\text{RPM} = 120 * 60 / 6$$

$$= 1200$$

# Examples

**A 40 MVar synchronous condenser operates on a 34.5 kV bus. The synchronous reactance is 150%. Estimate the field excitation to obtain a 30 to 40 MVar range of reactive power.**

# Examples

$$0.75 \leq Q \leq 1.0$$

$$0.75 \leq |I_a| \leq 1.0$$

$$V_t = E + jI_a x$$

$$1 \angle 0 = |E_f| \angle 0 + jI_a (1.5)$$

$$[1.0 - |E_f| \angle 0 = 1.5 |I_a| \angle (90^\circ + 90^\circ)]$$

$$\text{At } |I_a| = 0.75$$

$$1.0 - |E_f| = -1.5 |0.75|$$

$$|E_f| = 2.125$$



# Examples

$$V_t = E + jI_a x$$

$$1 \angle 0 = |E_f| \angle 0 + jI_a (1.5)$$

$$[1.0 - |E_f| \angle 0 = 1.5 |I_a| \angle (90^\circ + 90^\circ)]$$

$$\text{At } |I_a| = 1.00$$

$$1.0 - |E_f| = -1.5 |1.00|$$

$$|E_f| = 2.50$$

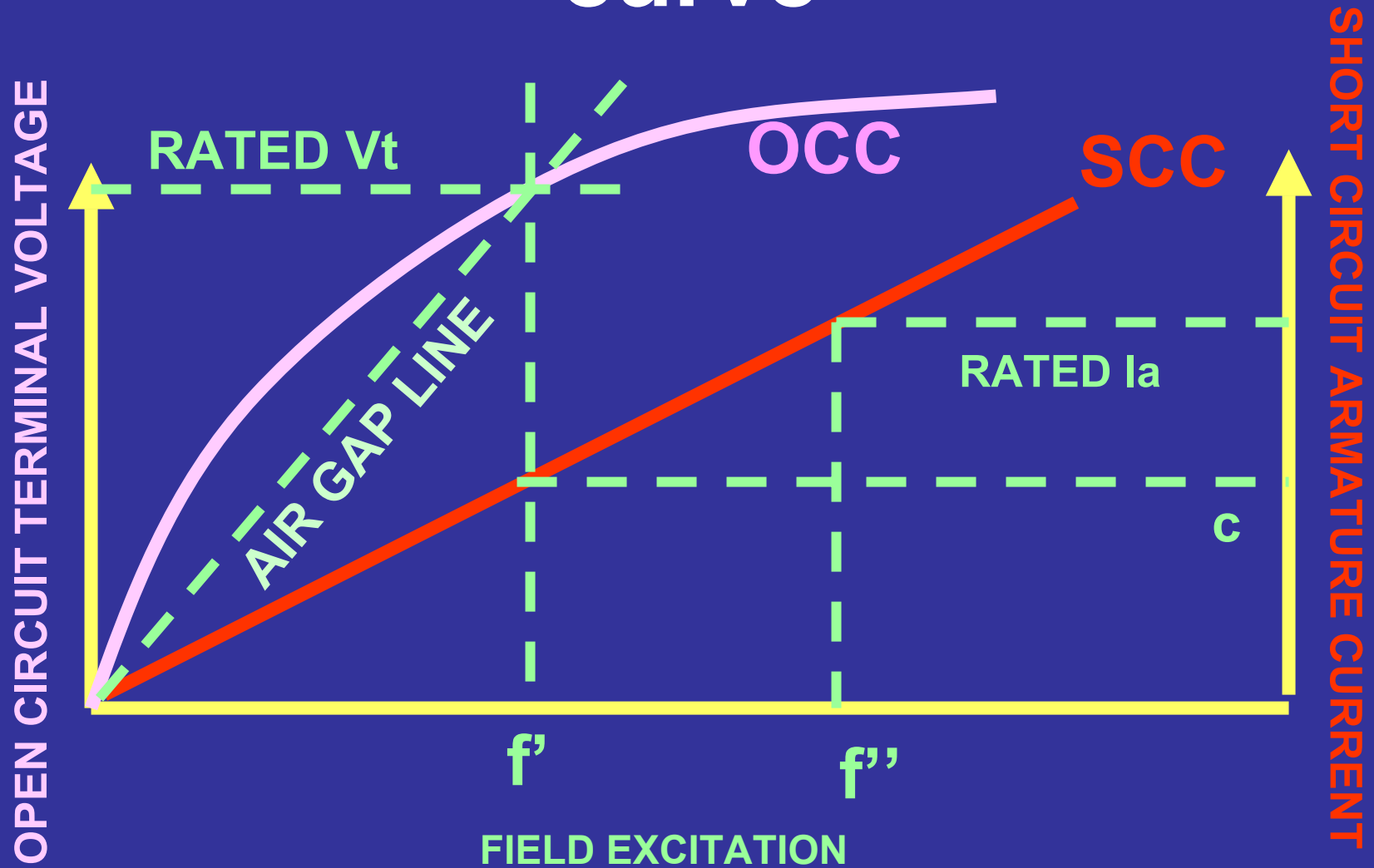
**Therefore the field excitation should be between 213% and 250 %**

# **SESSION 4**

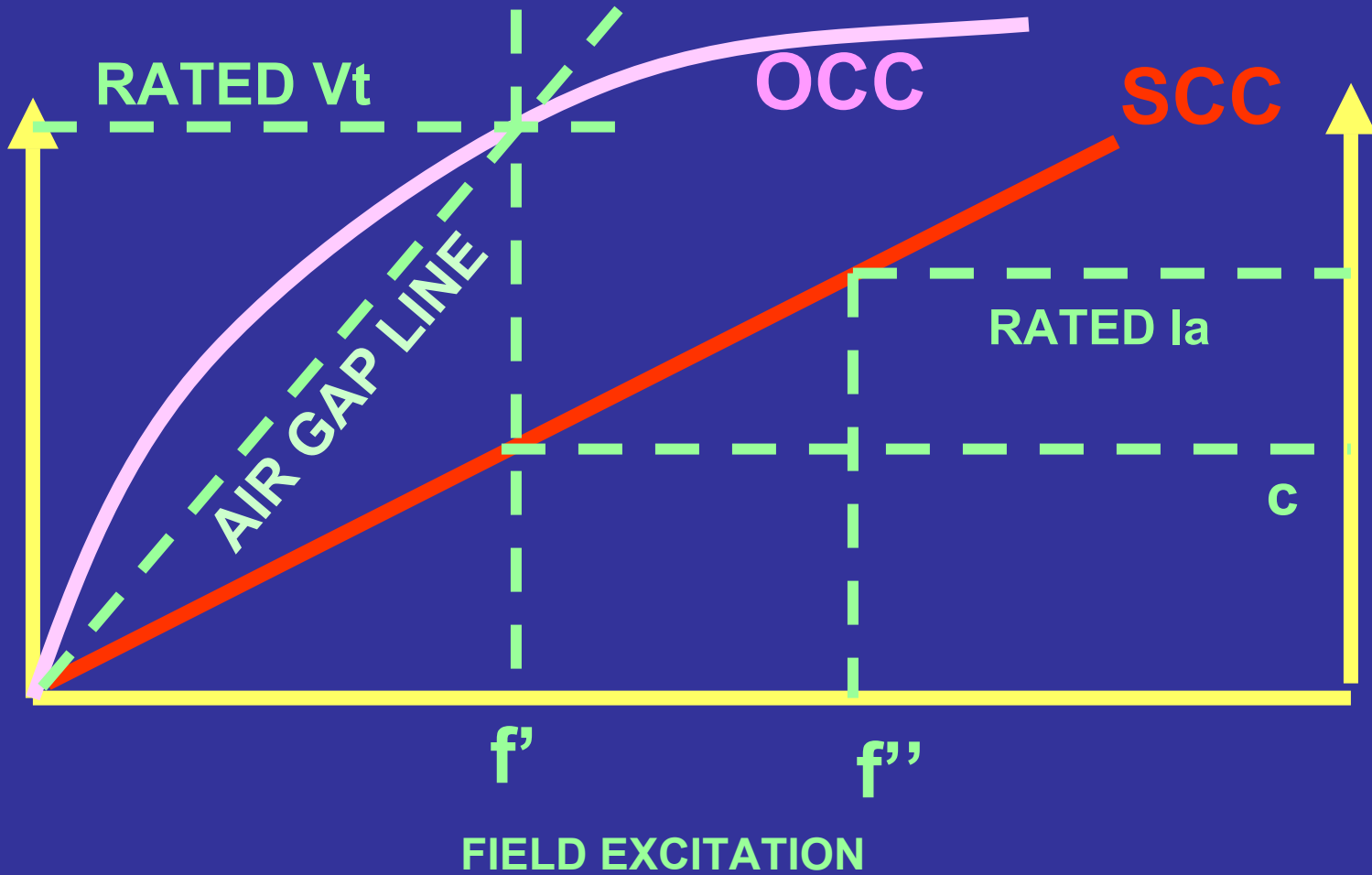
## **Synchronous machine models**

- **Saturation and the magnetization curve**
- **Park's transformation**
- **Transient and subtransient reactances, formulas for calculation**
- **Machine transients**

# Saturation and the magnetization curve



OPEN CIRCUIT TERMINAL VOLTAGE



SHORT CIRCUIT ARMATURE CURRENT

SYNCHRONOUS REACTANCE = SLOPE OF AIR GAP LINE

SHORT CIRCUIT RATIO =  $O_{f'}/O_{f''}$

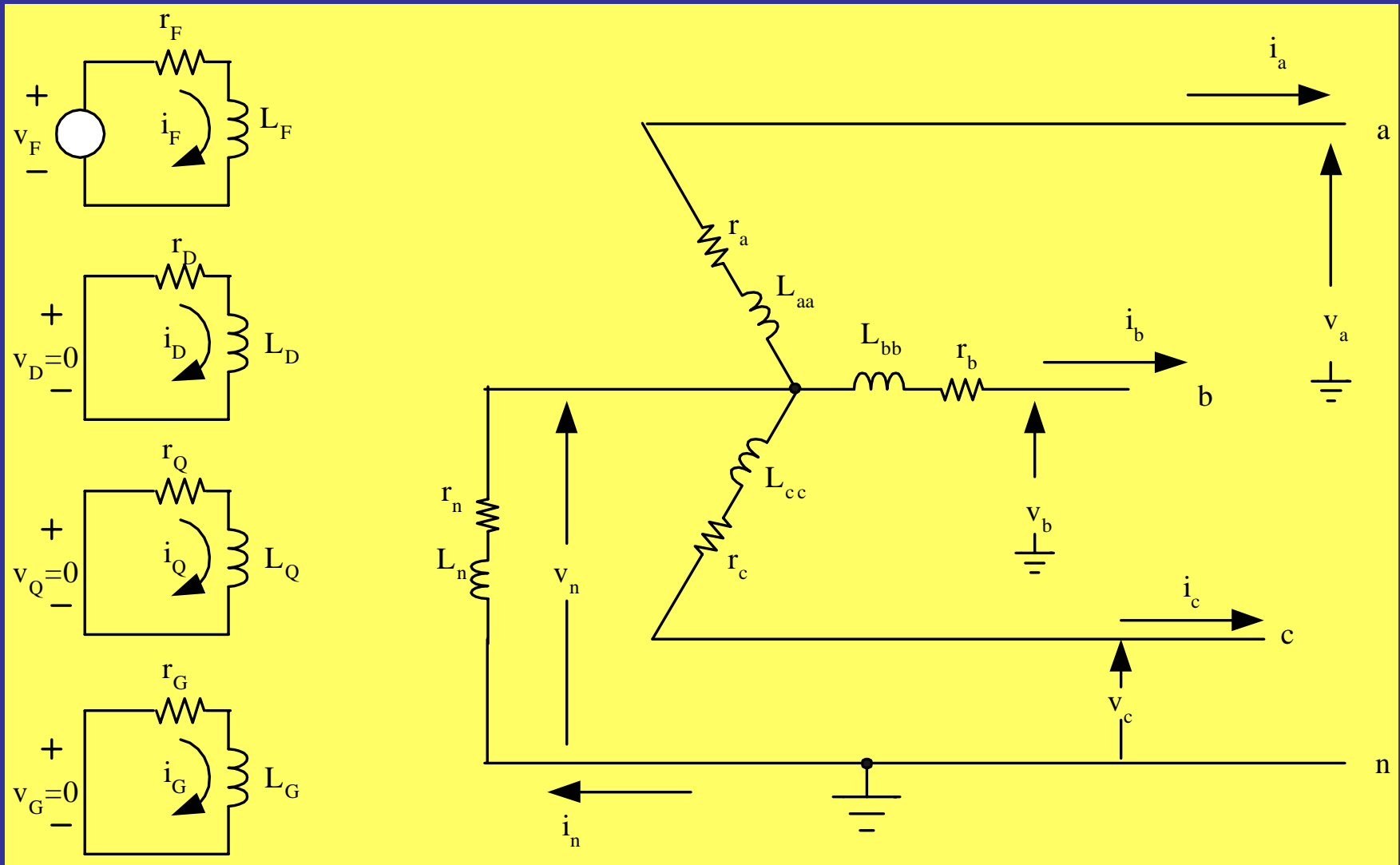
# Saturation and the magnetization curve

