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Seminar (UNI)

Reactivity measurements

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Abstract

In the seminar, measurements of the reactivity of nuclear reactors is presented. The theory of reactor kinetics is summarized with key problems exposed. These are then illustrated through the description of a few of the main reactivity worth measurement methods for power reactors.

Contents

1 Introduction

Nuclear power plants use nuclear fission to produce a considerable ammount of energy. A nuclear reactor (if we seek the simplest explanation) is sought to be as much a radiation-proof box as reasonablly achievable, where fision chain reaction could be controled and energy extracted. Since reactivity is a crucial reactor parameter that indicates changes in the output power, it is very important to have reliable methods to measure it.

2 Nuclear physics basics

2.1 Nuclear chain reaction

Heavy nuclei can be split into two or more fragments, which - as a result of the mass defect - together bind more energy than the sole heavy nuclei in the begining, and this energy is released during fission. This occurs naturaly (*spontaneous fission*) but can also be artificially induced by neutrons (*induced nuclear fission*). In addition to early mentioned nuclear fragments, fission also produces two or three new neutrons. On average we get ν neutrons per fission, ν having different values for different fission processes and for different incident neutron energies [1] (e.g. $\nu = 2.418$ for ^{235}U being split by a neutron with energy 0.0253 eV). These neutrons can be used to induce new fission reactions and thus start and maintain a *nuclear chain reaction*.

2.2 Neutron flux

When describing neutrons we use the quantity *neutron flux*, defined by the equation

$$\phi(\mathbf{r}, t) = v\, N(\mathbf{r}, t)\,. \tag{1}$$

With v being neutron speed and N their number density, the neutron flux gives the number of neutrons passing through a unit area in a time interval (neutron flux is a scalar quantity). The direction of passage is irrelevant - all directions are included.

It is wise to mention that both *neutron flux* $\phi(\mathbf{r}, t)$ and the *angular neutron flux*, $\varphi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) = v\, n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)$, exist as a quantity. Their symbols, ϕ and φ, however, are often misused. They can and should be told apart by angular dependence on $\hat{\mathbf{\Omega}}$ [2]. To further clarify: $\phi(\mathbf{r}, t) = \int \mathrm{d}E\, \phi(\mathbf{r}, E, t) = = \int \mathrm{d}E \int \mathrm{d}\hat{\mathbf{\Omega}}\, \varphi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)$, integrated over all E and $\hat{\mathbf{\Omega}}$.

On average, fission neutrons are "born" with energy of 2 MeV. But before they could induce fission on ^{235}U with considerable probability, they have to be slowed down (*i.e. moderated*) to lower energies. Regarding the neutron kinetic energy we therefore subdivide the neutron flux into three energy groups: *thermal* ($E < 1$ eV), *epithermal* (1 eV $< E <$ 100 keV) and *fast* neutron flux ($E >$ 100 keV).

Another key term has to be considered, namely the *delayed neutrons*. In contrast to *prompt neutrons*, which are produced at the very instant of nuclear fission event, it takes some time for the delayed ones to be emmited from certain fission fragments. These are called the delayed neutron *precursors*. Moreover, this delay is what makes the chain reaction control practically feasible [1].

2.3 Reactivity

Neutrons have already been introduced as a medium of maintaining the nuclear chain reaction. If their number is growing, then the frequency of nuclear fission events will also be increasing. Accordingly, if neutron numbers are in decline the chain reaction is fading. The quotient

$$k = \frac{\text{number of neutrons in generation } i+1}{\text{number of neutrons in preceeding generation } i} \tag{2}$$

is called *the multiplication factor*. Although counting the number of neutrons in a reactor is by all means impossible, this formulation can be used to further examine the time evolution of the neutron population (see text below eq. 4).

