# Parallel texture structures with cofactor zeros in lepton sector 

Weijian Wang<br>Department of Physics, North China Electric Power University, Baoding 071003, PR China

## A R T I C L E I N F O

## Article history:

Received 21 February 2014
Received in revised form 3 April 2014
Accepted 3 May 2014
Available online 9 May 2014
Editor: J. Hisano


#### Abstract

In this paper we investigate the parallel texture structures with cofactor zeros in the charged lepton and neutrino sectors. The textures cannot be obtained from arbitrary leptonic matrices by making weak basis transformations, which therefore have physical meaning. The 15 parallel textures are grouped as 4 classes where each class has the same physical implications. It is founded that one of them is not phenomenologically viable and another is equivalent to the texture zero structures extensively explored in previous literature. Thus we focus on the other two classes of parallel texture structures and study the their phenomenological implications. The constraints on the physical variables are obtained for each class, which is essential for the model selection and can be measured by future experiments. The model realization is illustrated in a radiated lepton mass model.


© 2014 Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/3.0/). Funded by SCOAP3.

## 1. Introduction

The discovery of neutrino oscillations has provided us with convincing evidences for massive neutrinos and leptonic flavor mixing with high degree of accuracy [1-3]. The recent measurement of large reactor mixing angle $\theta_{13}$ has not only opened the door for us to explore the leptonic CP violation and the mass hierarchy in the future experiments, but also highlighted the flavor puzzle of neutrino mass and mixing pattern which appears to be rather different from the distinct mass hierarchy and the small mixing angles shown by quarks. Although a full theory is still missing, several ideas have been proposed by reducing the number of free parameters of seesaw models [4] and introducing the specific structures into mass matrices to explain the observed leptonic mixing pattern. The models include texture zeros [5], hybrid textures [6,7], zero trace [8], zero determinant [9], vanishing minors [10-12], two traceless submatrices [13], equal elements or cofactors [14], hybrid $M_{v}^{-1}$ textures [15]. Among these models, the textures with zero elements or zero minors are particularly interesting because of their connection to the flavor symmetries and the stable behavior of running renormalization group. The phenomenological analysis of neutrino mass matrices with texture zeros or cofactor zeros in flavor basis have been widely investigated in earlier literature [5, 10-12].

However, there is no a priori requirement that the analysis must be done in flavor basis. The more general situation should be considered in the basis where both charged lepton mass ma-

[^0]trix $M_{l}$ and neutrino mass matrix $M_{v}$ are non-diagonal. In this spirit, the parallel Ansätze has been proposed where $M_{l}$ and $M_{v}$ have the same structure (we denote it "parallel texture structure"). A popular parallel texture structure appears as the Fritzsch-like model [16] with texture zeros in mass matrix and is firstly applied to understand the quark mixing pattern. Subsequently the idea is generalized to the lepton sector [17,18]. A systematic search on the parallel structures with texture zeros in lepton mass matrices is reported in Ref. [19]. It is shown that some sets of the texture zeros have no physical meaning by themselves, since they can be obtained by making suitable weak basis (WB) transformation from arbitrary mass matrix and leaving the gauge currents invariant. The minimal no trivial case is the four texture zeros model. Recently, a similar investigation is done in the context of parallel hybrid textures with one zero and two equal elements [20].

In this work, we study the parallel structures with two cofactor zeros in both $M_{\nu}$ and $M_{\nu}$. As shown in Ref. [11], the cofactor zeros in $M_{v}$ are generated by the type-I seesaw formula $M_{v}=-M^{D} M_{R}^{-1} M_{D}^{T}$ with texture zeros in $M_{D}$ and $M_{R}$. The cofactor zeros in $M_{l}$, on the other hand, seems to be rather unusual because no flavor symmetry directly leads to the cofactor zeros of Dirac mass matrix $M_{l}$. However, we will show that if we adopt the recent viewpoint proposed by Ma [21] that the radiated lepton mass originate from the one-loop diagram, a seesaw-like formula is possible for charged lepton masses and the cofactor zeros in $M_{l}$ can be realized. There exists $C_{6}^{2}=15$ logically possible patterns. Furthermore, we assume the mass matrices to be Hermitian and all neutrinos are massive, which indicates $\operatorname{det} M_{v} \neq 0$ and existence of $M_{v}^{-1}$. Thus the mass textures $M_{v}$ with cofactor zeros are equivalent to the $M_{v}^{-1}$ with texture zeros. As the texture zero case [19],
the 15 textures structures can be grouped into 4 classes with each class having the same physical implications. Among the 4 classes, we find that one of them is not viable phenomenologically and another class is equal to the matrices with texture zeros. Therefore we focus on the other two classes having not been studied before.

The paper is organized as follows. In Section 2, we discuss the classification of textures and relate them to the experimental results. In Section 3, we diagonalize the mass matrices, confront the numerical results with the experimental data and discuss their predictions. In Section 4, the realization of cofactor zeros in $M_{l}$ is discussed. A summary is given in Section 4.

## 2. Formalism

### 2.1. Weak basis equivalent classes

We assume the neutrinos to be Majorana fermions. The most general WB transformations leaving gauge currents invariant is given by
$M_{l} \rightarrow M_{l}^{\prime}=W^{\dagger} M_{l} W_{R} \quad M_{v} \rightarrow M_{v}^{\prime}=W^{T} M_{\nu} W$
where $W, W_{R}$ are $3 \times 3$ unitary matrices. Therefore the parallel texture with cofactor zeros located at different positions can be related by permutation matrix $P$ as the WB transformation
$M_{l}^{\prime}=P^{T} M_{l} P \quad M_{\nu}^{\prime}=P^{T} M_{\nu} P$
The permutation matrix $P$ changes the positions of cofactor zeros but preserves the parallel structures for both charged lepton and neutrino mass textures. It is noted that $P$ belongs to the group of 6 permutations and are isomorphic to $S_{3}$. Then the four cofactor zeros texture can be classified into 4 classes as following:

Class I:
$\left(\begin{array}{ccc}\Delta & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \times\end{array}\right) \quad\left(\begin{array}{ccc}\Delta & \Delta & \times \\ \Delta & \times & \times \\ \times & \times & \times\end{array}\right) \quad\left(\begin{array}{ccc}\times & \Delta & \times \\ \Delta & \Delta & \times \\ \times & \times & \times\end{array}\right)$
$\left(\begin{array}{ccc}\times & \times & \times \\ \times & \Delta & \Delta \\ \times & \Delta & \times\end{array}\right) \quad\left(\begin{array}{ccc}\times & \times & \Delta \\ \times & \times & \times \\ \Delta & \times & \Delta\end{array}\right) \quad\left(\begin{array}{ccc}\times & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \Delta\end{array}\right)$
Class II:
$\left(\begin{array}{ccc}\Delta & \times & \times \\ \times & \times & \Delta \\ \times & \Delta & \times\end{array}\right) \quad\left(\begin{array}{ccc}\times & \times & \Delta \\ \times & \Delta & \times \\ \Delta & \times & \times\end{array}\right) \quad\left(\begin{array}{ccc}\times & \Delta & \times \\ \triangle & \times & \times \\ \times & \times & \Delta\end{array}\right)$
Class III:
$\left(\begin{array}{ccc}\Delta & \times & \times \\ \times & \Delta & \times \\ \times & \times & \times\end{array}\right) \quad\left(\begin{array}{ccc}\Delta & \times & \times \\ \times & \times & \times \\ \times & \times & \Delta\end{array}\right) \quad\left(\begin{array}{ccc}\times & \times & \times \\ \times & \Delta & \times \\ \times & \times & \Delta\end{array}\right)$
Class IV:
$\left(\begin{array}{ccc}\times & \Delta & \Delta \\ \Delta & \times & \times \\ \Delta & \times & \times\end{array}\right) \quad\left(\begin{array}{ccc}\times & \Delta & \times \\ \Delta & \times & \Delta \\ \times & \Delta & \times\end{array}\right) \quad\left(\begin{array}{ccc}\times & \times & \Delta \\ \times & \times & \Delta \\ \Delta & \Delta & \times\end{array}\right)$
where " $\Delta$ " at $(i, j)$ position denotes the zero cofactor $C_{i j}=0$ while " $\times$ " stands for arbitrary element. Since $M_{l, v}$ with cofactor zeros is equivalent to $M_{l, v}^{-1}$ with zero elements, the classification given above is the same as the texture zero ones shown in Ref. [19] except for replacing " $\Delta$ " with " 0 ". Like the texture zero cases, the class IV leads to the decoupling of a generation of lepton from mixing and thus not experimentally viable. On the other hand, one
can easily check that the textures of class I correspond to the texture zero ones, which has already studied in previous literature [17-19]. As an example, for the first matrix of class I, we have

$$
\begin{align*}
M_{l, v}=\left(\begin{array}{ccc}
\triangle & \times & \triangle \\
\times & \times & \times \\
\triangle & \times & \times
\end{array}\right) & \Rightarrow M_{l, v}^{-1}=\left(\begin{array}{ccc}
0 & \times & 0 \\
\times & \times & \times \\
0 & \times & \times
\end{array}\right) \\
& \Rightarrow M_{l, v}=\left(\begin{array}{ccc}
\times & \times & \times \\
\times & 0 & 0 \\
\times & 0 & \times
\end{array}\right) \tag{7}
\end{align*}
$$

Therefore only class II and class III have no trivial physical implications.

### 2.2. Some useful notations

As we have mentioned, we only need to investigate two mass matrices respectively belonging to representations of class II and class III. In this work, we choose
$M_{l, v}^{I I}=\left(\begin{array}{ccc}\Delta & \times & \times \\ \times & \times & \triangle \\ \times & \Delta & \times\end{array}\right) \quad M_{l, v}^{I I I}=\left(\begin{array}{ccc}\Delta & \times & \times \\ \times & \triangle & \times \\ \times & \times & \times\end{array}\right)$
The charged leptonic mass texture $M_{l}$ is a complex Hermitian matrix and the Majorana neutrino mass texture $M_{v}$ is a complex symmetric matrix. They are diagonalized by unitary matrix $V_{l}$ and $V_{v}$
$M_{l}=V_{l} M_{l}^{D} V_{l}^{\dagger} \quad M_{v}=V_{v} M_{v}^{D} V_{v}^{T}$
where $M_{l}^{D}=\operatorname{Diag}\left(m_{e}, m_{\mu}, m_{\tau}\right), M_{\nu}^{D}=\operatorname{Diag}\left(m_{1}, m_{2}, m_{3}\right)$. The Pon-tecorvo-Maki-Nakagawa-Sakata matrix [22] $U_{P M N S}$ is given by
$U_{\text {PMNS }}=V_{l}^{\dagger} V_{v}$
and can be parameterized as

$$
\begin{align*}
& U_{P M N S} \\
& =U P_{v} \\
& =\left(\begin{array}{ccc}
c_{12} c_{13} & c_{13} s_{12} & s_{13} e^{-i \delta} \\
-s_{12} c_{23}-c_{12} s_{13} s_{23} e^{i \delta} & c_{12} c_{23}-s_{12} s_{13} s_{23} e^{i \delta} & c_{13} s_{23} \\
s_{23} s_{12}-c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23}-c_{23} s_{12} s_{13} e^{i \delta} & c_{13} c_{23}
\end{array}\right) \\
&  \tag{11}\\
& \\
& \times\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & e^{i \alpha} & 0 \\
0 & 0 & e^{i(\beta+\delta)}
\end{array}\right)
\end{align*}
$$