- **Saturation occurs because of the alignment of magnetic domains. When most of the domains align, the material saturates and no little further magnetization can occur**
- **Saturation is mainly a property of iron -- it does not manifest itself over a practical range of fluxes in air, plastic, or other non-ferrous materials**
- **The effect of saturation is to lower the synchronous reactance (to a 'saturated value')**

# Saturation and the magnetization curve

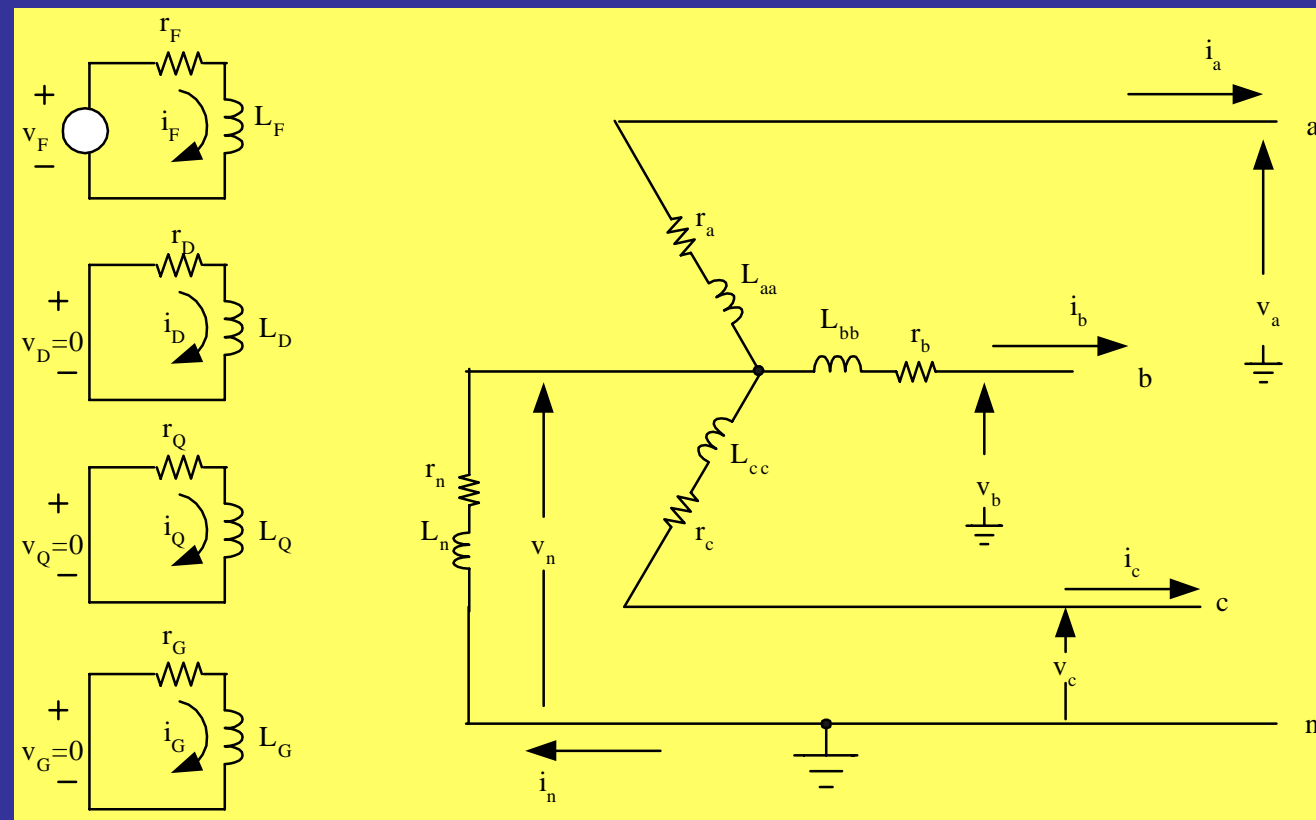
- Saturation may limit the performance of machines because of high air gap line voltage drop
- Saturation is often accompanied by hysteresis which results in losses in AC machines
- Saturation is not present in superconducting machines

# Transients and the dq transformation





# Transients and the dq transformation

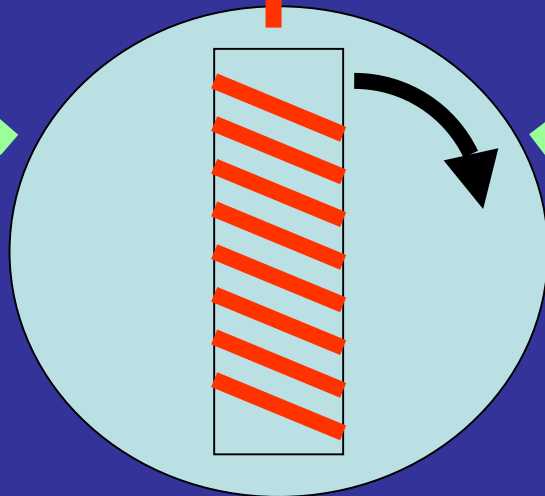


$$v = -ri - \dot{\lambda}$$

# Transients and the dq transformation

**d-axis**

THE BASIC IDEA IS TO WRITE THE VOLTAGE EQUATION AS IF THERE WERE ONLY A d-AXIS, AND AGAIN AS IF THERE WERE ONLY A q-AXIS



ROTATION

**q-axis**

$$v = -ri - \dot{\lambda}$$

# Transients and the dq transformation

$$v = -ri - \dot{\lambda}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ -v_D \\ -v_G \\ -v_Q \end{bmatrix} = - \begin{bmatrix} r_a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_a \\ \dot{\lambda}_b \\ \dot{\lambda}_c \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_G \\ \dot{\lambda}_Q \end{bmatrix}$$

# Transients and the dq transformation

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin \theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix}$$

**PARK'S TRANSFORMATION**

$$\theta = \omega_R t + \delta + \pi/2$$

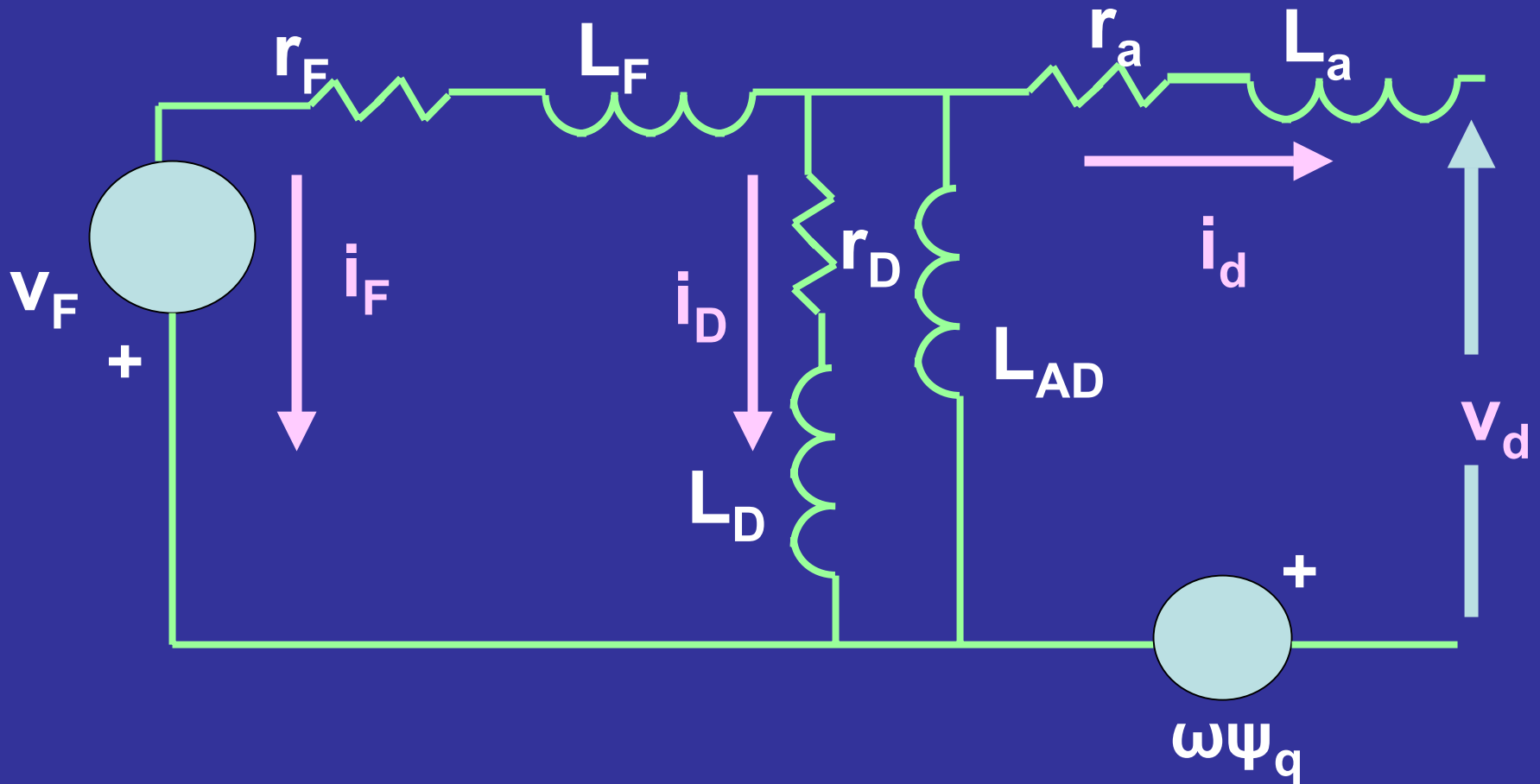
**BY APPLYING PARK'S TRANSFORMATION, THE TIME VARYING INDUCTANCES BECOME CONSTANTS**

# Transients and the dq transformation

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ -v_D \\ -v_G \\ -v_Q \end{bmatrix} = - \begin{bmatrix} r + 3r_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega(L_{AQ} + \ell_q) & 0 & 0 & \omega L_{AQ} & \omega L_{AQ} \\ 0 & -\omega(L_{AD} + \ell_d) & r & -\omega L_{AD} & -\omega L_{AD} & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix}$$

