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Distributed Coordinated Attitude Control for Multiple Rigid Bodies

Dai Gao[a]*, Jianting Lv[b], Benli Wang[a]

[a] *Research Center of Satellite Technology, Harbin Institute of Technology, Harbin, 150080, China*
[b] *School of Mathematical Science, Heilongjiang University, Harbin 150080, China*

Abstract

This paper addresses coordinated attitude control problem for multiple rigid bodies. Based on graph theory and Lyapunov stability theory, distributed coordinated attitude control laws are designed. The proposed control laws guarantee that each rigid body attains desired time-varying attitude and angular velocity while maintaining attitude synchronization with other rigid body. Furthermore, attitude tracking and synchronization without angular velocity measurements and with input constraints is also discussed.

Keywords: Distributed Control; Attitude Control; Output Feedback.

1. Introduction

Distributed coordinated attitude control problem for multiple rigid bodies has been the interest of many researchers. Different from centralized control schemes in which there is an object plays center body to control all the rigid bodies, there requires no central station in distributed control schemes. Therefore, when employing distributed coordinated control schemes, advantages such as flexible scalability, robustness and easy maintenance can be obtained.

Recently algebraic graph theory which was actively used in dealing with simple dynamic model such as single or double integrator dynamics [1-4] has been applied in analyzing formation flying spacecraft or rigid bodies in general [5-7]. In [8], distributed attitude synchronization problem for multiple spacecraft through local information exchange is studied. In [9], distributed attitude synchronization and tracking problem in the presence of a time-varying reference state is addressed. The coordinated attitude control law utilizing the Laplacian matrix of the associated communication graph of multiple rigid bodies was proposed in [10].

* Corresponding author. Tel.: +86-451-86413440; fax: +86-451-86416457.
E-mail address: mackingtosh@gmail.com.

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In [8-10], the control schemes adopted full-state feedback controls that utilize both attitude and angular velocity measurements. But the assumption of availability of the angular velocity measurement is not always satisfied because of either cost limitations or implementation considerations. Therefore, the study of attitude control without angular measurements has become an interested issue. In [11], a behavioral approach was used for attitude synchronization without angular velocity measurement. In [12], authors presented a solution to the problem of tracking relative attitude in a leader-follower spacecraft formation without angular velocity measurements. In [13], authors proposed a passivity based control law for distributed attitude synchronization under undirected communication graph.

Another important problem encountered in practice of the control system design is the control input constraints. Whenever the saturation occurs in the input of the control system, the system's dynamic performance goes bad even may cause the whole close loop instable. Coordinated attitude control for multiple rigid bodies is addressed in the circumstance that input constraints and angular velocity immeasurements exist simultaneously [14, 15].

In this paper, we propose distributed coordinated attitude control laws for multiple rigid bodies. First, full-state feedback attitude coordinated control law is designed that guarantee attitude synchronization and tracking. Second, we extend our results to solve the attitude synchronization without desired trajectory. And then, output feedback attitude coordinated control laws are designed without angular velocity measurements and in the presence of control input constraints. Throughout the paper, the communication flow among rigid bodies is assumed to be both fixed and undirected.

The rest of this paper is organized as follows. In Section 2, we introduce some background on the attitude dynamics of multiple rigid bodies and graph theory. In Section 3 main results on attitude synchronization are derived and finally conclusions follow in section 4.

2. Background and preliminaries

2.1. Rigid body attitude dynamics

Consider a group of N rigid body with the equations of motion given by

$$\boldsymbol{J}_i\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i^{\times}\boldsymbol{J}_i\boldsymbol{\omega}_i = \boldsymbol{u}_i \tag{1}$$

$$\dot{\boldsymbol{\sigma}}_i = \boldsymbol{G}(\boldsymbol{\sigma}_i)\boldsymbol{\omega}_i = \frac{1}{2}\left(\frac{1-\boldsymbol{\sigma}_i^{\mathrm{T}}\boldsymbol{\sigma}_i}{2}\boldsymbol{I}_3 + \boldsymbol{\sigma}_i^{\times} + \boldsymbol{\sigma}_i\boldsymbol{\sigma}_i^{\mathrm{T}}\right)\boldsymbol{\omega}_i \tag{2}$$

where $\boldsymbol{J}_i \in \mathbf{R}^{3\times 3}$ is the constant, positive-definite, symmetric inertia matrix; $\boldsymbol{\omega}_i \in \mathbf{R}^3$ is the angular velocity vector of the body frame with respect to the inertial frame, expressed in the body frame; $\boldsymbol{u}_i \in \mathbf{R}^3$ is the control torque; $\boldsymbol{\sigma}_i \in \mathbf{R}^3$ denotes the MRP that represents the orientation of the body frame with respect to the inertial frame.

