

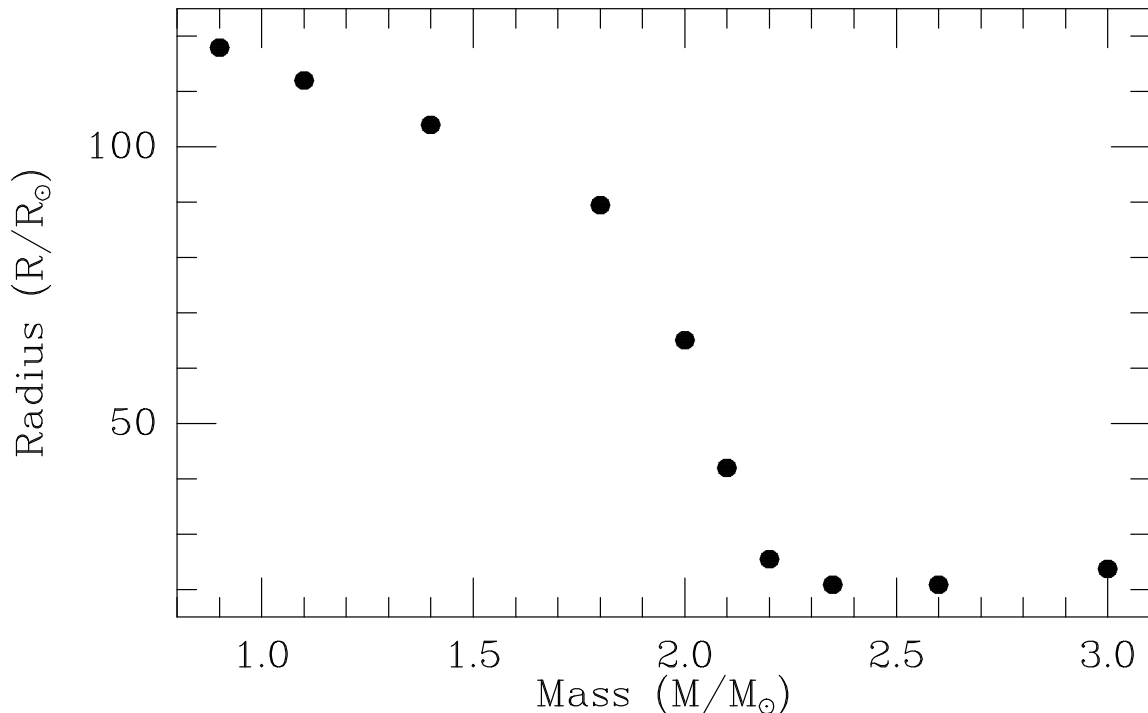
The Helium Flash

- When the temperature of a stellar core reaches $T \sim 10^8$ K, the star ignites helium. The mass of the core at this time (at least, for stars in the mass range between 0.8 and $2.3\mathcal{M}_\odot$) is

$$\mathcal{M}_c \approx 0.476 - 0.221(Y - 0.3) - 0.009(3 + \log Z) - 0.023(\mathcal{M} - 0.8)$$

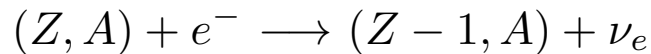
Note: the core mass at ignition is somewhat dependent on the star's initial helium abundance, weakly dependent on its initial metallicity, and surprisingly insensitive to the star's initial mass. In general, helium burning for all stars begins when the core reaches $\sim 0.45\mathcal{M}_\odot$.

- Stars with degenerate cores expand to $R \sim 100R_\odot$ before they ignite helium; the maximum size attained by stars that burn helium non-degenerately is $\sim 25R_\odot$. The transition region is $\mathcal{M} \gtrsim 2\mathcal{M}_\odot$.

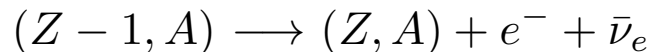


- When helium ignites degenerately, it does so in a thermonuclear runaway, called the “Helium Flash”. At maximum, the luminosity from this fusion is $\sim 10^{11} \mathcal{L}_{\odot}$, *i.e.*, like a supernovae! However, almost none of this energy reaches the surface; it is all absorbed in the expansion of the non-degenerate outer layers. As the flash proceeds, the degeneracy in the core is removed, and the core expands.

- Models of the helium flash are complicated and uncertain. It is likely that the actual ignition occurs off-center, due to the *Urca Process*. At high densities and/or temperatures of $T \gtrsim 10^8$ K, an isotope of atomic number Z and atomic mass A may undergo an electron capture



This new element then beta decays back to



The net result is energy is continually lost via neutrino emission. (And if you’re wondering what Urca means, it’s the name of a casino in Rio de Janiero.) Since, the Urca process works best where the matter is densest, the center of the core may actually be cooler than its surroundings, causing a lop-sided ignition.

- The high luminosity causes the region just outside the core to convect. However, models show that this inner convective zone does not merge with the outer convective zone; if it did, unprocessed material (hydrogen) would be mixed into the core, causing a number of unusual reactions. Similarly, the outward mixing would cause us to detect anomalous abundances in post-helium flash stars. This is not seen.

- After the flash, the core continues to quiescently burn helium, while hydrogen continues to burn in a shell above the core. Because the core is supported by the helium fusion, it is larger, and the pressure at the hydrogen burning shell is smaller. As a result, the luminosity of the star drops from its pre-helium flash luminosity to $\sim 100\mathcal{L}_\odot$. The restructuring of the star to this stable configuration occurs on a Kelvin-Helmholtz timescale.
- Stars more massive than $\sim 3\mathcal{M}_\odot$ ignite helium under non-degenerate conditions. This causes the core to expand, which decreases the pressure at the hydrogen-burning shell, which causes the shell luminosity to drop, which causes the outer layers to contract. Helium ignition occurs on the giant branch for intermediate mass stars, high mass stars ($\mathcal{M} \gtrsim 15\mathcal{M}_\odot$) can ignite helium in the Hertzsprung Gap, close to the main sequence.

Red Giant Branch Mass Loss

It is an observational fact that stars on the red giant undergo mass loss in the form of a slow (between 5 and 30 km s⁻¹) wind. The mechanism that drives this mass loss is unclear. The canonical hypothesis is that dust grains form in the atmosphere of cool stars, and these grains are accelerated outward via interactions with photons (radiation pressure). Interactions between the grains and gas atoms then transfer some of the grain's momentum to the gas, thereby causing the mass loss. There is very little evidence for this scenario, however, and the models run to date are almost certainly wrong.