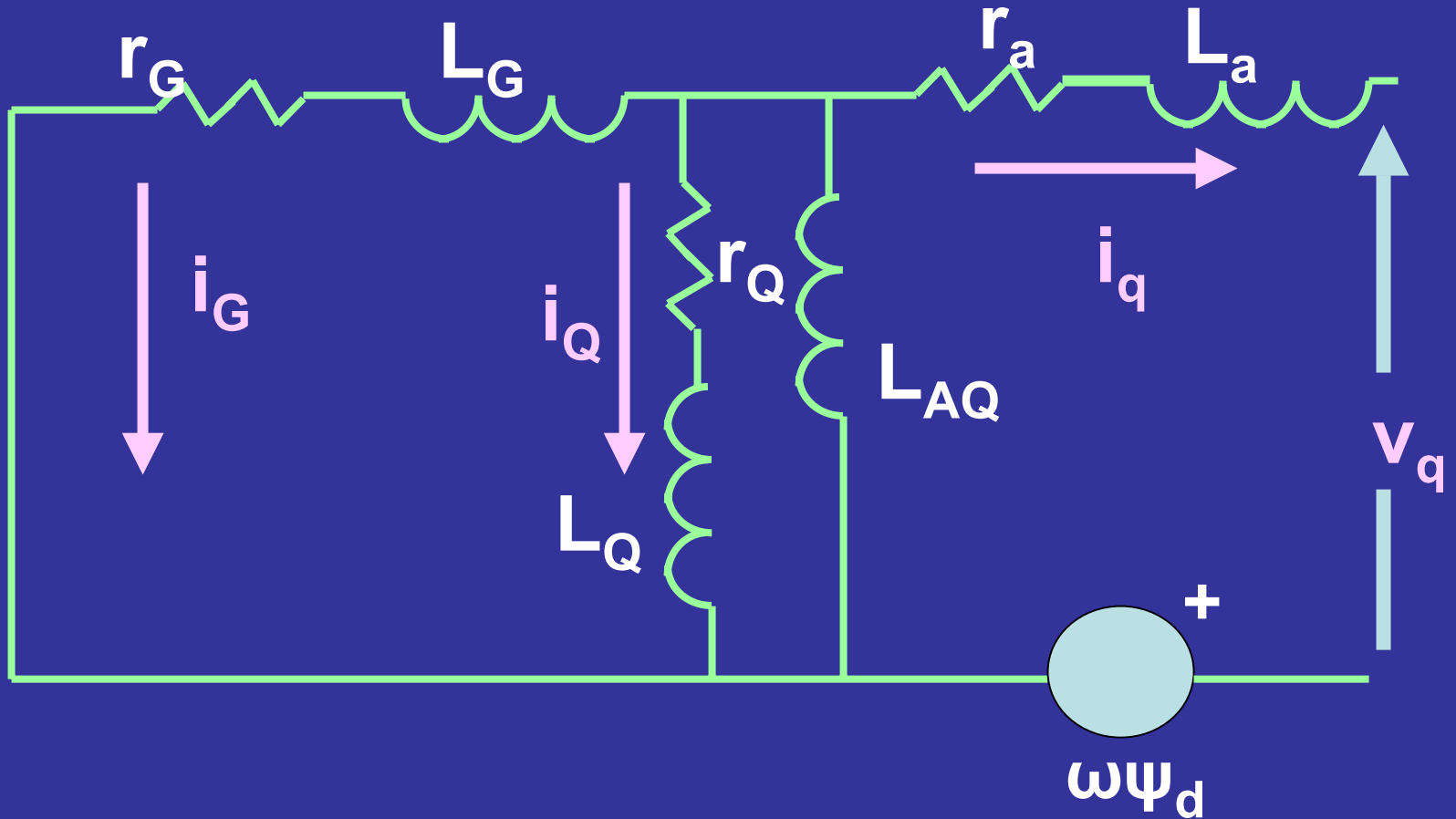
$$-\frac{1}{\omega_B} \cdot \begin{bmatrix} L_0 + 3L_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{AD} + \ell_d & 0 & L_{AD} & L_{AD} & 0 & 0 \\ 0 & 0 & L_{AQ} + \ell_q & 0 & 0 & L_{AQ} & L_{AQ} \\ 0 & L_{AD} & 0 & L_{AD} + \ell_F & L_{AD} & 0 & 0 \\ 0 & L_{AD} & 0 & L_{AD} & L_{AD} + \ell_D & 0 & 0 \\ 0 & 0 & L_{AQ} & 0 & 0 & L_{AQ} + \ell_G & L_{AQ} \\ 0 & 0 & L_{AQ} & 0 & 0 & L_{AQ} & L_{AQ} + \ell_Q \end{bmatrix} \begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_G \\ \dot{i}_Q \end{bmatrix}$$

# Machine reactances



d-axis equivalent circuit

# Machine reactances



q-axis equivalent circuit

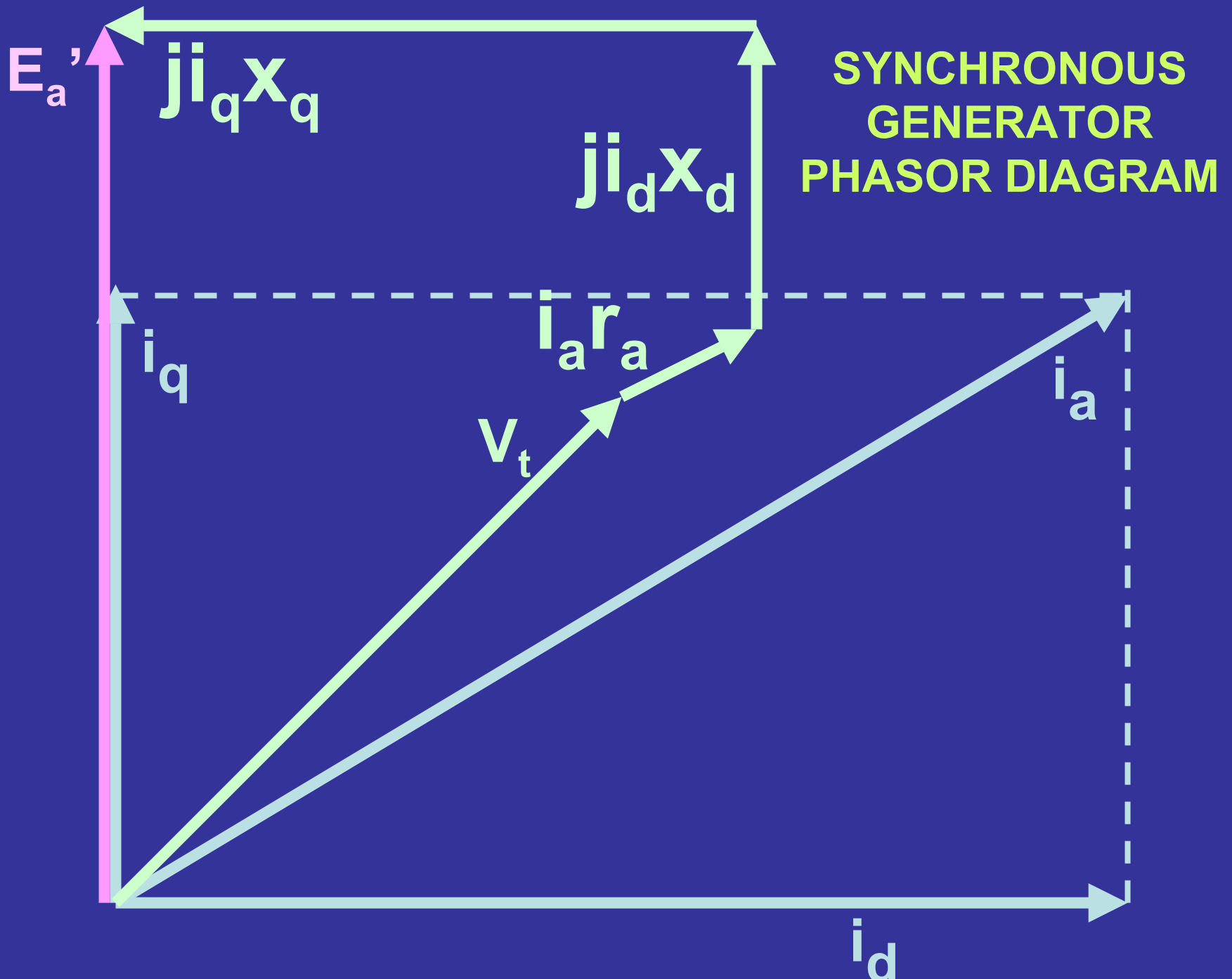
# Machine reactances

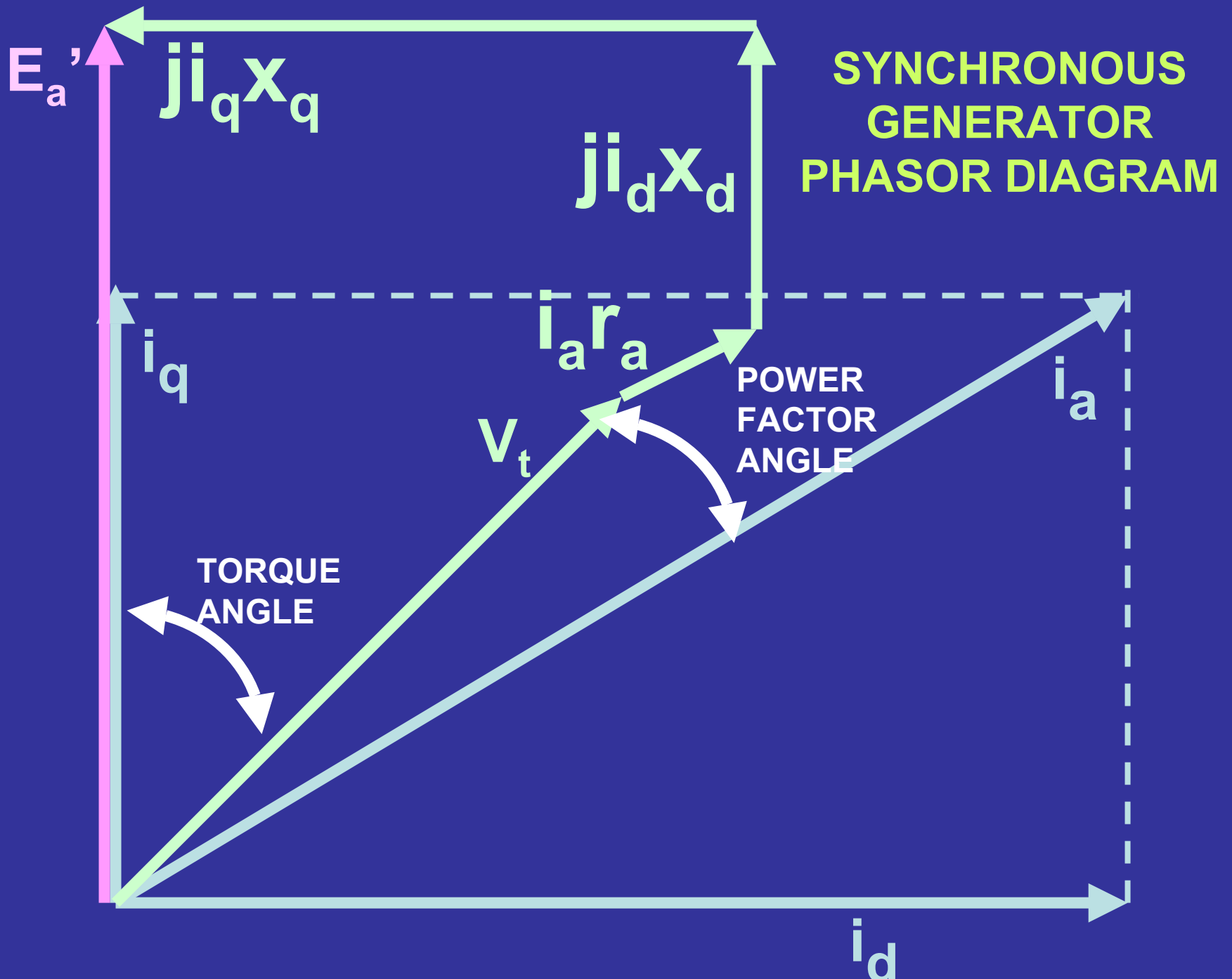
- These equivalent circuit parameters are traditionally obtained by a combination of manufacturers' design specifications and actual tests
- IEEE has a series of standardized tests for large generators that yield several time constants and equivalent circuit inductances
- Aging and saturation are not well accounted
- Change in operating point is not well accounted