Assume the desired trajectory is given by $\boldsymbol{\sigma}_{di}$ that represent the orientation of the desired frame and $\boldsymbol{\omega}_{di}$ that is the angular velocity of the desired frame.

The attitude tracking error are defined by $\delta\boldsymbol{\sigma}_i$ and angular velocity tracking error is given by

$$\delta\boldsymbol{\omega}_i = \boldsymbol{\omega}_i - \boldsymbol{R}(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di} \tag{3}$$

where $\boldsymbol{R}(\delta\boldsymbol{\sigma}_i) = \boldsymbol{R}(\boldsymbol{\sigma}_i)\boldsymbol{R}(\boldsymbol{\sigma}_{di})^T$ is the rotation matrix from the desired frame to the body frame.

The overall attitude tracking error dynamics is obtained by differentiating (3) with respect time as the following

$$\delta\dot{\boldsymbol{\sigma}}_i = \boldsymbol{G}(\delta\boldsymbol{\sigma}_i)\delta\boldsymbol{\omega}_i \tag{4}$$

$$\boldsymbol{J}_i\delta\dot{\boldsymbol{\omega}}_i = -\left(\delta\boldsymbol{\omega}_i + \boldsymbol{R}_i(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di}\right)^{\times}\boldsymbol{J}_i\left(\delta\boldsymbol{\omega}_i + \boldsymbol{R}_i(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di}\right) + \boldsymbol{J}_i\left(\delta\boldsymbol{\omega}_i^{\times}\boldsymbol{R}_i(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di} - \boldsymbol{R}_i(\delta\boldsymbol{\sigma}_i)\dot{\boldsymbol{\omega}}_{di}\right) + \boldsymbol{u}_i \tag{5}$$

The objective of our work is to design distributed control algorithms for multiple rigid bodies with or without angular velocity measurements and input constraints such that attitude synchronization and tracking can be guaranteed, i.e., $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_{di}$, $\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t) \to \boldsymbol{\sigma}_{dij}$, $\boldsymbol{\omega}_i \to \boldsymbol{\omega}_{di}$, $\boldsymbol{\omega}_i(t) - \boldsymbol{\omega}_j(t) \to \boldsymbol{\omega}_{dij}$, as $t \to \infty$, where $\boldsymbol{\sigma}_{dij}$ and $\boldsymbol{\omega}_{dij}$ are the desired relative attitude and relative angular velocity between the *ith* rigid body and the *jth* rigid body. Also we extend our results to solve the attitude synchronization without desired trajectory, i.e., $\boldsymbol{\sigma}_i(t) \to \boldsymbol{\sigma}_j(t) \to \boldsymbol{0}$, $\boldsymbol{\omega}_i(t) \to \boldsymbol{\omega}_j(t) \to \boldsymbol{0}$, as $t \to \infty$.

2.2. Graph theory

For an N rigid bodies system, there is information interchange among rigid bodies. We assume that information flow among rigid bodies is fixed and undirected and is described by the graph $G = \{\mathcal{V}, \mathcal{E}\}$. $\mathcal{V} = \{1, 2, \cdots, N\}$ is the set of nodes and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is set of edges. An edge (i, j) in an undirected graph denotes that node i and node j can obtain information from one another. The weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ of a graph G is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$. The Laplacian matrix $L = [l_{ij}] \in R^{n\times n}$ of a graph G is defined as $l_{ii} = \sum_{j\in N_i} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. A graph is called connected if for any two nodes there exists a set of edges that connect the two nodes.

