



# How to Solve the Zebra Problem

M.R.C. van Dongen

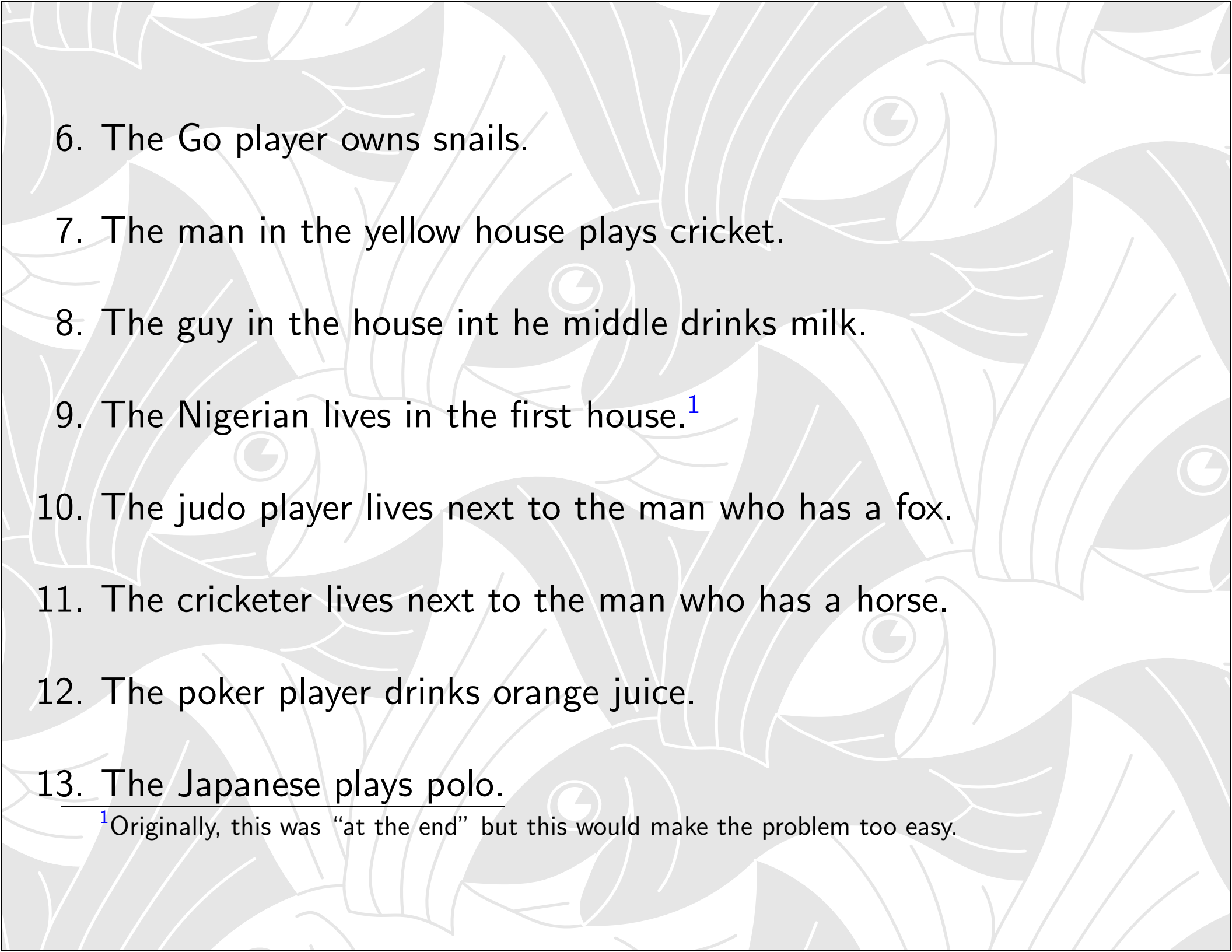
Delivered 15 June 2001. Re-designed 12 October 2002.

## Problem Formulation

There are five houses of different colours, inhabited by different nationals, with different pets, drinks, and sports.

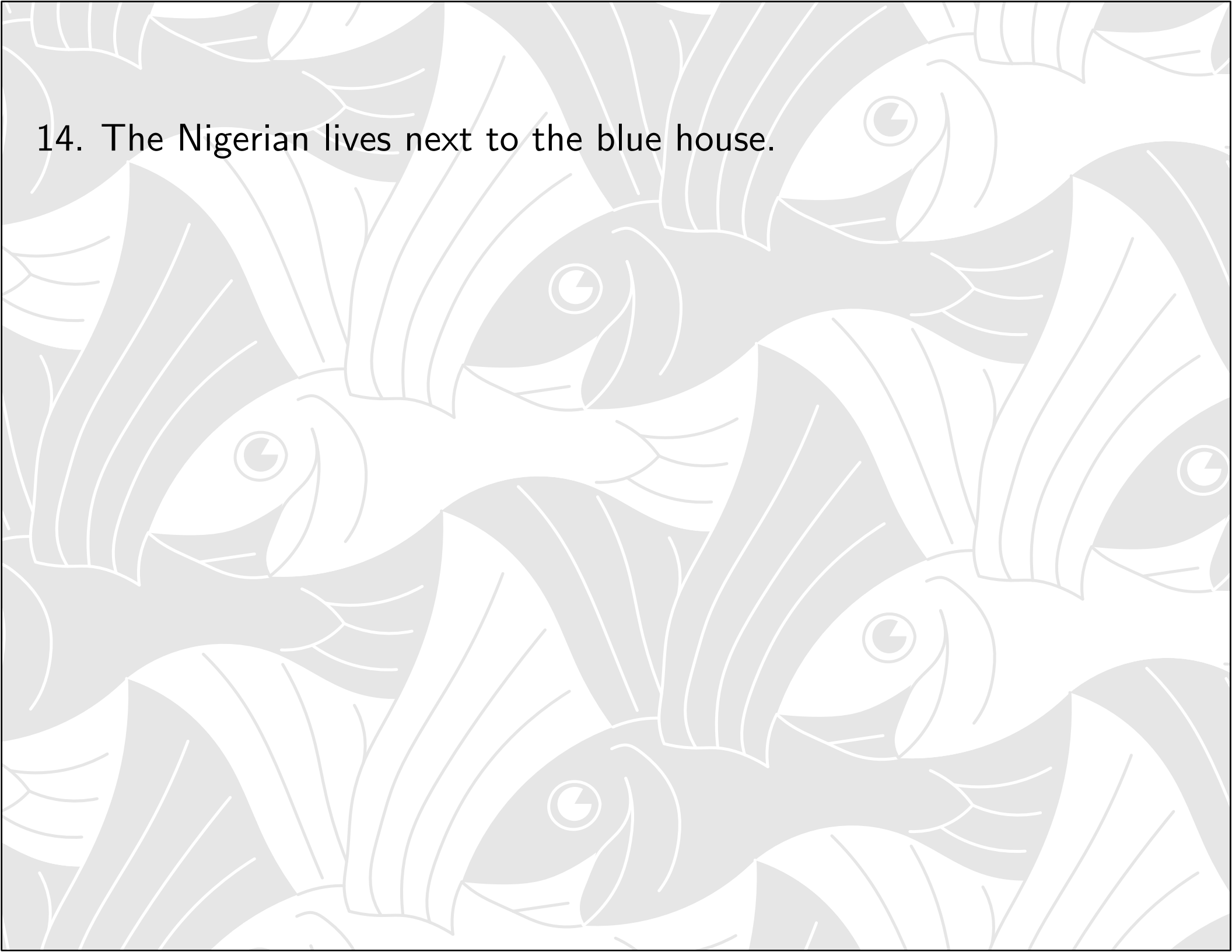
Furthermore, there are the following 14 additional constraints which I have changed for the occasion:

1. The Englishman lives in the red house.
2. The Spaniard owns a dog.
3. The man in the green house drinks coffee.
4. The Irishman drinks tea.
5. The green house is to the right of the ivory house.

- 
6. The Go player owns snails.
  7. The man in the yellow house plays cricket.
  8. The guy in the house in the middle drinks milk.
  9. The Nigerian lives in the first house.<sup>1</sup>
  10. The judo player lives next to the man who has a fox.
  11. The cricketer lives next to the man who has a horse.
  12. The poker player drinks orange juice.
  13. The Japanese plays polo.

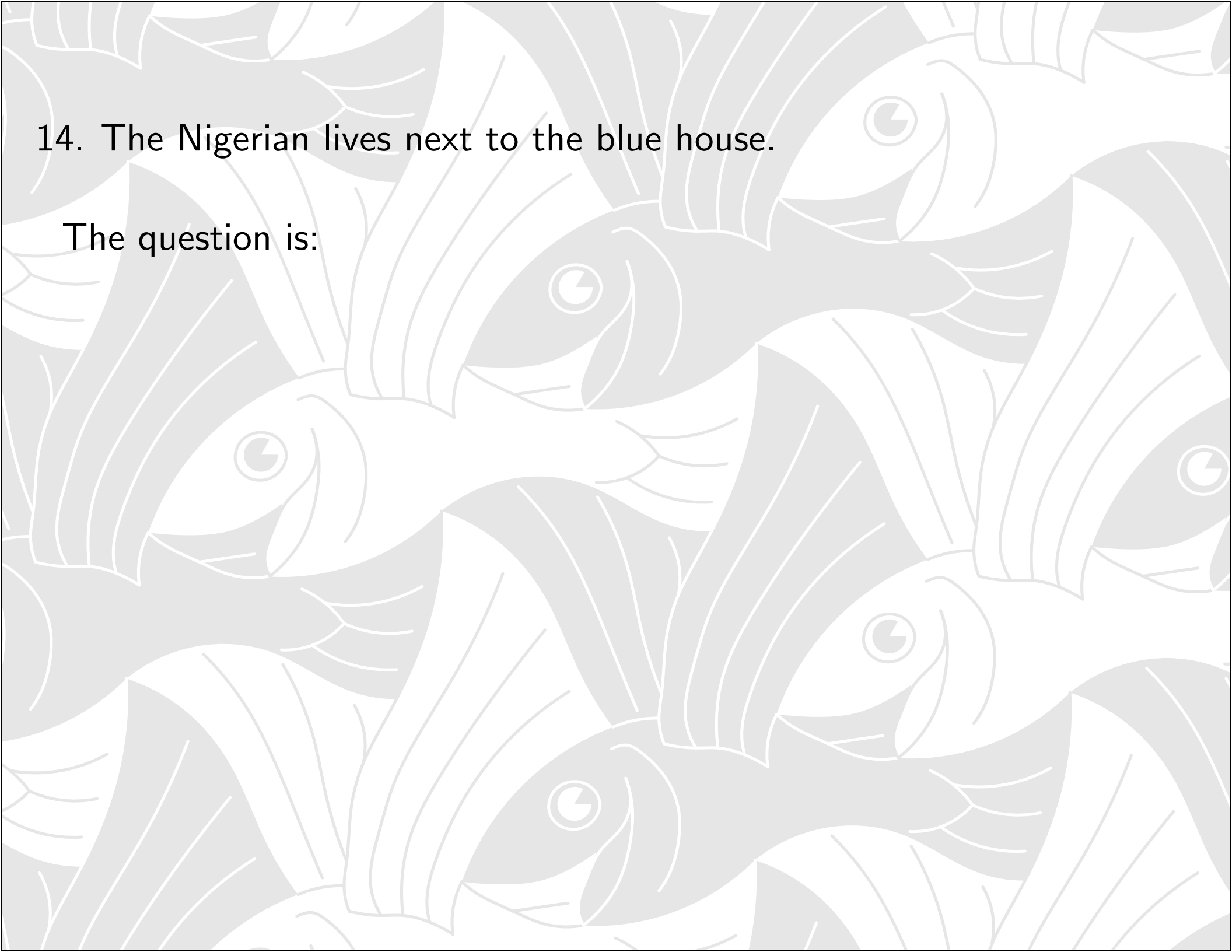
<sup>1</sup>Originally, this was “at the end” but this would make the problem too easy.

14. The Nigerian lives next to the blue house.



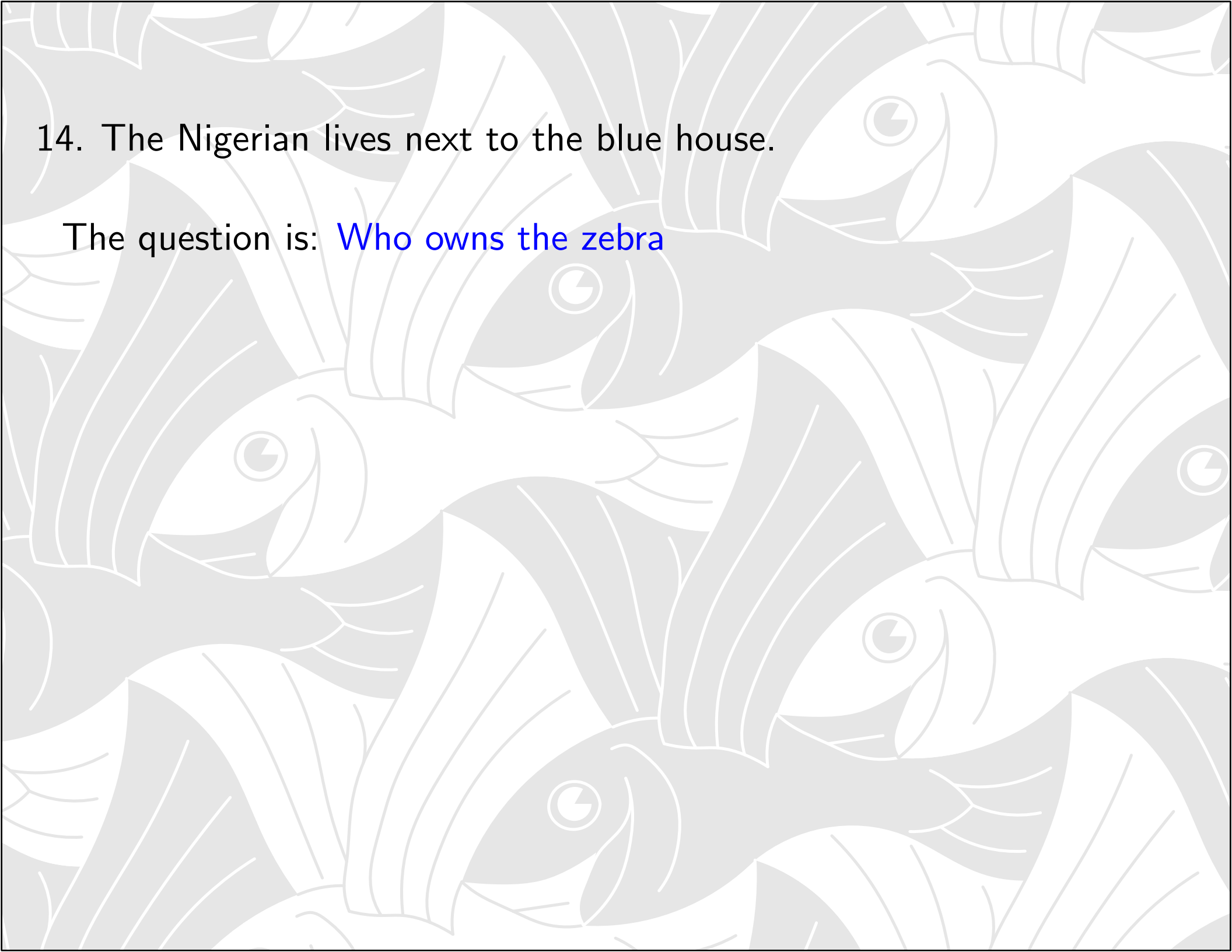
14. The Nigerian lives next to the blue house.

The question is:



14. The Nigerian lives next to the blue house.

The question is: **Who owns the zebra**





14. The Nigerian lives next to the blue house.

The question is: **Who owns the zebra and who drinks Guinness?**

## Some History



According to folklore, the Zebra Problem was designed by the English logician Charles Lutwidge Dodgson (a.k.a. Lewis Carroll. Born: 27 Jan 1832, Died: 14 Jan 1898).



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# Modeling the Problem

We can model the problem as a CSP. We number the houses (left to right) from 1 to 5. We then assign houses to things and we reduce the problem to the following:

- The number assigned to the person who drinks Guinness is the same as the number assigned to Guinness;
- The number assigned to the person who owns the zebra is the same as the number assigned to the zebra.

**nationalities** Englishman =  $A_1$ , Spaniard =  $A_2$ , Irishman =  $A_3$ ,  
Nigerian =  $A_4$ , Japanese =  $A_5$ .

**plays** go =  $B_1$ , cricket =  $B_2$ , judo =  $B_3$ , poker =  $B_4$ , polo =  $B_5$ .

**drinks** coffee =  $C_1$ , tea =  $C_2$ , milk =  $C_3$ , orange juice =  $C_4$ ,  
Guinness =  $C_5$ ,

**pets** dog =  $D_1$ , snails =  $D_2$ , fox =  $D_3$ , horse =  $D_4$ , zebra =  $D_5$ .

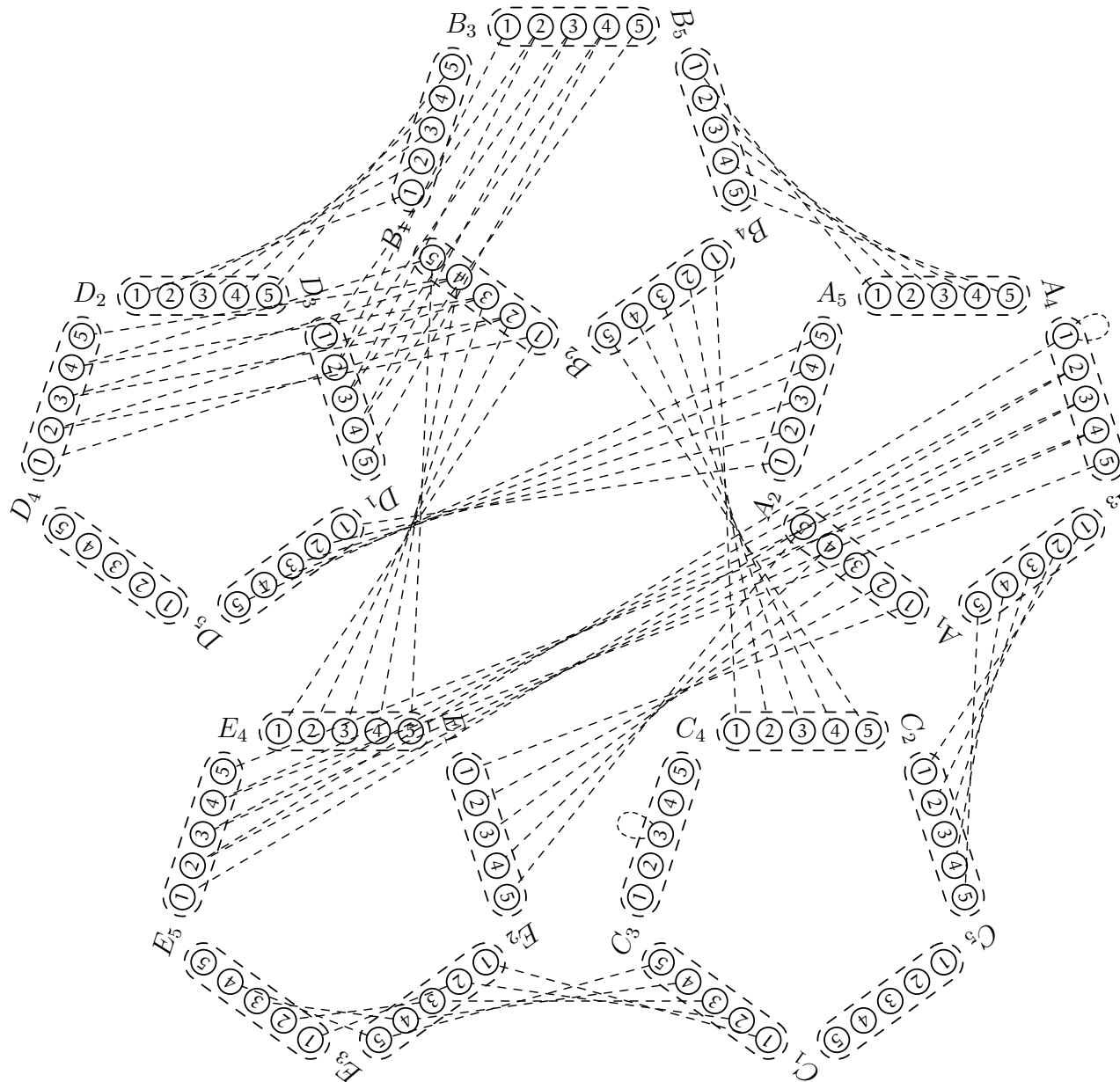
**colours** red =  $E_1$ , green =  $E_2$ , ivory =  $E_3$ , yellow =  $E_4$ , blue =  
 $E_5$ .

If  $X$  is a letter then  $X_i \neq X_j \iff i \neq j$ . Furthermore, we have:

1. The Englishman ( $A_1$ ) lives in the red ( $E_1$ ) house:  $A_1 = E_1$ .
2. The Spaniard ( $A_2$ ) owns a dog ( $D_1$ ):  $A_2 = D_1$ .
3. The man in the green ( $E_2$ ) house drinks coffee ( $C_1$ ):  $E_2 = C_1$ .
4. The Irishman ( $A_3$ ) drinks tea ( $C_2$ ):  $A_3 = C_2$ .
5. The green ( $E_2$ ) house is to the right of the ivory ( $E_3$ ) house:  
 $E_2 - E_3 = 1$ .
6. The Go ( $B_1$ ) player owns snails ( $D_2$ ):  $B_1 = D_2$ .
7. The man in the yellow ( $E_4$ ) house plays cricket ( $B_2$ ):  $E_4 = B_2$ .

