LINDSTRÖM LECTURE 2013

Ibn Sīnā on the foundations of logic

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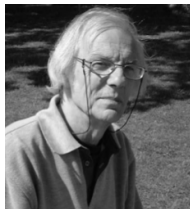
PER (PELLE) LINDSTRÖM, 1936–2009



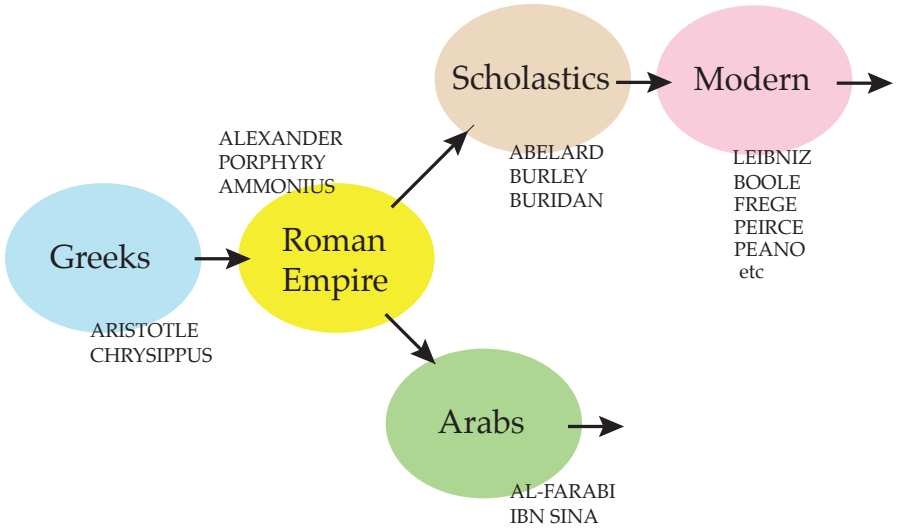
Though I never met him, his name was well-known already when I began research in the 1960s.
Among his many results, the one that will ensure his mention in every future history of logic is his abstract characterisation of first-order logic, which gave a new viewpoint for looking at the whole of logic.

I will introduce another logician with a broad and original view of the whole of logic — perhaps the first after Aristotle himself, and a logical theorist on the same level as Leibniz and Frege.

This is Ibn Sīnā, known to the medieval Latins as Avicenna. He was born in Afghanistan in 980 and died in Persia in 1037. He was bilingual in Arabic and Persian.

In past centuries he was famous for his medicine and his metaphysics.
His logic was unknown in the West until recently.

Ibn Sina, 980–1037



1. *Logic is a tool for checking the correctness of arguments*

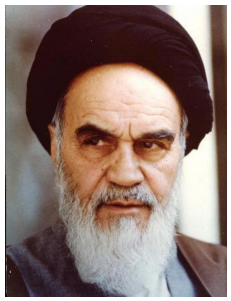
NB not (as some Aristotelians had claimed) a tool for distinguishing true from false.

Al-Ghazālī used Ibn Sīnā's viewpoint to reconcile Islam and logic.

Al-Ghazālī's defence of logic was rejected by Ibn Taymiyya, and hence by his followers in Salafi Islam (e.g. al-Qayda).

But in Ibn Sīnā's homeland of Iran, a logic based on Ibn Sīnā's work is still taught in the religious schools.

Ayatollah Khomeini to President Gorbachev in 1989:



You may command those scholars of yours who are well-versed in this field to study ... Ibn Sīnā, peace be upon [him]. It will then become clear that ... perception of general laws and concepts on which all reasoning rests is reached not by means of sensory experience but through rational argument.

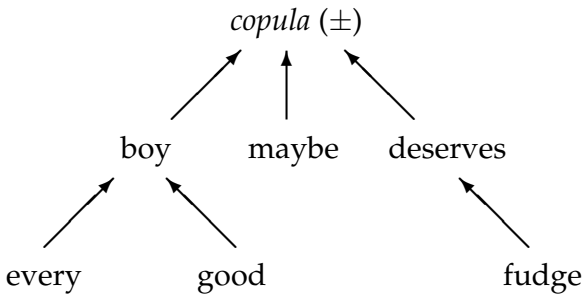
2. *Method: Analyse the argument down to single inference steps with two premises. Then parse the sentences.*

For Ibn Sīnā, analysis is a crucial part of logic for two reasons.