A convenient parameterization of the mass loss rate is the Reimers (1975) mass loss law

$$\dot{\mathcal{M}} \propto \frac{\mathcal{L}}{gR} \propto \frac{\mathcal{L}R}{\mathcal{M}} \quad (23.1)$$

Clearly, this formulation is reasonable: it says that the mass loss rate is directly proportional to luminosity (as you would expect if radiation pressure were involved), and inversely proportional to surface gravity (or, more precisely, the energy required to move mass to infinity). If \mathcal{M} , \mathcal{L} , and R are in solar units, then by scaling to the solar wind (and other stars), we get a constant of proportionality of $\sim 4 \times 10^{-13} \mathcal{M}_{\odot} \text{ yr}^{-1}$.

There are a number of alternatives to the Reimers law. For example, Volk & Kwok (1988) assumed that the mass-loss rate would also scale with the initial mass of the object

$$\dot{\mathcal{M}} = 1.8 \times \frac{\mathcal{M}_{\text{ZAMS}}}{8} \frac{\mathcal{L}R}{\mathcal{M}}$$

Bryan et al. (1990) when one step further, with

$$\dot{\mathcal{M}} = 1.5 \times 10^{-13} \eta \frac{\mathcal{L}R}{\mathcal{M}}$$

with

$$\eta = \mathcal{M}_{\text{ZAMS}}^2 - 10.6\mathcal{M}_{\text{ZAMS}} + 10.2$$

Alternatively, Schröder & Cuntz (2007) tried to take into account the impact of the mechanical energy associated with convection and various magnetic+accoustical waves. In their formulation

$$\dot{M} = \eta \frac{\mathcal{L}R}{\mathcal{M}} \cdot \left(\frac{T_{\text{eff}}}{4000} \right)^{3.5} \cdot \left(1 + \frac{g_{\odot}}{4300g} \right)$$

In general, the mass loss rates for most giant stars are very poorly known. In part, this is due to the difficulties in measuring accurate distances to giant stars. But it is also due to the problems of measuring the total mass of circumstellar envelopes. Thus, the error in the normalizations given above are at least a factor of a few. Note also that if grain formation is necessary for mass loss, then an additional factor should be included reflecting the inverse correlation between metallicity and grain density.

Mass loss rates for stars of $\sim 1\mathcal{M}_{\odot}$ can reach $\sim 10^{-8}\mathcal{M}_{\odot} \text{ yr}^{-1}$ at the tip of the giant branch. The total amount of mass lost during the giant branch phase can be $\sim 0.2 \mathcal{M}_{\odot}$. This rate is high enough so that a star at the tip of the giant branch will be surrounded by a circumstellar shell, which can redden and extinct the star. In the past, the geometry of this shell has been assumed to be spherical, but a large number of stars are now known to have circumstellar mass distributions that are bipolar. (This includes the ring around SN 1987A.)

Helium Burning in Higher Mass Stars

- When helium ignites on the giant branch under non-degenerate conditions, the immediate effect is to decrease the star's luminosity. Because the core has finite opacity, its nuclear energy is temporarily trapped; its temperature rises, and the region expands. This causes the temperature and density of the hydrogen burning shell to decrease. Since most of the star's luminosity is still coming from the shell, the luminosity of the star drops.
- Along with the drop in luminosity, the star also evolves blueward off the red giant branch. As the central core expands (due to the energy input from helium burning), the density and temperature in the hydrogen burning shell decreases. This decrease results in a decrease in the radiative support for the envelope, which causes the envelope to contract. The contracting envelope then maintains the energy generation of the shell by heating and compressing its hydrogen-rich matter.
- As the star evolves blueward, the increasing temperature in its interior decreases the opacity, and therefore decreases the size of the convective envelope. Between points 8 and 9, the envelope rapidly adjusts from convective to radiative energy transport.
- Because of its extreme temperature dependence, core ${}^4\text{He}$ burning always takes place in a small, convective region. This region will be a much smaller fraction of the star than the equivalent hydrogen core-burning region. Also, in contrast with the main-sequence core-burning, the size of the convective helium core increases (rather than decreases) with time.
- While helium is burning in the (convective) helium core, hydrogen is still burning in a shell around the (radiative) helium core. In fact, most of the star's luminosity still comes from the hydro-

gen shell. As the star evolves, the fraction of energy produced by He-burning will increase, but H-burning will still dominate. (Typically, helium will contribute somewhere between 6% and 50% of the star's luminosity.) Because the star has two sources of energy, fuel consumption arguments (such as those used to estimate main-sequence lifetimes) are difficult to apply.

- The dominant reaction will change with time; at first, most of the energy will be created via the triple-alpha process. However, as the abundance of ^{12}C increases, so will the importance of $^{12}\text{C}(^4\text{He}, \gamma)^{16}\text{O}$. The final $^{16}\text{O}/^{12}\text{C}$ and $^{20}\text{Ne}/^{16}\text{O}$ ratios depend on the stellar mass; higher mass stars burn helium at higher temperatures and thus have more ^{20}Ne and ^{16}O . Many applications assume the final $^{16}\text{O}/^{12}\text{C}$ ratio to be 50:50.

- As helium burning proceeds in the core, the abundance of ^{16}O increases due to the $^{12}\text{C}(^4\text{He}, \gamma)^{16}\text{O}$ reaction, which may, or may not come to equilibrium. At the same time, ^{18}O (which was formed from the burning of ^{14}N) reacts to form ^{22}Ne via $^{18}\text{O}(^4\text{He}, \gamma)^{22}\text{Ne}$. The equations governing the abundance of ^4He , ^{12}C , ^{16}O , and ^{20}Ne are straightforward.