3. Control Law Design

In this section, we consider distributed coordinated attitude control problem for multiple rigid bodies. We first develop full-state feedback control law and the result is stated in the theorem 1. Then we give the output feedback control law without angular velocity measurements and with input constraints. The main result is stated in the theorem 2.

Theorem 1: For the system given by (4)~(5), using the following control law

$$\begin{aligned}\boldsymbol{u}_i &= -k_{pi}\delta\boldsymbol{\sigma}_i - k_{di}\delta\boldsymbol{\omega}_i - \boldsymbol{G}^T(\delta\boldsymbol{\sigma}_i)\sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\sigma}_i - \delta\boldsymbol{\sigma}_j)\sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\omega}_i - \delta\boldsymbol{\omega}_j)\\ &\quad + (R(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di})^{\times}\boldsymbol{J}_i R(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di} + \boldsymbol{J}_i R(\delta\boldsymbol{\sigma}_i)\dot{\boldsymbol{\omega}}_{di}\end{aligned} \tag{6}$$

if the undirected graph is connected and control gains satisfy $k_{pi} \geq 3\sum_{j=1}^{N} a_{ij}$, then $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_{di}$, $\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t) \to \boldsymbol{\sigma}_{dij}$, $\boldsymbol{\omega}_i \to \boldsymbol{\omega}_{di}$, $\boldsymbol{\omega}_i(t) - \boldsymbol{\omega}_j(t) \to \boldsymbol{\omega}_{dij}$, as $t \to \infty$. Where a_{ij} is the $(i, j)th$ entry of the weighted adjacency matrix A associated with the communication graph G .

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{N}\left(\frac{1}{2}\delta\boldsymbol{\omega}_i^T\boldsymbol{J}_i\delta\boldsymbol{\omega}_i\right) + \sum_{i=1}^{N}\left(2k_{pi}\ln(1 + \delta\boldsymbol{\sigma}_i^T\delta\boldsymbol{\sigma}_i)\right) + \frac{1}{2}\delta\boldsymbol{\sigma}^T(L\otimes I_3)\delta\boldsymbol{\sigma} \tag{7}$$

where $\delta\boldsymbol{\sigma} = [\delta\boldsymbol{\sigma}_1^T, \delta\boldsymbol{\sigma}_2^T, \cdots \delta\boldsymbol{\sigma}_n^T]^T$.

The time derivative of V is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} (\delta\boldsymbol{\omega}_i^T \boldsymbol{J}_i \delta\dot{\boldsymbol{\omega}}_i) + \sum_{i=1}^{N} (\frac{4k_{pi}\boldsymbol{\sigma}_i^T \boldsymbol{G}(\delta\boldsymbol{\sigma}_i)\delta\boldsymbol{\omega}_i}{1+\delta\boldsymbol{\sigma}_i^2}) + \delta\boldsymbol{\sigma}^T (L\otimes I_3)\delta\dot{\boldsymbol{\sigma}} \\ &= \sum_{i=1}^{N} (-\delta\boldsymbol{\omega}_i^T ((R(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di})^{\times} \boldsymbol{J}_i R(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di} + \boldsymbol{J}_i R(\delta\boldsymbol{\sigma}_i)\dot{\boldsymbol{\omega}}_{di}) + \sum_{i=1}^{N} (k_{pi}\delta\boldsymbol{\sigma}_i^T \delta\boldsymbol{\omega}_i) \\ &+ \sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\sigma}_i - \delta\boldsymbol{\sigma}_j)^T \boldsymbol{G}(\delta\boldsymbol{\sigma}_i)\delta\boldsymbol{\omega}_i) + \sum_{i=1}^{N} (\delta\boldsymbol{\omega}_i^T \boldsymbol{u}_i) \end{aligned} \tag{8}$$

Substituting (6) into (8), we obtain

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^{N} k_{di}\delta\boldsymbol{\omega}_i^T \delta\boldsymbol{\omega}_i - \sum_{i=1}^{N} \delta\boldsymbol{\omega}_i^T \sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\omega}_i - \delta\boldsymbol{\omega}_j) \\ &= -\sum_{i=1}^{N} k_{di}\delta\boldsymbol{\omega}_i^T \delta\boldsymbol{\omega}_i - \delta\boldsymbol{\omega}^T (L\otimes I_3)\delta\boldsymbol{\omega} \end{aligned} \tag{9}$$

where $\delta\boldsymbol{\omega} = [\delta\boldsymbol{\omega}_1^T, \delta\boldsymbol{\omega}_2^T, \cdots, \delta\boldsymbol{\omega}_n^T]^T$.