where the abbreviations $s_{i j}=\sin \theta_{i j}$ and $c_{i j}=\cos \theta_{i j}$ are used. The $\alpha$ and $\beta$ in $P_{\nu}$ denote two Majorana CP-violating phases and $\delta$ in $U$ denotes the Dirac CP-violating phase. In order to facilitate our calculation, it is better to start from $M_{l}^{-1}$ rather than $M_{l}$. From (9), we get
$M_{l}^{-1}=V_{l}\left(M_{l}^{D}\right)^{-1} V_{l}^{\dagger}$
So the $V_{l}$ cannot only diagonalize the $M_{l}$ but also $M_{l}^{-1}$. Furthermore, we treat the Hermitian matrix $M_{l}^{-1}$ to be factorisable i.e.
$M_{l}^{-1}=K_{l}\left(M_{l}^{-1}\right)^{r} K_{l}^{\dagger}$
where $K_{l}$ is the unitary phase matrix and can be parameterized as $K_{l}=\operatorname{diag}\left(1, e^{i \phi_{1}}, e^{i \phi_{2}}\right)$. The $\left(M_{l}^{-1}\right)^{r}$ becomes a real symmetric matrix which can be diagonalized by real orthogonal matrix $O_{l}$. Then we have
$V_{l}=K_{l} O_{l}$
and
$U_{P M N S}=O_{l}^{T} K_{l}^{\dagger} V_{v}$
From (9), (10) and (15), the neutrino mass matrix $M_{\nu}$ is given by
$M_{\nu}=K_{l} V P_{\nu} M_{\nu}^{D} P_{\nu} V^{T} K_{l}^{\dagger}$
where $V \equiv O_{l} U$. From (16) the restriction of two cofactor zeros on $M_{\nu}$
$M_{\nu(p q)} M_{\nu(r s)}-M_{\nu(t u)} M_{\nu(v w)}=0$
$M_{\nu\left(p^{\prime} q^{\prime}\right)} M_{\nu\left(r^{\prime} s^{\prime}\right)}-M_{\nu\left(t^{\prime} u^{\prime}\right)} M_{\nu\left(v^{\prime} w^{\prime}\right)}=0$
induces two equations
$m_{1} m_{2} K_{3} e^{2 i \alpha}+m_{2} m_{3} K_{1} e^{2 i(\alpha+\beta+\delta)}+m_{3} m_{1} K_{2} e^{2 i(\beta+\delta)}=0$
$m_{1} m_{2} L_{3} e^{2 i \alpha}+m_{2} m_{3} L_{1} e^{2 i(\alpha+\beta+\delta)}+m_{3} m_{1} L_{2} e^{2 i(\beta+\delta)}=0$
where
$K_{i}=\left(V_{p j} V_{q j} V_{r k} V_{s k}-V_{t j} V_{u j} V_{v k} V_{w k}\right)+(j \leftrightarrow k)$
$L_{i}=\left(V_{p^{\prime} j} V_{q^{\prime} j} V_{r^{\prime} k} V_{s^{\prime} k}-V_{t^{\prime} j} V_{u^{\prime} j} V_{V^{\prime} k} V_{w^{\prime} k}\right)+(j \leftrightarrow k)$
with ( $i, j, k$ ) a cyclic permutation of (1, 2, 3). After solving Eq. (18) and (19), we arrive at
$\frac{m_{1}}{m_{2}} e^{-2 i \alpha}=\frac{K_{3} L_{1}-K_{1} L_{3}}{K_{2} L_{3}-K_{3} L_{2}}$
$\frac{m_{1}}{m_{3}} e^{-2 i \beta}=\frac{K_{2} L_{1}-K_{1} L_{2}}{K_{3} L_{2}-K_{2} L_{3}} e^{2 i \delta}$
With the help of Eqs. (22) and (23), we obtain the magnitudes of mass radios
$\rho=\left|\frac{m_{1}}{m_{3}} e^{-2 i \beta}\right|$
$\sigma=\left|\frac{m_{1}}{m_{2}} e^{-2 i \alpha}\right|$
as well as the two Majorana CP-violating phases
$\alpha=-\frac{1}{2} \arg \left(\frac{K_{3} L_{1}-K_{1} L_{3}}{K_{2} L_{3}-K_{3} L_{2}}\right)$
$\beta=-\frac{1}{2} \arg \left(\frac{K_{2} L_{1}-K_{1} L_{2}}{K_{3} L_{3}-K_{2} L_{3}} e^{2 i \delta}\right)$
The results of Eqs. (24), (25), (26) and (27) imply that the two mass ratios ( $\rho$ and $\sigma$ ) and two Majorana CP-violating phases ( $\alpha$ and $\beta$ ) are fully determined in terms of the real orthogonal matrix $O_{l}$ and $U\left(\theta_{12}, \theta_{23}, \theta_{13}\right.$ and $\left.\delta\right)$. The neutrino mass ratios $\rho$ and $\sigma$ are related to the ratios of two neutrino mass-squared ratios obtained from the solar and atmosphere oscillation experiments as
$R_{\nu} \equiv \frac{\delta m^{2}}{\Delta m^{2}}=\frac{2 \rho^{2}\left(1-\sigma^{2}\right)}{\left|2 \sigma^{2}-\rho^{2}-\rho^{2} \sigma^{2}\right|}$
and to the three neutrino mass as
$m_{2}=\sqrt{\frac{\delta m^{2}}{1-\sigma^{2}}} \quad m_{1}=\sigma m_{2} \quad m_{3}=\frac{m_{1}}{\rho}$
where $\delta m^{2} \equiv m_{2}^{2}-m_{1}^{2}$ and $\Delta m^{2} \equiv\left|m_{3}^{2}-\frac{1}{2}\left(m_{1}^{2}+m_{2}^{2}\right)\right|$. In the numerical analysis, we use the latest global-fit neutrino oscillation experimental data, at $3 \sigma$ confidential level, which is listed in Ref. [24]
$\sin ^{2} \theta_{12} / 10^{-1}=3.08_{-0.49}^{+0.51} \quad \sin ^{2} \theta_{23} / 10^{-1}=4.25_{-0.68}^{+2.16}$
$\sin ^{2} \theta_{13} / 10^{-2}=2.34_{-0.57}^{+0.63} \quad \delta m^{2} / 10^{-5}=7.54_{-0.55}^{+0.64} \mathrm{eV}^{2}$
$\Delta m^{2} / 10^{-3}=2.44_{-0.22}^{+0.22} \mathrm{eV}^{2}$
for normal hierarchy ( NH ) and
$\sin ^{2} \theta_{12} / 10^{-1}=3.08_{-0.49}^{+0.51} \quad \sin ^{2} \theta_{23} / 10^{-1}=4.25_{-0.74}^{+2.22}$
$\sin ^{2} \theta_{13} / 10^{-2}=2.34_{-0.61}^{+0.61} \quad \delta m^{2} / 10^{-5}=7.54_{-0.55}^{+0.64} \mathrm{eV}^{2}$
$\Delta m^{2} / 10^{-3}=2.40_{-0.23}^{+0.21} \mathrm{eV}^{2}$
for inverted hierarchy ( IH ). There is no constraint on the Dirac CPviolating phase $\delta$ at $3 \sigma$ level, however, the recent global fit tends to give $\delta \approx 1.40 \pi$. In neutrino oscillation experiments, CP violation effect is usually reflected by the Jarlskog rephasing invariant quantity [23] defined as
$J{ }_{C P}=s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^{2} \sin \delta$
The Majorana nature of neutrino can be determined if any signal of neutrinoless double decay is observed, implying the violation of leptonic number violation. The decay ratio is related to the effective of neutrino $m_{e e}$, which is written as
$m_{e e}=\left|m_{1} c_{12}^{2} c_{13}^{2}+m_{2} s_{12}^{2} c_{13}^{2} e^{2 i \alpha}+m_{3} s_{13}^{2} e^{2 i \beta}\right|$
Although a $3 \sigma$ result of $m_{e e}=(0.11-0.56) \mathrm{eV}$ is reported by the Heidelberg-Moscow Collaboration [25], this result is criticized in Ref. [26] and shall be checked by the forthcoming experiment. It is believed that the next generation $0 v \beta \beta$ experiments, with the sensitivity of $m_{e e}$ being up to 0.01 eV [27], will open the window to not only the absolute neutrino mass scale but also the Majorana-type CP violation. Besides the $0 \nu \beta \beta$ experiments, a more severe constraint was set from the recent cosmology observation. Recently, an upper bound on the sum of neutrino mass $\sum m_{i}<0.23 \mathrm{eV}$ is reported by Planck Collaboration [28] combined with the WMAP, high-resolution CMB and BAO experiments.