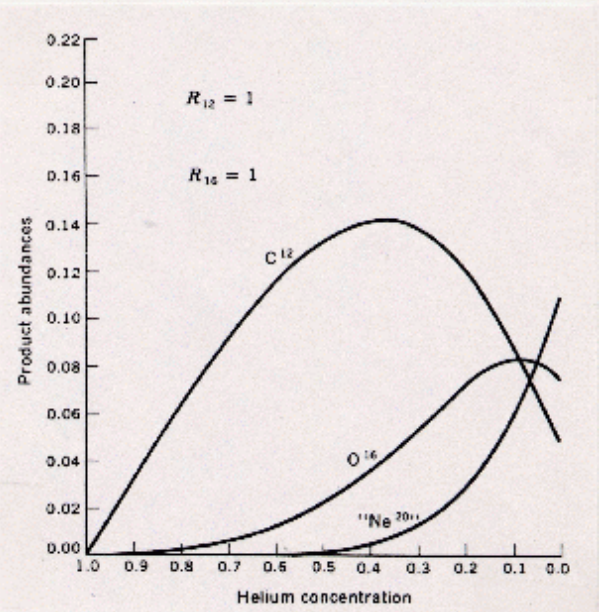
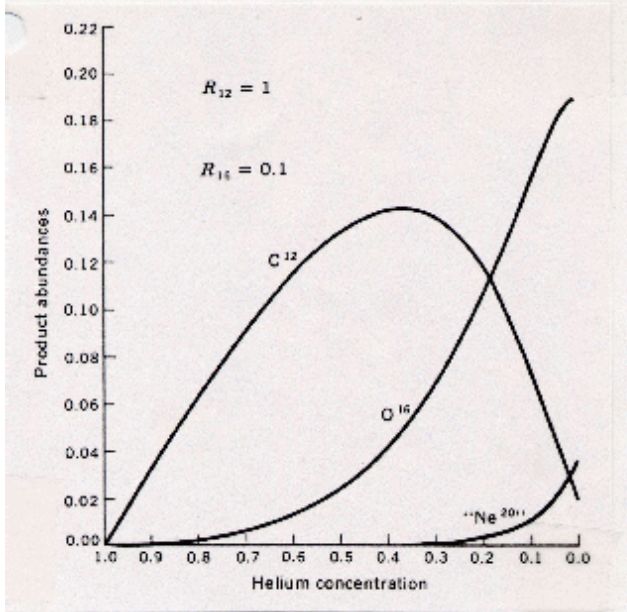
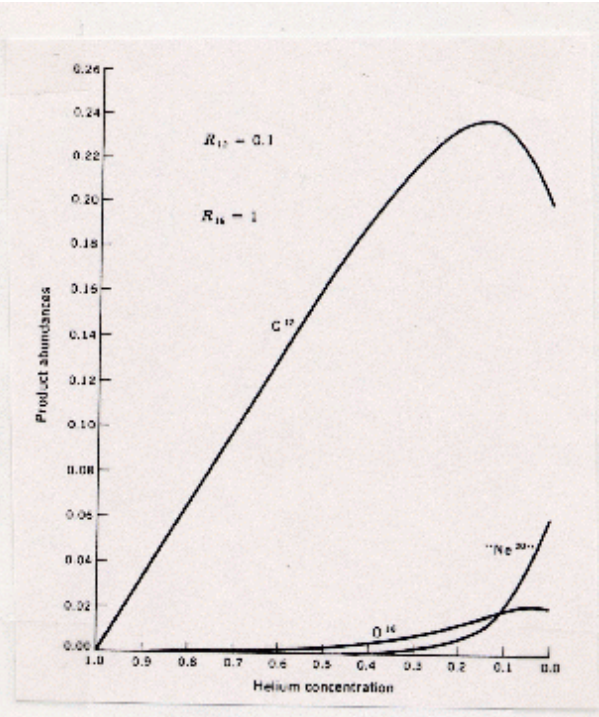
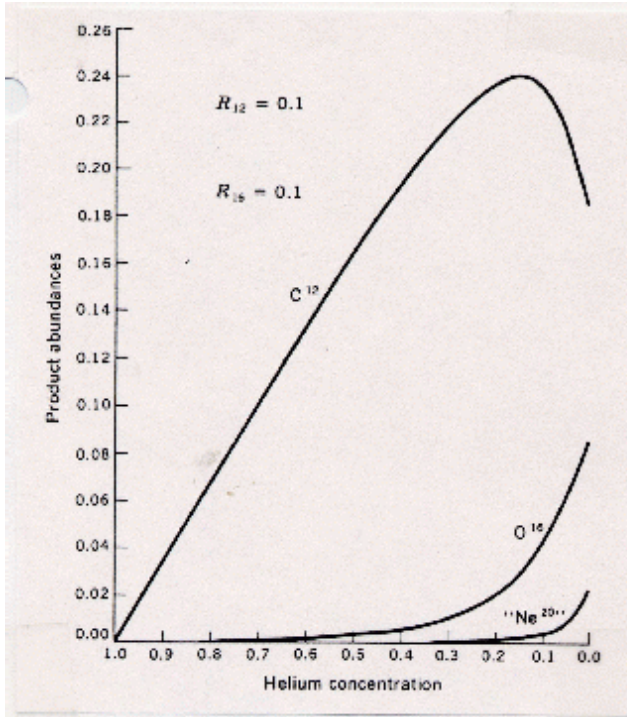
$$\frac{d^4\text{He}}{dt} = -3\lambda_{3\alpha}(^4\text{He})^3 - \lambda_{\alpha\text{C}}(^4\text{He})(^{12}\text{C}) - \lambda_{\alpha\text{O}}(^4\text{He})(^{16}\text{O})$$

$$\frac{d^{12}\text{C}}{dt} = 3\lambda_{3\alpha}(^4\text{He})^3 - \lambda_{\alpha\text{C}}(^4\text{He})(^{12}\text{C})$$

$$\frac{d^{16}\text{O}}{dt} = \lambda_{\alpha\text{C}}(^4\text{He})(^{12}\text{C}) - \lambda_{\alpha\text{O}}(^4\text{He})(^{16}\text{O})$$

$$\frac{d^{20}\text{Ne}}{dt} = \lambda_{\alpha\text{O}}(^4\text{He})(^{16}\text{O})$$

However, the results of these equations are sensitive to the (uncertain) values of the reaction rates.



The abundance of ^{12}C , ^{16}O , and ^{20}Ne as a function of helium consumed for different ratios of the reaction rates. These ratios are defined as