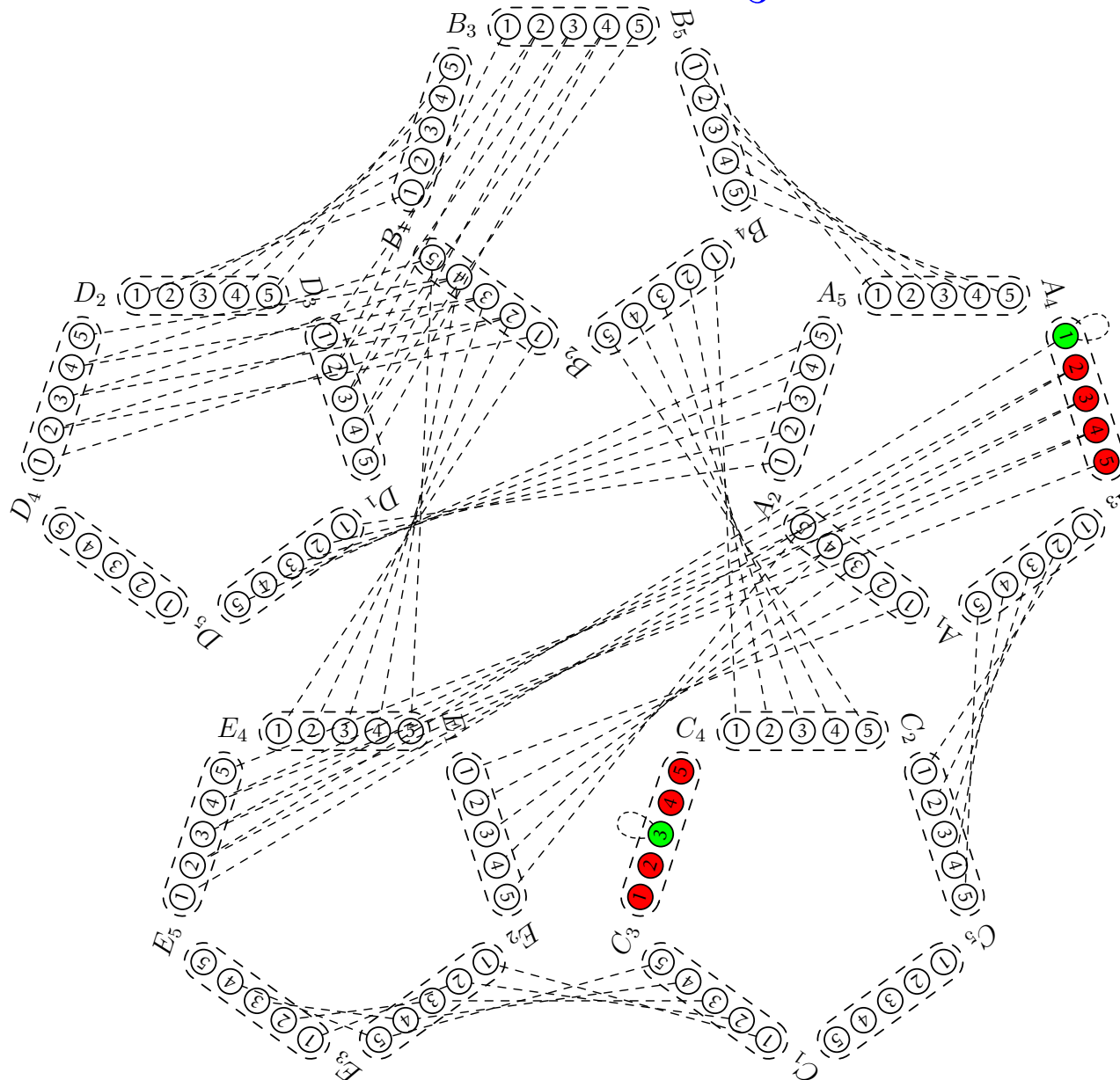
8. The guy in the house in the middle drinks milk ( $C_3$ ):  $C_3 = 3$ .
9. The Nigerian ( $A_4$ ) lives in the first house:  $A_4 = 1$ .
10. The Judo ( $B_3$ ) player lives next to the man who has a fox ( $D_3$ ):  
 $|B_3 - D_3| = 1$ .
11. The cricketer ( $B_2$ ) lives next to the man who has a horse ( $D_4$ ):  
 $|B_2 - D_4| = 1$ .
12. The poker ( $B_4$ ) player drinks orange juice ( $C_4$ ):  $B_4 = C_4$ .
13. The Japanese ( $A_5$ ) plays polo ( $B_5$ ):  $A_5 = B_5$ .
14. The Nigerian ( $A_4$ ) lives next to the blue ( $E_5$ ) house:  $|A_4 - E_5| = 1$ .

# Initial CSP.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

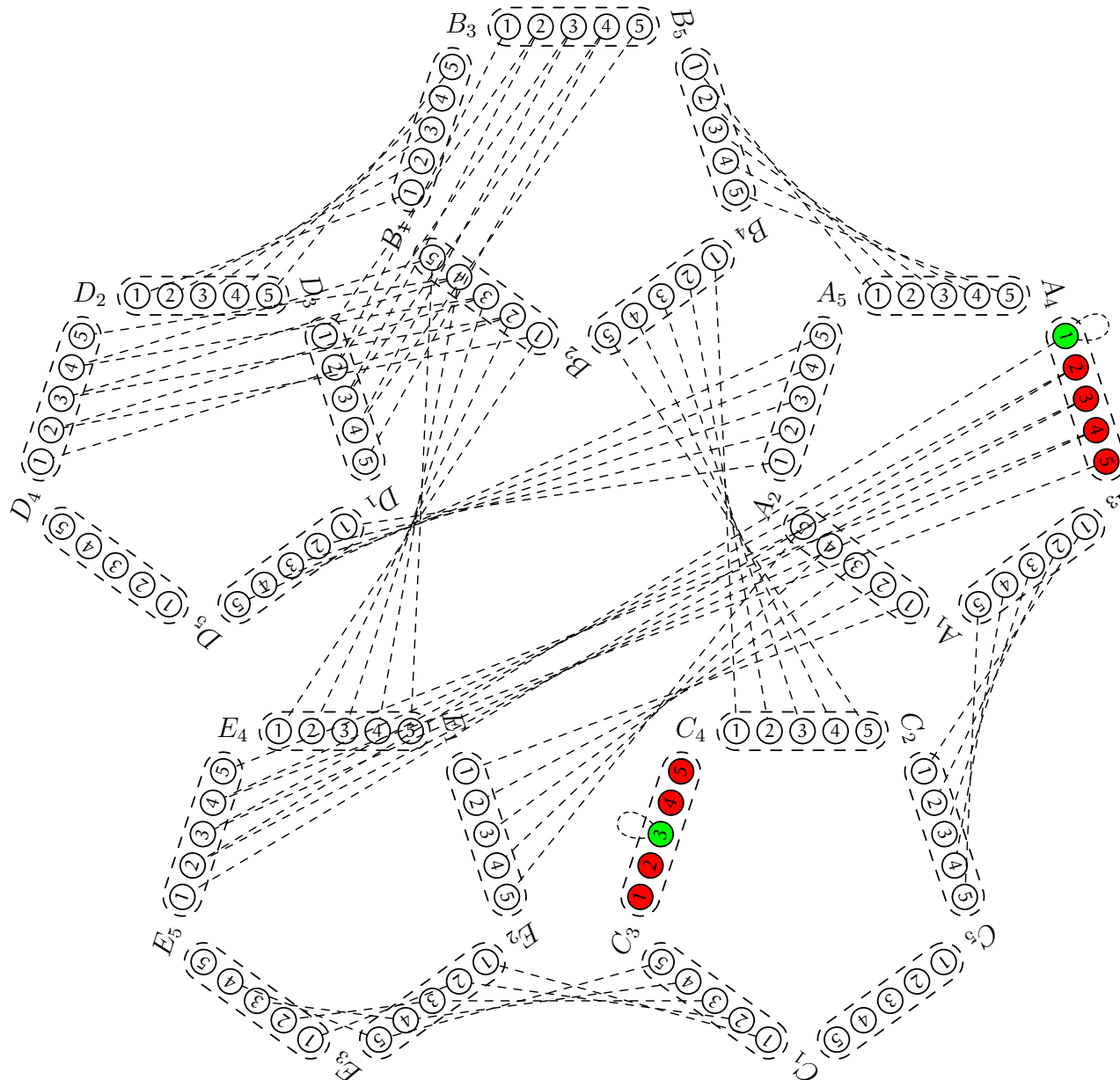
Because of the **unary** constraints (Rules 8 and 9)  $A_4$  must be 1 and  $C_3$  must be 3.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue



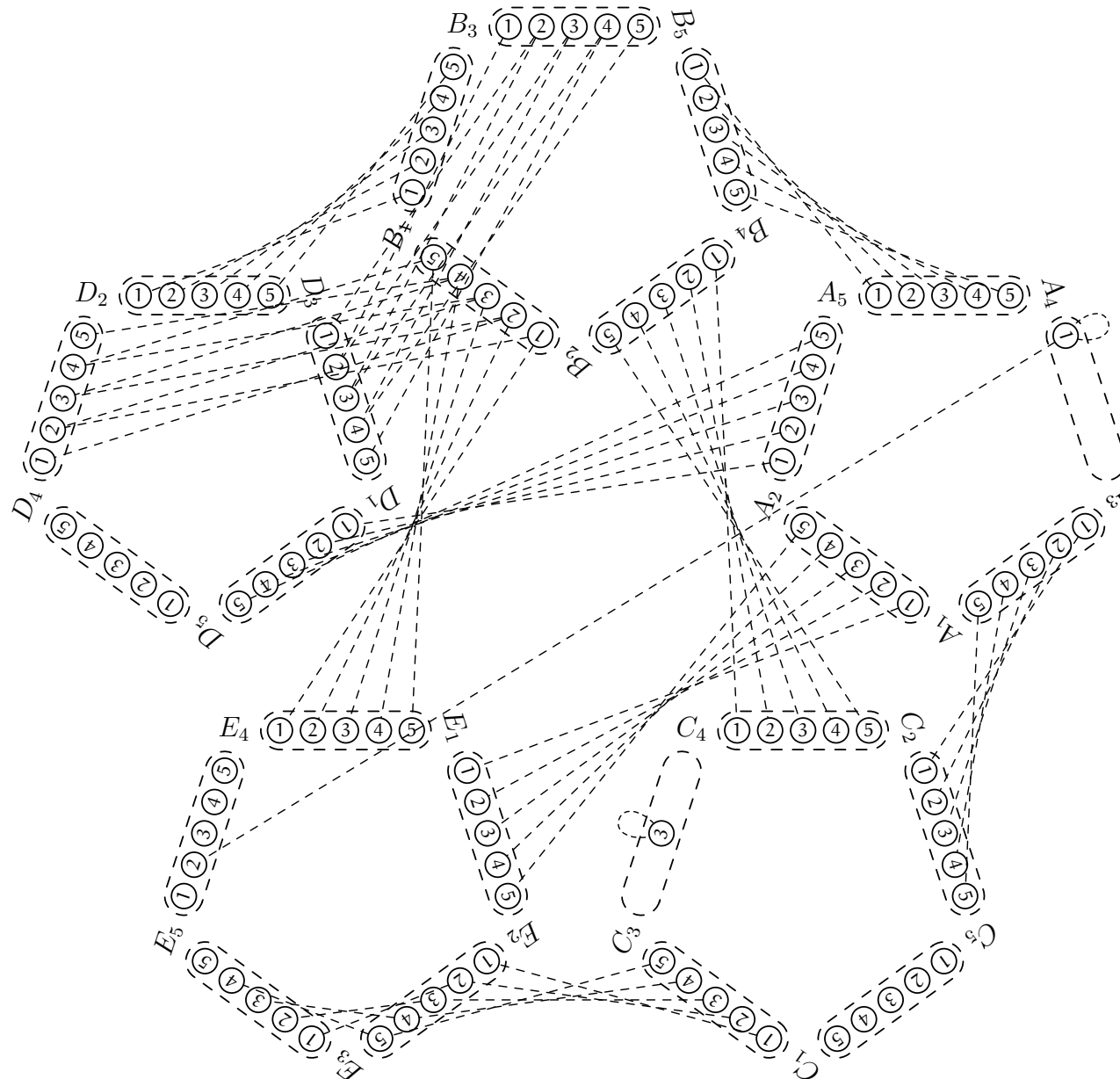
# We can remove the red values.



- A*<sub>1</sub> Englishman
- A*<sub>2</sub> Spaniard
- A*<sub>3</sub> Irishman
- A*<sub>4</sub> Nigerian
- A*<sub>5</sub> Japanese
- B*<sub>1</sub> go
- B*<sub>2</sub> cricket
- B*<sub>3</sub> judo
- B*<sub>4</sub> poker
- B*<sub>5</sub> polo
- C*<sub>1</sub> coffee
- C*<sub>2</sub> tea
- C*<sub>3</sub> milk
- C*<sub>4</sub> orange juice
- C*<sub>5</sub> Guinness
- D*<sub>1</sub> dog
- D*<sub>2</sub> snails
- D*<sub>3</sub> fox
- D*<sub>4</sub> horse
- D*<sub>5</sub> zebra
- E*<sub>1</sub> red
- E*<sub>2</sub> green
- E*<sub>3</sub> ivory
- E*<sub>4</sub> yellow
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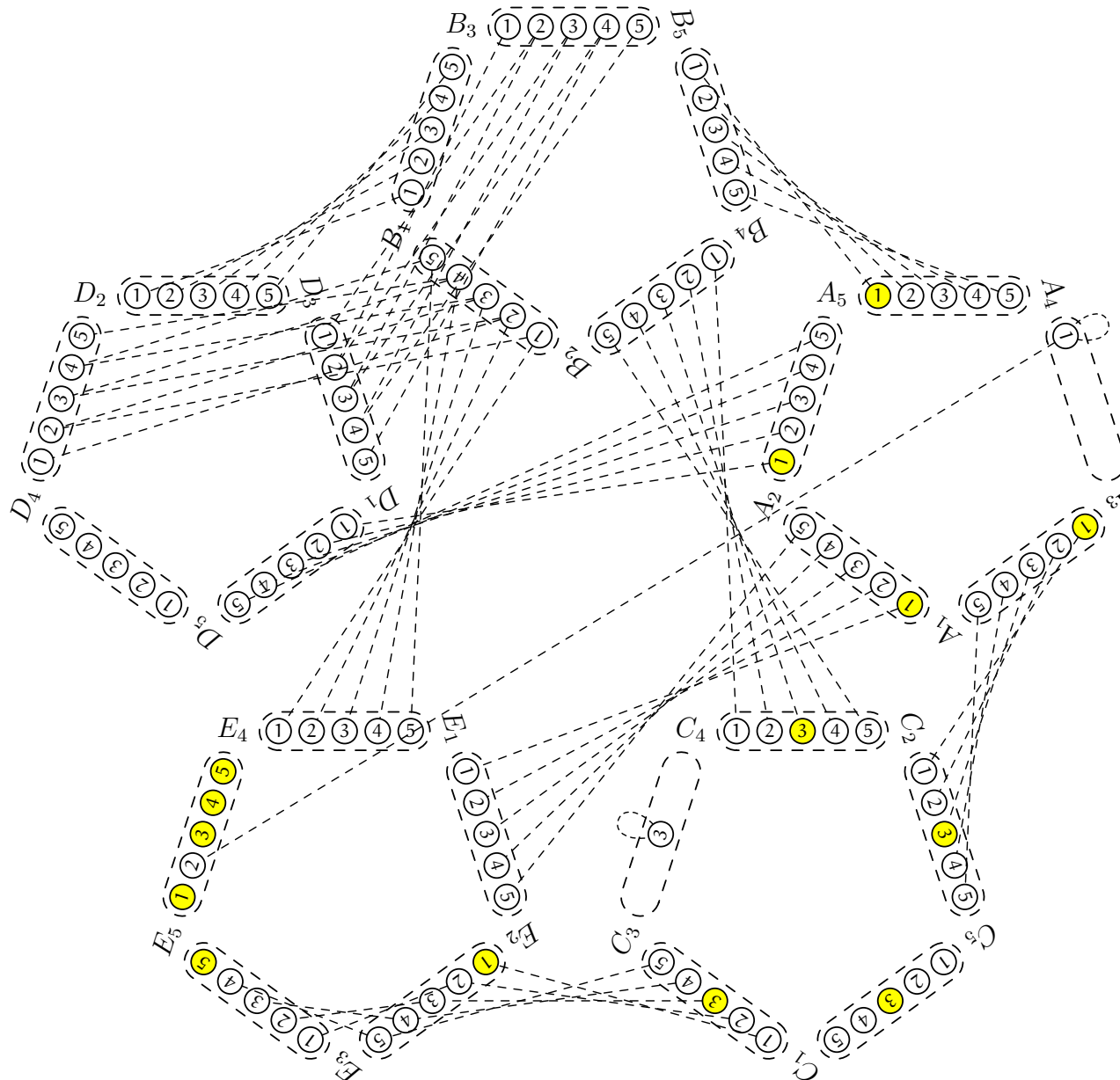


The resulting CSP is called **Node-Consistent**.



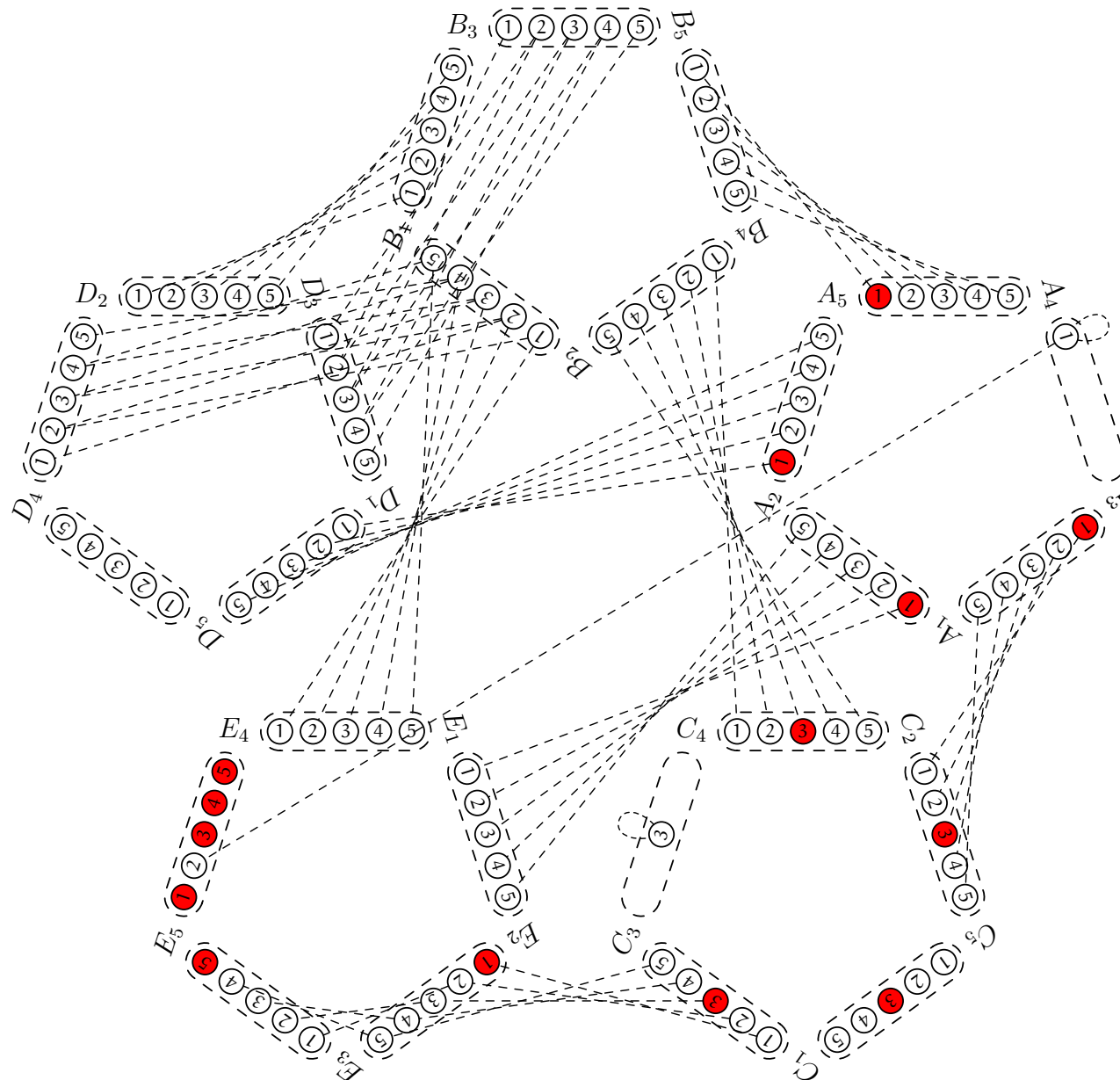
- $A_1$  Englishman
- $A_2$  Spaniard
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- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# Some Values have no Support.