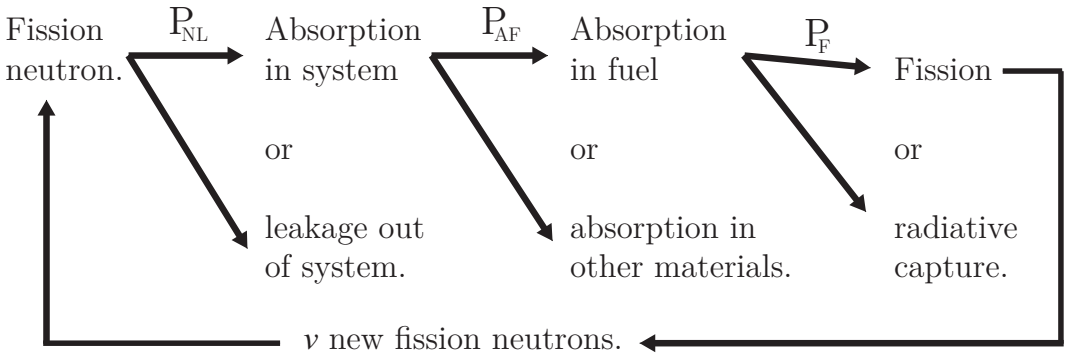

Figure 1: The scheme shows the possible events in the life of a neutron. Gradual approach suggests the use of conditional probability. Figure redrawn following [2].

Let us briefly forget about the delayed neutrons and have N_i neutrons at a certain time forming one generation. It can be schematically shown (figure 1) that the number of neutrons in the next generation, N_{i+1}, depends on a series of (conditional) probabilities [2]. A newly fission-born neutron is either absorbed in the reactor or leaks out of it. If absorbed in some other material than fission fuel, the neutron also does not contribute to the chain reaction. Fuel-absorbed neutron can cause fission or undergo radiative capture with subsequent gamma ray emission. But when a neutron causes fission, ν new fission neutrons are genereated (on average). The multiplication factor $k = N_{i+1}/N_i$ can be therefore rewritten as

$$k = \nu \, P_F \, P_{AF} \, P_{NL} \, , \tag{3}$$

where the factors are [2]:

P_{NL} - probability, that neutron will not leak out of system before absorption (*"non leakage"*);
P_{AF} - conditional probability that *if* neutron is absorbed, it will be absorbed in the fuel (*"absorption fuel"*);
P_F - conditional probability that *if* neutron is absorbed in fuel, it will induce a fission reaction (*"fission"*); and
ν - average number of new neutrons per fission.

Finally, we introduce the term *reactivity* through the equation

$$\rho = \frac{k-1}{k}\,. \tag{4}$$

We see that a negative reactivity, $\rho < 0$, indicates that in such a *subcritical* reactor the chain reaction is going to die out, in a *supercritical* reactor the reactivity is positive, $\rho > 0$, and the frequency of nuclear fission events grows exponentially, whereas a *critical* reactor with $\rho = 0$ operates at stationary (time independent) conditions. This types of behaviour can be observed in figure 2. Due to great impact on the reactor power, reactivity is measured in units of *pcm* (*per cent mille*, 10^{-5}).

For nuclear chain reaction control, various techniques are used, (e.g control rods, boration/dilution), but they all impact the same value of P_{AF}. For predictable results, their *reactivity worth* (impact on reactivity) needs to be measured.

3 Measurement of reactivity

The power output of a nuclear reactor is directly proportional to the number of neutrons and thus to the neutron flux. Usually the neutron flux is measured outside of the reactor core with an *ionisation chamber*. The latter detects ionised particles [3], so a type of conversion is needed inside the detector [4]. The two main reactions of converting neutrons to charged particles used are

$$\mathrm{n} + {}^{3}\mathrm{He} \rightarrow {}^{3}\mathrm{H} + {}^{1}\mathrm{H} \qquad \text{and} \tag{5a}$$

$$\mathrm{n} + {}^{10}\mathrm{B} \rightarrow {}^{7}\mathrm{Li} + {}^{4}\mathrm{He}\,. \tag{5b}$$

A similar neutron flux detector is a *fission chamber*. This is an ionisation chamber containing a layer of fissionable material (e.g. ^{235}U), where the neutron causes fission [5]. Again, charged fission fragments are detected. A fission chamber can be miniaturised for the use of neutron flux measurement inside the reactor core.

3.1 The point kinetics equations

Mathematically we describe the time (and other) dependence of neutron population with the so called *transport equation*, which is basically an equation of continuity for neutron flux [6, 7]. There can be no change in the number of neutrons in a unit volume other than that resulting from absorption, leakage, scattering or birth from some neutron sources. This exact equation is very difficult to solve, so we make approximations according to what needs to be calculated. Splitting the neutron flux's time and space dependence, $\phi = S(\mathbf{r}, E) \cdot T(t)$, leaves us - the extensive derivation can be found in [7] - with *the point kinetics equations* describing the time dependence of the neutron flux:

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = \frac{\rho - \beta}{\Lambda} T(t) + \sum_{i}^{I} \lambda_i C_i(t) + Q(t) \qquad \text{and} \tag{6a}$$

$$\frac{\mathrm{d}C_i(t)}{\mathrm{d}t} = \frac{\beta_i}{\Lambda} T(t) - \lambda_i C_i(t)\,, \qquad (i = 1,\, 2,\, \ldots I)\,. \tag{6b}$$

The quantities appearing can be summarized as follows:

$T(t)$	- neutron flux amplitude function;
$\rho(t)$	- reactivity (as a function of time, t);
β	- total average delayed neutrons fraction ($\beta = \sum_{i=1}^{I} \beta_i$);
Λ	- average (prompt) neutron lifetime;
λ_i	- decay constant of i^{th} group of delayed neutrons;
β_i	- average delayed neutron fraction of i^{th} group of delayed neutrons;
$C_i(t)$	- average delayed neutron precursor concentration of the i^{th} group;
$Q(t)$	- average neutron source magnitude (*the source term*).

The equation 6a represents the time evolution of the neutron flux amplitude function T (and thus reactor power P), while the equations 6b do the same for concentrations of delayed neutron precursors C_i. $I = 6$ groups of delayed neutrons are taken, distinguished by their decay constants λ_i. Table 1 gives summary of typical λ_i and β_i values. Recall that even though their total fraction has a value below one percent, $\beta = 0.0065$, the delayed neutrons enable us to control the chain reaction! Such is their importance.

If we neglect the source term $Q(t)$ in the equation 6a, we get a homogeneous system of equations. If then the reactivity ρ is constant, the system is solved with a solution in the form of

$$T(t) \propto \exp(t/\tau)\,, \tag{7}$$

which describes asimptotic behaviour of reactor power, illustrated in figure 2. The figure 3 shows detailed transiental behaviour, the so-called *promt*

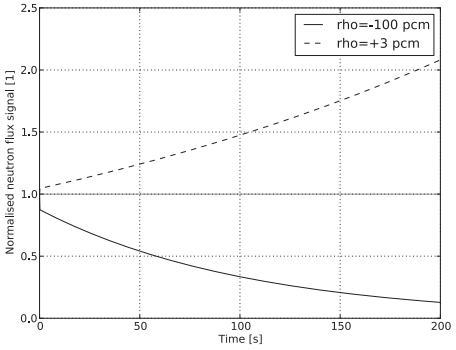

Figure 2: Figure shows a tipical (exponential) reactor power time evolution in a specific case. The plot was obtained by a calculation using solution to the point kinetics equations.

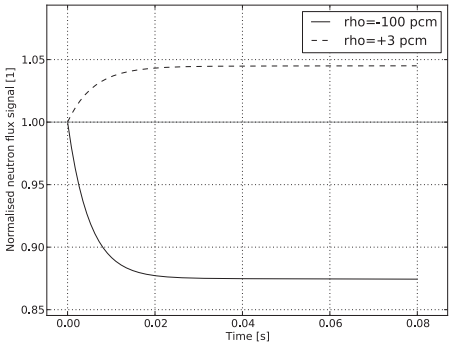

Figure 3: The detail from figure 2 shows the prompt jump/drop in reactor power. In both cases the reactivity changed from $\rho = 0$ instantaneously at time $t = 0$.

jump and *prompt drop*. The source term can be neglected only if the nuclear reactor operates at high or full power.

Note that in the point kinetics equations (6) the space dependence is totally neglected (reactor is regarded as a whole; hence the name *point* kinetics). Calculations would therefore be absolutely valid only if there would be no change in the relative neutron flux distribution. In general this is not true, but deviations are usually small and so the results are taken to be sufficiently accurate.

Group	Mean life [s]	Decay constant λ_i [s^{-1}]	Energy [keV]	fraction β_i
1	55.72	0.0124	250	0.000215
2	22.72	0.0305	560	0.001424
3	6.22	0.111	405	0.001274
4	2.30	0.301	450	0.002568
5	0.610	1.140	410	0.000748
6	0.230	3.01	-	0.000273

Table 1: The delayed neutrons data for ^{235}U thermal fission [1]. Data for more than twenty precursors is averaged into six "equivalent" groups [7]. Overall delayed neutron abundance is 0.0158 delayed neutrons per fission. The total delayed neutron fraction ($\beta = \sum_{i=1}^{I} \beta_i$) value is as low as $\beta = 0.0065$, but again: it enables the chain reaction control.