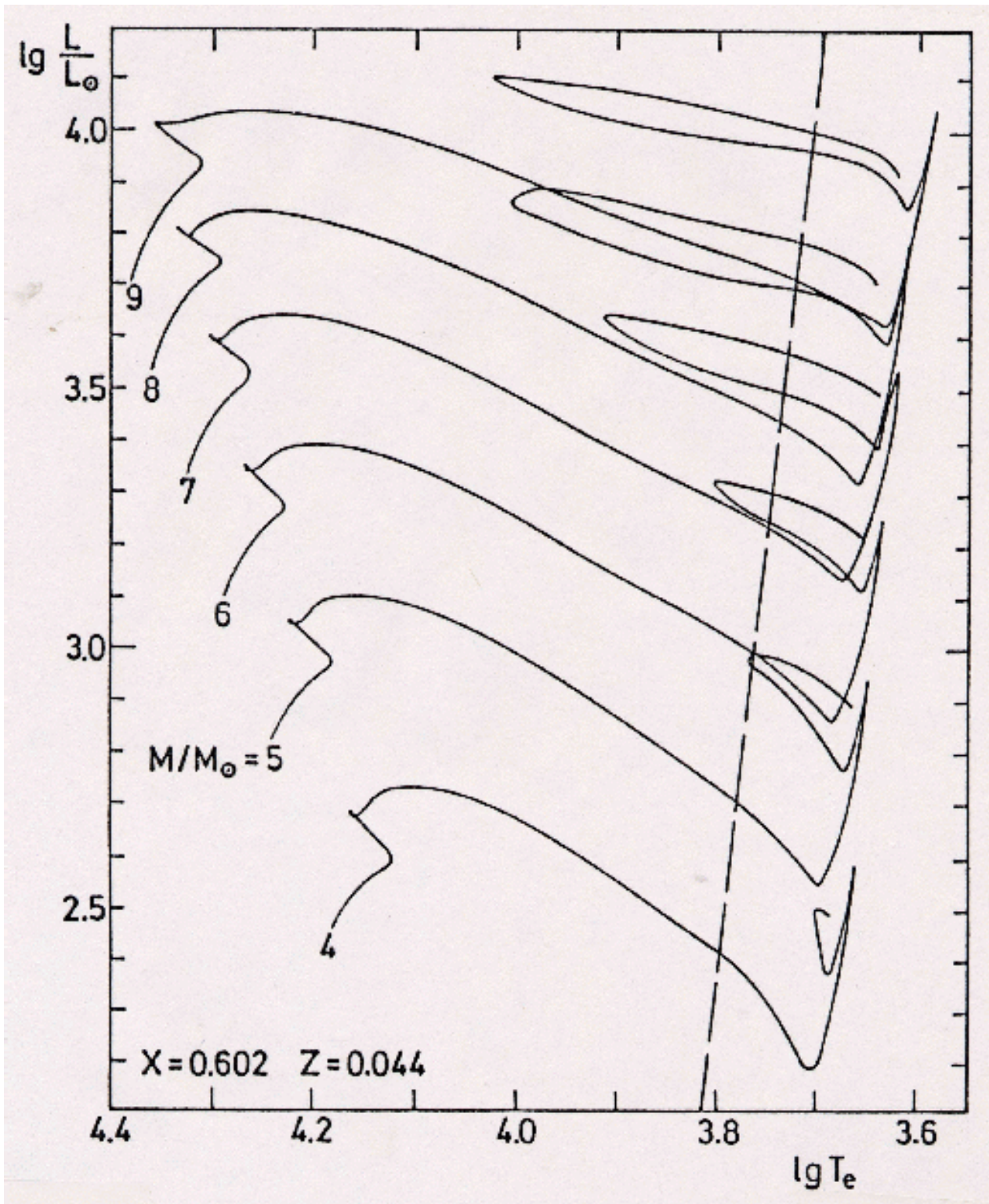
$$R_{12} = \frac{\lambda_{\alpha\text{C}}}{\lambda_{3\alpha}^4 \text{He}(0)} \quad R_{16} = \frac{\lambda_{\alpha\text{O}}}{\lambda_{3\alpha}^4 \text{He}(0)}$$

- During helium core burning, the radius of the helium core will remain approximately constant. The core will not contract until the helium fraction reaches $Y < 0.1$. Once it does, the star will restructure itself in a manner similar to that of the hydrogen-exhaustion phase. First, the stellar core will collapse, and a thick helium burning shell will form. The energy generated in this shell will push the adjacent material outward, thereby lowering the temperature and density of the hydrogen burning shell. As a result, the outer envelope will contract and the star will move blueward in the HR diagram. Shortly thereafter hydrogen burning will be extinguished, as the material becomes too diffuse to support fusion. When this happens, the outer envelope will expand again (driven by the helium shell luminosity), and the star will move back to the Hayashi line.

- During their post red-giant branch evolution, stars between 3 and $10 \mathcal{M}_{\odot}$ perform “loops” in the HR diagram. The size and timescale of the loop depends on the mass; high mass stars are more luminous, evolve more quickly, and have larger loops. Because this stage of evolution occurs on a nuclear timescale, there is a much higher probability of finding a star in these loops than finding a star in the Hertzsprung Gap. The precise nature of the loops, however, depend critically on the opacity, energy generation, metallicity, and theory of convection. Consequently, the modeling of these loops is extremely uncertain.

- The post red-giant branch loops go through the instability strip. Stars in the strip pulsate with periods of days or weeks (depending on their luminosity). These are δ Cepheid variables.

- Stars with $\mathcal{M} > 10 \mathcal{M}_{\odot}$ ignite helium before reaching the Hayashi line. Their evolution in the HR diagram is qualitatively different from that of lower mass stars, though their internal development is the same.



The evolution of high-mass stars in the HR diagram

Horizontal Branch Evolution

Low mass stars (those that undergo a helium flash) quickly change their structure and land on the zero-age horizontal branch (ZAHB). These stars burn helium to carbon non-degenerately in their core and also have a hydrogen-burning shell.

- Because of the large luminosity associated with helium burning, the central regions of horizontal branch stars are convective. A star with a helium core of $\sim 0.45 \mathcal{M}_\odot$ will have a convective helium-burning core of $\sim 0.1 \mathcal{M}_\odot$. The mass of this convective core depends only on the total mass of the helium core; it is not sensitive to the star's metallicity, total mass, or initial helium abundance. As the star evolves, the helium burning fraction of the core moves outward, and due to convective overshooting, the core will develop semi-convection at its boundary.

- Stars with large helium cores have larger convective core-burning regions. However, because an increase in the mass of the helium core strongly enhances the rate of helium burning, the lifetime of a horizontal branch star is inversely proportional to core mass. To first order, the lifetime of a horizontal branch star (in millions of years) is

$$\begin{aligned} \log t_{\text{HB}} &\approx 1.74 - 2.2(\mathcal{M}_c - 0.5) + \log f \\ &\approx 1.795 + 0.506(Y - 0.3) + 0.022(\log Z + 3) + \\ &\quad 0.077(\mathcal{M} - 0.8) + \log f \end{aligned}$$

where f is a factor that compensates for uncertainties in the handling of convective overshooting and semi-convection, $f \sim 2$. Note that the lifetime is entirely determined by the core mass (since this defines the amount of fuel available); the dependences on Y , Z , and \mathcal{M} come from the size of the helium core at helium ignition.