Note that $0 \le V(t) \le V(0) < \infty$. In addition, we can verify that $\ddot{V}$ is bounded. Using Barbalat's lemma [16], this implies that $\lim_{t\to\infty} \dot{V} = \boldsymbol{0}$. Therefore, it follows that $\delta\boldsymbol{\omega}_i \to \boldsymbol{0}$ and $\delta\boldsymbol{\omega}^T (L\otimes I_3)\delta\boldsymbol{\omega} \to \boldsymbol{0}$, as $t\to\infty$. Hence we have $\boldsymbol{\omega}_i \to \boldsymbol{\omega}_{di}$, $\boldsymbol{\omega}_i(t) - \boldsymbol{\omega}_j(t) \to \boldsymbol{\omega}_{dij}$.

Since $(L\otimes I_3)$ is positive semidefinite, if follows that $(L\otimes I_3)\delta\boldsymbol{\omega} \to \boldsymbol{0}$, which implies that $\delta\boldsymbol{\omega}_i \to \delta\boldsymbol{\omega}_j$. Therefore we can conclude that $\delta\boldsymbol{\omega}_i(t) \to \delta\boldsymbol{\omega}_j(t) \to \boldsymbol{0}$, or equivalently $\boldsymbol{\omega}_i \to \boldsymbol{\omega}_{di}$, $\boldsymbol{\omega}_i(t) - \boldsymbol{\omega}_j(t) \to \boldsymbol{\omega}_{dij}$.

Form (5) and (6), using the above results we get

$$-k_{pi}\delta\boldsymbol{\sigma}_i - \boldsymbol{G}^T(\delta\boldsymbol{\sigma}_i)\sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\sigma}_i - \delta\boldsymbol{\sigma}_j) \to 0 \tag{10}$$

Define as $p_{ii}(t) = k_{pi}[\boldsymbol{G}^{-1}(\delta\boldsymbol{\sigma})]^T + \sum_{j=1}^{N} a_{ij} I_3$, $p_{ij}(t) = -a_{ij} I_3$. We can rewrite (10) in matrix form as $P(t)\delta\boldsymbol{\sigma} \to \boldsymbol{0}$. Noting that $k_{pi} \ge 3\sum_{j=1}^{N} a_{ij}$, we see that $P(t)$ is strictly diagonally dominant and therefore has full rank, which implies that $\delta\boldsymbol{\sigma}(t) \to \boldsymbol{0}$. Therefore, we can conclude that $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_{di}$, $\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t) \to \boldsymbol{\sigma}_{dij}$ asymptotically.

In the following corollary, a distributed control algorithm is given to show that attitudes of multiple rigid bodies are able to converge to the origin without desired trajectory.

Corollary 1: For the system given by (1)~(2), using the following control law

$$\boldsymbol{u}_i = -k_{pi}\boldsymbol{\sigma}_i - k_{di}\boldsymbol{\omega}_i - \boldsymbol{G}^T(\boldsymbol{\sigma}_i)\sum_{j=1}^{N} a_{ij}(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\sum_{j=1}^{N} a_{ij}(\boldsymbol{\omega}_i - \boldsymbol{\omega}_j) \tag{11}$$

if the undirected graph G is connected and control gains satisfy $k_{pi} \geq 3\sum_{j=1}^{N} a_{ij}$, then $\boldsymbol{\sigma}_i(t) \to \boldsymbol{\sigma}_j(t) \to \boldsymbol{0}$, $\boldsymbol{\omega}_i(t) \to \boldsymbol{\omega}_j(t) \to \boldsymbol{0}$, as $t \to \infty$.