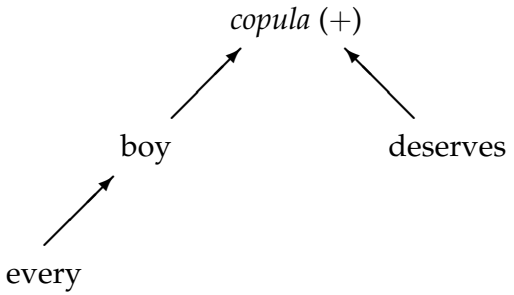
First, it's where the logician comes directly into contact with the intentions of the arguer.

Second, what we say is generally much less than we mean. (Examples to follow.)
So one key job of the logician is to bring unexpressed ideas to the surface.

Sample simple sentence, analysed à la Ibn Sīnā:



The next step is to cut down to the core (subject word, predicate word, quantifier, affirmative/negative):



The next two steps are purely algorithmic.
They test whether the argument has one of the forms of Aristotle's *predicative syllogisms*.

(1) *Rules of productivity* tell us whether the analysed premises do have a syllogistic consequence (i.e. whether they are *productive*).

(2) If yes, then *rules of following* tell us how to construct the conclusion.

(The rules above come from the Alexandrian school.)

Continuing the algorithmic idea, Ibn Sīnā tells us how to look systematically for further premises if the given premises are not enough to deduce the conclusion.

By doing this, Ibn Sīnā created the first *proof search algorithm* by a margin of at least 900 years,
and possibly the first ever search algorithm.

His algorithm is recursive, but at one point it calls on an oracle (since he is searching for proofs from all existing knowledge, not from a stated theory).

Unlike Al-Khwārizmī, he gives no correctness proof.

3. *Aristotle's syllogisms are not enough.*

Aristotle's predicative syllogisms deal only with very simple sentence forms.

Example:
Every animal breathes.
No tree breathes.
So no animal is a tree.

Ibn Sīnā says it's absurd to think we can do serious scientific reasoning with only sentences of these kinds. 'This is a stone', he says, mocking Aristotelian attempts at scientific logic.

Even simple sentences have implied extras. Thus:

Every human sleeps, *i.e. sleeps sometimes*.

Every horse is four-legged, *i.e. as long as it exists*.

Everybody travelling from Tehran to Baghdad goes through Kermanshah, *i.e. at least once during the journey*.

A writer moves his hand *i.e. all the time he is writing*.

Everything that breathes in breathes out, *i.e. . . .*

Etc. etc.

Ibn Sīnā brings to our notice *quantifiers over time or situation*, which often go hidden.

A similar but smaller class of sentences was noticed by Oscar Mitchell in C. S. Peirce's group at Johns Hopkins in 1883, and called *propositions of two dimensions*.

Peirce said Mitchell's analysis of these sentences led directly to Peirce's discovery of first-order logic in 1885.

So Ibn Sīnā had reached a potential turning-point in logic.

How to extend Aristotle's logic to a much wider range of arguments?

The later history of logic shows that people adopted two approaches:

The *New Arguments* approach is to find new valid argument forms for the new arguments.

The *Deeper Operations* approach is to look for simple and general logical operations that will generate new argument forms.

The New Arguments approach was popular both among the 14th century Latin scholastics (Burley, Buridan, Ockham), and among the Arabic successors of Ibn Sīnā (from Tūsī in 13th century to Turkish logicians in 19th century). No cross influence has been discovered as yet.

The Deeper Operations approach was also suggested by Burley and hinted at by Leibniz, but in the West it took off in the 19th century (Boole, Frege, Peirce, Peano).

And Ibn Sīnā? Many see him through the eyes of his successors, as a New Arguments man who took the first steps in a new system of modal and temporal logic.

I don't see this — I think he created no new system at all.

He did propose a way forward, which in principle would have placed him with the Deeper Operations group. But he had some important theoretical hangups that inhibited him, yet also mark him out as a deep thinker.

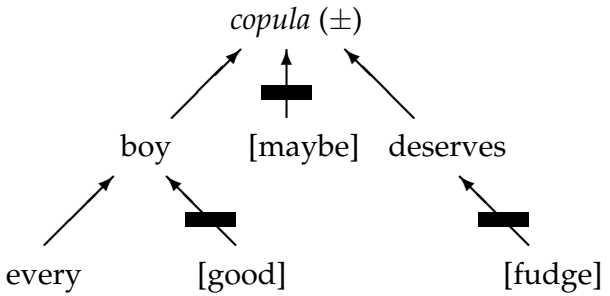
4. *'Taking care of the conditions'*

Ibn Sīnā believed, like Frege, that logic operates with meanings, and that it rests on analysis of how compound meanings are built up.

Like Frege, he believed that language gives an imperfect account of the structure of compound meanings.

Unlike Frege, he believed that we build up complex meanings largely by adding 'conditions'.