- The effective temperature of a ZAHB star depends principally on its envelope mass (especially when the mass of the envelope is small). Stars whose envelope mass is only $\sim 0.02 \mathcal{M}_{\odot}$ will be extremely blue, with $\log T_{\text{eff}} \gtrsim 4.3$. Stars with large envelope masses ($\sim 0.4 \mathcal{M}_{\odot}$) appear near the base of the red giant branch. The star's total mass or core mass does *not* affect the star's effective temperature, except through determining the envelope mass.
- Both helium-core burning and hydrogen shell-burning contribute to the luminosity of a horizontal branch star. In stars with small envelope masses, the luminosity from helium core-burning dominates; in those stars, the initial metallicity and helium abundance do not make any difference to the star's evolution. In stars with large envelope masses, the two luminosity sources are comparable.
- If the envelope mass of the star is large, the evolutionary timescale will be sensitive to the initial helium abundance. Larger values of Y cause the hydrogen shell to move outward quicker, depositing more mass on the helium core. Moreover, since the mass of the convective core depends on the helium core mass, the increased shell burning also increases the rate of core burning. The net result is stronger luminosity evolution, and a shorter evolutionary timescale.
- A larger total mass acts the same way as increased helium abundance: the hydrogen shell burns more rapidly, and this propagates into a larger core-burning luminosity.
- Higher helium abundances also cause the effective temperature of the star to increase by effectively moving the hydrogen burning shell closer to the surface.
- Globular clusters with extremely similar turnoff magnitudes can have very different structures to their horizontal branch. This is the “second parameter” problem. Presumably, clusters with many

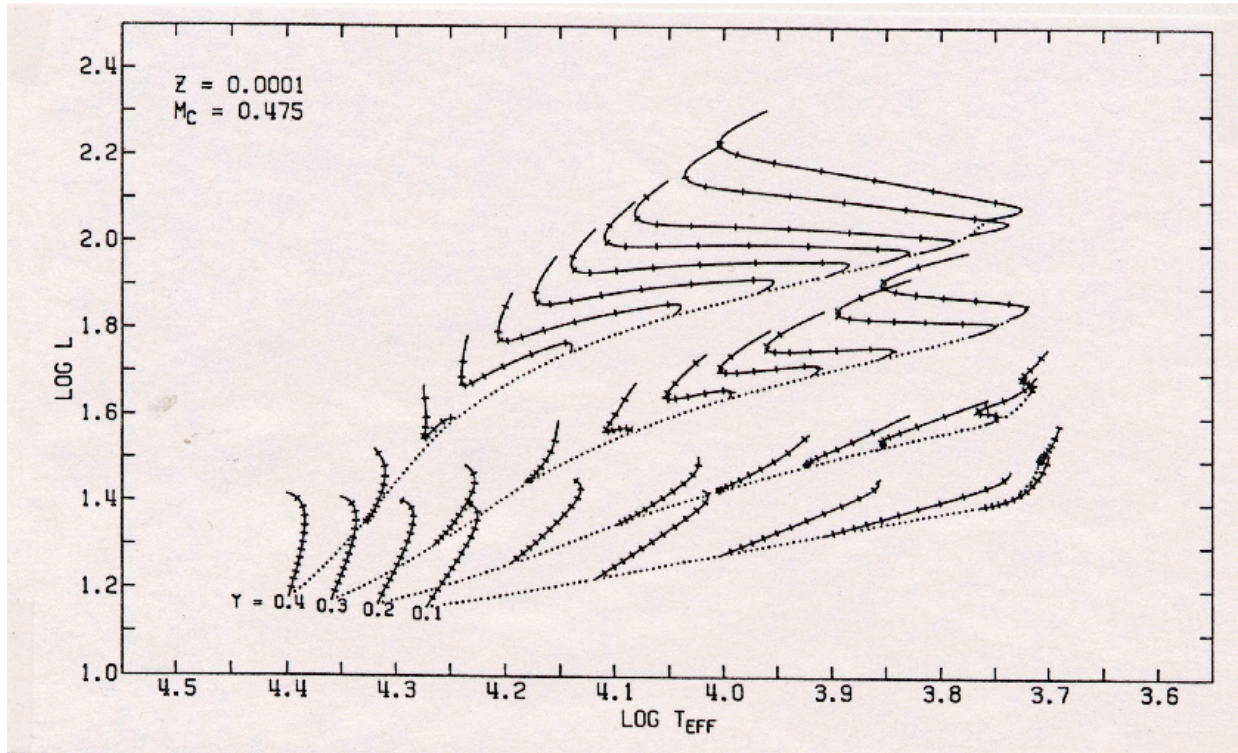
blue horizontal branch stars are dominated by stars with small envelopes, while clusters whose horizontal branch stars are in a “red clump” have large envelope mass stars. Are the core masses different due to age or metallicity? Are the total stellar masses different due to a different giant branch mass loss law? Is there something else at work?

- The “Instability Strip” passes through the horizontal branch. Stars within the strip pulsate with period between 12 and 24 hours, and are called RR Lyr stars. Because the structure of the horizontal branch can differ dramatically from cluster to cluster, the (luminosity specific) number of RR Lyr stars in a globular cluster can change by an order of magnitude, or more.

- In the observable HR diagram (*i.e.*, V vs. $B-V$), the extreme blue end of the horizontal branch turns downward, and becomes almost vertical. This is mostly a bolometric correct effect; for stars with extremely small envelope masses, most of the luminosity comes out in the ultraviolet.

- Stars with *extremely* small envelope masses may run out of hydrogen in their shell before helium core burning is completed. When this happens, the star can evolve directly from the blue end of the horizontal branch to the white dwarf phase. There is some evidence that these *AGB manqué* stars exist in some old, metal-rich open clusters.

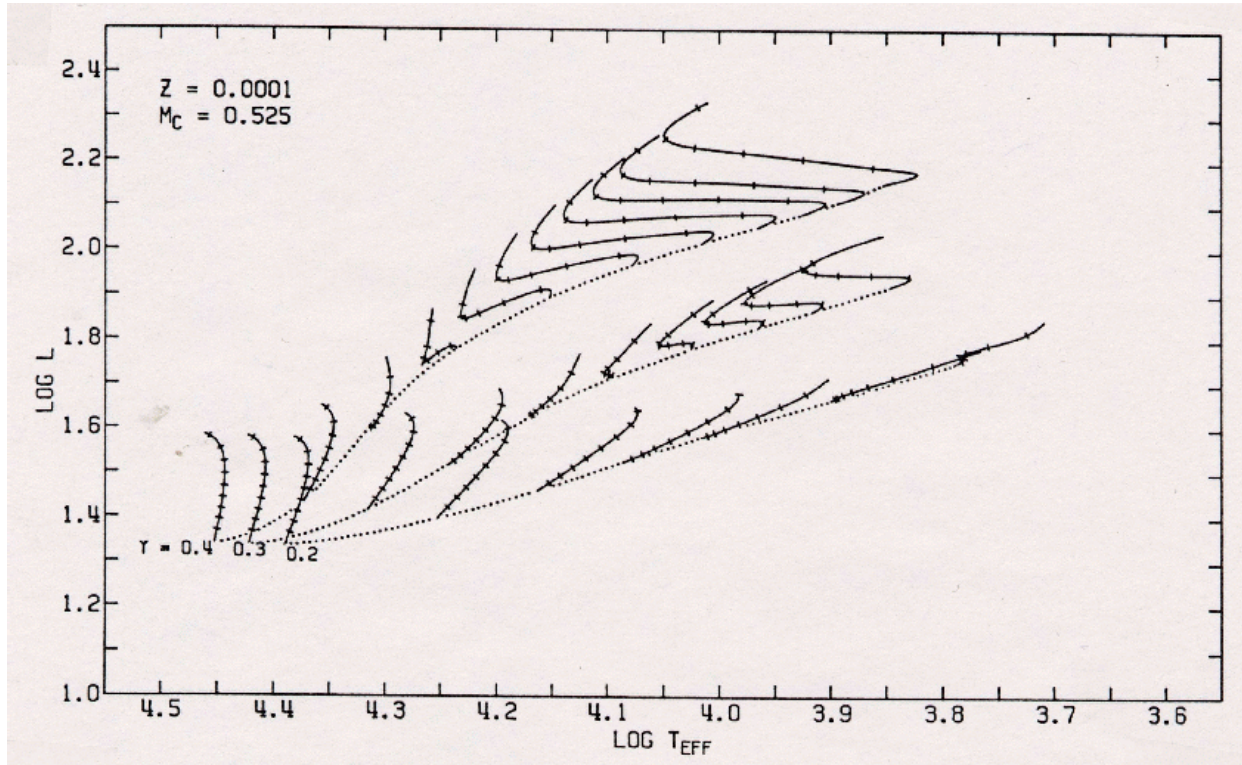
- When core helium burning ceases, the central core of the star will contract, and helium will begin burning in a thick shell. The luminosity from this shell will cause the region outside of it to expand. When this happens, the temperature and density of the hydrogen shell will decrease, and hydrogen burning will be extinguished.