3.2 Neutron flux redistribution factors

Control rods are *by definition* a big neutron absorber, meaning that they have decisive impact on the neutron flux spatial distribution. This is especially true for small reactors, such as "Jozef Stefan" Institute's (JSI) TRIGA nuclear reactor, where the four control rods (labels T, S, R and C on figure 4) represent the main chain reaction control mechanism [8]. Similarly the control rods also interact betwen each other, the phenomenon beign known as *the shadow effect* [1].

While measuring the neutron flux in a detector at a certain point outside the reactor core, we would not mind if the neutron flux imbalances could somehow be taken into account and eliminated. This is achieved using the neutron flux *redistribution factors*. In the ideal case the signal $T(x)$ on the detector at a place x is directly proportional to the nuclear reactor power P:

$$P(t) = K_x T(t, x)\,, \tag{8}$$

K_x being a predetermined coefficient ($K_V \approx 7.03 \pm 0.16$ kW/μA) [8]. Focusing on a certain detector - let it be the detector V - the notation for spatial dependence on x is hereinafter omitted. If the control rod C is inserted to a certain degree, the detector V will give us lower neutron flux signal than the detector L. This undervalued shaded signal T_m is then adjusted with

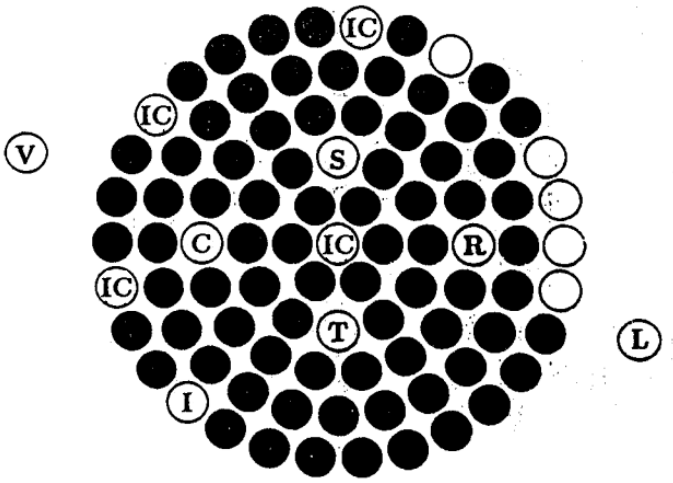

Figure 4: Diagram of the TRIGA reactor core at JSI. Control rod positions are T, S, R and C. Locations of fuel elements are black, dummy graphite elements are left blank, I is the neutron source, while positions marked with IC represent iradiation channels. V and L are the detectors. Figure taken from reference [8].

perturbation factors p_Y according to the equation

$$T(t) = \frac{T_m(t)}{\prod_Y p_Y}\,, \tag{9}$$

where the symbol Y stands for a certain control rod symbol. We then write

$$p_Y(l) = 1 + (f_Y - 1)\, g_Y(l)\,, \tag{10}$$

using the *flux redistribution factor* f_Y as a measure of absolute magnitude of correction. It is defined as a ratio of thermal neutron flux for fully inserted control rod and that of a core where all rods are withdrawn, $f_Y = F_Y/F_0$. The interpolation function $g_Y(l)$ then gives the insertion depth (l) dependence and tells whether the correction is to be used or not: $g_Y \in [0, 1]$. The values in between are calculated from experience-and-measurements based [8] equation

$$g_Y(l) = \frac{\rho(l)}{W_Y}\,, \tag{11}$$

which is a ratio between current control rod induced reactivity $\rho(l)$ and its *total* (i.e. integrated) *reactivity worth* W_Y. Previously omitted spatial dependence notation implies that flux redistribution factors f_Y need to be known for each detector (and, of course, for each control rod).

The described procedure ensures sufficiently enough that the signals from neutron detectors are indeed propotional to reactor power throughout the reactivity measurements.

4 Reactivity measurement methods

Suppose that we have a reliable and a quick way to calculate reactivity out of neutron flux time evolution, we can now finally measure the change in reactivity if we move the control rod a few steps (i.e. measure its reactivity worth).

Reactivity can be determined simply from the exponential behaviour of the neutron flux (see figure 2) by measuring the neutron flux *dubling time* using a common stopwatch. Indeed it was done so in the past, but the results we not quite precise [9]. Developement of a compact digital processing system named the *Digital Meter of Reactivity* (or shortly *DMR*) greatly simplified reactivity worth measurements and also made way for a new reactivity worth measurement method, the *rod-insertion method* discussed below [9, 10]. DRM calculates reactivity with greatly improoved precision from the

point kinetics equations (6), including multiple delayed neutron groups and the (otherwise normally neglected) source term $Q(t)$.