## 3. Phenomenological implications of parallel cofactor zero textures

### 3.1. Class II

In this section, we study the phenomenological implications of class II. The factorisable formation of inverse charged leptonic matrix $\left(M_{l}^{-1}\right)^{r}$ is parameterized as
$\left(M_{l}^{-1}\right)_{I I}^{r}=\left(\begin{array}{lll}0 & a & c \\ a & b & 0 \\ c & 0 & d\end{array}\right)$
and can be diagonalized by an orthogonal matrix $O_{l}$
$O_{l}^{T}\left(M_{l}^{-1}\right)_{I I}^{r} O_{l}=\operatorname{diag}\left(m_{e}^{-1},-m_{\mu}^{-1}, m_{\tau}^{-1}\right)$
where the coefficients $a, c, d$ are real and positive; The $m_{e}, m_{\mu}$ and $m_{\tau}$ denote the mass eigenvalues of charged leptons for three generations. The minus sign in (35) has been introduced to facilitate the analytical calculation and has no physical meaning since the charged lepton is Dirac fermions. Using the invariant $\operatorname{Tr}\left(M_{l}^{-1}\right)^{r}$, $\operatorname{Det}\left(M_{l}^{-1}\right)^{r}$ and $\operatorname{Tr}\left(M_{l}^{-1}\right)^{r^{2}}$ the nonzero elements of $\left(M_{l}^{-1}\right)^{r}$ can be expressed in terms of three mass eigenvalues $m_{e}, m_{\mu}, m_{\tau}$ and $d$
$a=\sqrt{-\frac{\left(m_{e}^{-1}-m_{\mu}^{-1}-d\right)\left(m_{e}^{-1}+m_{\tau}^{-1}-d\right)\left(-m_{\mu}^{-1}+m_{\tau}^{-1}-d\right)}{m_{e}^{-1}-m_{\tau}^{-1}+m_{\tau}^{-1}-2 d}}$


Fig. 1. The correlation plots for class II(NH).
$b=m_{e}^{-1}-m_{\mu}^{-1}+m_{\tau}^{-1}-d$
$c=\sqrt{\frac{\left(d-m_{e}^{-1}\right)\left(d+m_{\mu}^{-1}\right)\left(d-m_{\tau}^{-1}\right)}{m_{e}^{-1}-m_{\tau}^{-1}+m_{\tau}^{-1}-2 d}}$
where the parameter $d$ is allowed in the range of $0<d<m_{\tau}^{-1}$ and $m_{e}^{-1}-m_{\tau}^{-1}<d<m_{e}^{-1}$. Then the $O_{l}$ can be easily constructed as
$O_{l}=\left(\begin{array}{ccc}\frac{\left(b-m_{e}^{-1}\right)\left(d-m_{e}^{-1}\right)}{N_{1}} & \frac{\left(b+m_{\mu}^{-1}\right)\left(d+m_{\mu}^{-1}\right)}{N_{2}} & \frac{\left(b-m_{\tau}^{-1}\right)\left(d-m_{\tau}^{-1}\right)}{N_{3}} \\ -\frac{a\left(d-m_{e}^{-1}\right)}{N_{1}} & -\frac{a\left(d+m_{\mu}^{-1}\right)}{N_{2}} & -\frac{a\left(d-m_{\tau}^{-1}\right)}{N_{3}} \\ -\frac{c\left(b-m_{e}^{-1}\right)}{N_{3}} & -\frac{c\left(b+m_{\mu}^{-1}\right)}{N_{3}} & -\frac{c\left(b-m_{\tau}^{-1}\right)}{N_{3}}\end{array}\right)$
where the $a, b$ and $c$ in (39) are given in (36), (37) and (38); The $N_{1}, N_{2}$ and $N_{3}$ are the normalized coefficients given by
$N_{1}^{2}=\left(b-m_{e}^{-1}\right)^{2}\left(d-m_{e}^{-1}\right)^{2}+a^{2}\left(d-m_{e}^{-1}\right)^{2}+c^{2}\left(b-m_{\tau}^{-1}\right)^{2}$
$N_{2}^{2}=\left(b+m_{\mu}^{-1}\right)^{2}\left(d+m_{\mu}^{-1}\right)^{2}+a^{2}\left(d+m_{\mu}^{-1}\right)^{2}+c^{2}\left(b+m_{\mu}^{-1}\right)^{2}$
$N_{3}^{2}=\left(b-m_{\tau}^{-1}\right)^{2}\left(d-m_{\tau}^{-1}\right)^{2}+a^{2}\left(d-m_{\tau}^{-1}\right)^{2}+c^{2}\left(b-m_{\tau}^{-1}\right)^{2}$