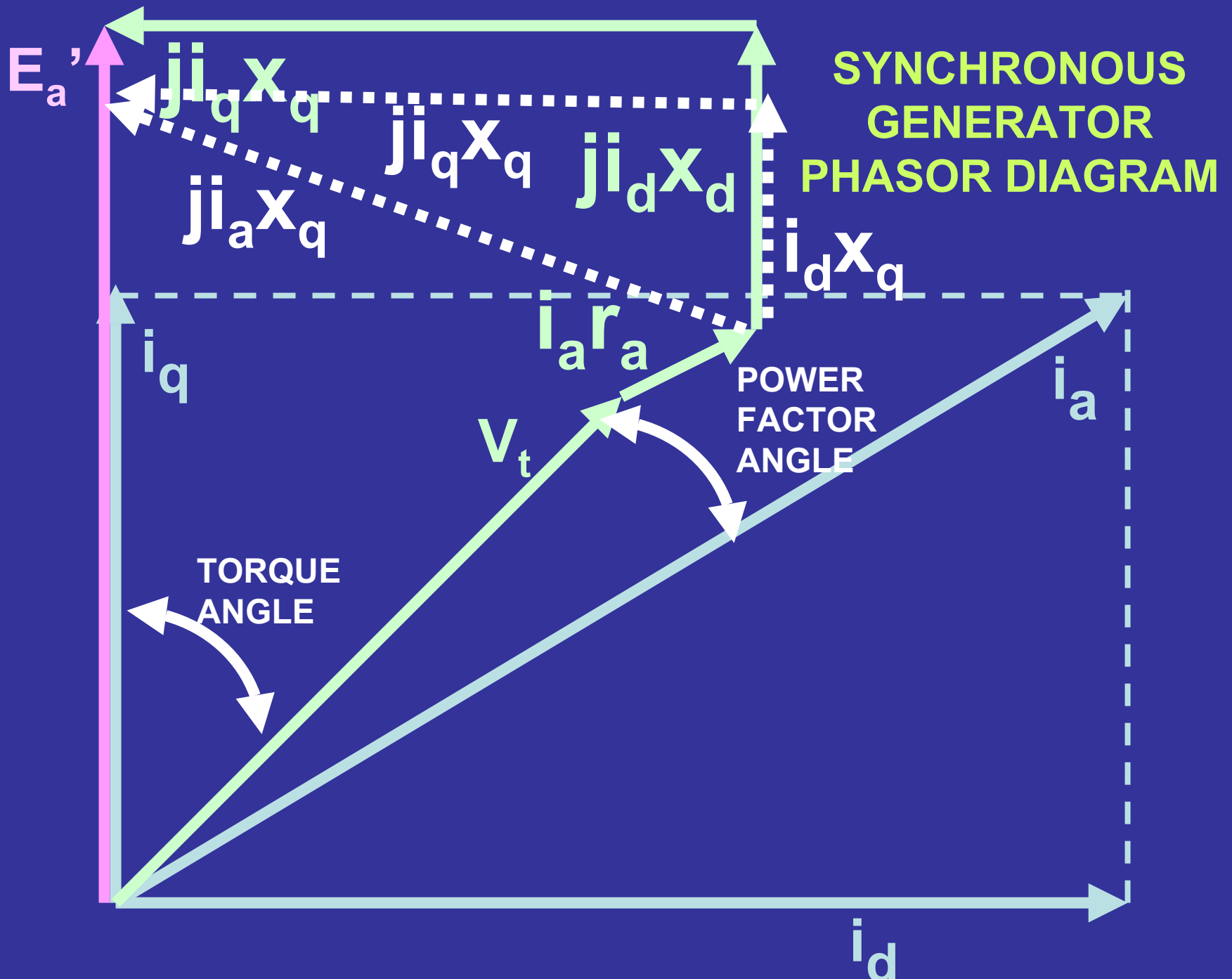


# Machine transient and subtransient reactances

Subtransient direct axis inductance	$L_d''$	$L_d - \frac{L_D L_{AD}^2 + L_F L_{AD}^2 - 2L_{AD}^3}{L_F L_D - L_{AD}^2}$
Transient direct axis inductance	$L_d'$	$L_d - \frac{L_{AD}^2}{L_F}$
Subtransient open circuit time constant in the direct axis	$\tau_{do}''$	$\frac{L_D L_F - L_{AD}^2}{\omega_B r_D L_F}$
Transient open circuit time constant in the direct axis	$\tau_{do}'$	$\frac{L_F}{\omega_B r_F}$
Subtransient short circuit time constant in the direct axis	$\tau_d''$	$\frac{L_d''}{L_d'} \tau_{do}''$
Transient short circuit time constant in the direct axis	$\tau_d'$	$\frac{L_d'}{L_d} \tau_{do}'$







# Machine transient and subtransient reactances

The usual procedure is that IEEE standardized tests are used to obtain inductances and time constants. Then using the formulas, circuit inductances and resistances can be solved.

***TESTS***

***TIME CONSTANTS  
INDUCTANCES***

***EQUIVALENT  
CIRCUIT  
PARAMETERS***



# Transient calculations

- Transients in dynamic systems are calculated as solutions of differential equations
- The usual solution approach is a numerical solution of  $(dX/dt) = AX + bu$
- Most numerical solutions relate to the approximation of  $dX/dt$  as  $(\Delta X)/(\Delta t)$
- Solutions are iterative in the sense that the given initial condition is used to obtain  $X$  at time  $t = h$ ; then  $X(h)$  is used to obtain  $X(2h)$ , etc.
- Popular solution methods include Matlab toolboxes, EMTP, ETMSP, PSpice
- The computer solutions could be used to compare with actual field measurements. And if there are discrepancies, the computer model could be updated to obtain better agreement – and hence a more accurate model.

# SESSION 5

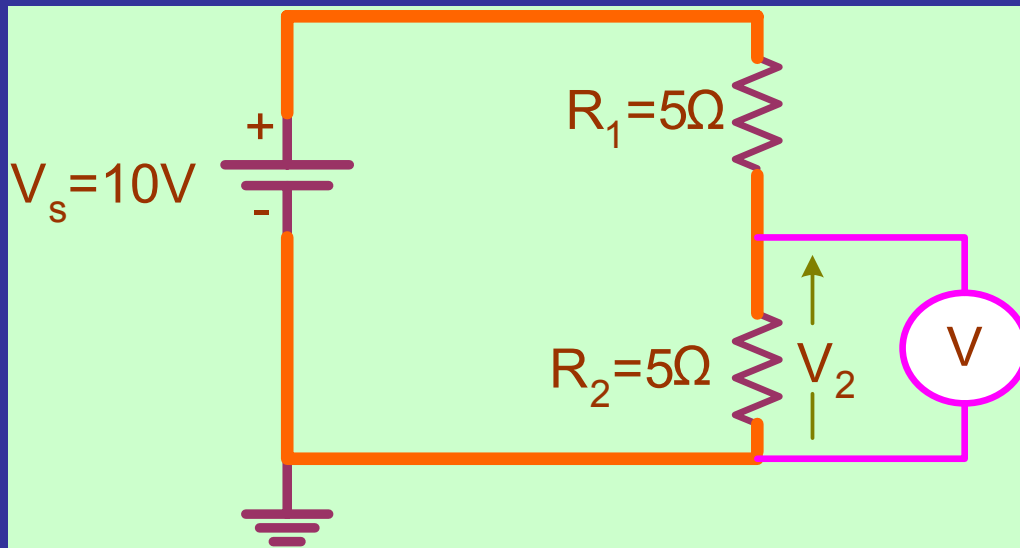
**State estimation applied to  
synchronous generators**

# **Session topics:**

- Basics of state estimation**
- Application to synchronous generators**
- Demonstration of software to identify synchronous generator parameters**



# BASICS OF STATE ESTIMATION



It is desired to measure the voltage across  $R_2$

Assume we have two voltmeters:  $A$  and  $B$   
Measure the voltage across  $R_2$  with both voltmeters

$$\begin{aligned} V_a &= 5.1 \text{ V} \\ V_b &= 4.7 \text{ V} \end{aligned}$$

Since the two measurements do not agree but are close to each other, average the result to estimate  $V_2$

$$V_2 = \frac{V_a + V_b}{2} = \frac{5.1 + 4.7}{2} = 4.9 \text{ V}$$

# BASICS OF STATE ESTIMATION

Now assume that we have a third voltmeter  $C$

Let the measurement from  $C$  be  $V_c = 15 \text{ V}$

Clearly this measurement is not reliable

Simple approach: disregard  $V_c$  and estimate  $V_2$  from  $V_a$  and  $V_b$

Another approach: Use *weighted state estimation*

This means, assign appropriate weights to each of the three measurements according to the confidence that the user has to each instrument.

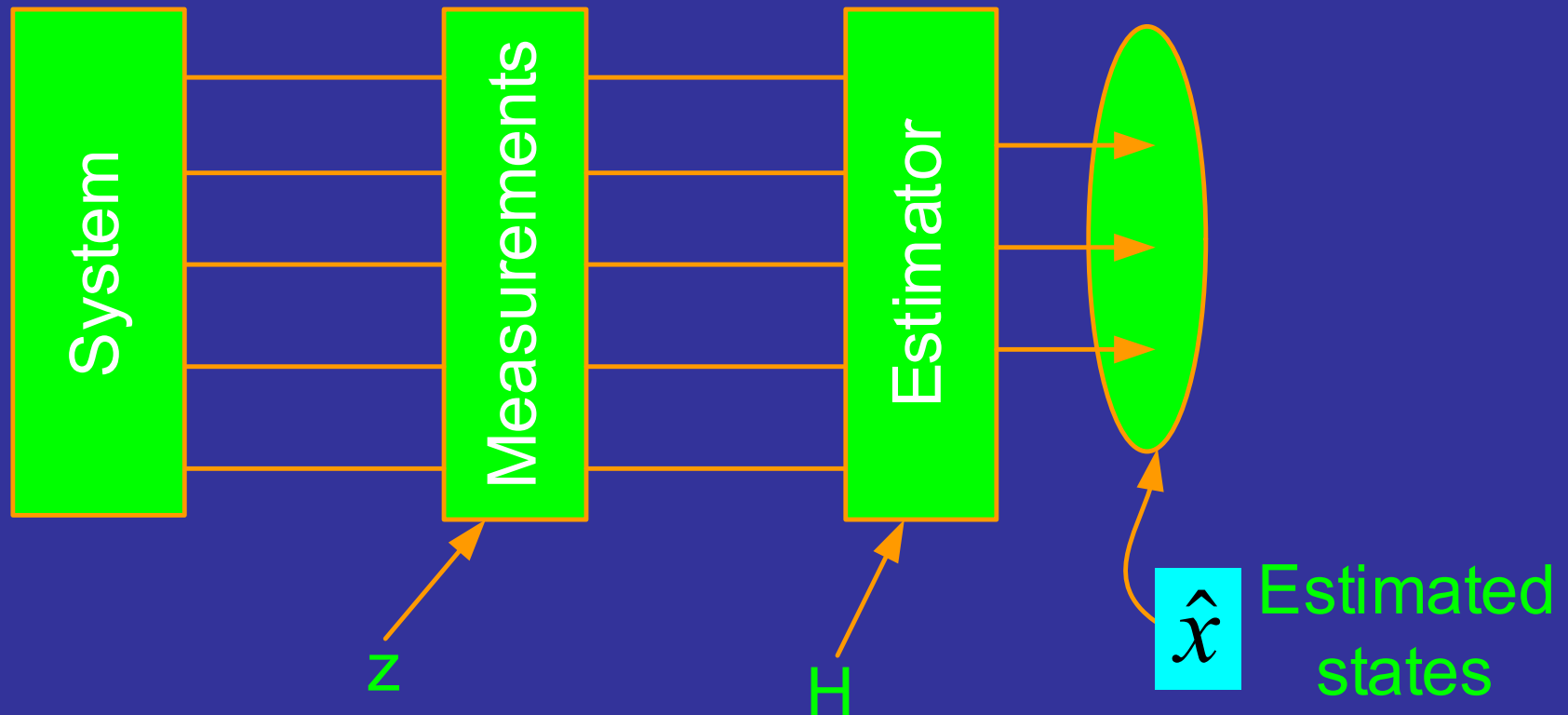
For example, give the following weights:

- if  $B$  is the best instrument give it a weight of 20
- give a weight of 18 to  $A$
- give a weight of 1 to  $C$  since it is not reliable

$$\Rightarrow V_2 = \frac{5.1 \times 18 + 4.7 \times 20 + 15 \times 1}{39} = 5.15$$

# BASICS OF STATE ESTIMATION

**Definition:** State estimation is the process of assigning a value to an unknown system state variable, using measurements from the system under study. Knowledge of the system configuration and of the accuracy of the measuring instruments is used in this process.