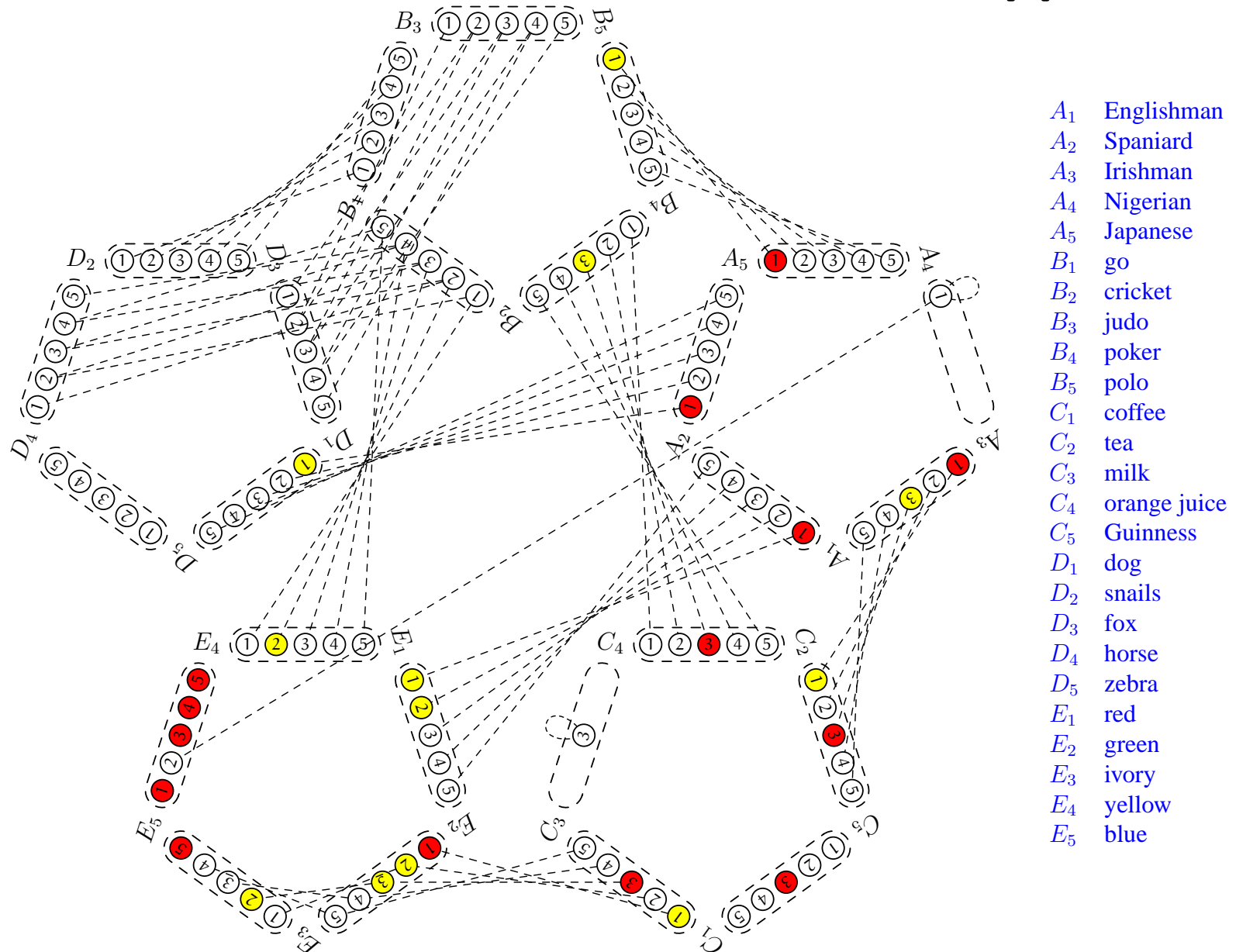
- A<sub>1</sub> Englishman
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- E<sub>1</sub> red
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- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

Let's mark them for removal and colour them red.

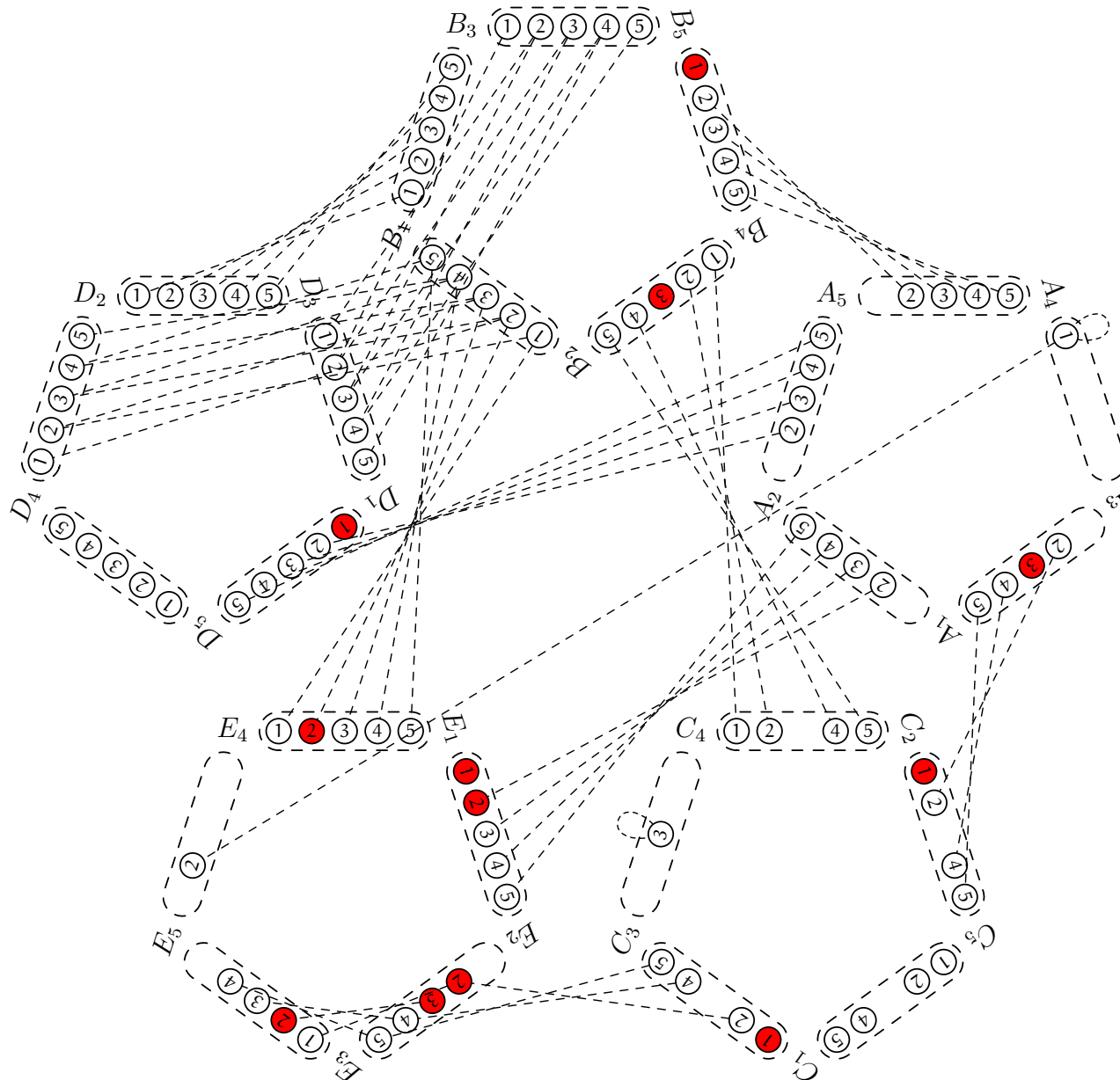


- A*<sub>1</sub> Englishman
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- E*<sub>2</sub> green
- E*<sub>3</sub> ivory
- E*<sub>4</sub> yellow
- E*<sub>5</sub> blue

If we **propagate** the consequences of removing the red values, more values will lose support.

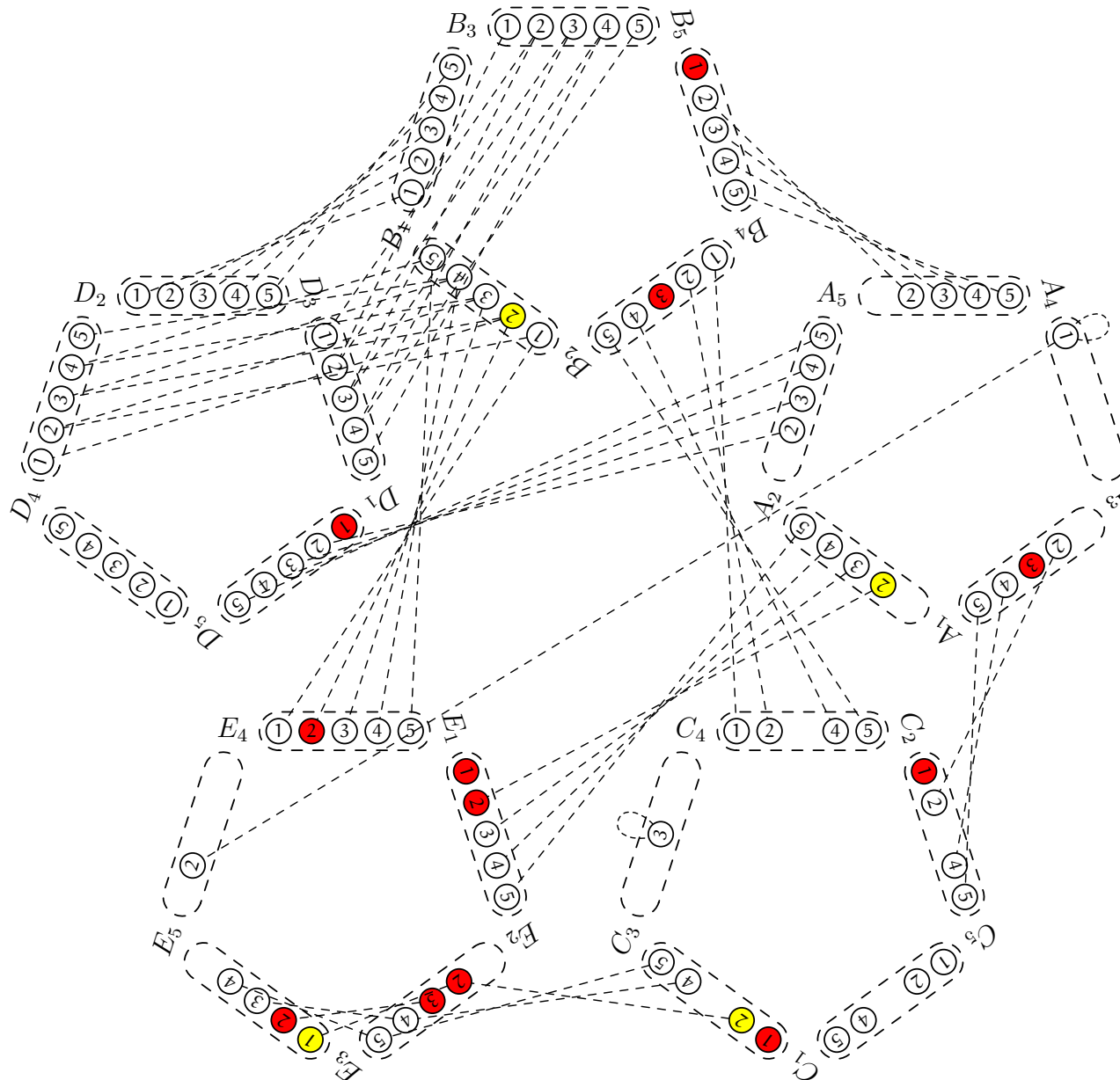


# Remove



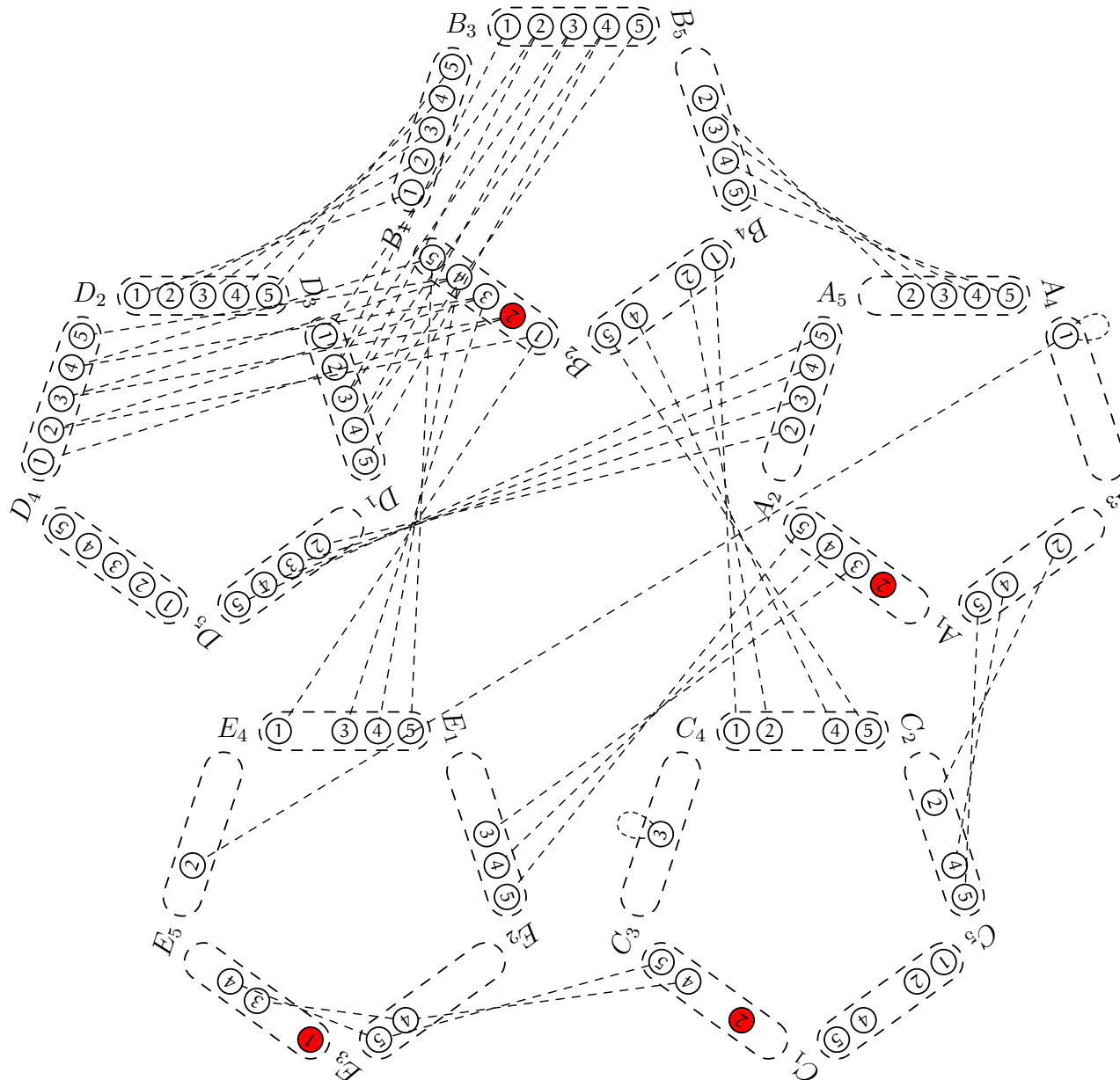
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- E<sub>4</sub> yellow
- E<sub>5</sub> blue

# and Propagate.



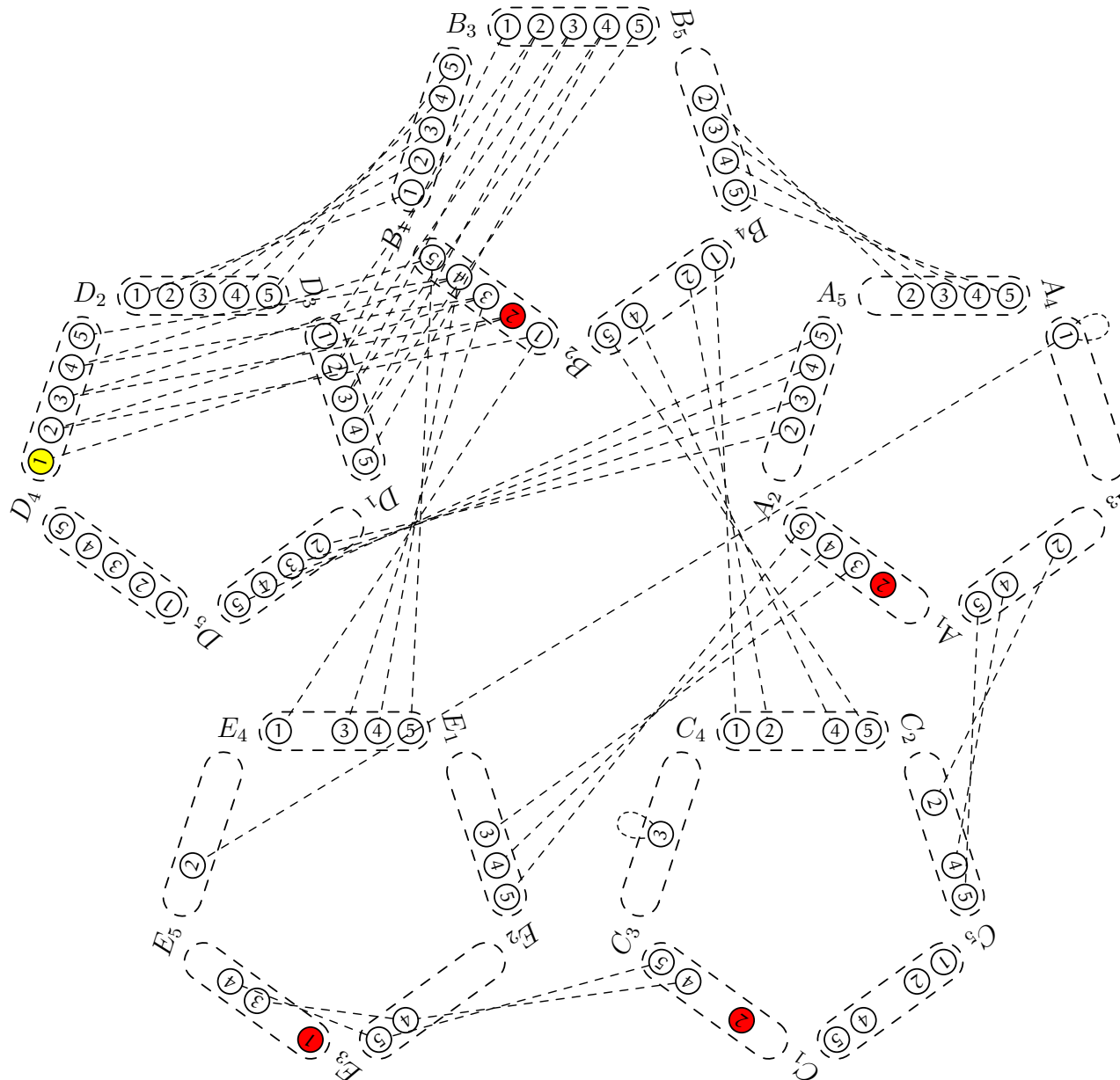
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# More Propagation.



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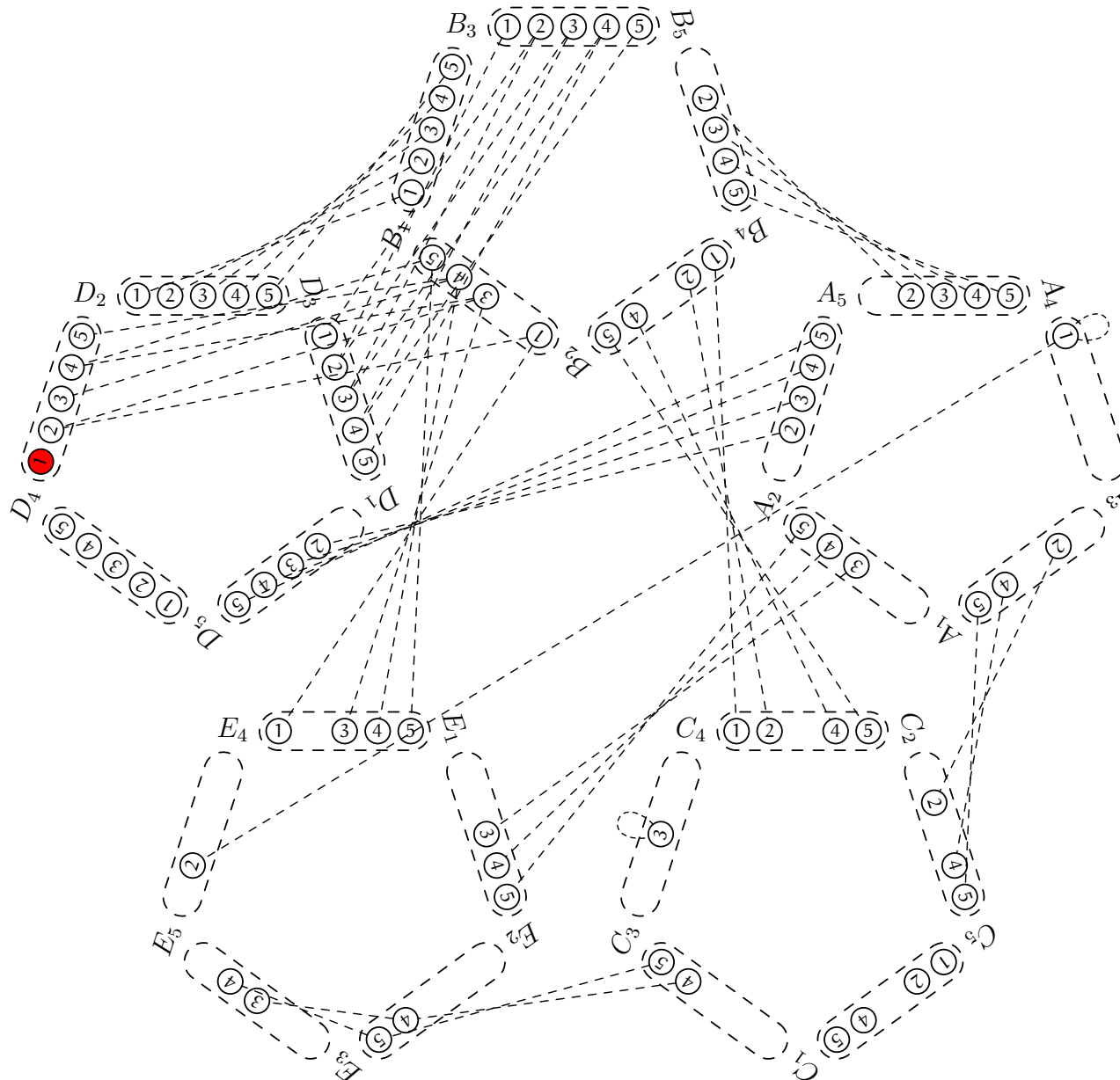
# More Values will lose Support.