Substitute the $O_{l}$ we obtained into (39) to (24), (25), (26), (27) and (28), the ratio of mass squared difference can be expressed via eight parameters i.e. three mixing angles ( $\theta_{12}, \theta_{23}, \theta_{13}$ ), one Dirac CP-violating phase $\delta$, three charged lepton mass ( $m_{e}, m_{\mu}, m_{\tau}$ ) and a parameter $d$. Here we choose the three charged leptonic masses at the electroweak scale $\left(\mu \simeq M_{Z}\right)$ i.e. [29]
$m_{e}=0.486570154 \mathrm{MeV} \quad m_{\mu}=102.7181377 \mathrm{MeV}$
$m_{\tau}=1746.17 \mathrm{MeV}$

In the numerical analysis, we randomly vary the three mixing angles $\left(\theta_{12}, \theta_{23}, \theta_{13}\right)$ in their $3 \sigma$ range and parameter $d$ in its proper range. Up to now, no bound was set on Dirac CP-violating phase $\delta$ at $3 \sigma$ level, so we vary it randomly in the range of $[0,2 \pi]$. Using Eq. (28), the mass-squared difference ratio $R_{v}$ is determined. Then the input parameters are empirically acceptable when the $R_{v}$ falls inside the $3 \sigma$ range of experimental data, otherwise they are excluded. Finally, we get the value of neutrino mass and Majorana CP -violating phases $\alpha$ and $\beta$ though Eqs. (24), (25). Since we have already obtained the absolute neutrino mass $m_{1,2,3}$, the further constraint from cosmology should be considered. In this work, we set the upper bound on the sum of neutrino mass $\sum m_{i}$ less than 0.23 eV .

We present the numerical results of class II in Fig. 1 for the NH and in Fig. 2 for the IH. One can see from the figures that different mass spectra exhibit different correlations between physical variables. For the NH case, the Dirac CP-violating phase $\delta$ is highly restricted in the range of $60^{\circ} \sim 70^{\circ}$ and leads to the Jarlskog rephasing invariant $\left|J_{C P}\right|>0.02$ which is promising to be detected in the future long baseline neutrino oscillation experiments. On the other hand, there exists a bound of $\theta_{23}>48^{\circ}$. Although accepted at $3 \sigma$ level, this result is phenomenologically ruled out at $2 \sigma$ level since recent experiments tend to give $\theta_{23}<\pi / 4$. We obtain the bound on the lightest neutrino mass $M_{1}, 0.025 \mathrm{eV}<m_{1}<0.075 \mathrm{eV}$ and the effective Majorana neutrino mass $m_{e e} 0.04 \mathrm{eV}<m_{e e}<0.10 \mathrm{eV}$ which reaches the accuracy of future neutrinoless double beta decay $(0 \nu \beta \beta)$ experiments. The correlation between $\alpha$ and $\beta$ is also illustrated that the small range is allowed at $3 \sigma$ level. For the IH case, the three mixing angles $\theta_{12}, \theta_{23}$, and $\theta_{13}$ are fully covered the $3 \sigma$ range while the constrained Dirac CP-violated phase $\delta$ lies in the range of $70^{\circ} \sim 290^{\circ}$, leading to the $\left|J_{C P}\right| \sim(0)-(0.04)$. Interestingly, $m_{e e}$ and the lightest neutrino mass $m_{3}$ exhibit a strong dependence on $\delta$. Such correlations are essential for the model selection and could be tested by experiments. There also exists a bound of $0.005 \mathrm{eV}<m_{e e}<0.095 \mathrm{eV}$ which could be in principle tested by future $0 \nu \beta \beta$ experiments. The Majorana phase $\alpha$ is cov-


Fig. 2. The correlation plots for class II (IH).
ered in the whole range of $-90^{\circ} \sim 90^{\circ}$ while $\beta$ is constrained in the range of $-25^{\circ} \sim 25^{\circ}$.

### 3.2. Class III

Let's consider another class of textures which is phenomenologically interesting. In the factorisable case, the real matrix $\left(M_{l}^{-1}\right)^{r}$ is parameterized as
$\left(M_{l}^{-1}\right)_{I I I}^{r}=\left(\begin{array}{lll}0 & a & b \\ a & 0 & c \\ b & c & d\end{array}\right)$
where $a, b, c$ and $d$ are real numbers. Without loss of generality, the parameters $b, c$ are set to be positive. The matrix can be diagonalized by the orthogonal matrix $O_{l}$
$O_{l}^{T}\left(M_{l}^{-1}\right)_{I I I}^{r} O_{l}=\operatorname{diag}\left(m_{e}^{-1},-m_{\mu}^{-1}, m_{\tau}^{-1}\right)$
Different from class II, we choose $a$ as the free parameter since the trace of $\left(M_{l}^{-1}\right)^{r}$ has already fixed $d$ to be
$d=m_{e}^{-1}-m_{\mu}^{-1}+m_{\tau}^{-1}$
Using the invariant $\operatorname{Det}\left(M_{l}^{-1}\right)^{r}$ and $\operatorname{Tr}\left(M_{l}^{-1}\right)^{r^{2}}$, the parameters $b$, $c$ can be expressed by three charged leptonic mass eigenvalues ( $m_{e}, m_{\mu}, m_{\tau}$ ) and $a$

$$
\begin{align*}
(b \pm c)^{2}= & -\left(-m_{e}^{-1} m_{\mu}^{-1}+m_{e}^{-1} m_{\tau}^{-1}-m_{\mu}^{-1} m_{\tau}^{-1}\right)-a^{2} \\
& \pm \frac{a\left(m_{e}^{-1}-m_{\mu}^{-1}+m_{\tau}^{-1}\right)-m_{e}^{-1} m_{\mu}^{-1} m_{\tau}^{-1}}{a} \tag{47}
\end{align*}
$$