# EXAMPLE 1

Assume that it is desired to estimate two states (variables)  
Three measurements are obtained, which form the following equations

$$\begin{aligned}x_1 + x_2 &= 3.1 \\2x_1 - x_2 &= 0.2 \\x_1 - 3x_2 &= -4.8\end{aligned}$$

In matrix form:

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3.1 \\ 0.2 \\ -4.8 \end{bmatrix}$$

Process matrix  
3x2

2 states  
2x1 vector

3 measurements  
3x1 vector

The matrix equation is of the form  $Hx = z$

# EXAMPLE 1

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3.1 \\ 0.2 \\ -4.8 \end{bmatrix}$$

Number of measurements:  $n=3$

Number of states:  $m=2$

Since  $n > m$ , the system is overdetermined

Hence there is no unique solution

The solution is not unique since in general it is not possible to satisfy all the equations exactly for any choice of the unknowns.

A solution should be selected such that the error in satisfying each equation is minimum.

This error is called the residual of the solution and can be computed by,

$$r = z - H\hat{x}$$

$\hat{x}$  : the vector of the estimated parameters

The residual will be calculated later

# EXAMPLE 1

There are many ways to minimize the residual  $r$

One of the most popular is the least squares method, which in effect minimizes the length (Euclidean norm) of the residual  $r$ .

This method results in a simple formula to calculate the estimated parameters

Given the system is of the form  $Hx=z$ , the vector of the estimated parameters is given by,

$$\hat{x} = (H^T H)^{-1} H^T z = H^+ z$$

$H^+$  is called the pseudoinverse of  $H$

# EXAMPLE 1

$$\begin{matrix} \mathbf{H} & & \mathbf{x} & & \mathbf{z} \\ \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -3 \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} & = & \begin{bmatrix} 3.1 \\ 0.2 \\ -4.8 \end{bmatrix} \end{matrix}$$

Substitute  $H$  and  $z$  in  $\hat{x} = (H^T H)^{-1} H^T z$   
and solve for the unknown states

$$\hat{x} = \left[ \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -3 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 3.1 \\ 0.2 \\ -4.8 \end{bmatrix}$$

$$\Rightarrow \hat{x} = \begin{bmatrix} 6 & -4 \\ -4 & 11 \end{bmatrix}^{-1} \begin{bmatrix} -1.3 \\ 17.3 \end{bmatrix} = \begin{bmatrix} 0.22 & 0.08 \\ 0.08 & 0.12 \end{bmatrix} \begin{bmatrix} -1.3 \\ 17.3 \end{bmatrix}$$

$$\Rightarrow \hat{x} = \begin{bmatrix} 1.098 \\ 1.972 \end{bmatrix}$$

# EXAMPLE 1

To see how much error we have in the estimated parameters, we need to calculate the residual in a least squares sense

$$J = r^T r = (z - H\hat{x})^T (z - H\hat{x})$$

$$r = H\hat{x} - z = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1.098 \\ 1.972 \end{bmatrix} - \begin{bmatrix} 3.1 \\ 0.2 \\ -4.8 \end{bmatrix} = \begin{bmatrix} -0.03 \\ 0.224 \\ -0.018 \end{bmatrix}$$

$$\Rightarrow J = \begin{bmatrix} -0.03 & 0.224 & -0.018 \end{bmatrix} \begin{bmatrix} -0.03 \\ 0.224 \\ -0.018 \end{bmatrix} = 0.0514$$



# WHY ARE ESTIMATORS NEEDED?

In power systems the state variables are typically the voltage magnitudes and the relative phase angles at the nodes of the system.

The available measurements may be voltage magnitudes, current, real power, or reactive power.

The estimator uses these noisy, imperfect measurements to produce a best estimate for the desired states.

# WHY ARE ESTIMATORS NEEDED?

It is not economical to have measurement devices at every node of the system

The measurement devices are subject to errors



If errors are small, these errors may go undetected



If errors are large, the output would be useless

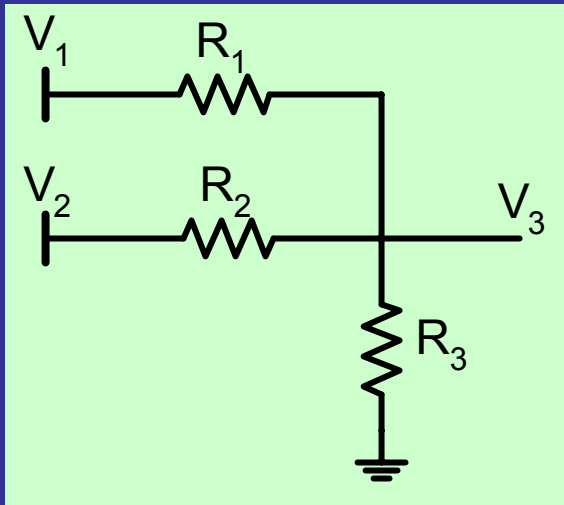
There are periods when the communication channels do not operate. Therefore, the system operator would not have any information about some part of the network.

# HOW DOES THE ESTIMATOR HELP?

**An estimator may:**

- **reduce the amount of noise in the measurements**
- **detect and smooth out small errors in readings**
- **detect and reject measurements with gross errors**
- **fill in missing measurements**
- **estimate states that otherwise are difficult to measure**

# EXAMPLE 2



Assume we have a network configuration as in the figure on the left. Assume that measurements are available for  $V_1$ ,  $V_2$ , and  $V_3$ . Find a relationship for  $V_3$  that has the following form:  $V_3 = aV_1 + bV_2 + c$

Available measurements

$V_1$	$V_2$	$V_3$
7.1	0	3.1
8.3	3.2	2.3
10.4	5.1	1.4
0	9.1	4.0

This is clearly an estimation problem with three unknowns ( $a$ ,  $b$ ,  $c$ ), and four measurements. Therefore it is an overdetermined estimation problem.

Now, it is necessary to express the estimation problem mathematically

# EXAMPLE 2

Substitute the measurements obtained in the desired model

$$V_3 = aV_1 + bV_2 + c$$

$$3.1 = 7.1a + 0b + c$$

$$2.3 = 8.3a + 3.2b + c$$

$$1.4 = 10.4a + 5.1b + c$$

$$4.0 = 0a + 9.1b + c$$

$V_1$	$V_2$	$V_3$
7.1	0	3.1
8.3	3.2	2.3
10.4	5.1	1.4
0	9.1	4.0

In matrix form,

$$\begin{bmatrix} 7.1 & 0 & 1 \\ 8.3 & 3.2 & 1 \\ 10.4 & 5.1 & 1 \\ 0 & 9.1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3.1 \\ 2.3 \\ 1.4 \\ 4.0 \end{bmatrix}$$

$H$

$x$

$z$

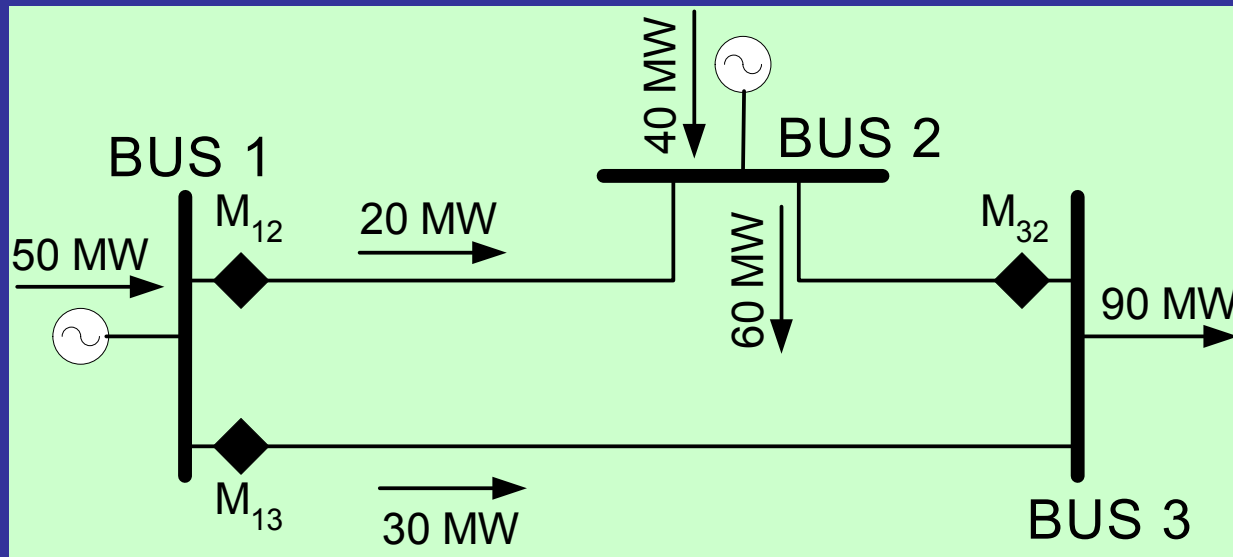
As in example 1, we can solve this matrix equation by taking the pseudoinverse of the  $H$  matrix

$$\Rightarrow \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = \begin{bmatrix} 7.1 & 0 & 1 \\ 8.3 & 3.2 & 1 \\ 10.4 & 5.1 & 1 \\ 0 & 9.1 & 1 \end{bmatrix}^+ \begin{bmatrix} 3.1 \\ 2.3 \\ 1.4 \\ 4.0 \end{bmatrix} = \begin{bmatrix} -0.3028 \\ -0.1374 \\ 5.2505 \end{bmatrix}$$

# EXAMPLE 3

Let's work out another example:

Estimate the relative phase angles at the buses of the figure below



**Given:**

$$X_{12} = 0.2 \text{ p.u.}$$

$$X_{13} = 0.4 \text{ p.u.}$$

$$X_{23} = 0.1 \text{ p.u.}$$

**System base: 100 MVA**

# EXAMPLE 3

## SOLUTION

Let bus 1 be the reference bus

$$\vartheta_1 = 0$$

From the measurements:

$$M_{12} = 20 \text{ MW} = 0.2 \text{ p.u.}$$

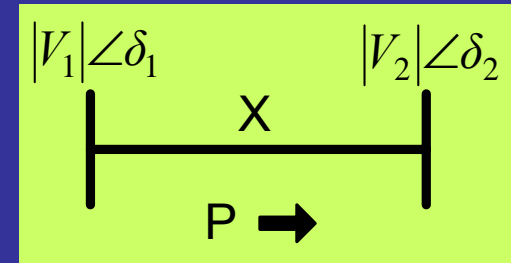
$$M_{13} = 30 \text{ MW} = 0.3 \text{ p.u.}$$

$$M_{32} = -60 \text{ MW} = -0.6 \text{ p.u.}$$

The line flows are given by,

$$f_{ab} = \frac{1}{X_{ab}} (\vartheta_a - \vartheta_b) = M_{ab}$$

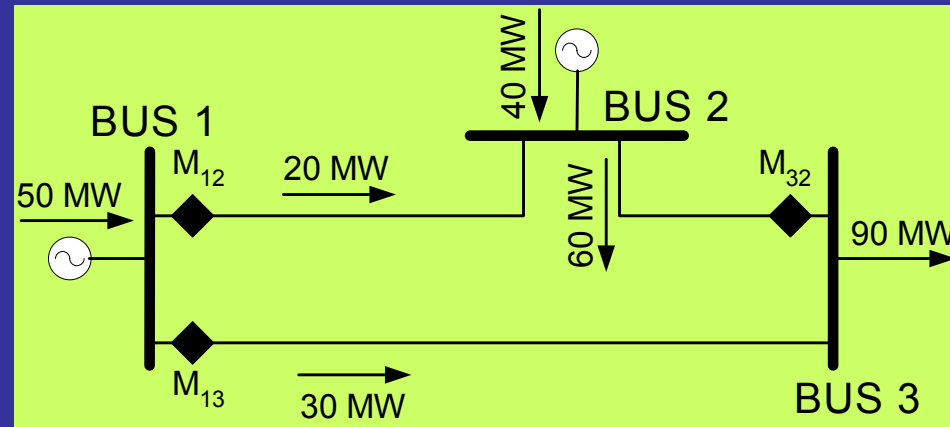
The above formula can be shown considering a simple two bus arrangement



$$P \approx \frac{|V_1||V_2|\sin(\delta_1 - \delta_2)}{X}$$

Since  $V_1$  and  $V_2$  are approximately 1 p.u., and the angle  $\delta_1 - \delta_2$  is small,  $P$  can be obtained as,

$$P \approx \frac{\delta_1 - \delta_2}{X}$$



# EXAMPLE 3

Hence,  $f_{12} = \frac{1}{X_{12}}(\vartheta_1 - \vartheta_2) = \frac{1}{0.2}(0 - \vartheta_2) = -5\vartheta_2 = 0.2$

$$f_{13} = \frac{1}{X_{13}}(\vartheta_1 - \vartheta_3) = \frac{1}{0.4}(0 - \vartheta_3) = -2.5\vartheta_3 = 0.3$$

$$f_{32} = \frac{1}{X_{32}}(\vartheta_3 - \vartheta_2) = \frac{1}{0.1}(\vartheta_3 - \vartheta_2) = 10\vartheta_3 - 10\vartheta_2 = -0.6$$

In matrix form,

$$\begin{bmatrix} -5 & 0 \\ 0 & -2.5 \\ -10 & 10 \end{bmatrix} \begin{bmatrix} \vartheta_2 \\ \vartheta_3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.3 \\ -0.6 \end{bmatrix}$$

This is again of the form  $Hx = z$ , and is solved as in example 1:

$$\hat{\vartheta} = \begin{bmatrix} -5 & 0 & -10 \\ 0 & -2.5 & 10 \end{bmatrix} \begin{bmatrix} -5 & 0 \\ 0 & -2.5 \\ -10 & 10 \end{bmatrix}^{-1} \begin{bmatrix} -5 & 0 & -10 \\ 0 & -2.5 & 10 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.3 \\ -0.6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \hat{\vartheta}_2 \\ \hat{\vartheta}_3 \end{bmatrix} = \begin{bmatrix} -0.0438 \\ -0.1048 \end{bmatrix} \text{rad}$$



# APPLICATION OF STATE ESTIMATION TO SYNCHRONOUS GENERATORS

- Need to know the operating parameters of generators to
  - perform studies
  - study behavior of the system at various operating levels
  - perform postmortem analysis
- Meet requirements for machine testing (e.g. NERC)
- To reestablish machine parameters after a repair
- Fault identification / signature analysis
- Incipient event identification

# APPLICATION OF STATE ESTIMATION TO SYNCHRONOUS GENERATORS

## Problems:

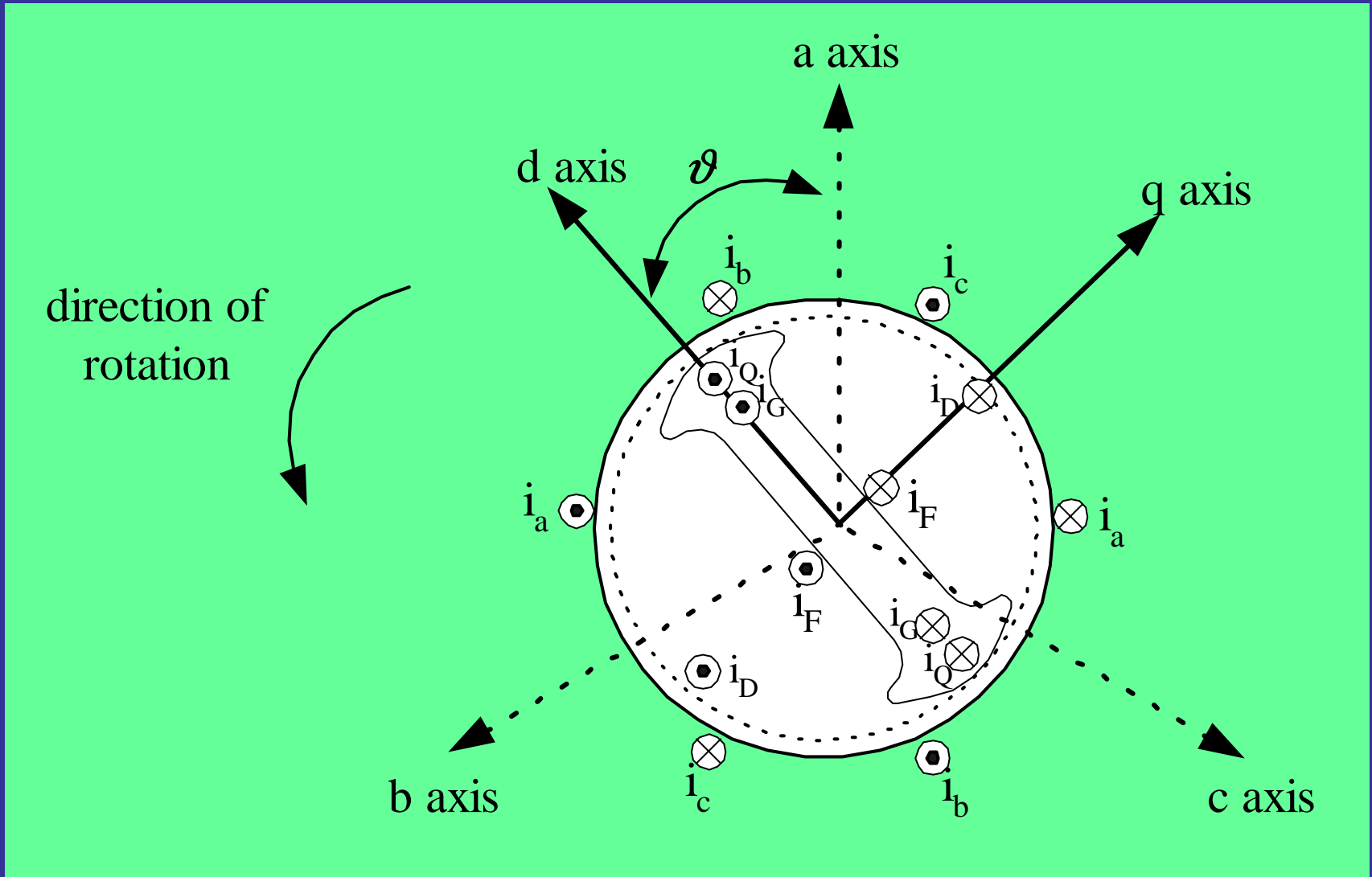
Generator parameters change with operating point, aging  
Cannot measure parameters while generator is committed  
Cannot afford to decommit unit in order to measure its parameters

## Solution:

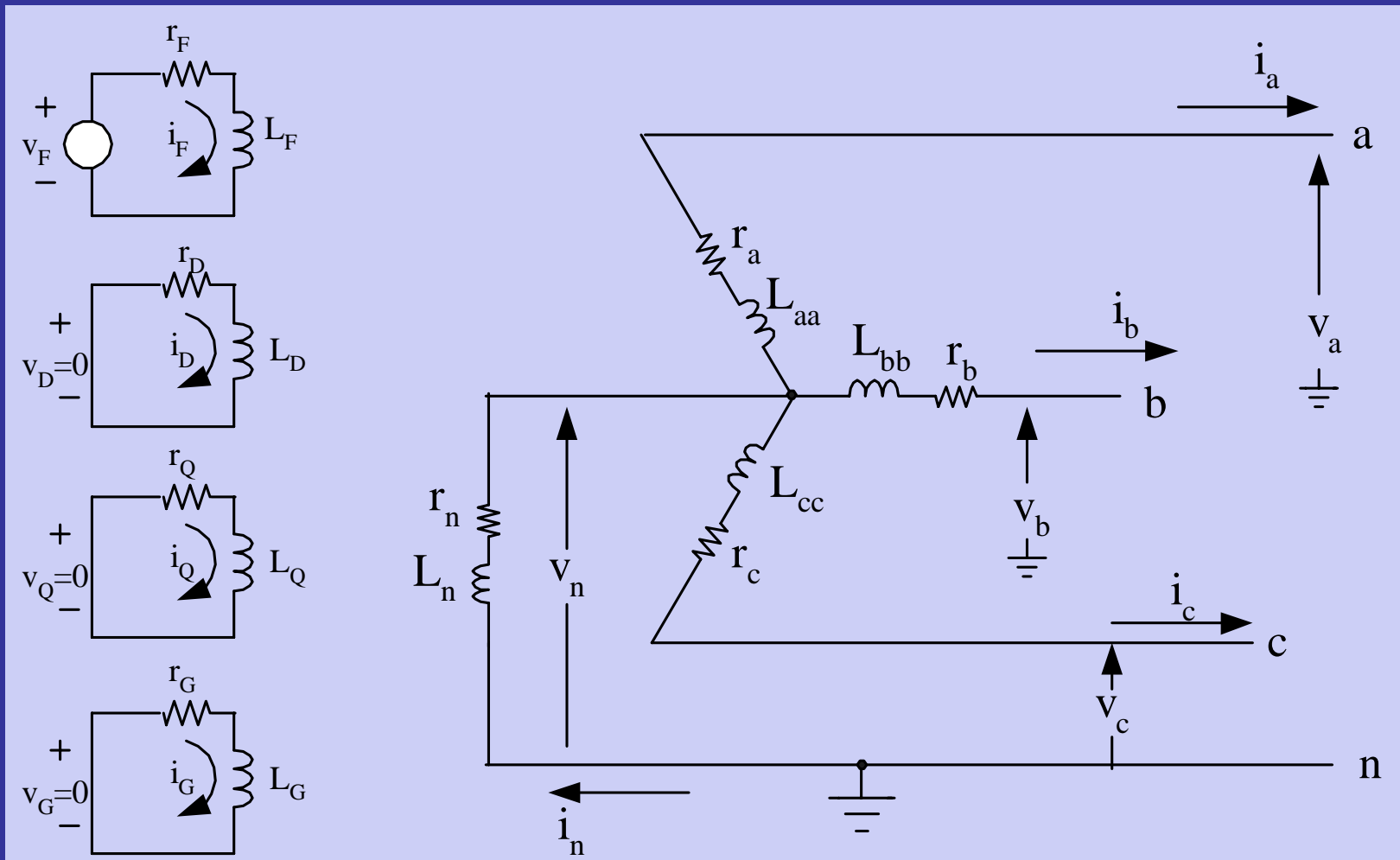
Use available terminal measurements, knowledge of the model of the generator, and state estimation, to approximate the required parameters