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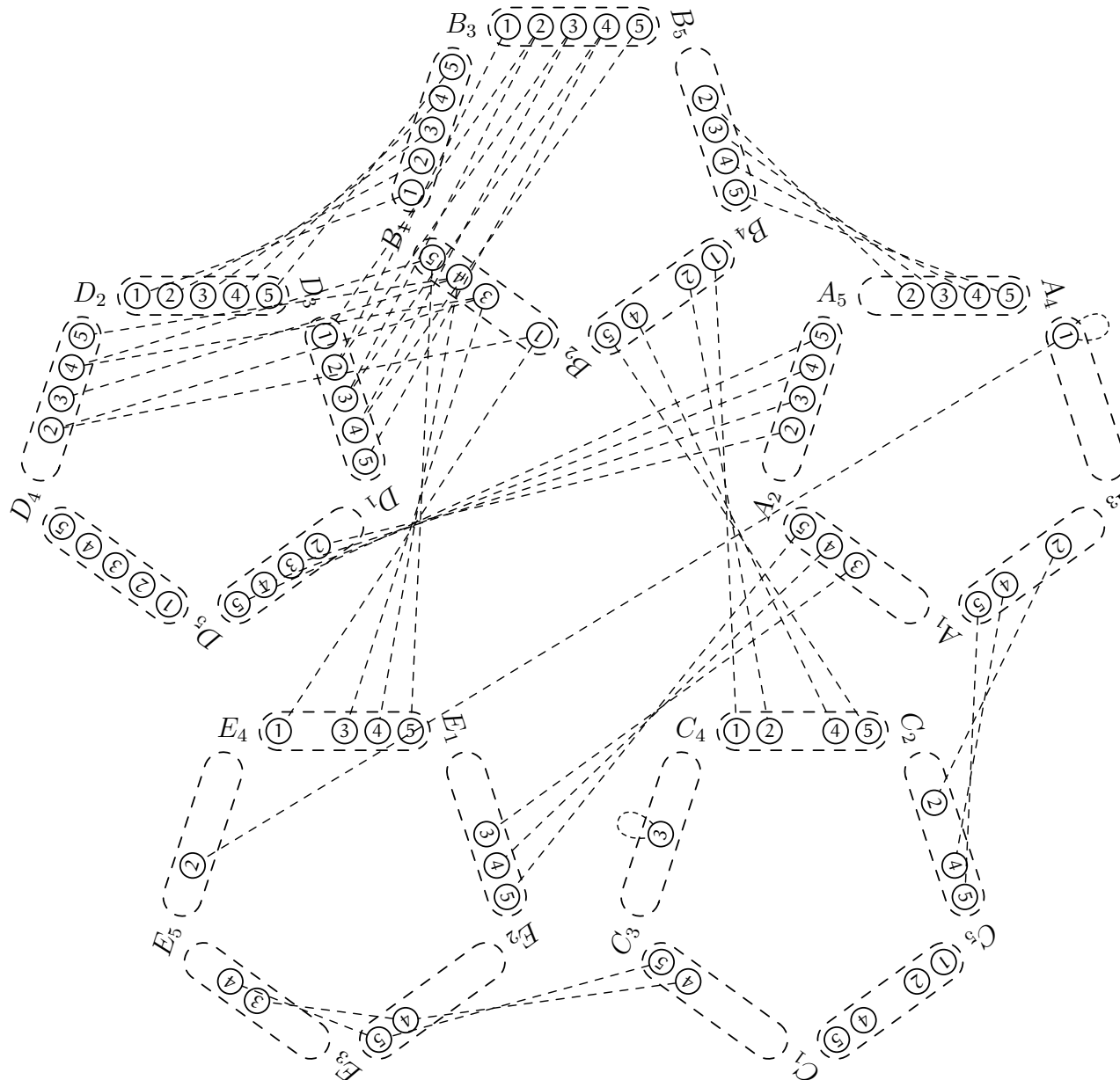


# More Propagation. . .



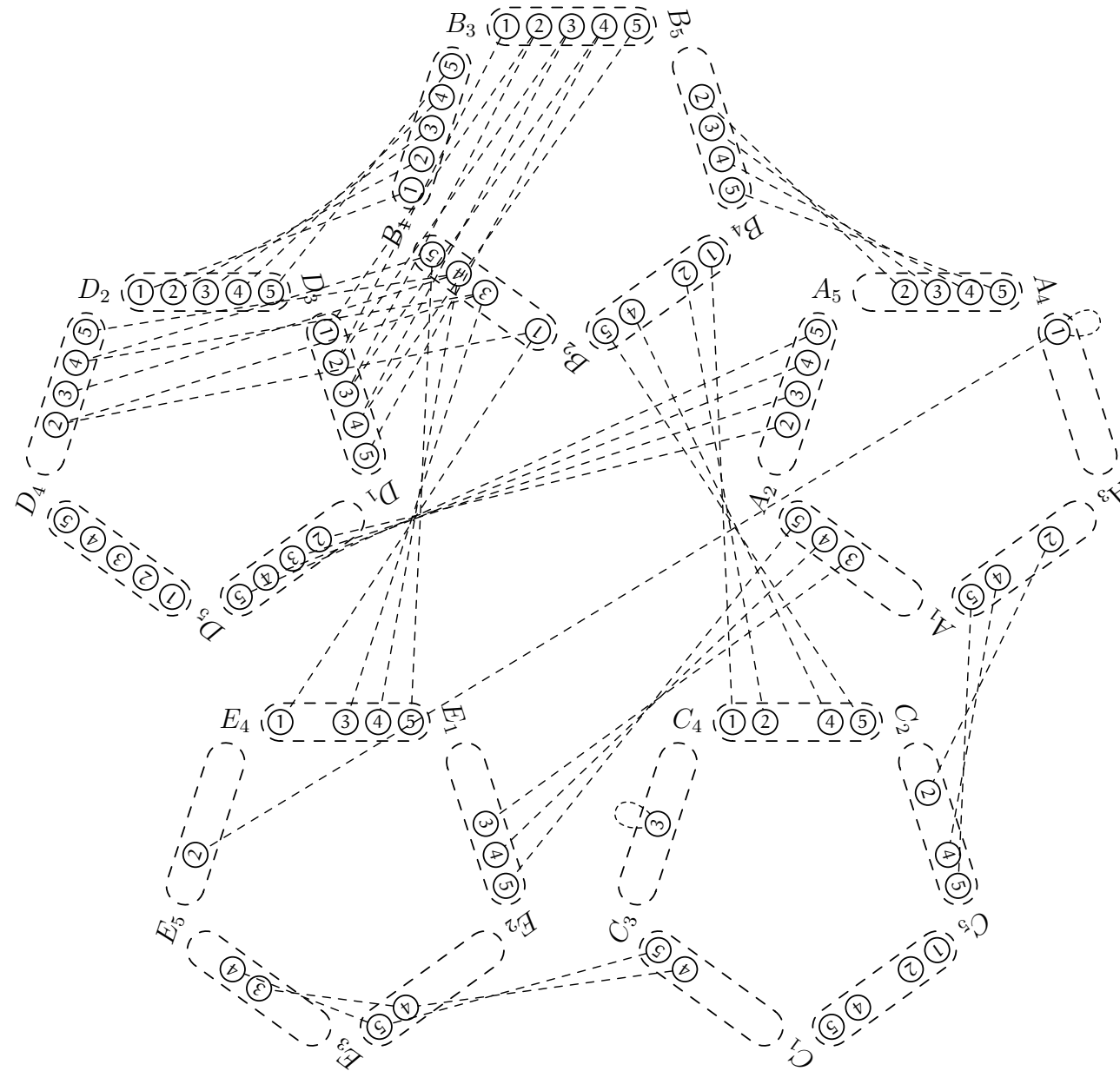
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- E*<sub>4</sub> yellow
- E*<sub>5</sub> blue

# We have reached a Fix-point



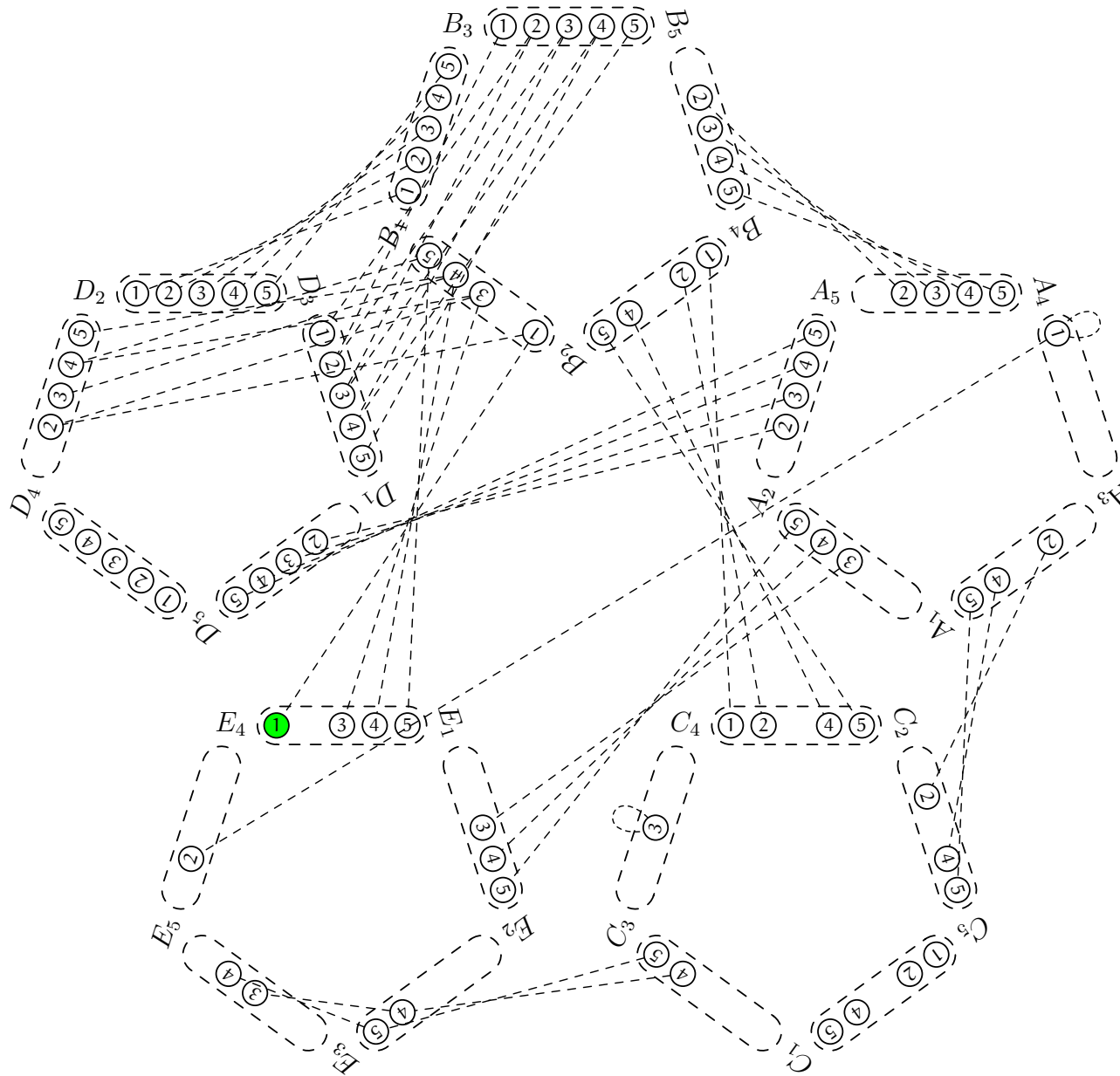
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The resulting CSP is called **Arc-consistent**.



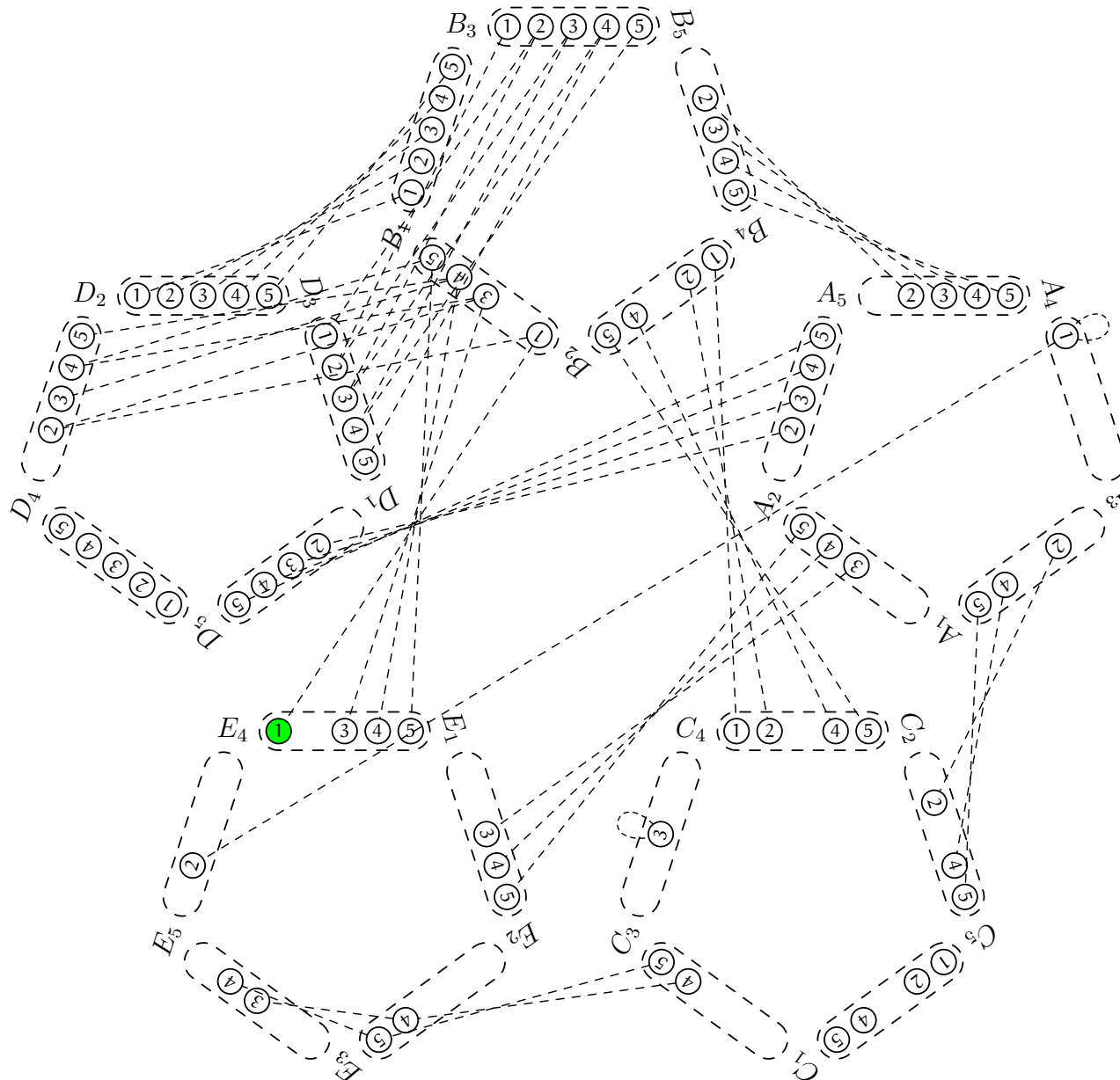
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- E<sub>4</sub> yellow
- E<sub>5</sub> blue

$E_4$ 's domain contains 1.



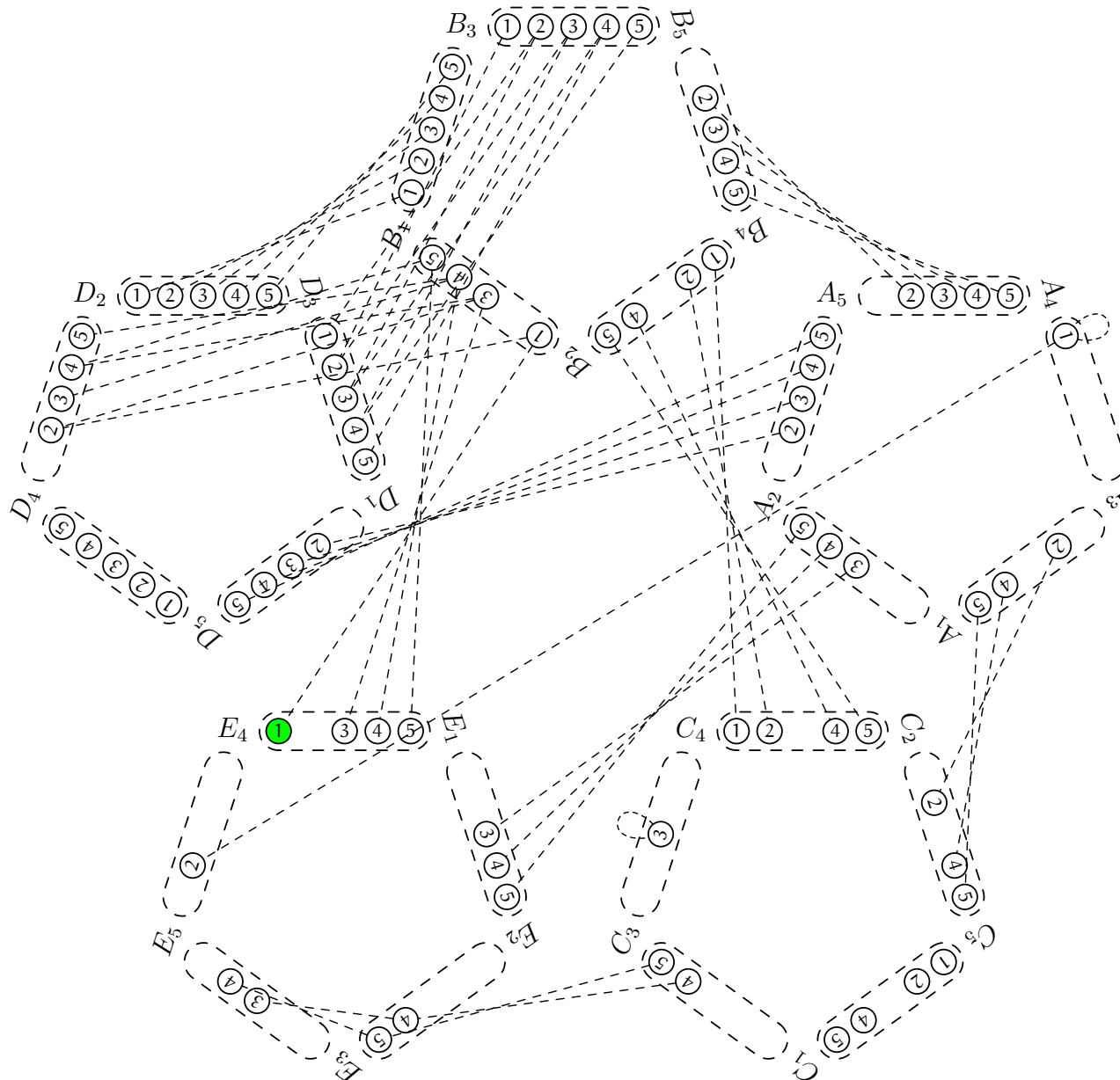
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The domains of the other  $E_i$  do not contain 1.