With the help of Eq. (46) and Eq. (47), one can construct the diagonalized matrix $O_{l}$ to be
$\left(M_{l}^{-1}\right)_{I I I}^{r}=\left(\begin{array}{ccc}\frac{O(11)}{N_{1}} & \frac{O(12)}{N_{2}} & \frac{O(13)}{N_{3}} \\ \frac{O(21)}{N_{1}} & \frac{O(22)}{N_{2}} & \frac{O(23)}{N_{3}} \\ \frac{O(31)}{N_{1}} & \frac{O(32)}{N_{2}} & \frac{O(33)}{N_{3}}\end{array}\right)$
where

$$
\begin{align*}
& O(11)=a m_{e}^{-1}\left(b m_{e}^{-1}+c a^{-1}\right)+b m_{e}^{-1}\left(a m_{e}^{-1}-m_{e} a^{-1}\right) \\
& O(12)=a m_{e}^{-1}\left(-b m_{\mu}^{-1}+c a^{-1}\right)-b m_{\mu}^{-1}\left(-a m_{\mu}^{-1}+m_{\mu} a^{-1}\right) \\
& O(13)=a m_{\tau}^{-1}\left(b m_{\tau}^{-1}+c a^{-1}\right)+b m_{\tau}^{-1}\left(a m_{\tau}^{-1}-m_{\tau} a^{-1}\right) \\
& O(21)=b m_{e}^{-1}+c a^{-1} \\
& O(22)=-b m_{\mu}^{-1}+c a^{-1} \\
& O(23)=b m_{\tau}^{-1}+c a^{-1} \\
& O(31)=a m_{e}^{-1}-m_{e} a^{-1} \\
& O(32)=-a m_{\mu}^{-1}+m_{\mu} a^{-1} \\
& O(33)=a m_{\tau}^{-1}-m_{\tau} a^{-1} \tag{49}
\end{align*}
$$

and the normalized coefficients are given by
$N_{1}^{2}=O(11)^{2}+O(21)^{2}+O(31)^{2}$
$N_{2}^{2}=O(12)^{2}+O(22)^{2}+O(32)^{2}$
$N_{3}^{2}=O(13)^{2}+O(23)^{2}+O(33)^{2}$
From the condition that $b, c$ are real and positive, we have the free parameter $a$ allowed in the range of
$-\left(\frac{m_{e}^{-1} m_{\mu}^{-1} m_{\tau}}{m_{e}^{-1}-m_{\mu}^{-1}+m_{\tau}^{-1}}\right)^{\frac{1}{2}}<a<0$
or
$\left(\frac{m_{e}^{-1} m_{\mu}^{-1} m_{\tau}}{m_{e}^{-1}-m_{\mu}^{-1}+m_{\tau}^{-1}}\right)^{\frac{1}{2}}$

$$
\begin{equation*}
<a<\left(m_{e}^{-1} m_{\mu}^{-1}+m_{\mu} m_{\tau}-m_{e}^{-1} m_{\tau}^{-1}\right)^{\frac{1}{2}} \tag{52}
\end{equation*}
$$



Fig. 3. The correlation plots for class III (NH).







Fig. 4. The correlation plots for class III (IH).

We present the allowed region for class III in Fig. 3 and Fig. 4. For NH case, no bound is set on the Dirac CP-violating phase $\delta$. However, the numerical result shows a strong preference for the $\delta$ in the range of $0^{\circ} \sim 70^{\circ}\left(290^{\circ} \sim 360^{\circ}\right)$. In this region, the $0.04 \mathrm{eV}<$ $m_{e e}<0.20 \mathrm{eV}$ is obtained unless $\delta$ is fine-tuned around $0^{\circ}\left(180^{\circ}\right)$ where there exists a possibility for $m_{e e} \simeq 0 \mathrm{eV}$. On the other hand,
when $\delta$ are located at $70^{\circ} \sim 290^{\circ}$, we have a highly suppressed $m_{e e} \simeq 0 \mathrm{eV}$, implying the underlying cancellation of three neutrino masses in $m_{e e}$. There also exists the lower bound on the lightest neutrino mass $m_{1}>0.05 \mathrm{eV}$. Although $\alpha$ and $\beta$ are covered their whole range, $\alpha$ is dominantly located at around $0^{\circ}$. For IH case, we get the Dirac CP-violated phase $\delta$ constrained to the range of


Fig. 5. The one-loop diagram for generating radiated charged lepton masses.
$110^{\circ} \sim 250^{\circ}$. No constrained parameter space is obtained for three mixing angles, leading to the $\left|J_{C P}\right| \sim(0)-(0.035)$. Similar to the class II, there exists interesting correlations between $\delta$, the lightest neutrino mass $m_{3}$ and the effective Majorana neutrino mass $m_{e e}$. The Majorana phase $\beta$ is restricted in the range of $-18^{\circ} \sim 18^{\circ}$. In particular, we obtain the bound $0.01 \mathrm{eV}<m_{e e}<0.05 \mathrm{eV}$ for $110^{\circ}<\delta<160^{\circ}\left(200^{\circ}<\delta<250^{\circ}\right)$, the values in the scope of the accuracy of $0 \nu \beta \beta$ experiment near the future.

## 4. Cofactor zeros in charged lepton matrices

One reminds the type-I seesaw mechanism as $M_{v}=$ $-M_{D} M_{R}^{-1} M_{D}^{T}$. Then the cofactor zeros of $M_{\nu}$ are attributed to the texture zeros in $M_{D}$ and $M_{R}$. Generally, this can be easily realized by Abelian $Z_{n}$ flavor symmetry [11,30]. Can the cofactor zeros in $M_{l}$ arise using the same way? At the tree level, it is obviously impossible. At the loop level, the answer is yes! Here we adopt the model proposed by Ma [21], consisting of the SM extended by adding three Dirac singlet neutral fermion $N_{k}(k=1,2,3)$, a doublet scalar $\left(\eta^{+}, \eta^{0}\right)$ and a charged singlet $\sigma^{+}$. In Ma's model, the particles transform under the proper $U(1)_{D}$ gauge symmetry and $A_{4}$ flavor symmetry. Here, we choose the $Z_{2}^{(A)}$ instead of $A_{4}$ flavor symmetry under which $N_{k},\left(\eta^{+}, \eta^{0}\right)$ and $\sigma^{+}$are odd. To forbid the tree level Dirac lepton mass, another $Z_{2}^{(B)}$ symmetry is imposed such that $l_{R}$ and $\sigma^{+}$are odd while others are even. Actually, the flavor symmetry we propose is the same as the one in Ref. [31] where the Dirac neutrino mass is generated at one-loop level. The allowed Yukawa interactions are $y_{i j} \bar{N}_{i R}\left(l_{j L} \eta^{+}-v_{j L} \eta^{0}\right)$ and $h_{i j} \bar{l}_{i R} N_{j L} \sigma^{-}$. The $Z_{2}^{(B)}$ is allowed to be softly broken by the trilinear term $\mu\left(\eta^{+} \phi^{0}-\eta^{0} \phi^{+}\right) \sigma^{-}$with the SM vacuum expectation $v=\left\langle\phi^{0}\right\rangle$. The one-loop charged lepton mass is thus generated as shown in Fig. 5, the result being