In the following text few of the main methods of measuring the control rod reactivity worth are listed and explained. Optionally, reactivity of a certain reactor core (and control rod) configuration can be calculated using simulation by the *Monte Carlo method* [11], which will not be explained here due to the seminar's lenght limitations.

4.1 Rod swap

Rod swap is perhaps the most intuitive mehtod of reactivity worth measurement. We start with a critical ractor, where all but one control rods are fully withdrawn out of the reactor core. Criticality is achieved using the remaining control rod which is to have its reactivity worth measured (*measured rod*).

By moving a reference control rod a few steps into the reactor core, an amount of negative reactivity is inserted and the neutron flux signal begins its exponential decline. Now the measured rod is withdrawn until reactor becomes slightly supercritical and the value of reactivity is again recorded. The two rods are then swapped step by step in a previously described manner, yielding a graph of differential and integral reactivity worth as a function of measured rod position. By exchange of positive and negative reactivity the nuclear reactor is kept at a relatively constant power.

This method encounters a problem, if the *excess reactivity* (reactivity that needs to be compensated when the reactor is not operational, and all control rods are withdrawn) is to low. In such case the initial criticality is achieved by slight withdrawal of the measured rod, so its reactivity worth can not be measured for its full lenght.

4.2 Boron exchange

Chemical element boron, especially ^{10}B, is a significant neutron absorber and is used in the control rods. It can also be used in the form of water solution (supposing water is the main reactor collant). Following this method, one starts with a critical reactor with all rods withdrawn and the criticality achieved with boric acid water solution. When the measured control rod is inserted a few steps, the reactivity change is now compensated with a decrease of boron concentration in reactor core [10] (figure 5). Again, the reactor is kept almost critical.

This method is time consuming and produces a significant amount of waste collant and boric acid, but on the upside yields very accurate results.

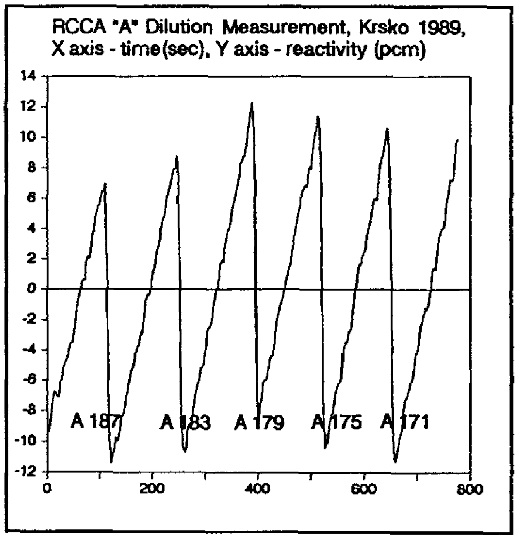

Figure 5: Boron dilution reactivity worth measurement time progress for *Rod Control Cluster Assembly* (RCCA) "A" in Krško nuclear power plant [10]. Sharp reactivity drop represents RCCA repositioning, while boron dilution results in the slow changes.

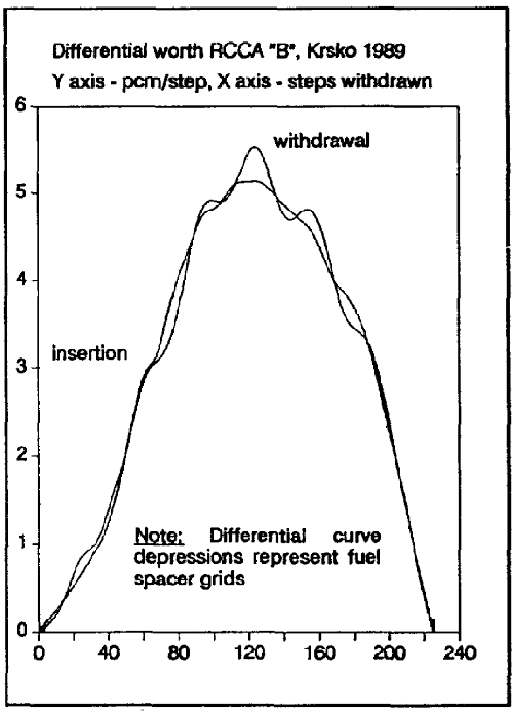

Figure 6: Differential reactivity worth for RCCA "B" obtained with use of the rod-insertion method [10]. This method also works for rod withdrawal.