Recall how we checked an argument by first removing conditions:



Ibn Sīnā advises that we should first check the argument with the conditions removed, and if it works, then we should put the conditions back and see whether they affect the validity.

How do we tell? Ibn Sīnā believes this procedure allows us to call on our instinctive sense of validity. If we are not sure, we can build up our intuition by 'testing' on a large number of similar cases. (He uses the same word for 'testing' the effectiveness of medicines.)

He has some impressive examples — for example he uses conditions to explain making and discharging assumptions, anticipating some modern approaches.

He deliberately doesn't build up a logical theory for non-Aristotelian arguments, because he thinks all logical justification has to be in terms of basic intuitions.

Example: he would never justify an argument

$$\phi, \text{ therefore } \psi$$

by showing that if ϕ was true in a situation S then ψ would have to be true in S too. This is because

$$'\phi \text{ is true in situation } S'$$

is got from ϕ by adding a condition 'in situation S'.
If we can't see that the argument works with just ϕ, how could we see that it works with the condition added?

5. *'Becoming a predicate'*

Our last topic is Ibn Sīnā's 'subject-term of logic'.
This is widely discussed in the encyclopedias, but most of them get it quite wrong.

Ibn Sīnā: Every science starts with a term for the kinds of entity that it studies. This is its 'subject-term'.
(Cf. in early modern logic, the symbol for the domain.)

The standard account is that for Ibn Sīnā, logic studies logical notions like 'subject', 'predicate'. This account is uninteresting and not at all what Ibn Sīnā actually said.

In his late work *Easterners* he defines the subject-term of logic as:

> meanings, in the context of their being subject to composition through which an idea is made available in our minds which was not in our minds before.

He explains that new ideas take two forms:

> (1) new concepts defined in terms of old ones;
> (2) new propositions shown to be true by inference from old ones.

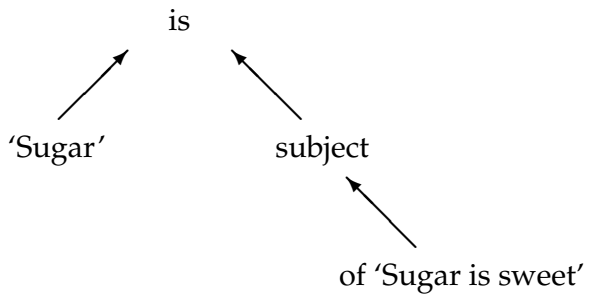
So logic has two parallel streams, one dealing with definition and one dealing with inference.

Elsewhere he explains that a meaning can only become a subject or a predicate by being put into a compound meaning.
He calls this being taken in ‘second existence’.

He is right: a meaning (or a word with that meaning) can only be the subject of a proposition p, it’s meaningless to talk of a word on its own being a subject.
But why ‘second existence’?

What follows is partly speculation, but I’m confident that it moves in the right circle of ideas.

Take the sentence



Here “of ‘Sugar is sweet’ ” is a condition added to the predicate term.

Ibn Sīnā seems to be asking — a kind of question he regularly does ask — whether the condition could be added to the subject term instead.

Compare an example he discusses in several places:

Zayd — sings on Tuesdays.

Zayd-on-Tuesdays — sings.

Modern version:

$$A \to (C \to B)$$

$$(A \times C) \to B$$

In other words, is there a kind of object that we can literally say ‘is a subject’ as opposed to ‘is a subject of such-and-such’?

For modern logicians the answer is obvious:
an *occurrence* of the word in a certain context.

In a terminology that Ibn Sīnā builds up through his logical writings, a word ‘becomes’ a subject by having the rest of a proposition attached to it.
This subject is in one sense the same entity as the word (it inherits properties like being a noun),
but in another sense a different one (it has new properties like being subject).

Taking a word ‘in second existence’ is taking this second entity.

The relation between the two entities is for Ibn Sīnā an ontological question at the heart of the foundations of logic.

I would rather call it methodological, and I strongly suspect that much of Ibn Sīnā’s metaphysical writings has similar methodological purposes.
It could lead to a great demystification if this could be proved.

The idea that we need to build up a framework of special kinds of entity as a basis for logic is completely new with Ibn Sīnā.

It was a total mystery to his contemporaries,
and may have helped to discourage the Latins from reading his logic.

But today we can sympathise more,
given the foundational work of the early 20th century,
and the kinds of object that appear in object-oriented programming.

Today for the first time we have texts of Ibn Sīnā and a perspective that allows us to take their contents seriously.

For this reason I hope people will investigate Ibn Sīnā’s view of the foundations of logic more closely.

I believe it will turn out to be deeply insightful but with some very unfamiliar perspectives,
and no doubt some plain mistakes and confusions.

Ibn Sina, 980–1037