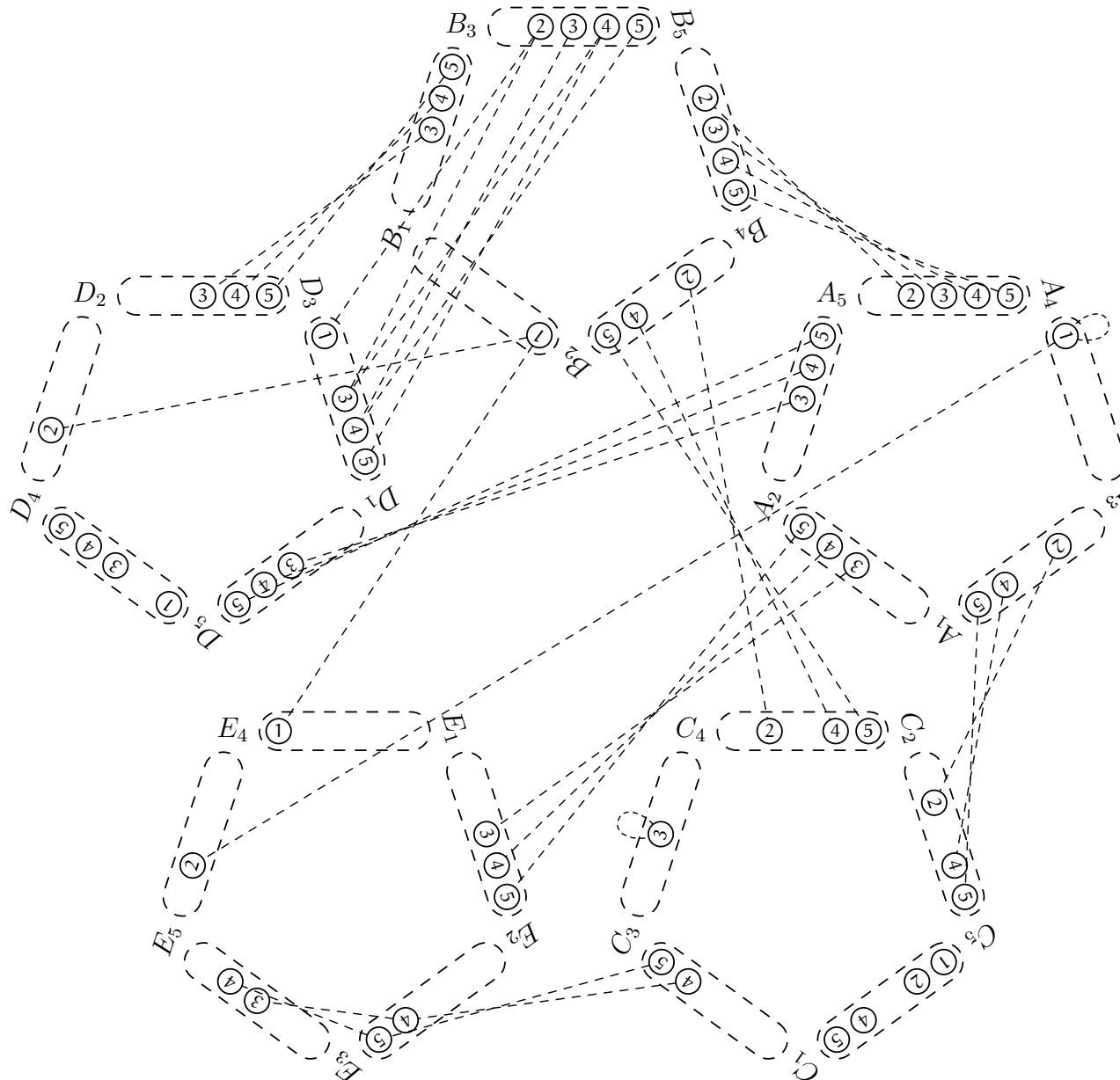
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- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
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- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

$E_4$  must be 1.



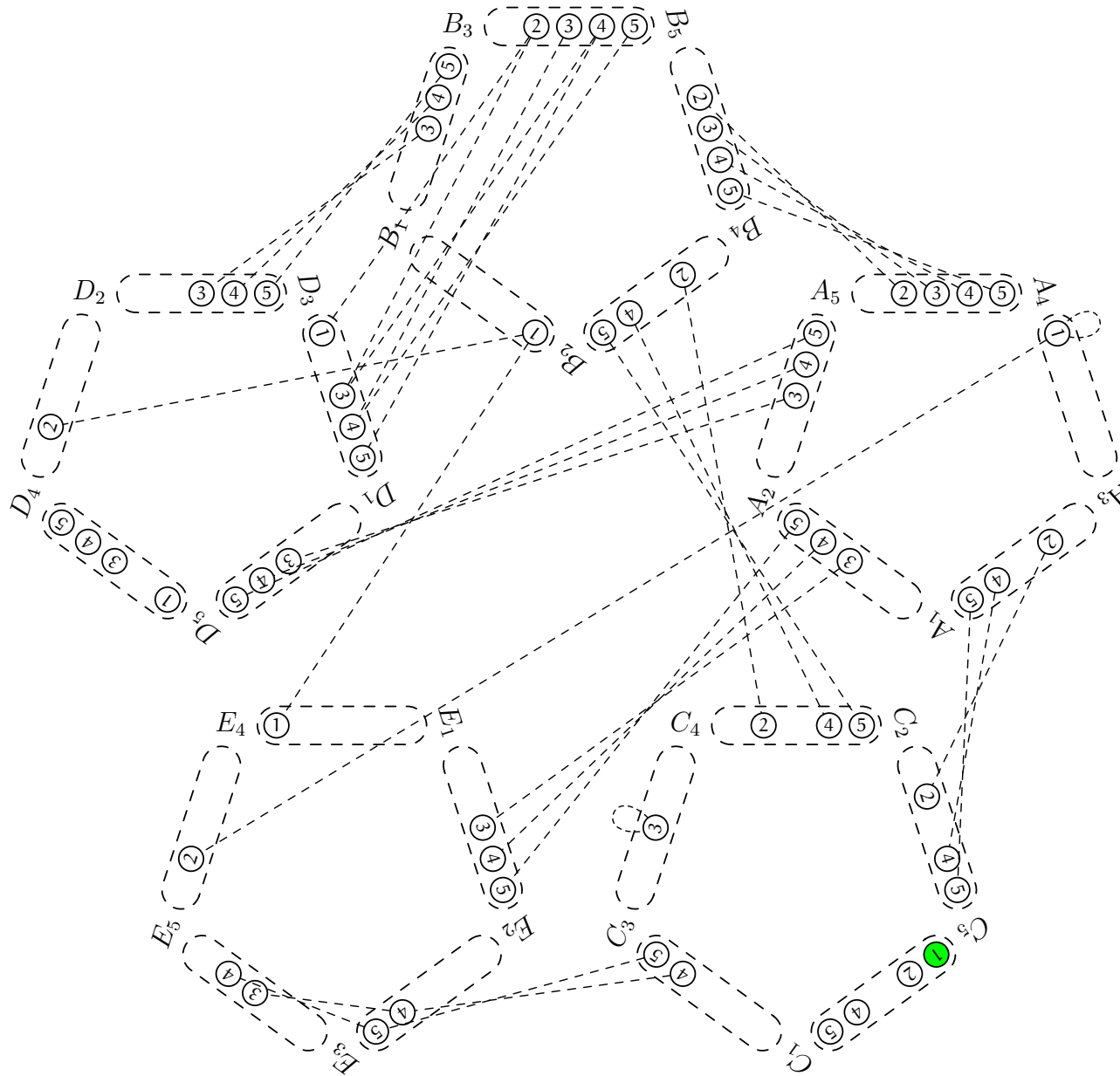
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# After Assignment $E_4 = 1$ and Arc-Consistency.



- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

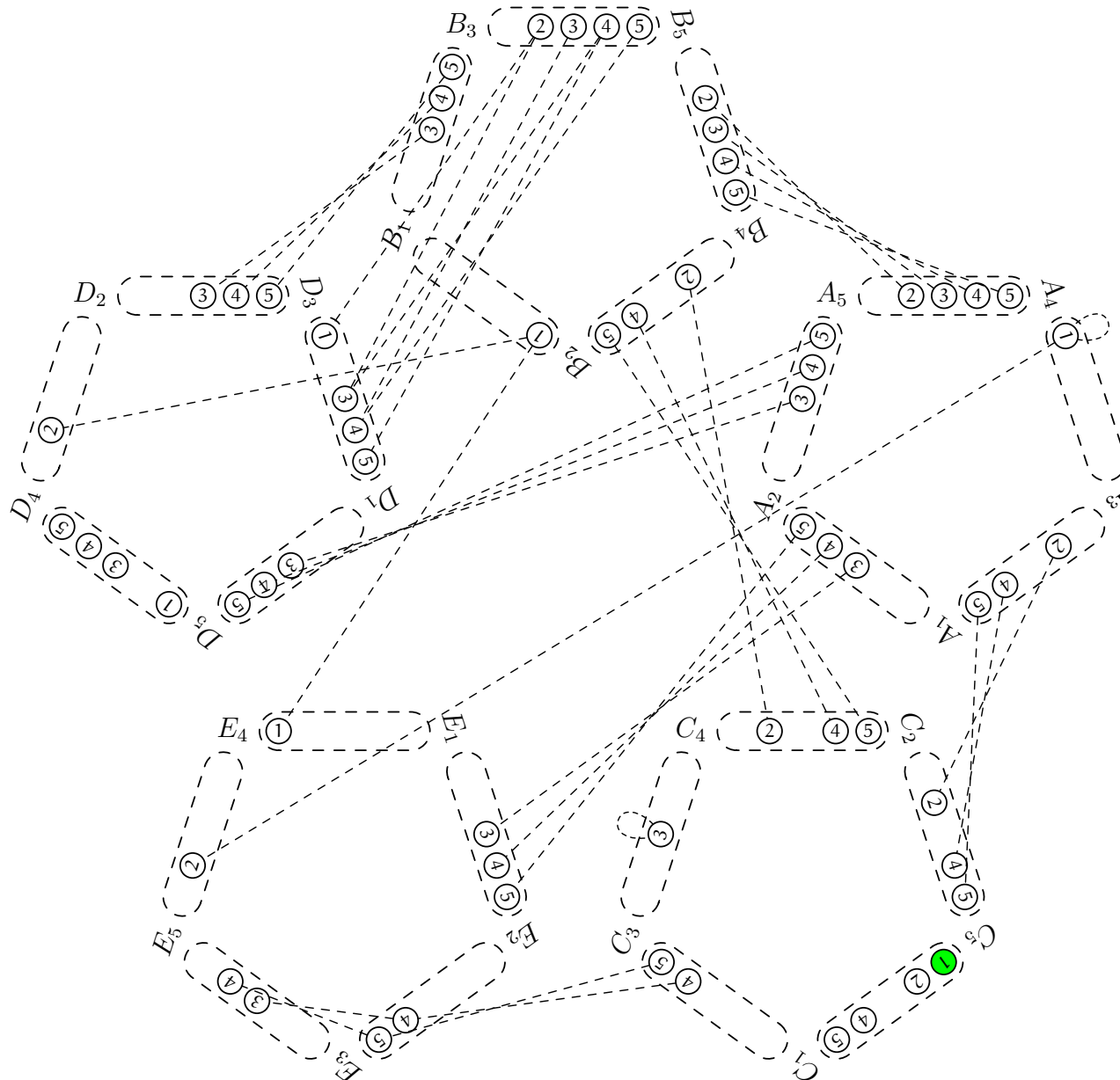
# $C_5$ 's domain contains 1.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

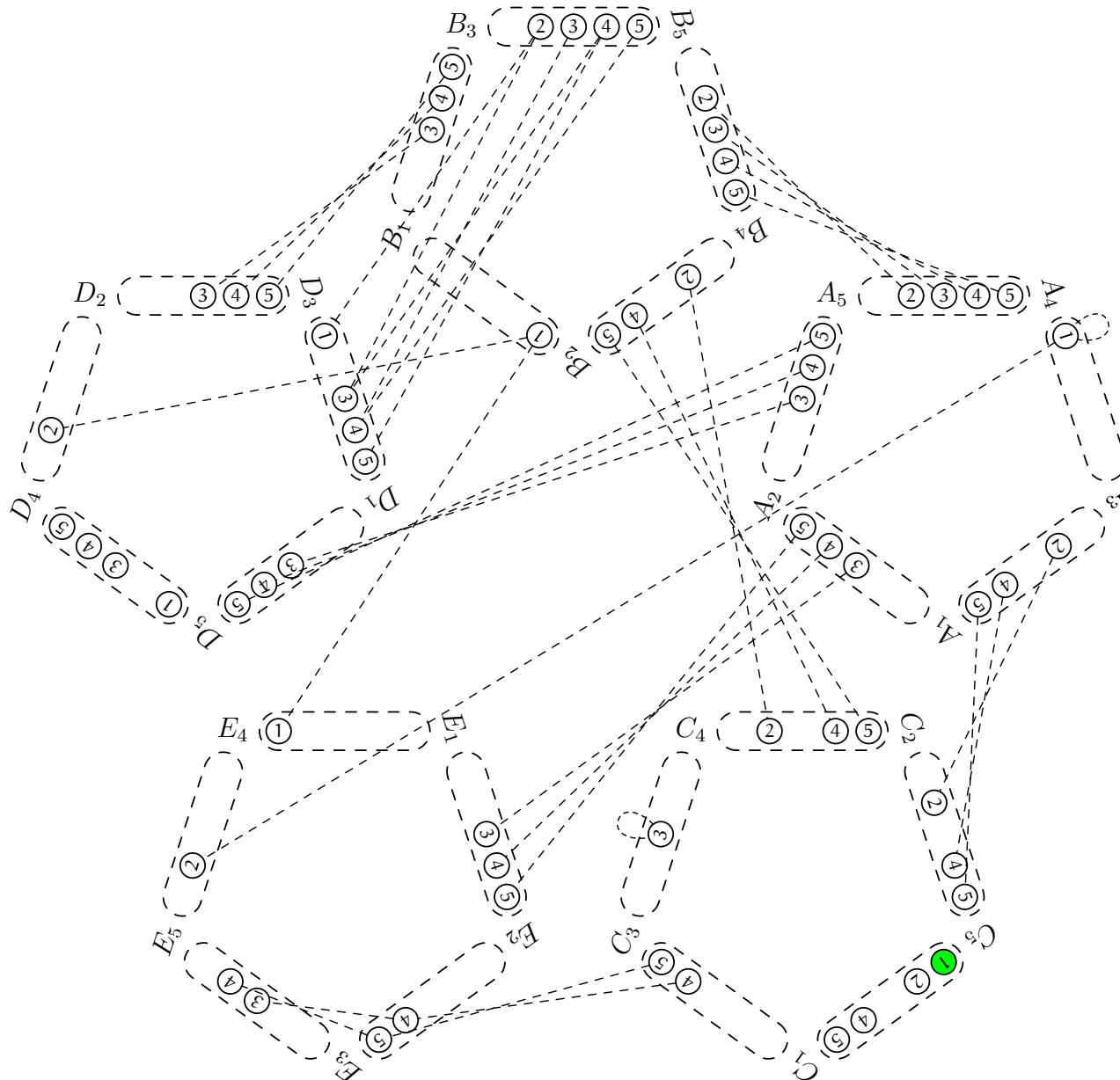


The domains of the other  $C_i$  do not contain 1.



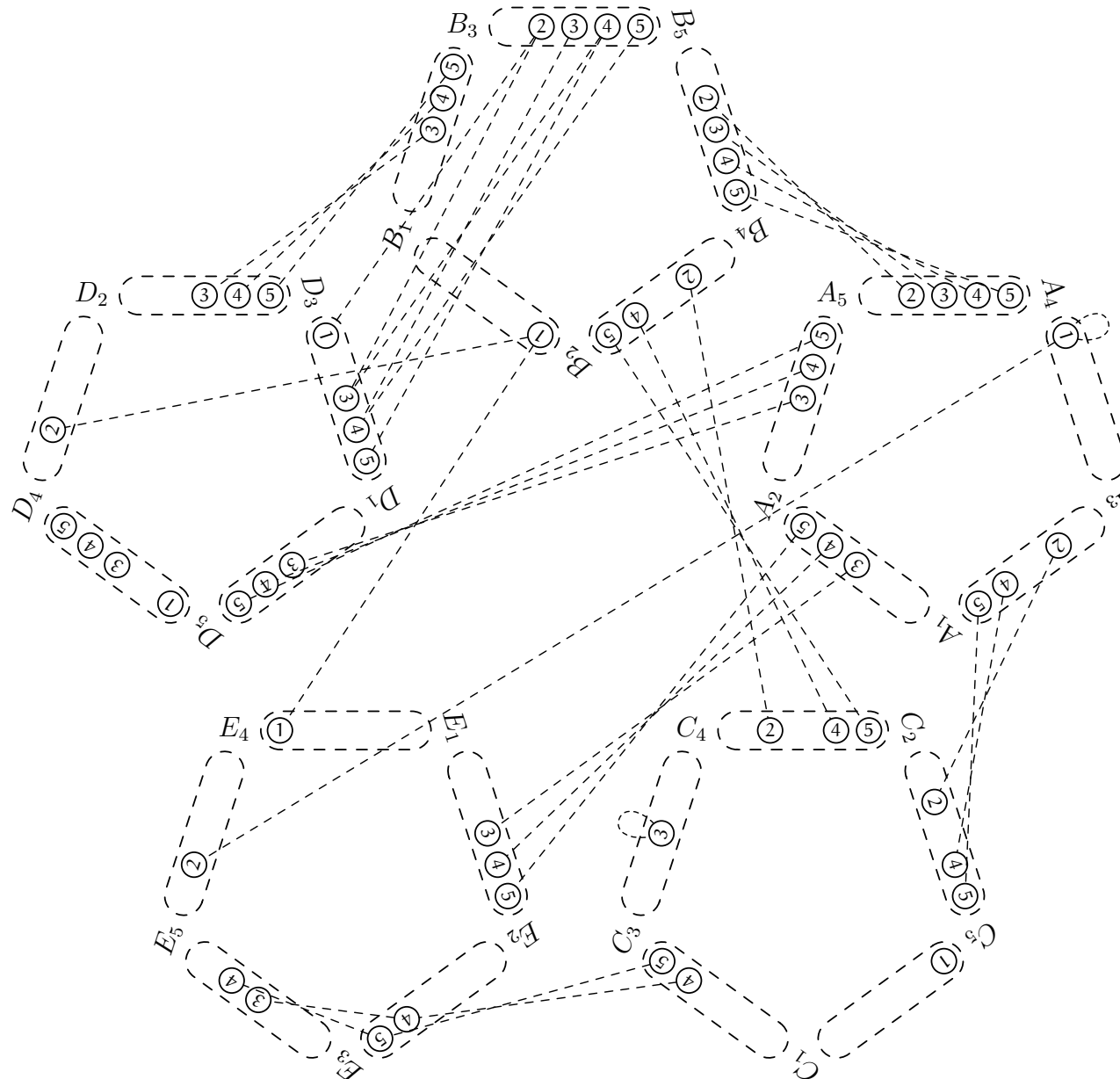
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

$C_5$  must be 1.



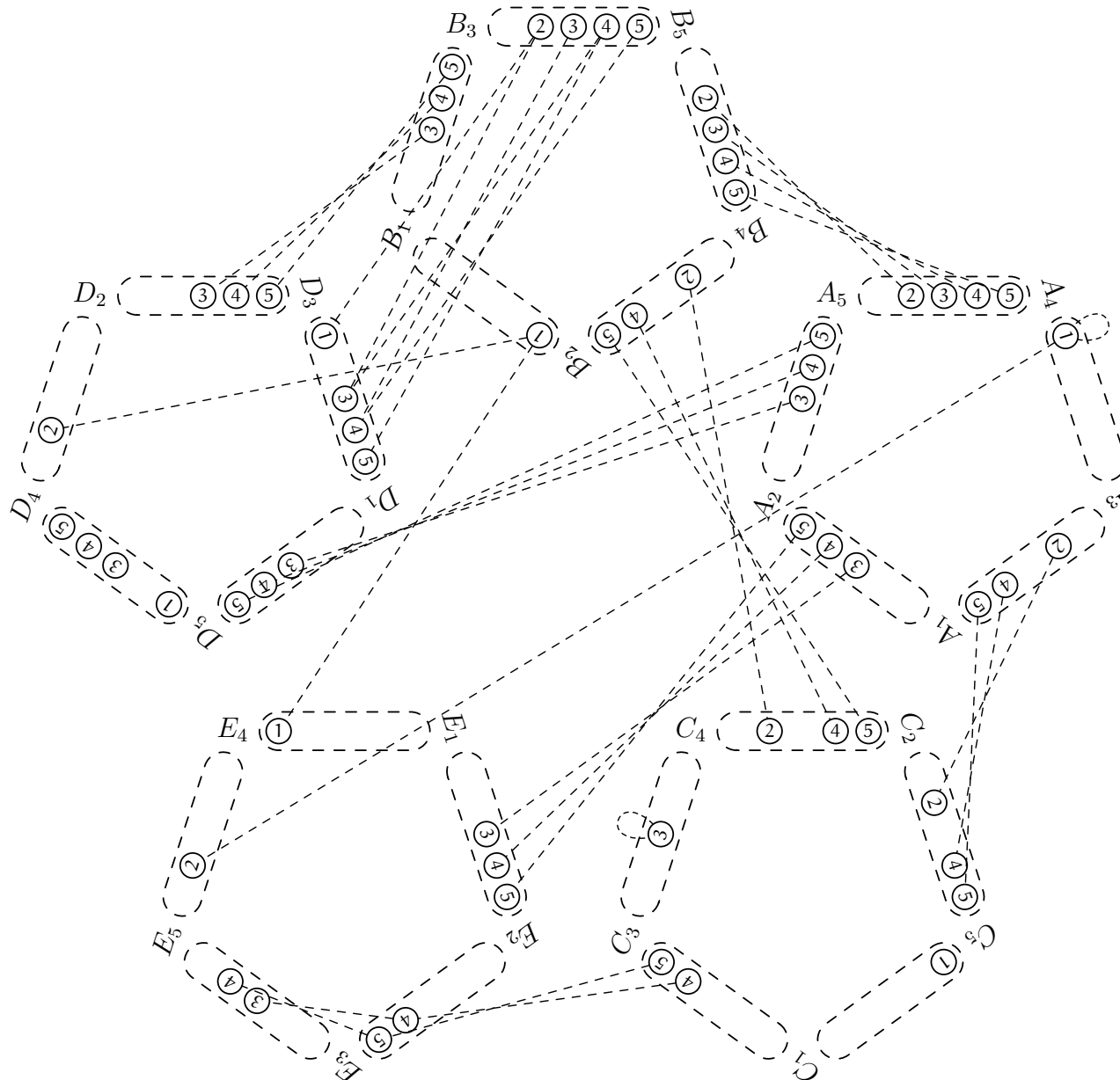
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