$$
\begin{align*}
\left(M_{l}\right)_{i j}= & \frac{\sin 2 \theta}{32 \pi^{2}} \sum_{k} y_{i k} M_{k}\left[\frac{m_{1}^{2}}{m_{1}^{2}-M_{k}^{2}} \ln \left(\frac{m_{1}^{2}}{M_{k}^{2}}\right)\right. \\
& \left.-\frac{m_{2}^{2}}{m_{2}^{2}-M_{k}^{2}} \ln \left(\frac{m_{2}^{2}}{M_{k}^{2}}\right)\right] h_{k j}^{\dagger} \tag{53}
\end{align*}
$$

The $m_{1,2}$ and $\theta$ denote the eigenvalues and the mixing angle of mass squared texture
$\left(\begin{array}{ll}m_{\sigma}^{2} & \mu v \\ \mu v & m_{\eta}^{2}\end{array}\right)$
with
$m_{1,2}^{2}=\frac{1}{2}\left[m_{\eta}^{2}+m_{\sigma}^{2} \mp \sqrt{\left(m_{\eta}^{2}-m_{\sigma}^{2}\right)^{2}+4 \mu^{2} v^{2}}\right]$
and
$\sin 2 \theta=\frac{2 \mu^{2} v^{2}}{\sqrt{\left(m_{\eta}^{2}-m_{\sigma}^{2}\right)^{2}+4 \mu^{2} v^{2}}}$
For $M_{k} \gg m_{1,2}$, Eq. (53) is simplified as
$\left(M_{l}\right)_{i j} \simeq \frac{\sin 2 \theta}{32 \pi^{2}} m_{1}^{2} \sum_{k} F\left(\frac{m_{1}^{2}}{m_{2}^{2}}, \frac{M_{k}^{2}}{m_{1}^{2}}\right) y_{i k} \frac{1}{M_{k}} h_{k j}^{\dagger}$
with
$F(x, y) \equiv x \ln \left(\frac{y}{x}\right)+\ln y$
Following the same strategy of Ref. [32], the $F\left(\frac{m_{1}^{2}}{m_{2}^{2}}, \frac{M_{k}^{2}}{m_{1}^{2}}\right)$ is treated as a constant at leading order if three $M_{k}$ are assumed to be nearly degenerated. Then we get
$M_{l} \sim m_{1} y\left(M_{N}\right)_{\text {diag }}^{-1} h^{\dagger}$
On the other hand, if we assume $m_{\eta} \simeq m_{\sigma} \simeq M_{k}$ and note $\mu v \ll$ $M_{k}^{2}$, then
$\left(M_{l}\right)_{i j} \simeq \frac{\mu v}{16 \pi^{2}} \sum_{k} y_{i k} \frac{1}{M_{k}} h_{k j}^{\dagger} \sim \mu v y\left(M_{N}\right)_{\text {diag }}^{-1} h^{\dagger}$
The expression also appears in [33] where the Majorana neutrino mass is generated at one-loop level. From (59) and (60), the charged leptons acquire the radiated masses via the seesaw-like mechanism and masses of heavy Dirac neutral particles $N_{k}$ play the role of seesaw scale.

Consider now the weak basis where the mass matrix of $M_{N}$ is not diagonal. It is obvious that, working in the context of the seesaw-like mechanism with a diagonal Dirac matrices $y$ and $h$, the vanishing cofactors in the charged lepton mass matrix are equivalent to texture zeros in the heavy Dirac fermion mass matrix $M_{N}$. As having done in neutrino sector, the texture zeros in $y$, $h$, and $M_{N}$ are easily achieved by introducing extra $Z_{n}$ flavor symmetries. Form Eq. (60), it is clear that the seesaw-like scale $M_{N}$ is reduced to TeV by the smallness of factor $\mu v / 16 \pi^{2}$ originated from softly broken $Z_{2}^{(B)}$.

## 5. Conclusion and discussion

In this work, we have studied the parallel structures with cofactor zeros in lepton mass matrices. These matrices cannot obtained from arbitrary Hermitian texture by making WB transformations. Using the permutation transformation, the 15 possible textures are grouped into 4 classes where the matrices in each class lead to the same physical implications. Among the 4 classes, one of them is not compatible with experimental results and another is equivalent to the texture zero structures explored extensively in previous literature. We focus on the other two classes (class II and class III). Using the new results from the neutrino oscillation and cosmology experiments, a systematic and phenomenological analysis are proposed for each class and mass hierarchy. We have demonstrated that some predictions for the atmosphere mixing angle $\theta_{23}$, the Dirac CP-violating phase $\delta$ and the Majorana effective neutrino mass $m_{e e}$ are rather interesting and deserve to be explored in the future experiments. We also demonstrate how the cofactor zeros arise in a seesaw-like model where charged lepton mass are generated at one-loop level. We expect that a cooperation between phenomenological study and the flavor symmetry study will finally help us real the structure of leptonic texture.

## Acknowledgements

The author would like to thank Z.Z. Xing for the useful discussion during this work and J. Zhang for his kind help in plotting Fig. 5.