Horizontal branch mass tracks for

Y	M_T/M_\odot
0.10	0.50, 0.54, 0.58, 0.62, 0.66, 0.90
0.20	0.50, 0.54, 0.58, 0.62, 0.66, 0.74, 0.90
0.30	0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.90
0.40	0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.82 0.90, 1.02

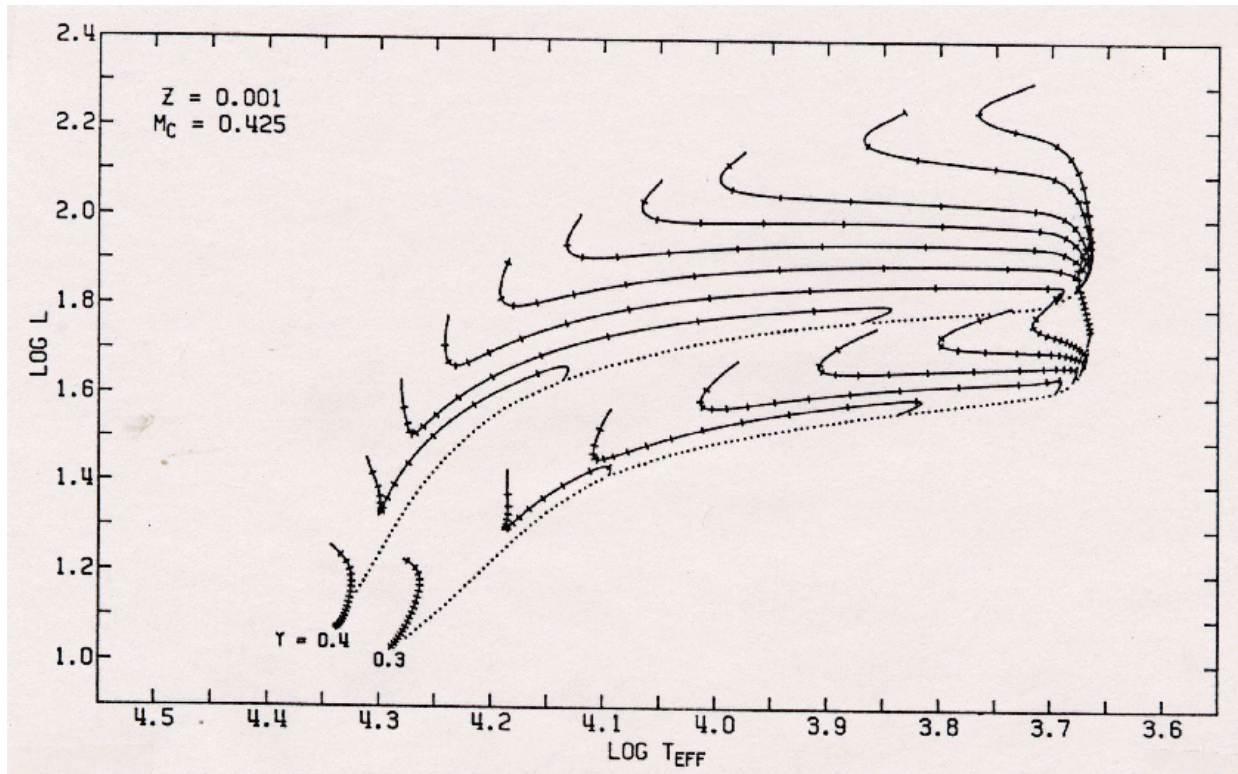
Tick marks are 10^7 years.



Horizontal branch mass tracks for

Y	M_T/M_\odot
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0.20	0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.90
0.30	0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.82, 0.90
0.40	0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.82, 0.86 0.90, 0.94, 0.98, 1.06

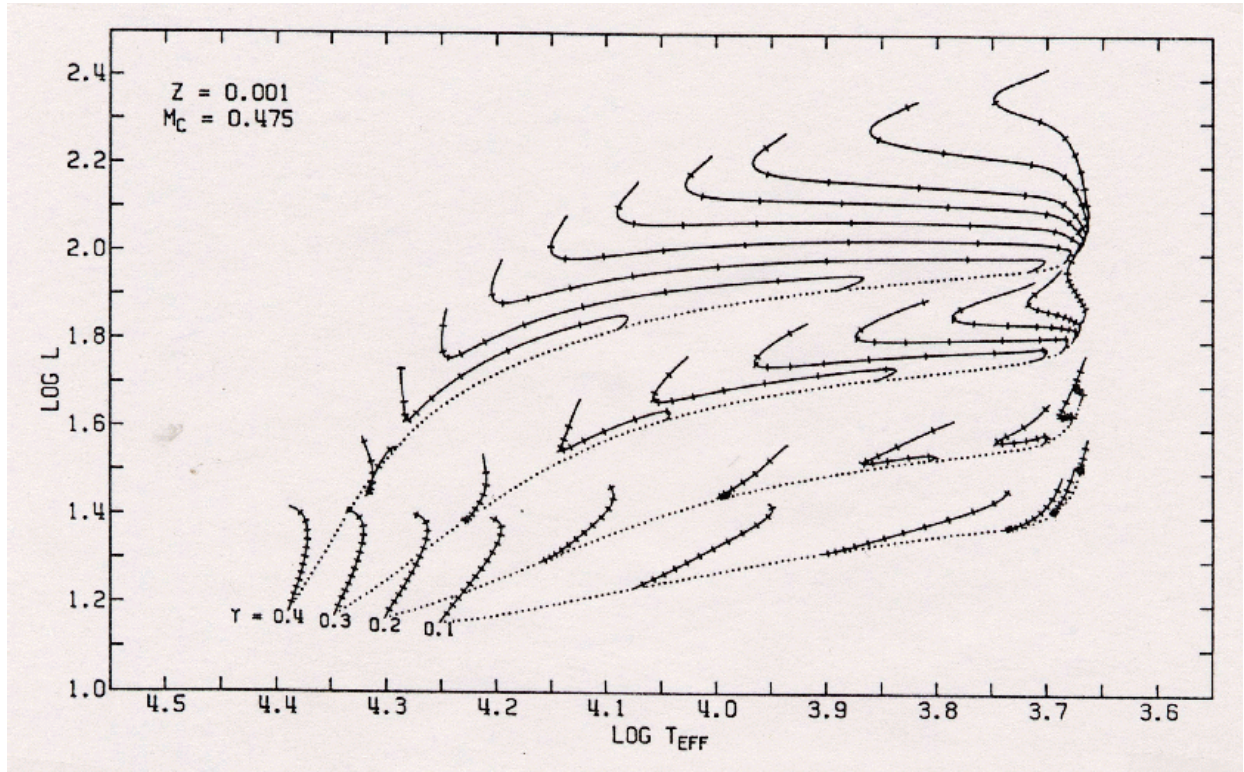
Tick marks are 10^7 years.