To do that, it is necessary to develop a model for the synchronous generator

# SYNCHRONOUS GENERATOR REPRESENTATION



# SYNCHRONOUS GENERATOR MODEL



Schematic diagram of a synchronous generator

# DEVELOPMENT OF SYNCHRONOUS GENERATOR MODEL

$$\mathbf{v} = -r\mathbf{i} - \dot{\boldsymbol{\lambda}} - \mathbf{v}_n$$

 $\Rightarrow$ 

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ -v_D \\ -v_G \\ -v_Q \end{bmatrix} = - \begin{bmatrix} r_a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_a \\ \dot{\lambda}_b \\ \dot{\lambda}_c \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_G \\ \dot{\lambda}_Q \end{bmatrix} + \begin{bmatrix} v_n \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_G \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aG} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bG} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cG} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FG} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DG} & L_{DQ} \\ L_{Ga} & L_{Gb} & L_{Gc} & L_{GF} & L_{GD} & L_{GG} & L_{GQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QG} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix}$$

 $\Rightarrow$ 

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_D \\ \lambda_G \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_G & kM_Q \\ 0 & kM_F & 0 & L_F & M_X & 0 & 0 \\ 0 & kM_D & 0 & M_X & L_D & 0 & 0 \\ 0 & 0 & kM_G & 0 & 0 & L_G & M_Y \\ 0 & 0 & kM_Q & 0 & 0 & M_Y & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix}$$

# DEVELOPMENT OF SYNCHRONOUS GENERATOR MODEL

$$\lambda_{0dq} = P\lambda_{abc}$$

$$i_{0dq} = Pi_{abc}$$

$$v_{0dq} = Pv_{abc}$$

$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos \vartheta & \cos(\vartheta - 2\pi/3) & \cos(\vartheta + 2\pi/3) \\ \sin \vartheta & \sin(\vartheta - 2\pi/3) & \sin(\vartheta + 2\pi/3) \end{bmatrix}$$

**Resulting model:**

$$\begin{bmatrix} v_{0dq\ 3 \times 1} \\ v_{FDGQ\ 4 \times 1} \end{bmatrix} = -[R_{7 \times 7}] \begin{bmatrix} i_{0dq\ 3 \times 1} \\ i_{FDGQ\ 4 \times 1} \end{bmatrix} - [L_{7 \times 7}] \begin{bmatrix} \dot{i}_{0dq\ 3 \times 1} \\ \dot{i}_{FDGQ\ 4 \times 1} \end{bmatrix}$$

# SYNCHRONOUS GENERATOR MODEL

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ -v_D \\ -v_G \\ -v_Q \end{bmatrix} = - \begin{bmatrix} r + 3r_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega(L_{AQ} + \ell_q) & 0 & 0 & \omega L_{AQ} & \omega L_{AQ} \\ 0 & -\omega(L_{AD} + \ell_d) & r & -\omega L_{AD} & -\omega L_{AD} & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix}$$
  

$$-\frac{1}{\omega_B} \begin{bmatrix} L_0 + 3L_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{AD} + \ell_d & 0 & L_{AD} & L_{AD} & 0 & 0 \\ 0 & 0 & L_{AQ} + \ell_q & 0 & 0 & L_{AQ} & L_{AQ} \\ 0 & L_{AD} & 0 & L_{AD} + \ell_F & L_{AD} & 0 & 0 \\ 0 & L_{AD} & 0 & L_{AD} & L_{AD} + \ell_D & 0 & 0 \\ 0 & 0 & L_{AQ} & 0 & 0 & L_{AQ} + \ell_G & L_{AQ} \\ 0 & 0 & L_{AQ} & 0 & 0 & L_{AQ} & L_{AQ} + \ell_Q \end{bmatrix} \begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_G \\ \dot{i}_Q \end{bmatrix}$$

# MODEL DISCUSSION

After the development of the model it is necessary to carefully examine the available information about the system, find out what is known in the model, what is unknown and needs to be calculated or assumed, and what is desired to be estimated.

For the synchronous generator case,

## Measured/Known

line-to-line terminal voltages

line currents

field voltage (for an exciter with brushes)

field current (for an exciter with brushes)

## Unknown

damper currents

current derivatives

Finally, some of the parameters need to be estimated through state estimation, while the other parameters need to be calculated from manufacturer's data



# STATE ESTIMATOR CONFIGURATION

## EXAMPLE

Estimate  $L_{AD}$ ,  $L_{AQ}$ , and  $r_F$

- Calculate current derivatives by using
- Rearrange system in the form  $Hx = z$

$$i(t) \approx \frac{i(t + \Delta t) - i(t)}{\Delta t}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\omega_B}(i'_d + i'_F + i'_D) & \omega(i_q + i_G + i_Q) & 0 \\ -\omega(i_d + i_F + i_D) & \frac{1}{\omega_B}(i'_q + i'_G + i'_Q) & 0 \\ \frac{1}{\omega_B}(i'_d + i'_F + i'_D) & 0 & i_F \end{bmatrix} \begin{bmatrix} L_{AD} \\ L_{AQ} \\ r_F \end{bmatrix} = - \begin{bmatrix} r + 3r_n & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega l_q & 0 & 0 & 0 \\ 0 & -\omega l_d & r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_G \\ i_Q \end{bmatrix}$$

$$-\frac{1}{\omega_B} \begin{bmatrix} L_0 + 3L_n & 0 & 0 & 0 & 0 & 0 \\ 0 & l_d & 0 & 0 & 0 & 0 \\ 0 & 0 & l_q & 0 & 0 & 0 \\ 0 & 0 & 0 & l_F & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_G \\ \dot{i}_Q \end{bmatrix} = \begin{bmatrix} V_0 \\ V_d \\ V_q \\ -V_F \\ -V_D \\ -V_G \\ -V_Q \end{bmatrix}$$

# DEMONSTRATION OF PROTOTYPE APPLICATION FOR PARAMETER ESTIMATION

- Prototype application developed in Visual C++
- Portable, independent application
- Runs under Windows
- Purpose: Read measurements from DFR and use manufacturer's data to estimate generator parameters

# **SESSION 6**

## **Machine instrumentation**

## **Session topics:**

- **Digital Fault Recorders (DFRs)**
- **Calculation of torque angle**

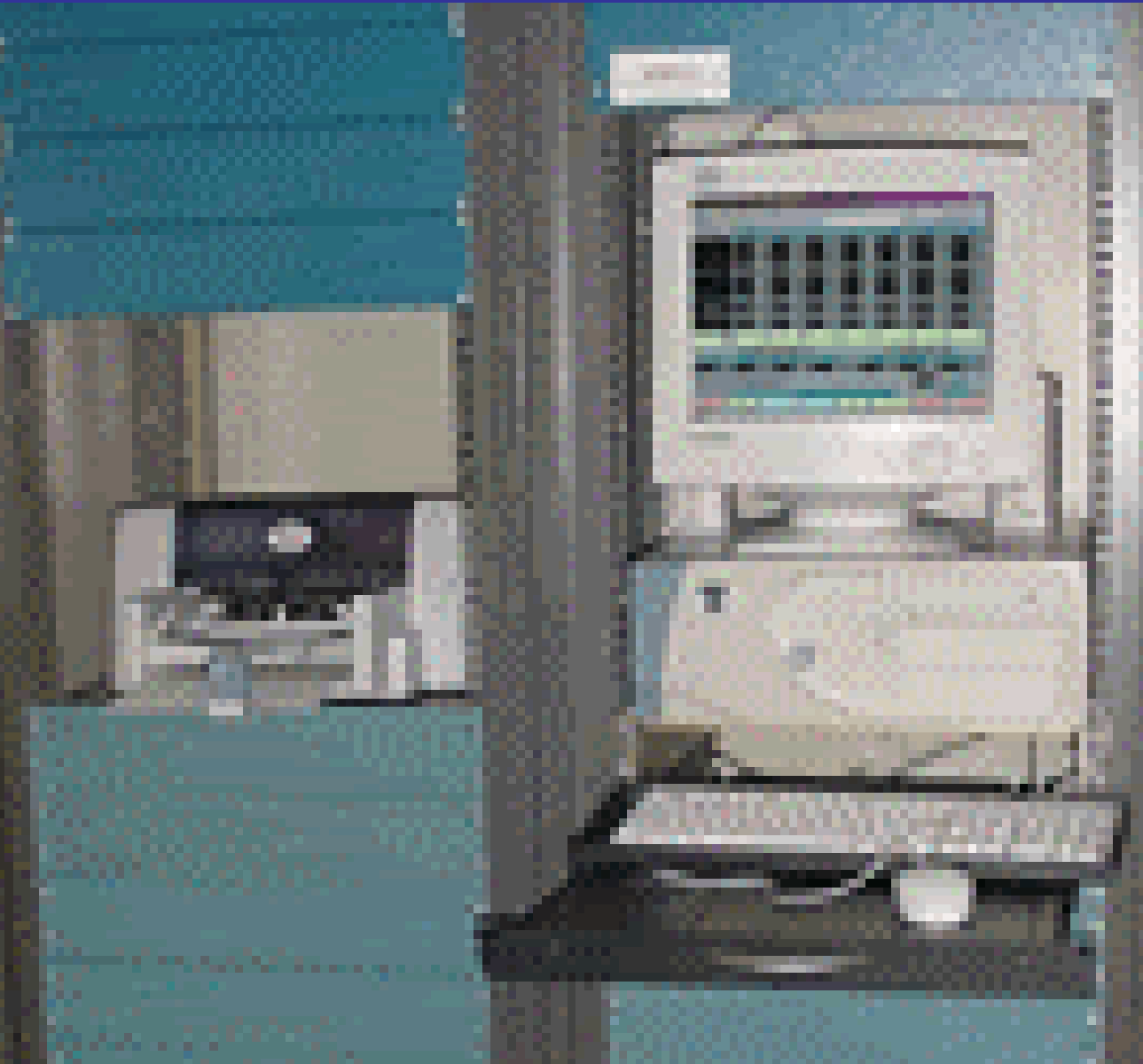
# DIGITAL FAULT RECORDERS (DFRs)

A DFR is effectively a data acquisition system that is used to monitor the performance of generation and transmission equipment.

It is predominantly utilized to monitor system performance during stressed conditions. For example, if a lightning strikes a transmission line, the fault recognition by protective relays and the fault clearance by circuit breakers takes only about 50 to 83 ms.