## References

[1] Q.R. Ahmad, et al., SNO Collaboration, Phys. Rev. Lett. 89 (2002) 011301; K. Eguchi, et al., KamLAND Collaboration, Phys. Rev. Lett. 90 (2003) 021802; M.H. Ahn, et al., K2K Collaboration, Phys. Rev. Lett. 90 (2003) 041801.
[2] F.P. An, et al., DAYA-BAY Collaboration, Phys. Rev. Lett. 108 (2012) 171803.
[3] J.K. Ahn, et al., RENO Collaboration, Phys. Rev. D 108 (2012) 191802.
[4] H. Fritzsch, M. Gell-Mann, P. Minkowski, Phys. Lett. B 59 (1975) 256; P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of Workshop on Unified Theory and the Baryon Number of the Universe, KEK, Tsukuba, 1979, p. 95;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D.Z. Freeman (Eds.), Supergravity, North-Holland, Amsterdam, 1979, p. 315;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912;
J. Schechter, J.W.F. Valle, Phys. Rev. D 22 (1980) 2227;
J. Schechter, J.W.F. Valle, Phys. Rev. D 25 (1982) 774.
[5] P.H. Frampton, S.L. Glashow, D. Marfatia, Phys. Lett. B 536 (2002) 79; Z-z. Xing, Phys. Lett. B 530 (2002) 159;
M. Randhawa, G. Ahuja, M. Gupta, Phys. Lett. B 643 (2006) 175;
A. Merle, W. Rodejohann, Phys. Rev. D 73 (2006) 073012;
S. Dev, S. Kumar, S. Verma, S. Gupta, Phys. Rev. D 76 (2007) 013002;
S. Dev, S. Kumar, S. Verma, S. Gupta, Nucl. Phys. B 784 (2007) 103;
G. Ahuja, S. Kumar, M. Randhawa, M. Gupta, S. Dev, Phys. Rev. D 76 (2007) 013006;
S. Dev, S. Kumar, Mod. Phys. Lett. A 22 (2007) 1401;
S. Kumar, Phys. Rev. D 84 (2011) 077301;
P.O. Ludl, S. Morisi, E. Peinado, Nucl. Phys. B 857 (2012) 411;
W. Grimus, P.O. Ludl, arXiv:1208.4515;
D. Meloni, G. Blankenburg, Nucl. Phys. B 867 (2013) 749;
H. Fritzsch, Z.-z. Xing, S. Zhou, J. High Energy Phys. 09 (2011) 083.
[6] S. Kaneko, H. Sawanaka, M. Tanimoto, J. High Energy Phys. 08 (2005) 073; S. Dev, S. Verma, S. Gupta, Phys. Lett. B 687 (2010) 53;
S. Goswami, S. Khan, A. Watanable, Phys. Lett. B 687 (2010) 53; W. Grimus, P.O. Ludl, arXiv:1208.4515.
[7] J.-Y. Liu, S. Zhou, Phys. Rev. D 87 (2013) 093010.
[8] X.-G. He, A. Zee, Phys. Rev. D 68 (2003) 037302.
[9] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, T. Yanagida, Phys. Lett. B 562 (2003) 265;
B.C. Chauhan, J. Pulido, M. Picariello, Phys. Rev. D 73 (2006) 053003.
[10] L. Lavoura, Phys. Lett. B 609 (2005) 317;
E.I. Lashin, N. Chamoun, Phys. Rev. D 78 (2008) 073002;
E.I. Lashin, N. Chamoun, Phys. Rev. D 80 (2009) 093004.
[11] S. Dev, S. Gupta, R.R. Gautam, Mod. Phys. Lett. A 26 (2011) 501; S. Dev, S. Gupta, R.R. Gautam, L. Singh, Phys. Lett. B 706 (2011) 168;
T. Araki, J. Heeck, J. Kubo, J. High Energy Phys. 07 (2012) 083;
S. Verma, Nucl. Phys. B 854 (2012) 340;
S. Dev, R.R. Gautam, L. Singh, arXiv:1309.4219.
[12] S. Dev, S. Verma, S. Gupta, R.R. Gautam, Phys. Rev. D 81 (2010) 053010; J. Liao, D. Marfatia, K. Whisnant, arXiv:1311.2639.
[13] H.A. Alhendi, E.I. Lashin, A.A. Mudlej, Phys. Rev. D 77 (2008) 013009.
[14] S. Dev, R.R. Gautam, L. Singh, Phys. Rev. D 87 (2013) 073011.
[15] S. Dev, R.R. Gautam, L. Singh, Phys. Rev. D 88 (2013) 033008; W. Wang, Eur. Phys. J. C 73 (2013) 2551.
[16] H. Fritzsch, Phys. Lett. B 73 (1978) 317; H. Fritzsch, Nucl. Phys. B 155 (1979) 189.
[17] Z.Z. Xing, Phys. Lett. B 550 (2002) 178; Z.Z. Xing, S. Zhou, Phys. Lett. B 593 (2004) 156; S. Zhou, Z.Z. Xing, Eur. Phys. J. C 38 (2005) 495; G. Ahuja, M. Gupta, M. Randhawa, R. Verma, Phys. Rev. D 79 (2009) 093006.
[18] Y.L. Zhou, Phys. Rev. D 86 (2012) 093011.
[19] G.C. Branco, D. Emmannuel-Costa, R. González Felipe, H. Serôdio, Phys. Lett. B 670 (2009) 340.
[20] S. Dev, S. Gupta, R.R. Gautam, Phys. Rev. D 82 (2010) 073015.
[21] E. Ma, Phys. Rev. Lett. 112 (2014) 091801.
[22] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33 (1957) 549; Z. Maki, M. Nakagawa, N. Sakata, Prog. Theor. Phys. 28 (1962) 870.
[23] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039.
[24] F. Copazzi, G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, arXiv:1312. 2878.
[25] H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney, I.V. Krivosheina, Mod. Phys. Lett. A 16 (2001) 2409.
[26] C.E. Aalseth, et al., Mod. Phys. Lett. A 17 (2002) 1475; F. Feruglio, A. Strumia, F. Vissani, Nucl. Phys. B 637 (2002) 345.
[27] S.M. Bilenky, C. Giunti, Mod. Phys. Lett. A 16 (2012) 1230015.
[28] P.A.R. Ade, et al., Planck Collaboration, arXiv:1303.5076.
[29] Z.Z. Xing, H. Zhang, S. Zhou, Phys. Rev. D 86 (2012) 013013.
[30] W. Grimus, A.S. Joshipura, L. Lavoura, M. Tanimoto, Eur. Phys. J. C 36 (2004) 227.
[31] E. Ma, Phys. Rev. D 86 (2012) 033007.
[32] P.H. Gu, U. Sarker, Phys. Rev. D 77 (2008) 105031.
[33] E. Ma, Phys. Rev. D 73 (2006) 077301.


[^0]:    E-mail address: wjnwang96@gmail.com.