Horizontal branch mass tracks for

Y	M_T/M_\odot
0.10	0.50, 0.54, 0.58, 0.62, 0.70, 0.90
0.20	0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.78, 0.90
0.30	0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.82, 0.90
0.40	0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.82 0.90, 0.98

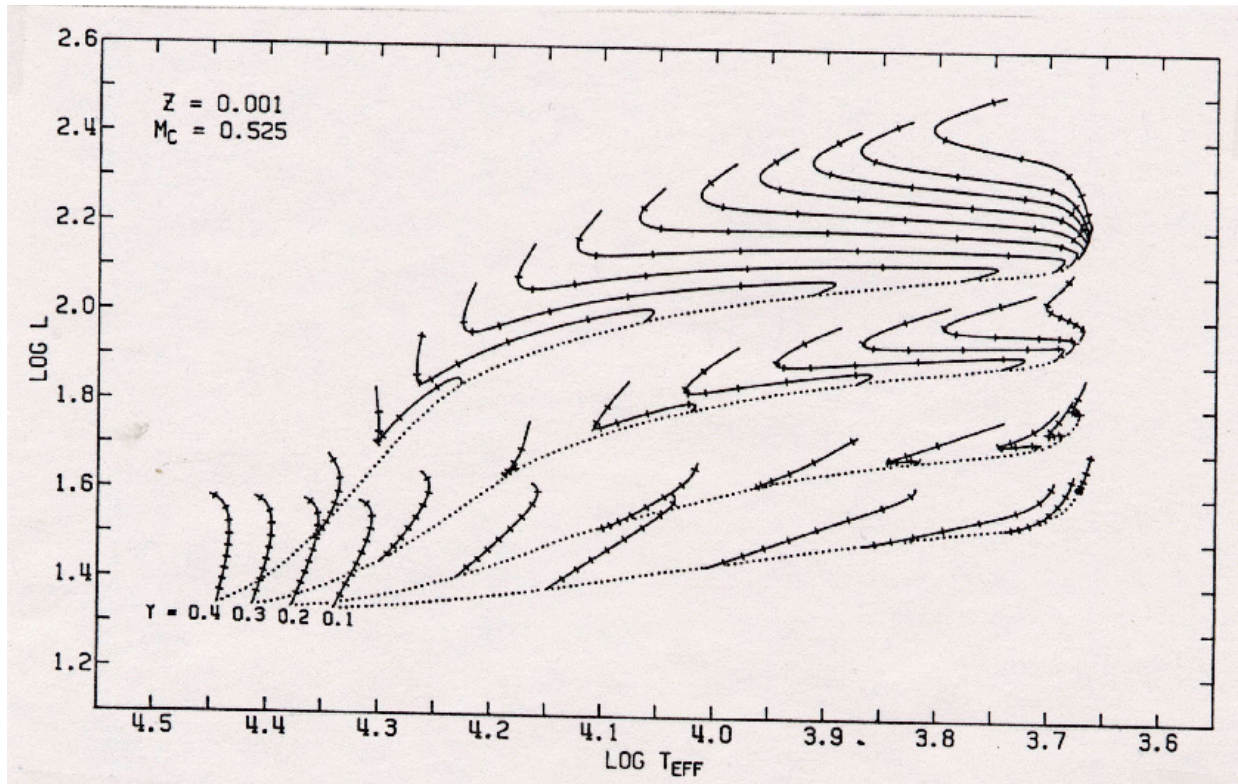
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Horizontal branch mass tracks for

Y	M_T/M_\odot
0.20	0.54, 0.58, 0.62, 0.66, 0.70, 0.78, 0.90
0.30	0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.82, 0.90
0.40	0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.78, 0.82, 0.86 0.90, 0.98