4.3 Rod-insertion

In contrast to the two previous methods, the rod-insertion method does not require the reactor to remain critical. In combination with the Digital Meter of Reactivity, this method calculates the integral and differential reactivity worth curve during the post-processing. The "hardware part" of the measurement itself is to insert the measured control rod into the reactor core at a constant rate, during which the neutron flux data is stored [10]. Note that since no reactivity compensation is needed, the reactor goes deeply subcritical and eventually the neutron flux signal is indistinguishable from the signal background. Therefore a way of background elimination has had to be implemented [9]. Rod-insertion method's results can be seen on figures 6 and 7.

Introduction of the rod-insertion method signifficantly reduced the time needed to perform reactivity worth measurements. This, allong with the advandage of eliminating boron waste, can be read from the table 2. Since its introduction around the year of 1989, the rod-insertion method became

main reactivity worth measurement method for Krško Nuclear power plant [9].

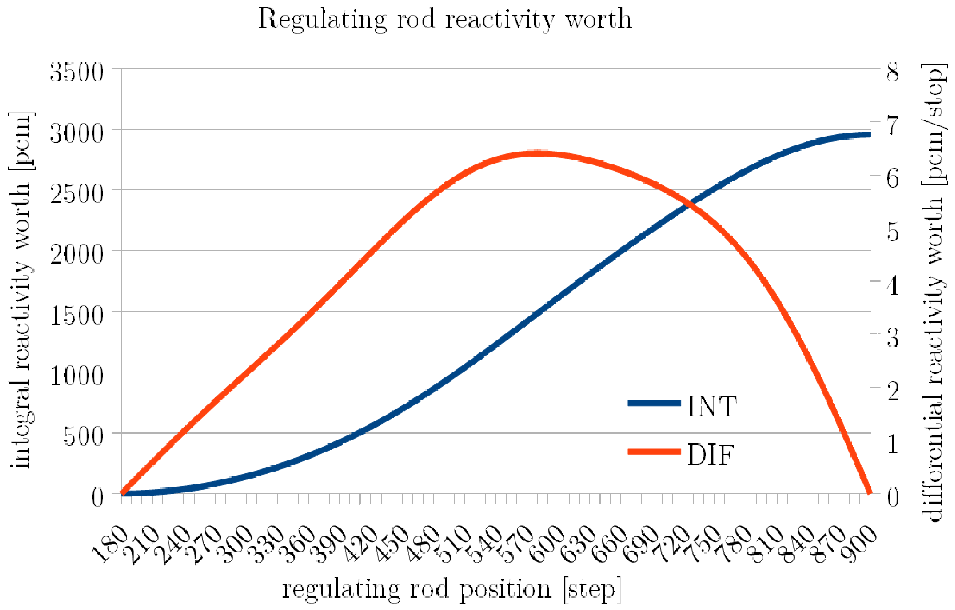

Figure 7: A typical control rod reactivity worth as a function of insertion depth. Both differential and integral curves are plotted (right y-axis values correspond to the differential curve). Note that differential worth curve has a maximum, which indicates that the control rod has most impact on the neutron flux in the middle of the reactor core height. Values were obtained with DMR during rod-insertion method for regulating rod on JSI TRIGA reactor, 15. january 2013.

method	time required [h]	total change in boron concentration [ppm]	waste water [m^3]	waste boron [m^3]
boron exchange	48	490	45	9
rod swap	12	166	16	4.5
rod-insertion	1 or 2	0	0	0

Table 2: Comparison between the three control rod worth measurement methods discussed. These approximate values were measured in 1989 during the start-up physics tests at Krško NPP (nuclear power plant) [12].

5 Conclusion

In reactor physics the multiplication factor k (and thus reactivity ρ) play a crucial role. Nuclear chain reaction control is possible because of the delayed neutrons. It is esential to have reliable methods of measuring the reactivity worth of the control mechanisms. For control rods three reactivity worth measurement methods were presented, namely the *Rod swap*, *Boron exchange* and *Rod-insertion* method. Knowledge of the physical background reveals their advantages and disadvantages and also show possible improvements. Briefly mentioned Monte Carlo calcualtions may help us towards the goal of even more accurate, precise and compact reactivity measurement methods.

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