# After Assignment $C_5 = 1$ and Arc-Consistency.



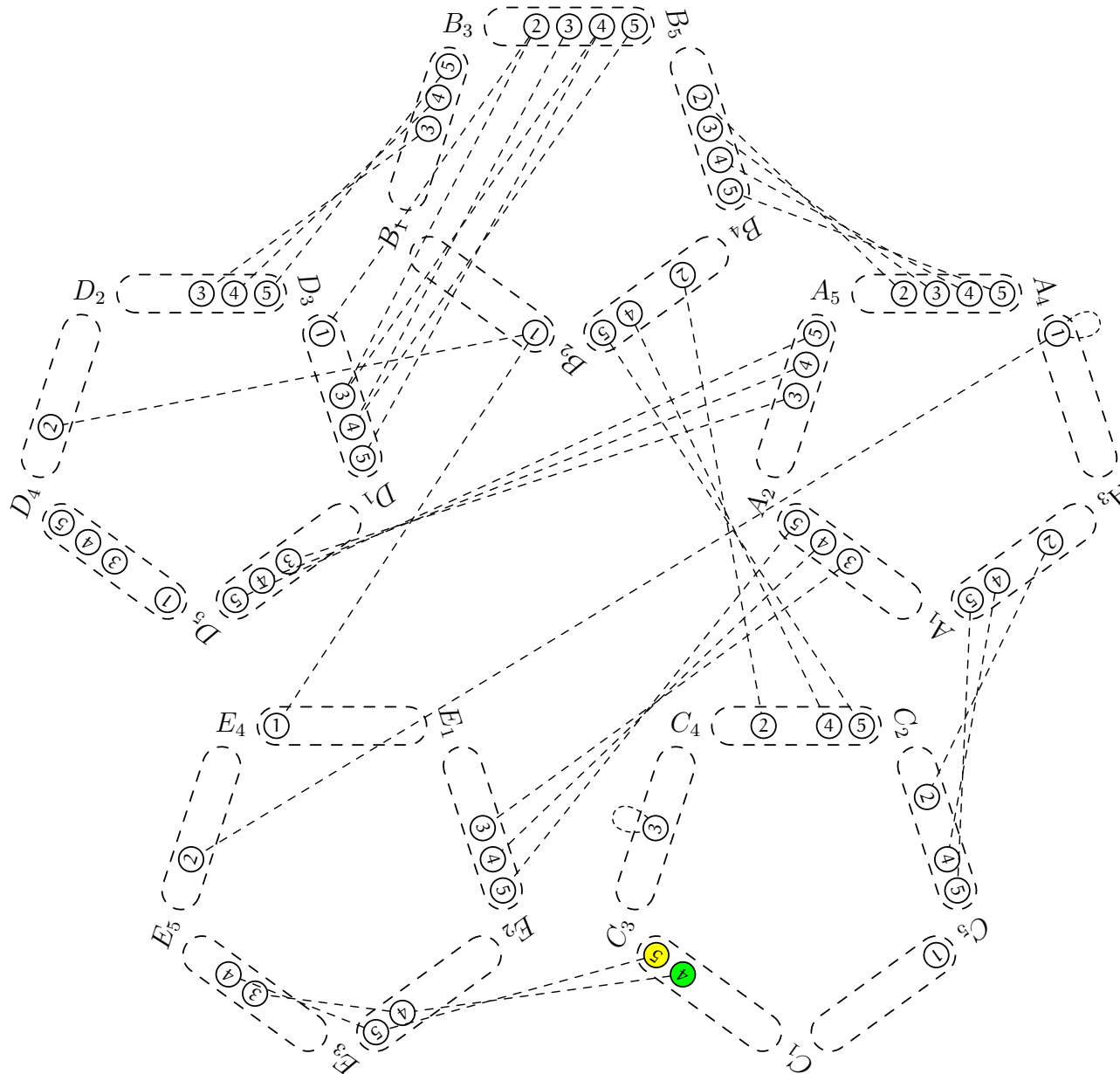
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

# Start **MAC-Search** (Maintain Arc-Consistency).



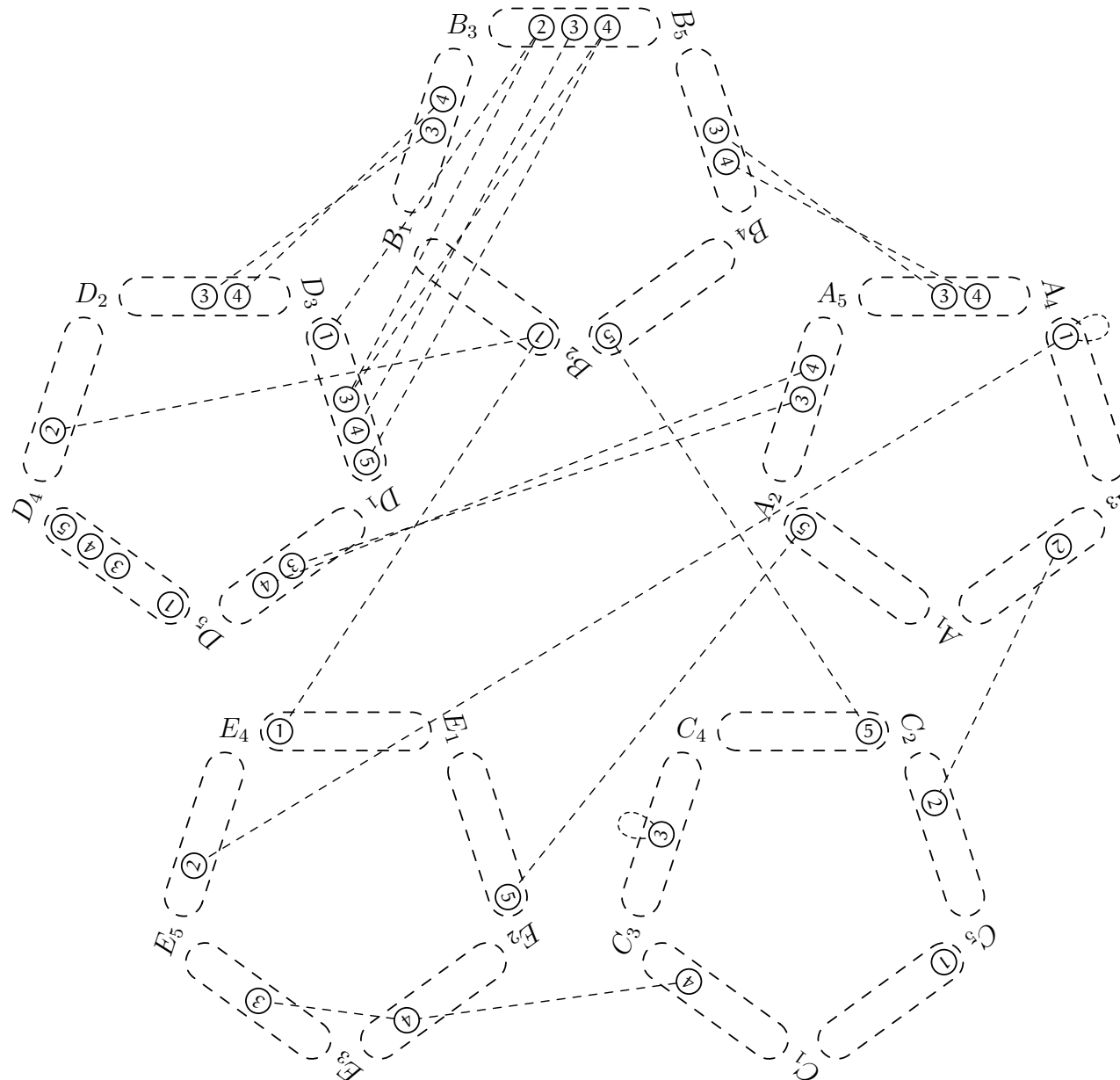
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# Select $C_1$ as Current Variable.



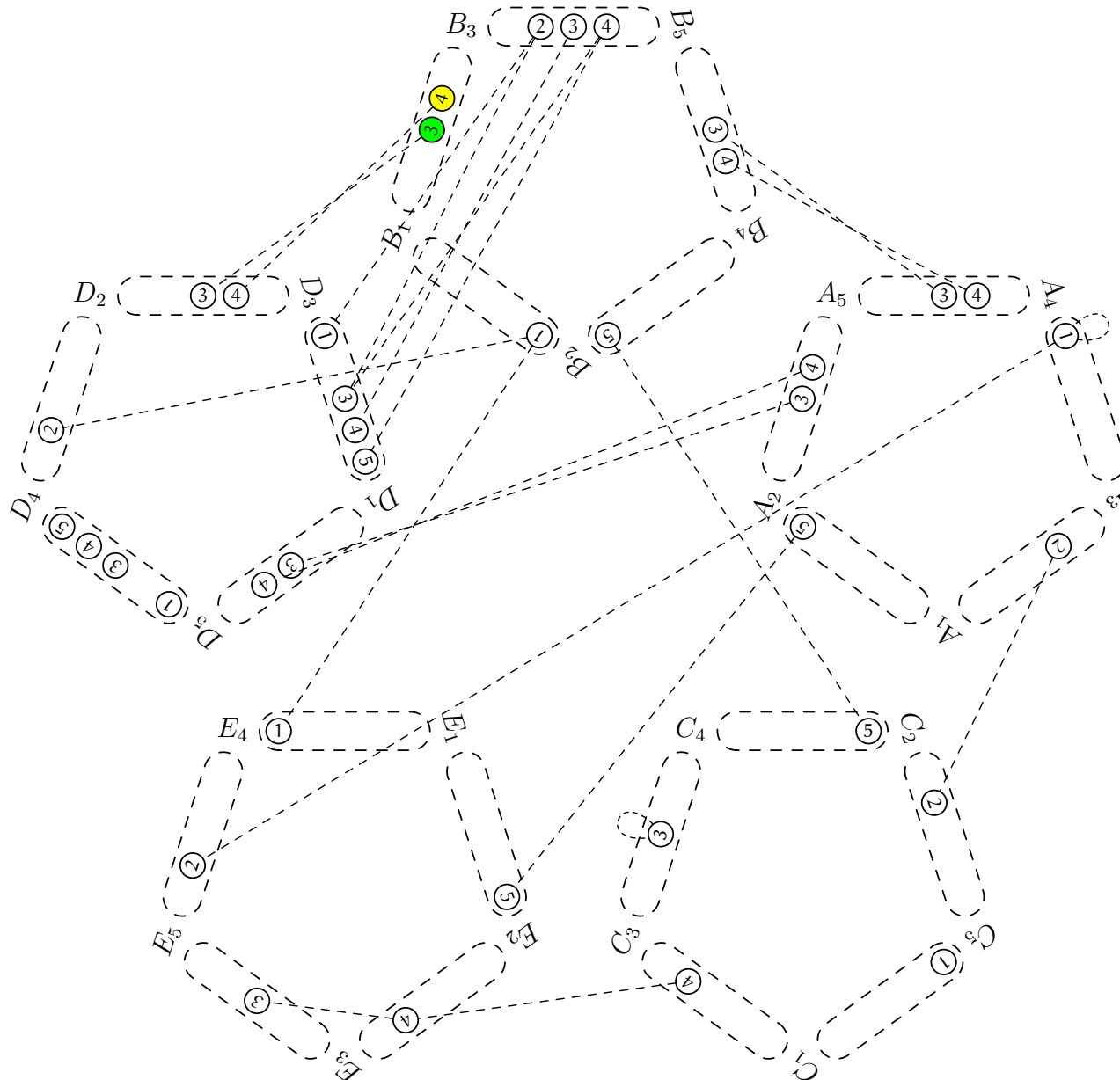
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# After Assignment $C_1 = 4$ and Arc-Consistency.



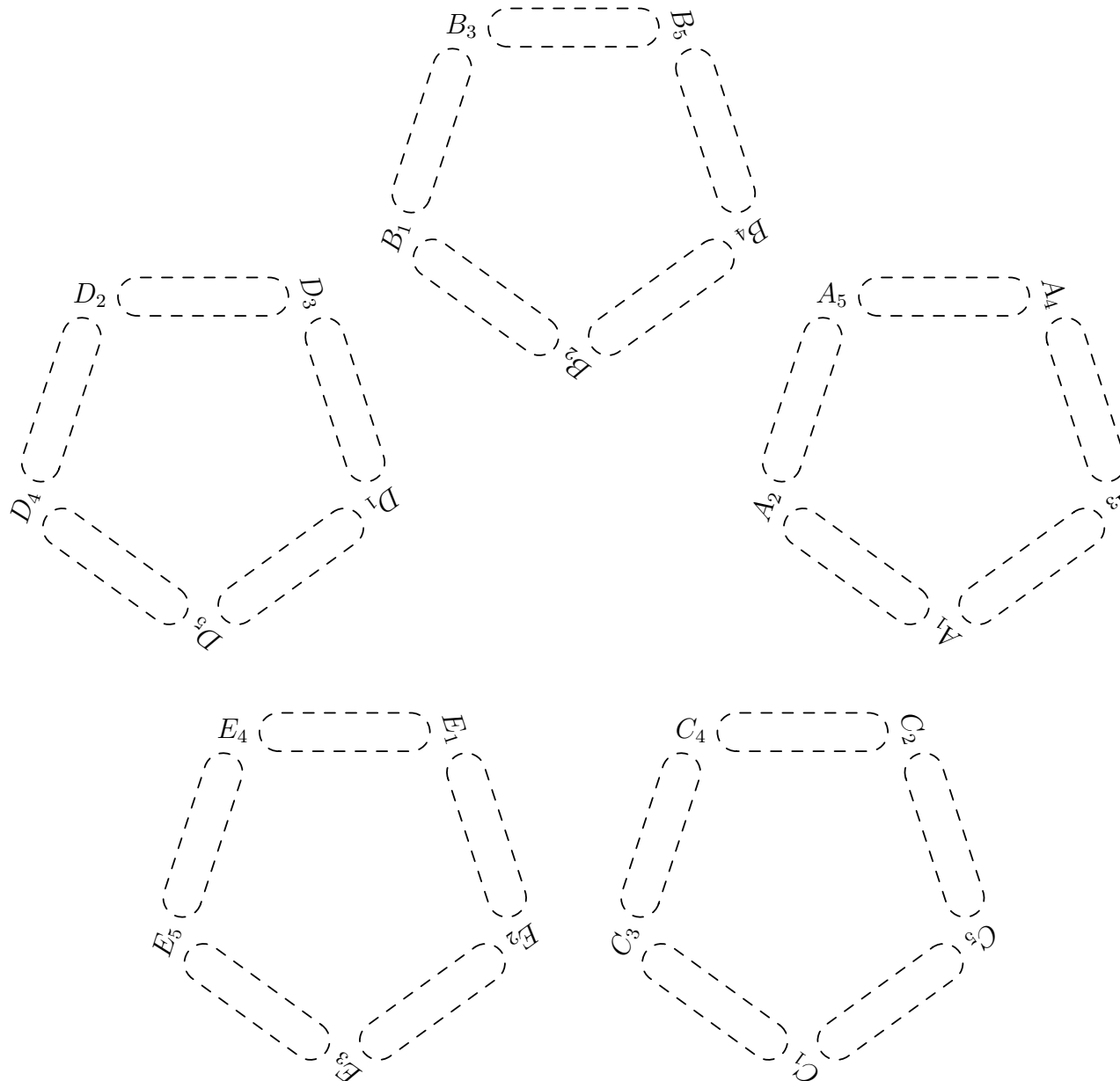
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

Select  $B_1$  as Current Variable.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

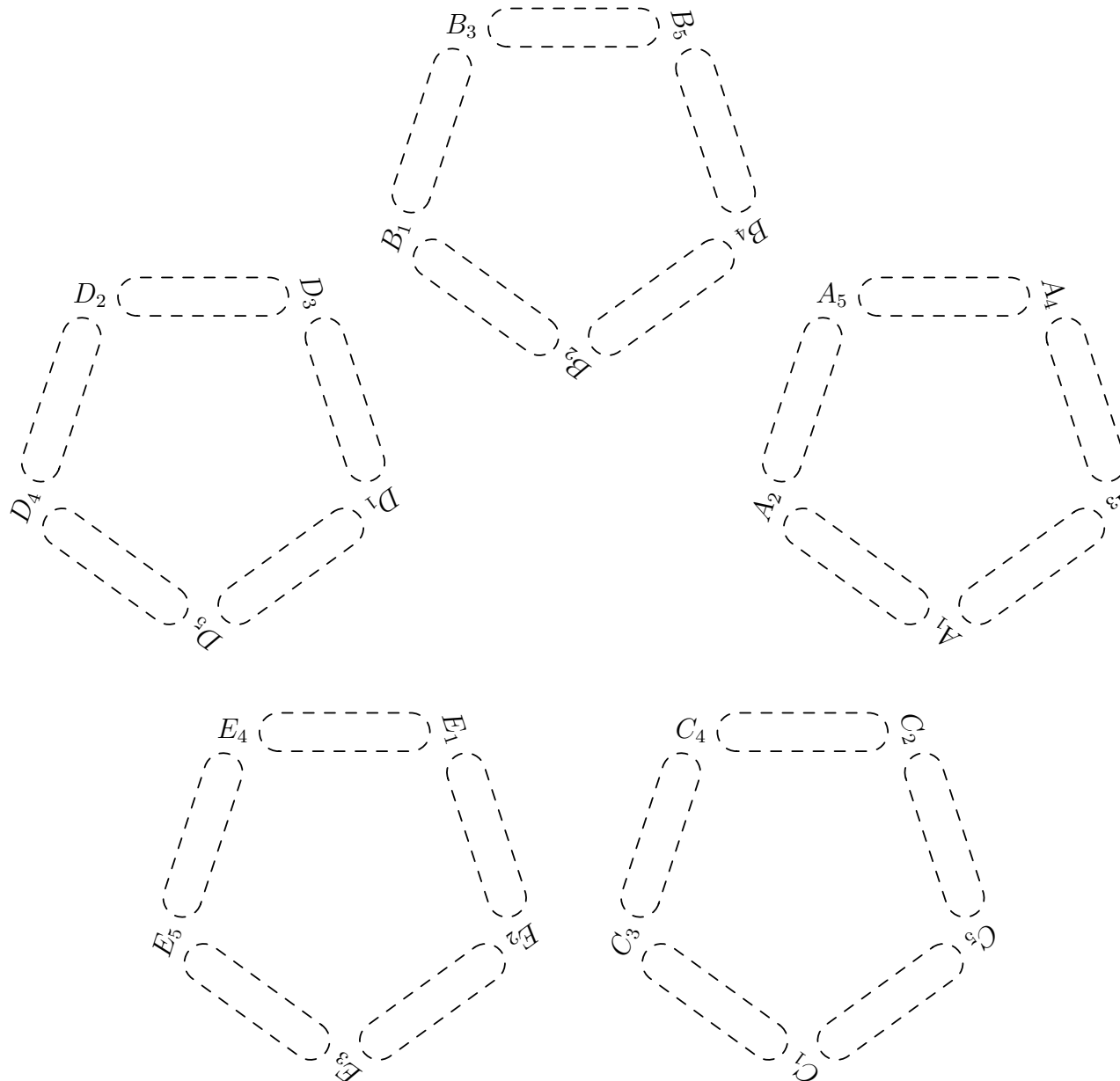
# After Assignment $B_1 = 3$ and Arc-Consistency.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

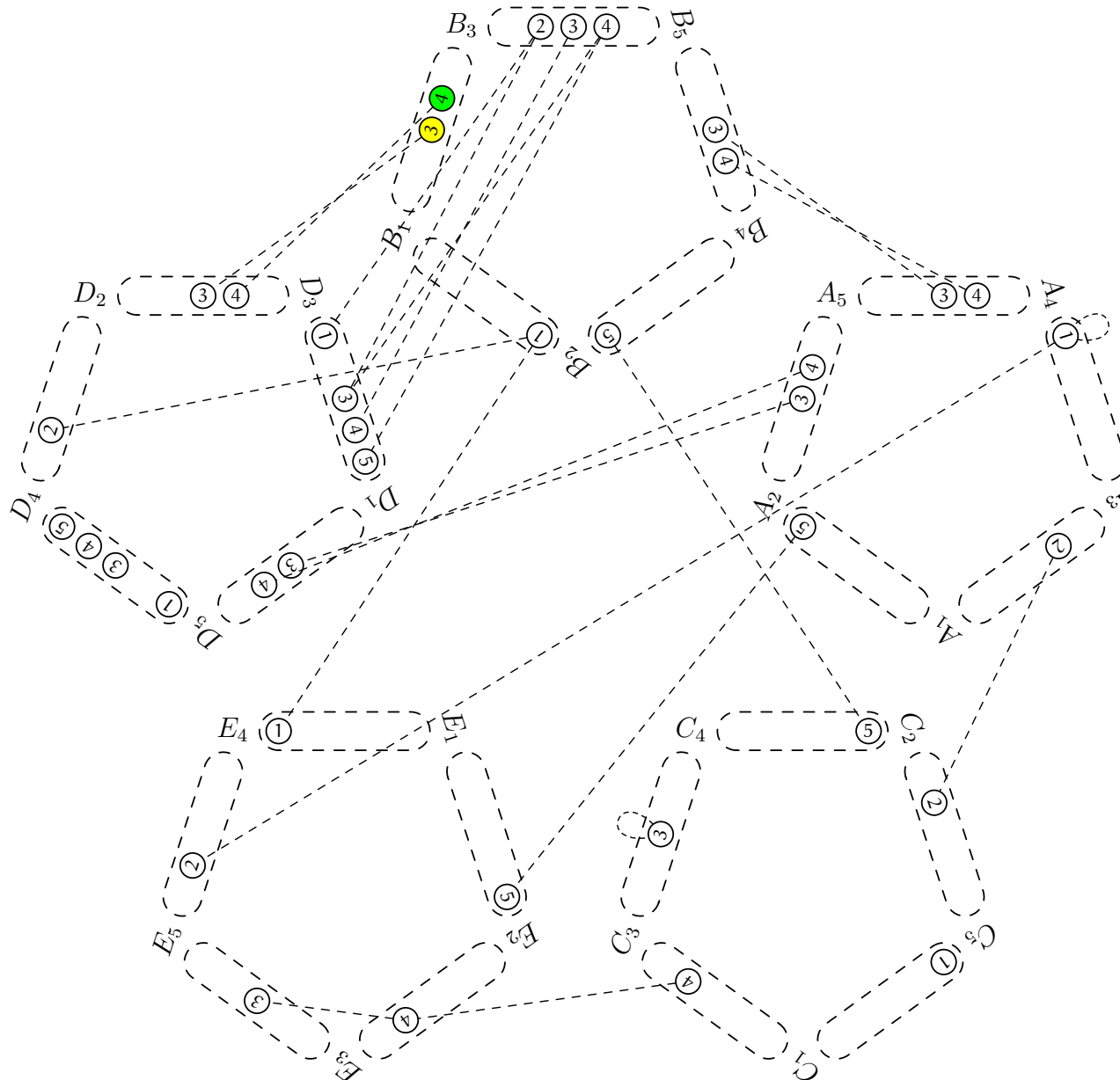


All domains are empty. We must backtrack on  $B_1$ .