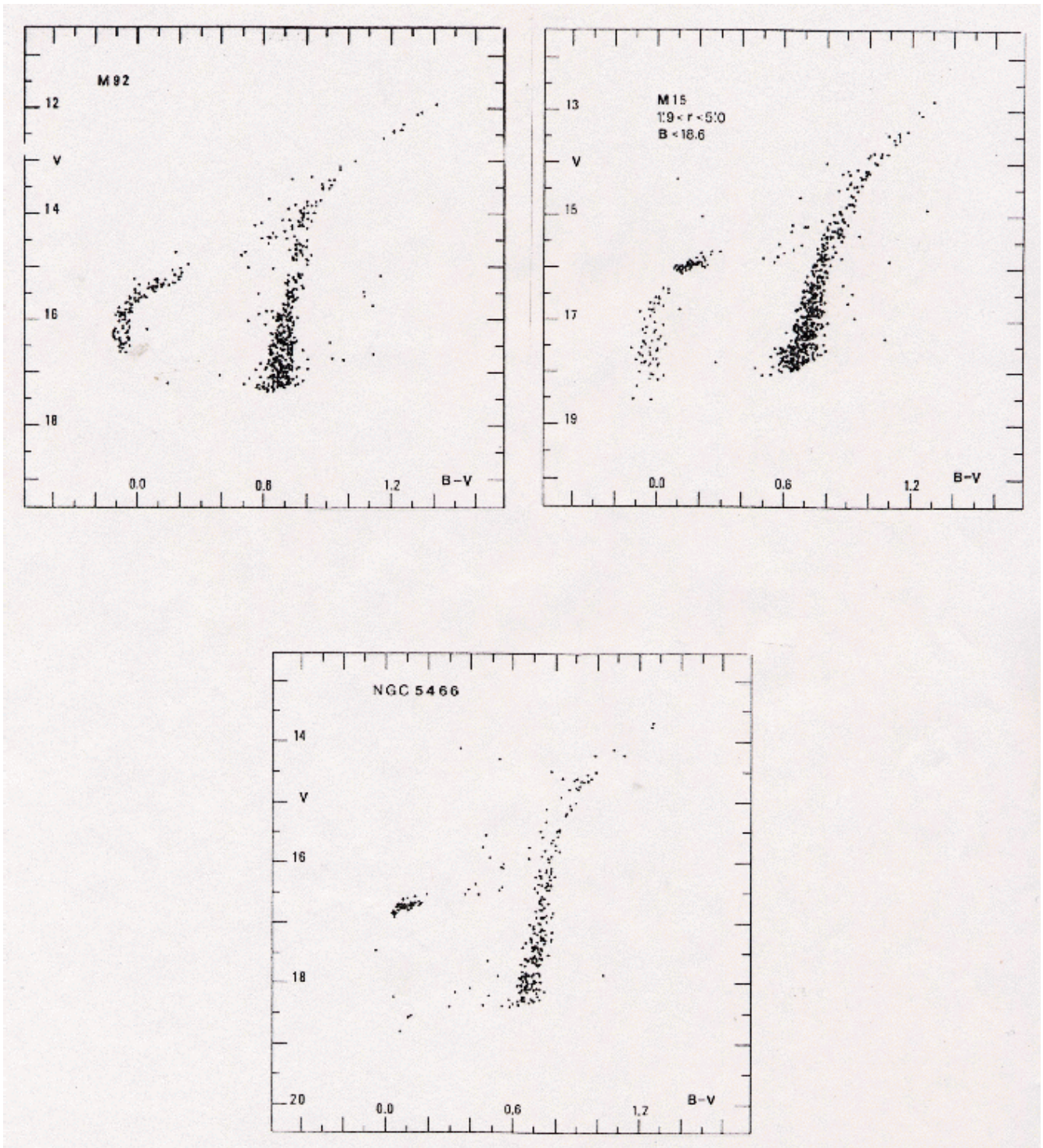
Tick marks are 10^7 years.



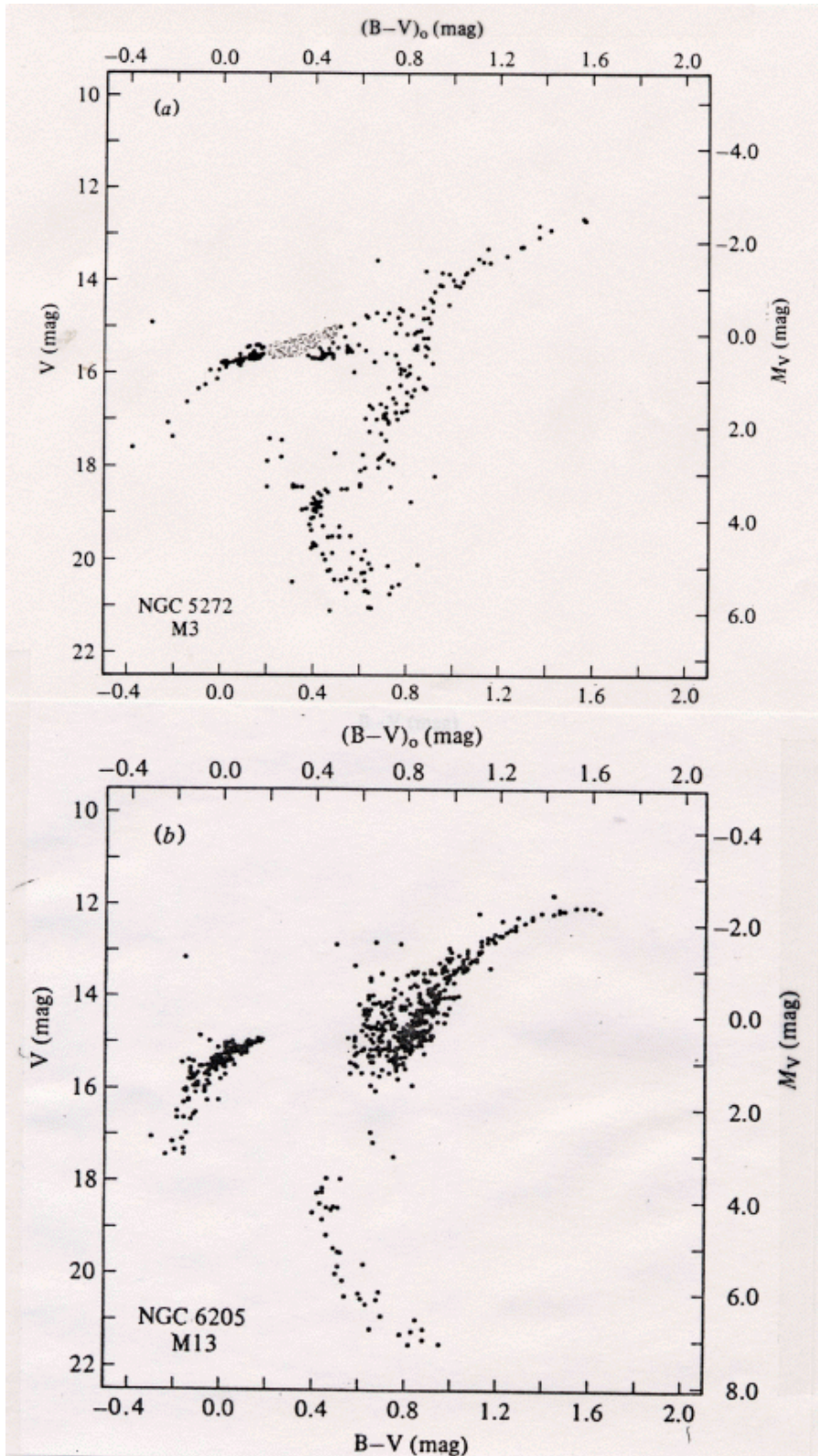
Horizontal branch mass tracks for

Y	\mathcal{M}_T/M_{\odot}
0.30	0.46, 0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.90
0.40	0.46, 0.50, 0.54, 0.58, 0.62, 0.66, 0.70, 0.74, 0.82 0.90

Tick marks are 10^7 years.



HR diagrams for 3 globular clusters. Not the differences in the horizontal branch.



HR diagrams for 2 globular clusters, with RR Lyrae stars plotted as small points. Note the vast difference in the number of RR Lyrae stars in the two clusters.