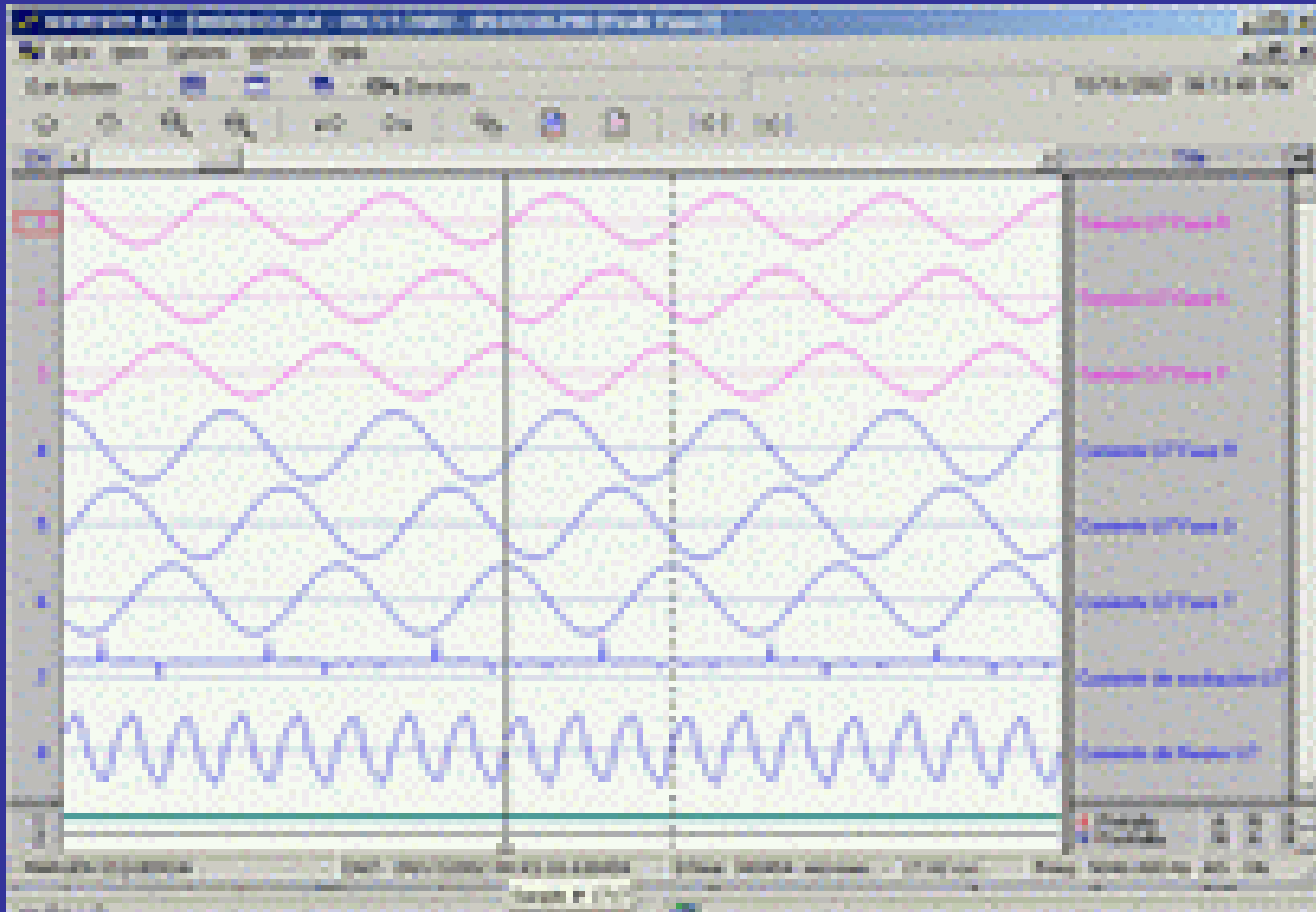
This process is too fast for human intervention. Therefore, the DFR saves a record of the desired signals (e.g. power and current), and transmits this record to the central offices over a modem, where a utility engineer can perform post-event analysis to determine if the relays, circuit breakers and other equipment functioned properly.

# DIGITAL FAULT RECORDERS (DFRs)



The DFR sends the measured signals to a central pc station through a modem

# DIGITAL FAULT RECORDERS (DFRs)



Typical graphics window showing a snapshot of the measured signals

# TYPICAL DFR SPECIFICATIONS

Data files are stored in COMTRADE IEEE format

The DFR can be configured to create transient records and continuous records

Can be used during disturbances, abnormal conditions, and normal conditions

## Typical specifications:

Analog channels: 8, 16, 24, or 32

Digital channels: 16, 32, 48, or 64

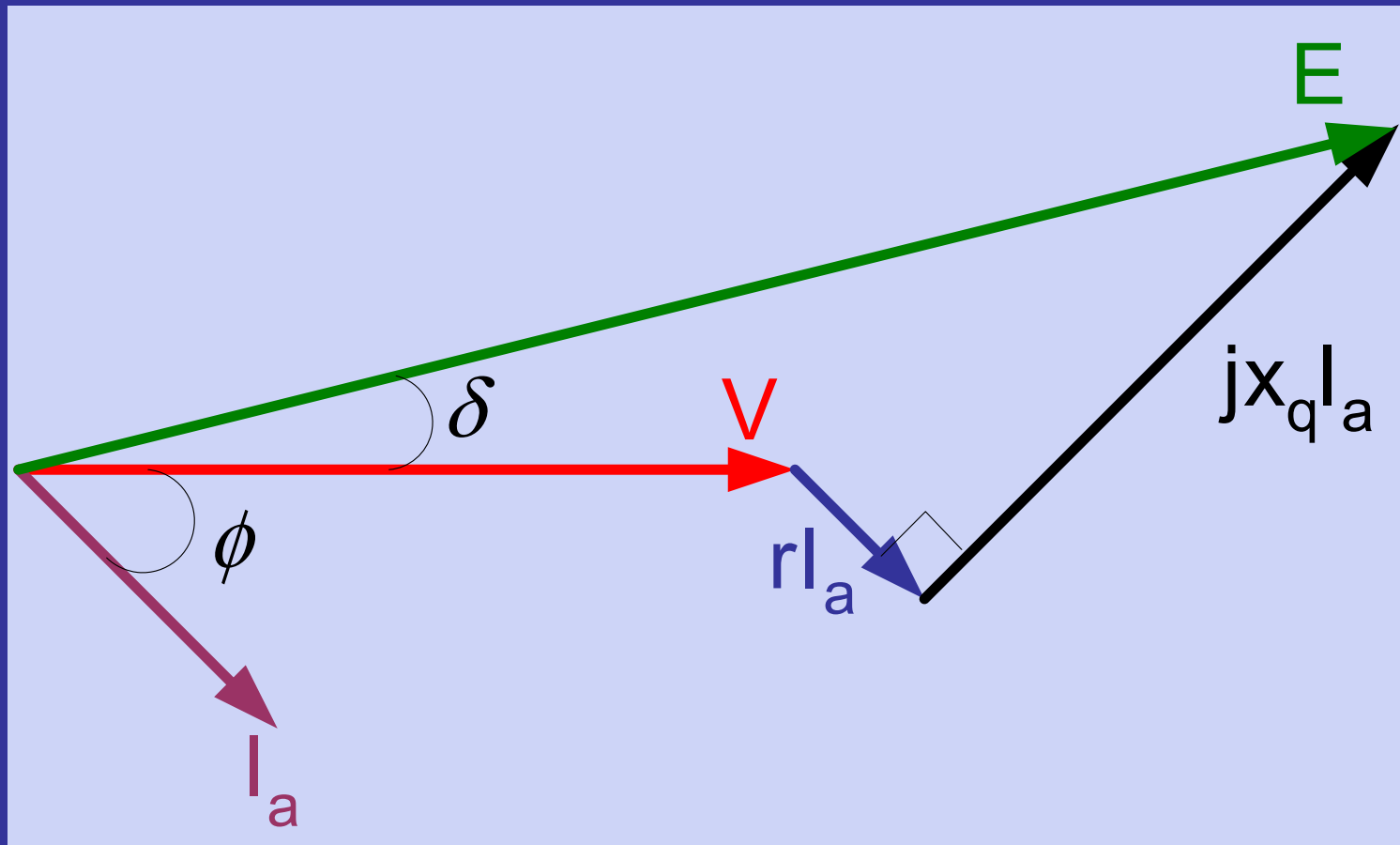
Sample rate: 24-192 samples/min

Operating voltage: 48VDC, 125VDC, 250VDC, 120VAC



# CALCULATION OF TORQUE ANGLE

The torque angle  $\delta$  is defined as the angle between the machine emf  $E$  and the terminal voltage  $V$  as shown in the phasor diagram



# CALCULATION OF TORQUE ANGLE

The torque angle can be calculated in different ways depending on what information is available

Two ways to calculate the torque angle will be shown:

1. Using line to line voltages and line currents (stator frame of reference)
2. Using voltages and currents in the rotor frame of reference (0dq quantities)

# CALCULATION OF TORQUE ANGLE IN THE STATOR REFERENCE FRAME

## Known quantities:

Line to line voltages ( $v_{ab}$ ,  $v_{bc}$ ,  $v_{ca}$ )

Line currents ( $i_a$ ,  $i_b$ ,  $i_c$ )

## Procedure:

1. Calculate phase voltages

$$v_a = \frac{1}{3}(v_{ab} - v_{ca})$$
$$v_b = \frac{1}{3}(-v_{ab} + v_{bc})$$
$$v_c = \frac{1}{3}(-v_{bc} + v_{ca})$$

2. Calculate three phase active and reactive power

$$P = v_{ab}i_a - v_{bc}i_c$$

$$Q = (v_{ab}i_c + v_{bc}i_a + v_{ca}i_b) / \sqrt{3}$$

3. Calculate the power factor

$$\varphi = \tan^{-1} \frac{Q}{P}$$

# CALCULATION OF TORQUE ANGLE IN THE STATOR REFERENCE FRAME

4. Calculate the voltage angle for each phase  
For a balanced 3-phase system,

$$v_a = v_m \cos \theta$$

$$v_b = v_m \cos(\theta - 120)$$

$$v_c = v_m \cos(\theta + 120)$$

For phase *a*, use phases *b* and *c*

$$v_b = v_m \cos \theta \cos 120 + v_m \sin \theta \sin 120 = -\frac{1}{2}v_m \cos \theta + \frac{\sqrt{3}}{2}v_m \sin \theta$$

$$v_c = v_m \cos \theta \cos 120 - v_m \sin \theta \sin 120 = -\frac{1}{2}v_m \cos \theta - \frac{\sqrt{3}}{2}v_m \sin \theta$$

It can be observed that,

$$v_b + v_c = -v_m \cos \theta$$

$$v_b - v_c = \sqrt{3}v_m \sin \theta$$

$$\Rightarrow \frac{v_b + v_c}{v_b - v_c} = -\frac{1}{\sqrt{3}} \cot \theta$$

$$\Rightarrow \tan \theta = -\frac{1}{\sqrt{3}} \frac{v_b - v_c}{v_b + v_c}$$

# CALCULATION OF TORQUE ANGLE IN THE STATOR REFERENCE FRAME

The angles for the other phases can be calculated in a similar fashion. The angles for all phases are given by,

$$\theta_{v_a} = \tan^{-1} \left( \frac{-1}{\sqrt{3}} \frac{(v_b - v_c)}{(v_b + v_c)} \right)$$

$$\theta_{v_b} = \tan^{-1} \left( \frac{-1}{\sqrt{3}} \frac{(v_c - v_a)}{(v_c + v_a)} \right)$$

$$\theta_{v_c} = \tan^{-1} \left( \frac{-1}{\sqrt{3}} \frac{(v_a - v_b)}{(v_a + v_b)} \right)$$

5. Find the angle of  $i_a$

Using the above procedure,

$$\theta_{i_a} = \tan^{-1} \left( \frac{-1}{\sqrt{3}} \frac{(i_b - i_c)}{(i_b + i_c)} \right)$$

# CALCULATION OF TORQUE ANGLE IN THE STATOR REFERENCE FRAME

6. Calculate the instantaneous line to neutral rms voltage for phase a

$$V_{t_{rms}} = -\frac{v_{bc} + v_{ca}}{\sqrt{2}\sqrt{3}\cos\theta_{v_a}}$$

7. Calculate the machine generated emf

$$\bar{E}_t = V_t + (r + jx_q)I_a = |E_t|\angle\delta$$

$\delta$  is the torque angle

# CALCULATION OF TORQUE ANGLE IN THE ROTOR REFERENCE FRAME

## Known quantities:

0dq voltages ( $v_0, v_d, v_q$ )

0dq currents ( $i_0, i_d, i_q$ )

## Procedure:

1. Calculate the active and reactive power

$$P = v_d i_d + v_q i_q$$

$$Q = v_q i_d - v_d i_q$$

2. Calculate the terminal voltage

$$\bar{E}_t = V_d + jV_q = |E_t| \angle \gamma$$

# CALCULATION OF TORQUE ANGLE IN THE ROTOR REFERENCE FRAME

## 3. Calculate the terminal current

$$I_t = \frac{\sqrt{P^2 + Q^2}}{|E_t|}$$

## 4. Calculate the power factor angle

$$\varphi = \cos^{-1}\left(\frac{P}{E_t I_t}\right)$$

## 5. Calculate the torque angle

$$\delta = \tan^{-1} \frac{(x_q I_t \cos \varphi - r I_t \sin \varphi)}{(E_t + r I_t \cos \varphi + x_q I_t \sin \varphi)}$$



# **SESSION 7**

**Question and answer  
session**