Using the same proof procedure as for Theorem 1, it can been proved that $\boldsymbol{\sigma}_i(t) \to \boldsymbol{\sigma}_j(t - T_{ji}) \to \boldsymbol{0}$, $\boldsymbol{\omega}_i(t) \to \boldsymbol{\omega}_j(t - T_{ji}) \to \boldsymbol{0}$, as $t \to \infty$.

Next we give the bounded output feedback control algorithm without angular velocity measurements and with input constraints. To facilitate the control laws design we make the following definition and assumptions.

Definition 1: A saturation function is denoted by strictly increasing continuously differentiable function with the properties: (1) $sat(0) = 0$ and $\boldsymbol{x}sat(\boldsymbol{x}) > 0$ for $\boldsymbol{x} \neq 0$; (2) $|sat(\boldsymbol{x})| \leq 1$ for $\boldsymbol{x} \in \mathbb{R}$; (3) $\dfrac{\partial sat(\boldsymbol{x})}{\partial \boldsymbol{x}} \geq 0$.

Assumption 1: For the inertia matrix $\boldsymbol{J}_i$, there exists $J_{Mi} > 0$, such that $\| \boldsymbol{J}_i \| \leq J_{Mi}$.

Assumption 2: $\boldsymbol{\omega}_{di}$ and $\dot{\boldsymbol{\omega}}_{di}$ are bounded, there exists $\upsilon_1 > 0$, $\upsilon_2 > 0$, such that $| \boldsymbol{\omega}_{di} \| \leq \upsilon_1$, $| \dot{\boldsymbol{\omega}}_{di} \| \leq \upsilon_2$.

Theorem 2: For the system given by (4)~(5), using the following control law

$$\begin{aligned}\boldsymbol{u}_i = &-\boldsymbol{G}^T(\delta\boldsymbol{\sigma}_i)\sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\sigma}_i - \delta\boldsymbol{\sigma}_j) - \boldsymbol{G}^T(\delta\boldsymbol{\sigma}_i)\left(k_{pi}\delta\boldsymbol{\sigma}_i + k_{di} sat(-\boldsymbol{A}_{mi}\boldsymbol{z}_i + \delta\boldsymbol{\sigma}_i)\right) \\ &+ (R(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di})^{\times} \boldsymbol{J}_i R(\delta\boldsymbol{\sigma}_i)\boldsymbol{\omega}_{di} + \boldsymbol{J}_i R(\delta\boldsymbol{\sigma}_i)\dot{\boldsymbol{\omega}}_{di}\end{aligned} \tag{12}$$

$$\dot{\boldsymbol{z}}_i = -\boldsymbol{A}_{mi}\boldsymbol{z}_i + \delta\boldsymbol{\sigma}_i \tag{13}$$

if the undirected graph G is connected and control gains satisfy

$$\frac{1}{2}\sum_{j=1}^{N} a_{ij} + \frac{1}{2}k_{pi} + \frac{\sqrt{3}}{2}k_{di} + J_{Mi}(\upsilon_1^2 + \upsilon_2) \leq u_{Mi} \tag{14}$$

then $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_{di}$, $\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t) \to \boldsymbol{\sigma}_{dij}$, $\boldsymbol{\omega}_i \to \boldsymbol{\omega}_{di}$, $\boldsymbol{\omega}_i(t) - \boldsymbol{\omega}_j(t) \to \boldsymbol{\omega}_{dij}$, $t \to \infty$, $\boldsymbol{u}_i \in \Omega_u = \{\boldsymbol{u}_i : \| \boldsymbol{u}_i \| \leq u_{Mi}\}$. Where $\boldsymbol{z}_i \in \mathbf{R}^3$, $k_{pi}, k_{di}, \boldsymbol{A}_{mi} > 0$, and $sat(\boldsymbol{\zeta}) = [sat(\zeta_1)\ sat(\zeta_2)\ sat(\zeta_3)]^T \in \mathbf{R}^3$ for $\boldsymbol{\zeta} \in \mathbf{R}^3$.