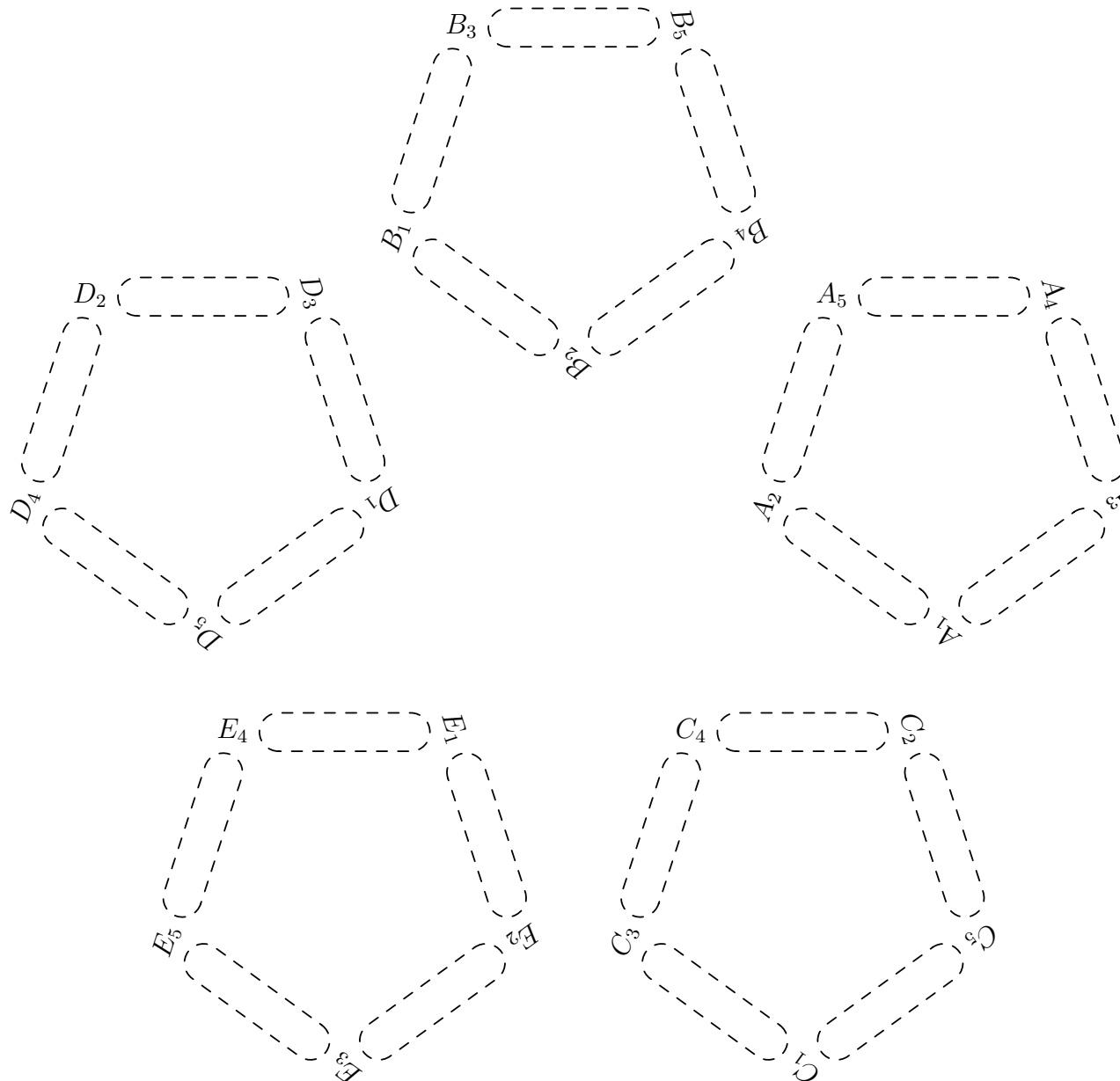
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# Next Assignment to $B_1$ .



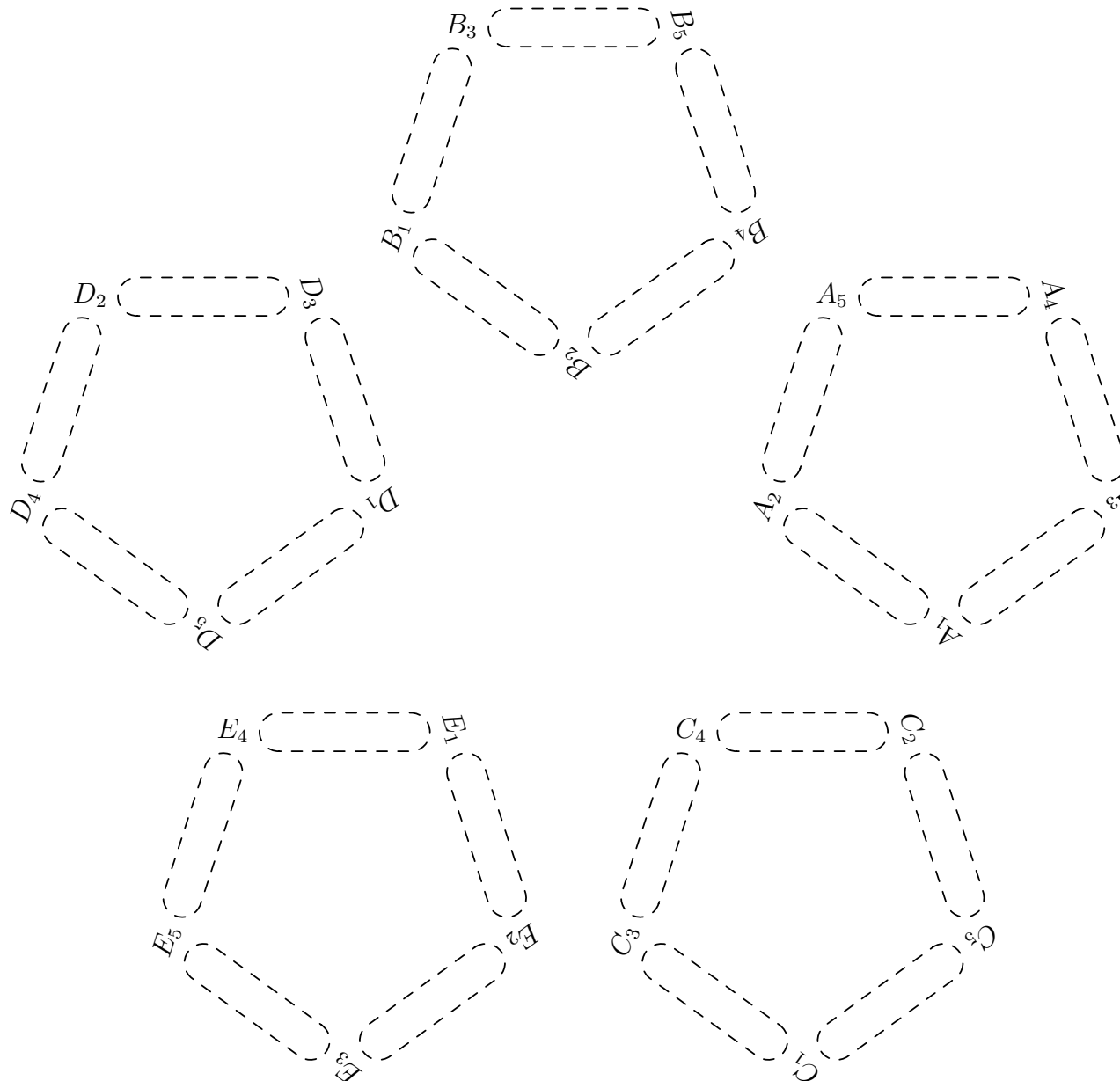
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# After Assignment $B_1 = 4$ and Arc-Consistency.



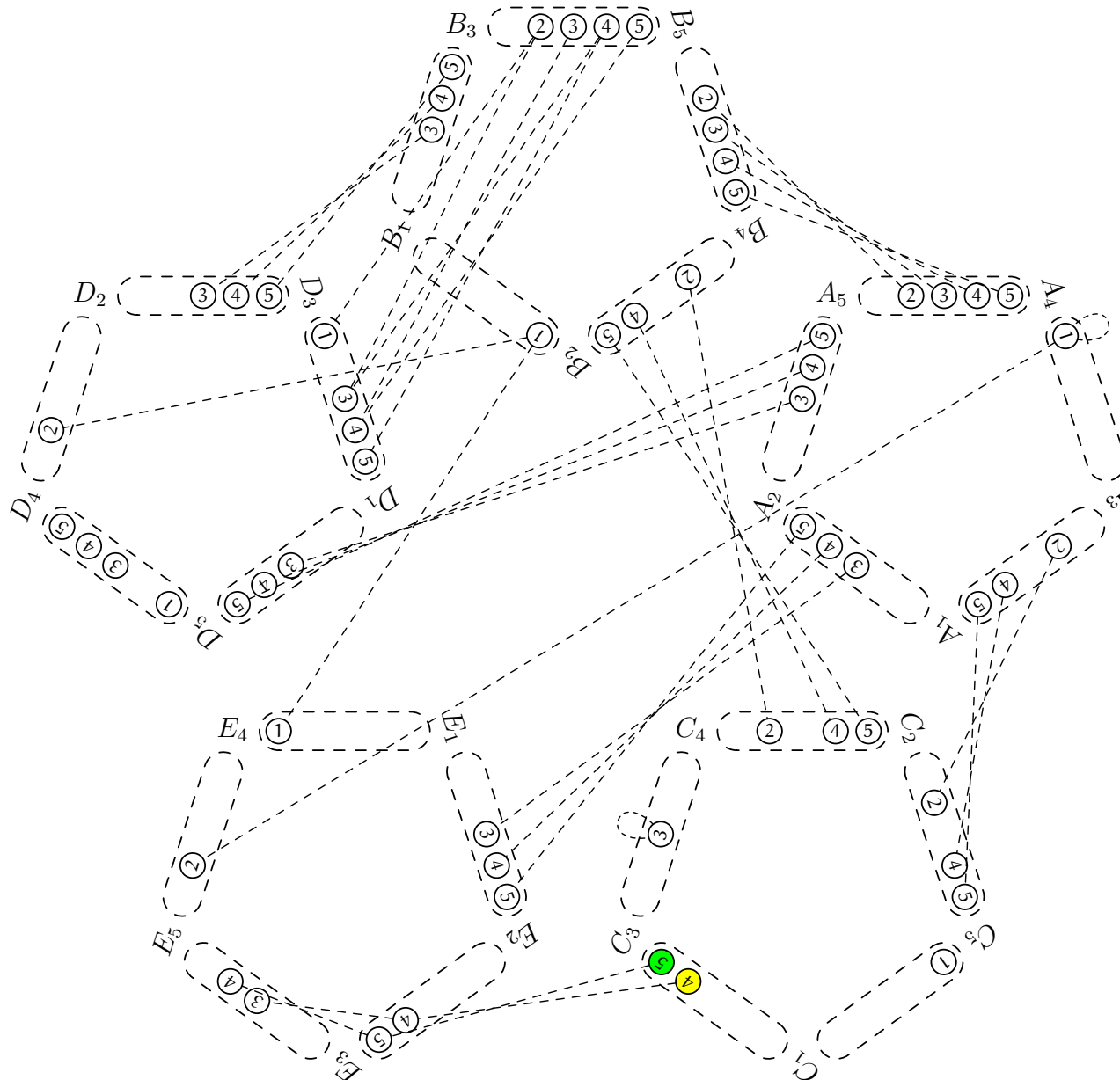
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

We must backtrack on  $C_1$ .



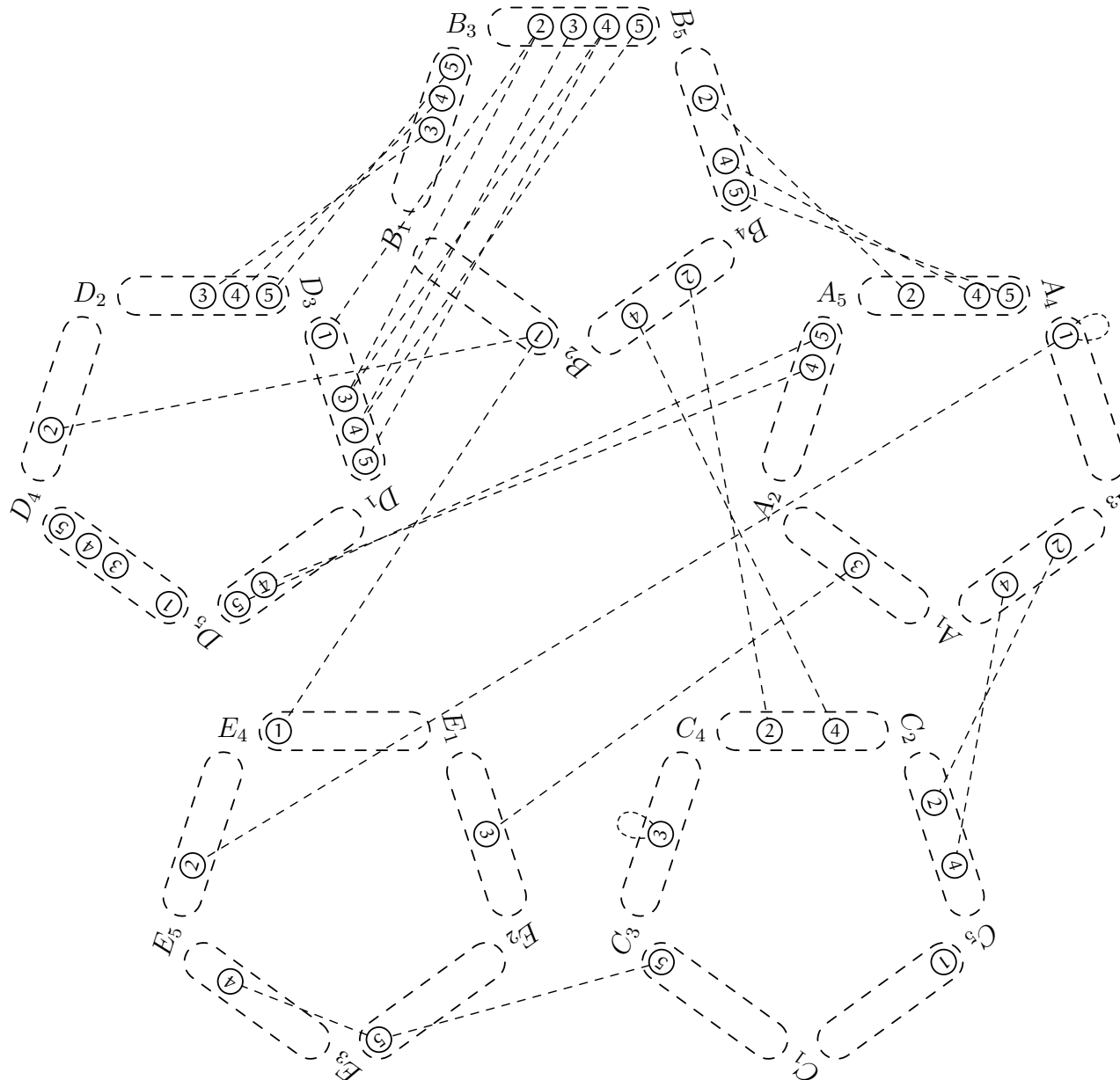
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# Next Assignment to $C_1$ .



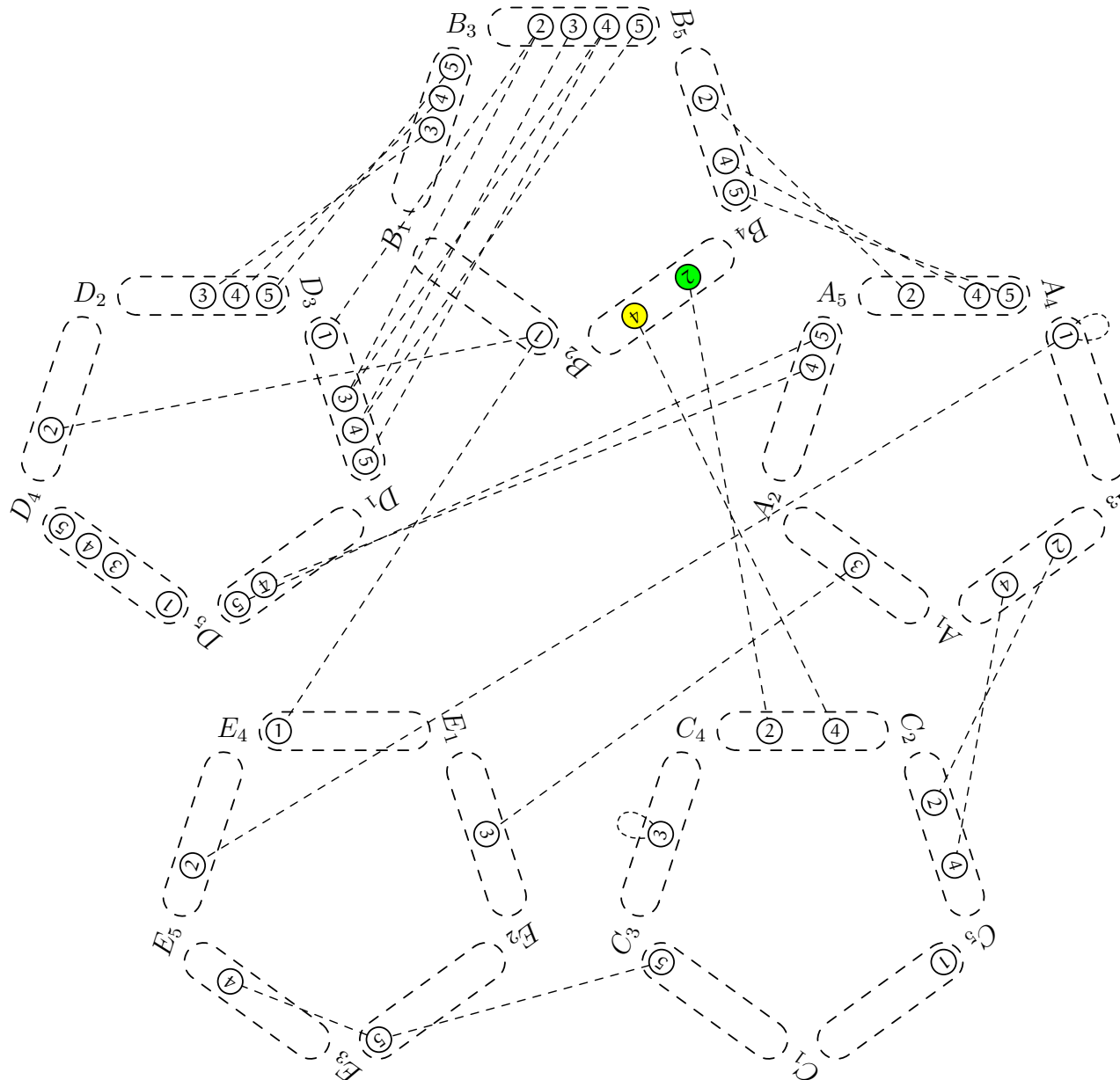
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