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{N}(\frac{1}{2}k_{pi}\delta\boldsymbol{\sigma}_i^T\delta\boldsymbol{\sigma}_i) + \sum_{i=1}^{N}(\frac{1}{2}\delta\boldsymbol{\omega}_i^T\boldsymbol{J}_i\delta\boldsymbol{\omega}_i) + \sum_{i=1}^{N}(k_{di}\sum_{l=1}^{3}\int_0^{\dot{z}_{il}} sat(x)dx) + \frac{1}{2}\delta\boldsymbol{\sigma}^T(L \otimes I_3)\delta\boldsymbol{\sigma} \tag{15}$$

The time derivative of V is given by

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}(k_{pi}\delta\boldsymbol{\sigma}_i^T\delta\dot{\boldsymbol{\sigma}}_i) + \sum_{i=1}^{N}(\delta\boldsymbol{\omega}_i^T\boldsymbol{J}_i\delta\dot{\boldsymbol{\omega}}_i) + \sum_{i=1}^{N}(k_{di}\ddot{\boldsymbol{z}}_i^T sat(\dot{\boldsymbol{z}}_i)) + \delta\boldsymbol{\sigma}^T(L \otimes I_3)\delta\dot{\boldsymbol{\sigma}} \\ &= \sum_{i=1}^{N}(k_{pi}\delta\boldsymbol{\sigma}_i^T\delta\dot{\boldsymbol{\sigma}}_i) + \sum_{i=1}^{N}(\delta\boldsymbol{\omega}_i^T\boldsymbol{u}_i) - \sum_{i=1}^{N}(\delta\boldsymbol{\omega}_i^T((\boldsymbol{R}\boldsymbol{\omega}_{di})^{\times}\boldsymbol{J}\boldsymbol{R}\boldsymbol{\omega}_{di} + \boldsymbol{J}\boldsymbol{R}\dot{\boldsymbol{\omega}}_{di})) \\ &\quad + \sum_{i=1}^{N}(k_{di}(-\boldsymbol{A}_{mi}\dot{\boldsymbol{z}}_i + \delta\dot{\boldsymbol{\sigma}}_i)^T sat(\dot{\boldsymbol{z}}_i)) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\delta\boldsymbol{\sigma}_i - \delta\boldsymbol{\sigma}_j)\boldsymbol{G}^T(\delta\boldsymbol{\sigma}_i)\delta\boldsymbol{\omega}_i\end{aligned} \tag{16}$$

Substituting (12) and (13) into (16), we obtain

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^{N}(k_{di}\delta\dot{\boldsymbol{\sigma}}_i^T sat(\dot{\boldsymbol{z}}_i)) + \sum_{i=1}^{N}(-\boldsymbol{A}_{mi}k_{di}\dot{\boldsymbol{z}}_i^T sat(\dot{\boldsymbol{z}}_i) + k_{di}\delta\dot{\boldsymbol{\sigma}}_i^T sat(\dot{\boldsymbol{z}}_i)) \\ &= -\sum_{i=1}^{N}\boldsymbol{A}_{mi}k_{di}\dot{\boldsymbol{z}}_i^T sat(\dot{\boldsymbol{z}}_i) \le 0 \end{aligned} \tag{17}$$

Because $V \ge 0$ and $\dot{V} \le 0$, $\delta\boldsymbol{\sigma}_i$ and $\dot{\boldsymbol{z}}_i$ are all uniformly bounded. It is easy to establish that $\dddot{\boldsymbol{z}}_i$ is uniformly bounded, in other words, uniform continuity for $\ddot{\boldsymbol{z}}_i$. Since $\lim_{t\to\infty}\dot{\boldsymbol{z}}_i = \boldsymbol{0}$, and using Barbalat's lemma, we obtain $\lim_{t\to\infty}\ddot{\boldsymbol{z}}_i = \boldsymbol{0}$.

Also $\ddot{\boldsymbol{z}}_i = \boldsymbol{A}_{mi}\dot{\boldsymbol{z}}_i + \delta\dot{\boldsymbol{\sigma}}_i$. We can get $\delta\dot{\boldsymbol{\sigma}}_i \to \boldsymbol{0}$. From (4) implies $\lim_{t\to\infty}\delta\boldsymbol{\omega}_i \to \mathbf{0}$ since $G(\delta\boldsymbol{\sigma}_i)$ is nonsingular for all $\delta\boldsymbol{\sigma}_i$. Furthermore, through Barbalat's lemma we conclude that $\delta\dot{\boldsymbol{\omega}}_i \to \mathbf{0}$.

Using the above results, the error dynamics (5), with (12) reduces to

$$k_{pi}\delta\boldsymbol{\sigma}_i + \sum_{j=1}^{N}a_{ij}(\delta\boldsymbol{\sigma}_i - \delta\boldsymbol{\sigma}_j) \to \mathbf{0} \tag{18}$$

Note that (18) can be written in matrix form as

$$(k_{pi}I_n \otimes I_3 + L \otimes I_3)\delta\boldsymbol{\sigma} \to \mathbf{0} \tag{19}$$

We see that is strictly diagonally dominant and therefore is full rank, which in turn implies that $\delta\boldsymbol{\sigma}_i \to \mathbf{0}$. Finally we can conclude that $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_{di}$, $\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t) \to \boldsymbol{\sigma}_{dij}$, $\boldsymbol{\omega}_i \to \boldsymbol{\omega}_{di}$, and $\boldsymbol{\omega}_i(t) - \boldsymbol{\omega}_j(t) \to \boldsymbol{\omega}_{dij}$, as $t \to \infty$.

On the other hand, from $\|\delta\boldsymbol{\sigma}_i\| \le 1$, $\|\boldsymbol{G}\| = \sqrt{\lambda_{\max}(\boldsymbol{G}^T\boldsymbol{G})} = \dfrac{1+\delta\boldsymbol{\sigma}_i^2}{4} \le \dfrac{1}{2}$, and assumptions 1~2, we can get $\|\boldsymbol{u}_i\| \le \dfrac{1}{2}\sum_{i=1}^{N}a_{ij} + \dfrac{1}{2}k_{pi} + \dfrac{\sqrt{3}}{2}k_{di} + J_{Mi}(\upsilon_1^2 + \upsilon_2)$. Finally we can conclude that $\boldsymbol{u}_i \in \Omega_u$ if condition (14) holds.

In order to prove that attitudes of rigid bodies are able to converge to the origin without desired trajectory, we proposed the control algorithm stated the following corollary.

Corollary 2: For the system given by (1)~(2), using the following control law

$$\boldsymbol{u}_i = -\boldsymbol{G}^T(\boldsymbol{\sigma}_i)\sum_{j=1}^{N}a_{ij}(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) - \boldsymbol{G}^T(\boldsymbol{\sigma}_i)\left(k_{pi}\boldsymbol{\sigma}_i + k_{di}sat(-\boldsymbol{A}_{mi}\boldsymbol{z}_i + \boldsymbol{\sigma}_i)\right) \tag{20}$$

$$\dot{\boldsymbol{z}}_i = -\boldsymbol{A}_{mi}\boldsymbol{z}_i + \boldsymbol{\sigma}_i \tag{21}$$

if the undirected graph G is connected and control gains satisfy $\dfrac{1}{2}\sum_{j=1}^{N}a_{ij} + \dfrac{1}{2}k_{pi} + \dfrac{\sqrt{3}}{2}k_{di} \le u_{Mi}$, then $\boldsymbol{\sigma}_i(t) \to \boldsymbol{\sigma}_j(t) \to \boldsymbol{0}$, $\boldsymbol{\omega}_i(t) \to \boldsymbol{\omega}_j(t) \to \boldsymbol{0}$, as $t \to \infty$, $\boldsymbol{u}_i \in \Omega_u = \{\boldsymbol{u}_i : \|\boldsymbol{u}_i\| \le u_{Mi}\}$. Where $\boldsymbol{z}_i \in \mathbb{R}^3$, $k_{pi}, k_{di}, \boldsymbol{A}_{mi} > 0$.

The proof procedure of Corollary 2 is the same as for Theorem 2, therefore we will not go into.

4. Conclusion

We address the problem of coordinated attitude control for multiple rigid bodies. Distributed coordinated attitude control schemes are presented to guarantee attitude synchronization and tracking, one is the state feedback control scheme, and the other is the output feedback control scheme. And we extend both results when no desired trajectory required. The global asymptotic stability of closed-loop system is shown through Lyapunov analysis.

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