# After Assignment $C_1 = 5$ and Arc-Consistency.



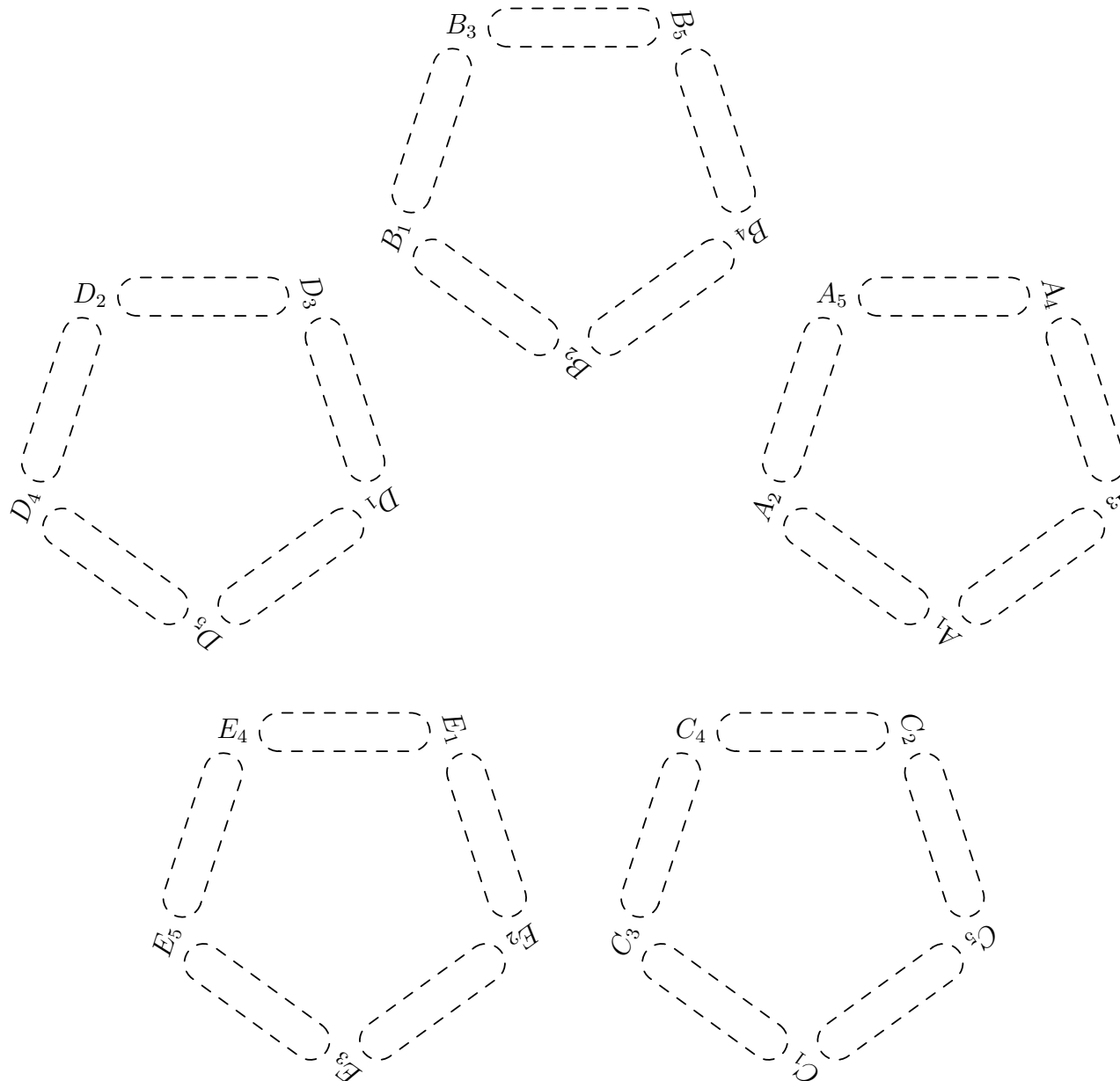
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

Select  $B_4$  as Current Variable.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

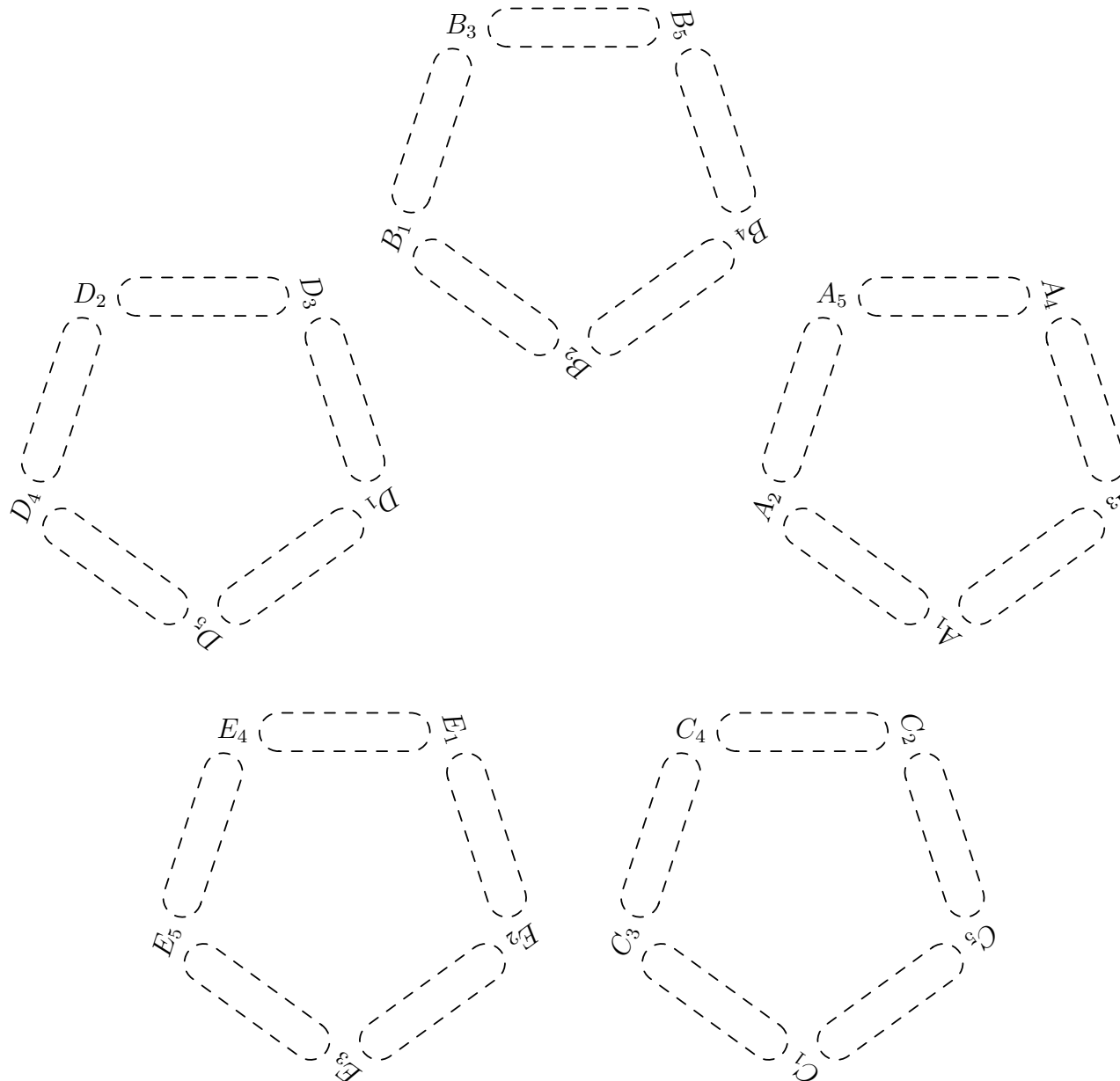
# After Assignment $B_4 = 2$ and Arc-Consistency.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

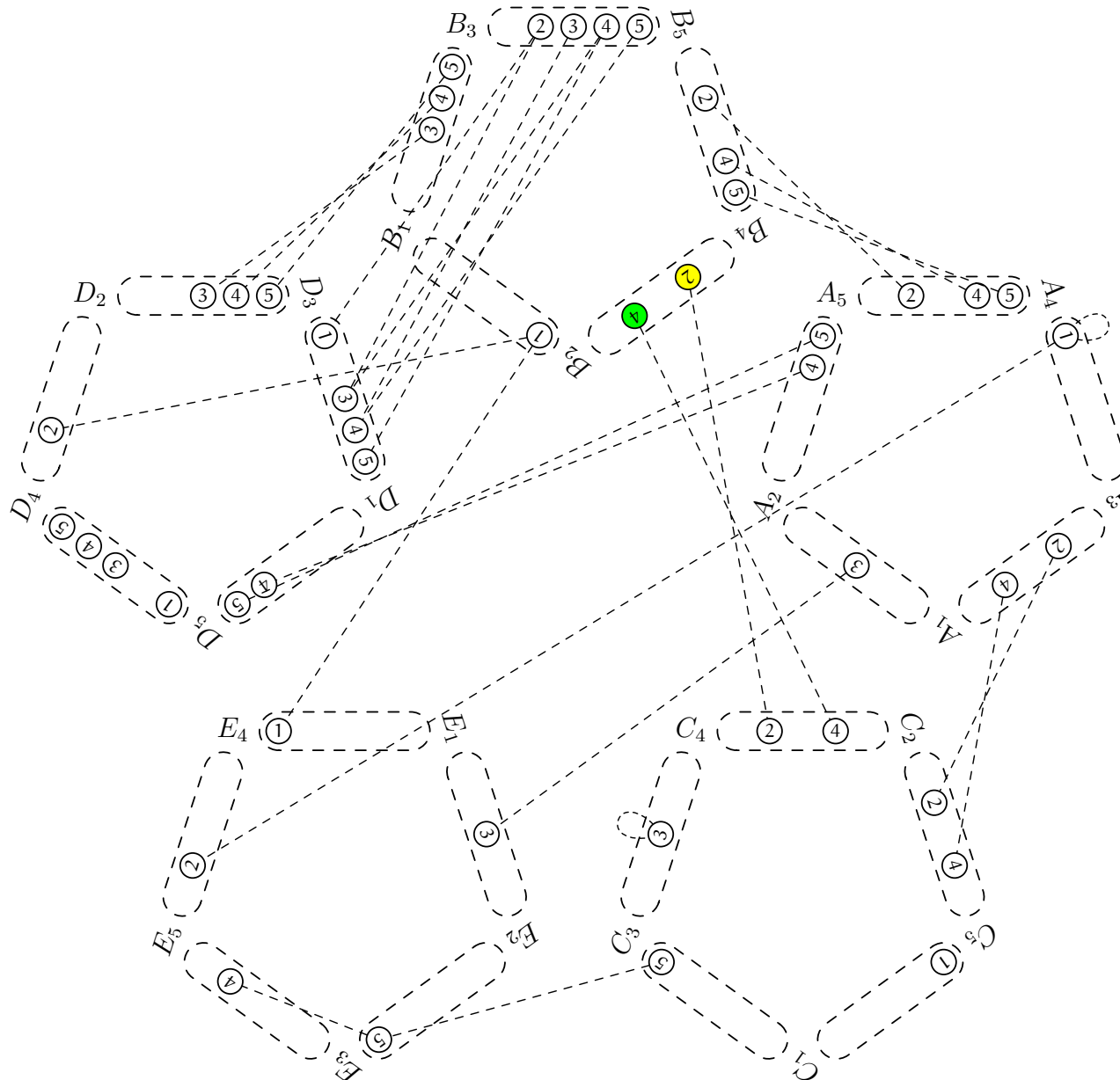


# Backtrack on $B_4$ .



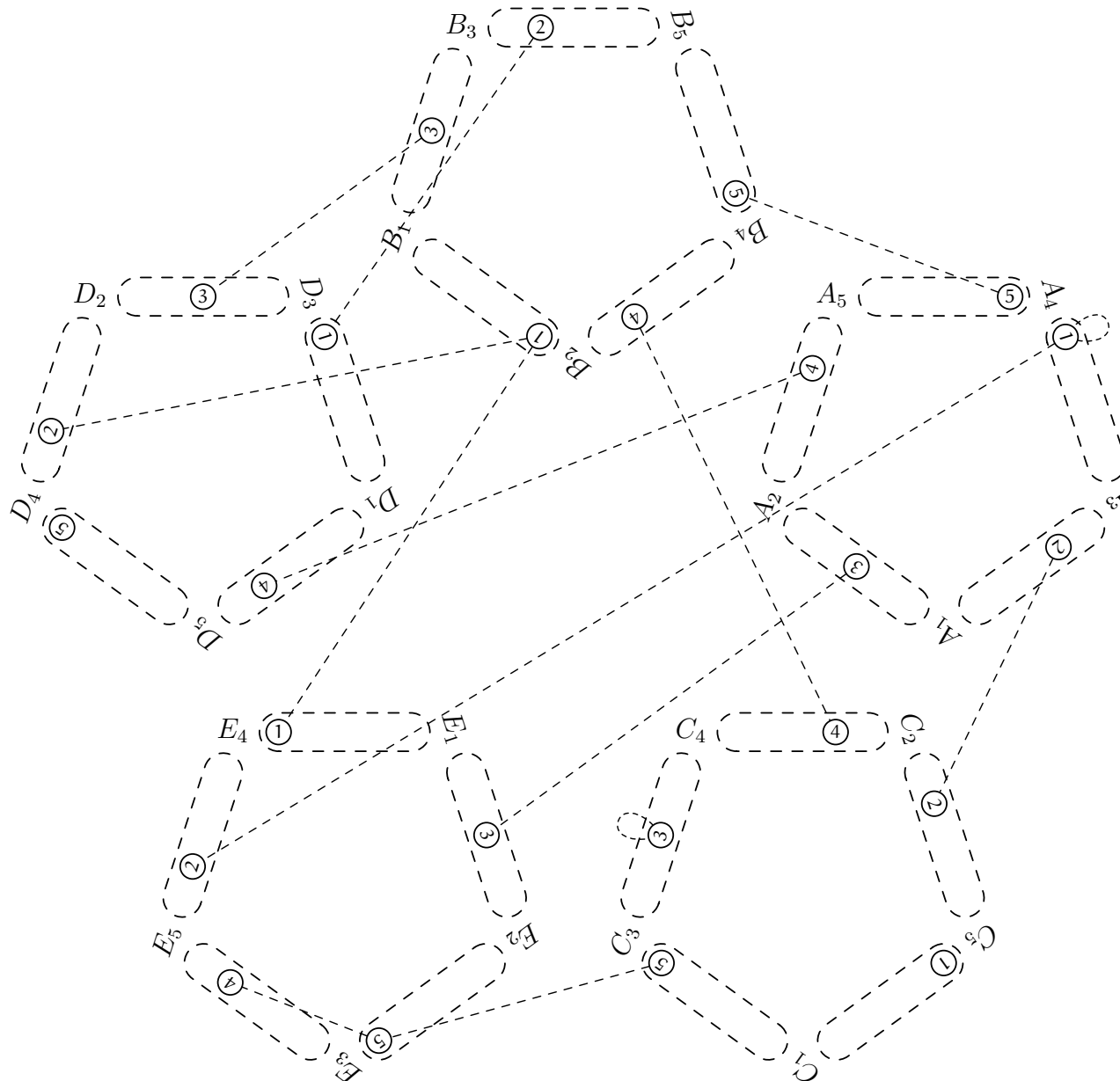
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# Next Assignment to $B_4$ .



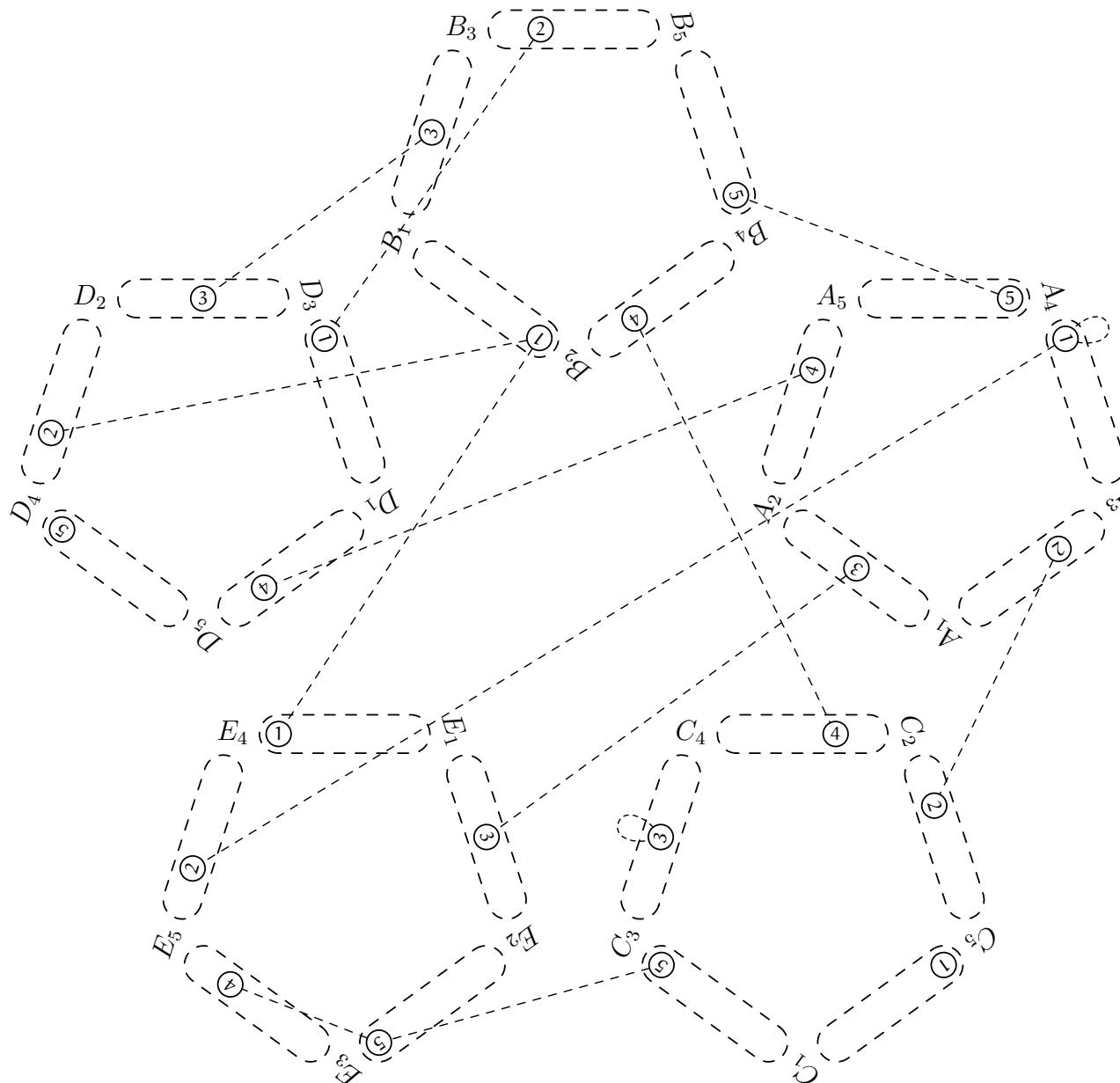
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# After Assignment $B_4 = 4$ and Arc-Consistency.



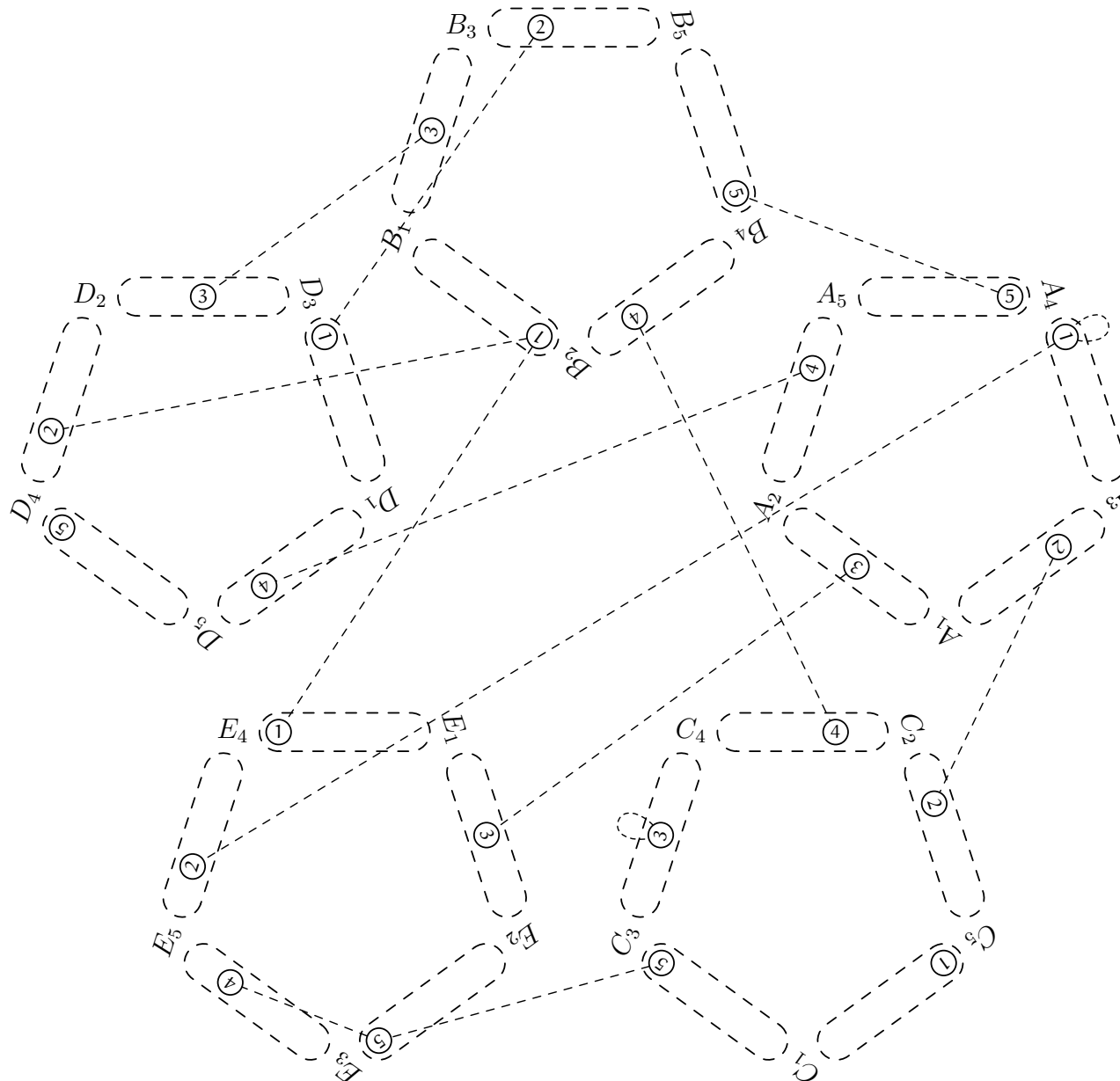
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

# All domains are singletons.



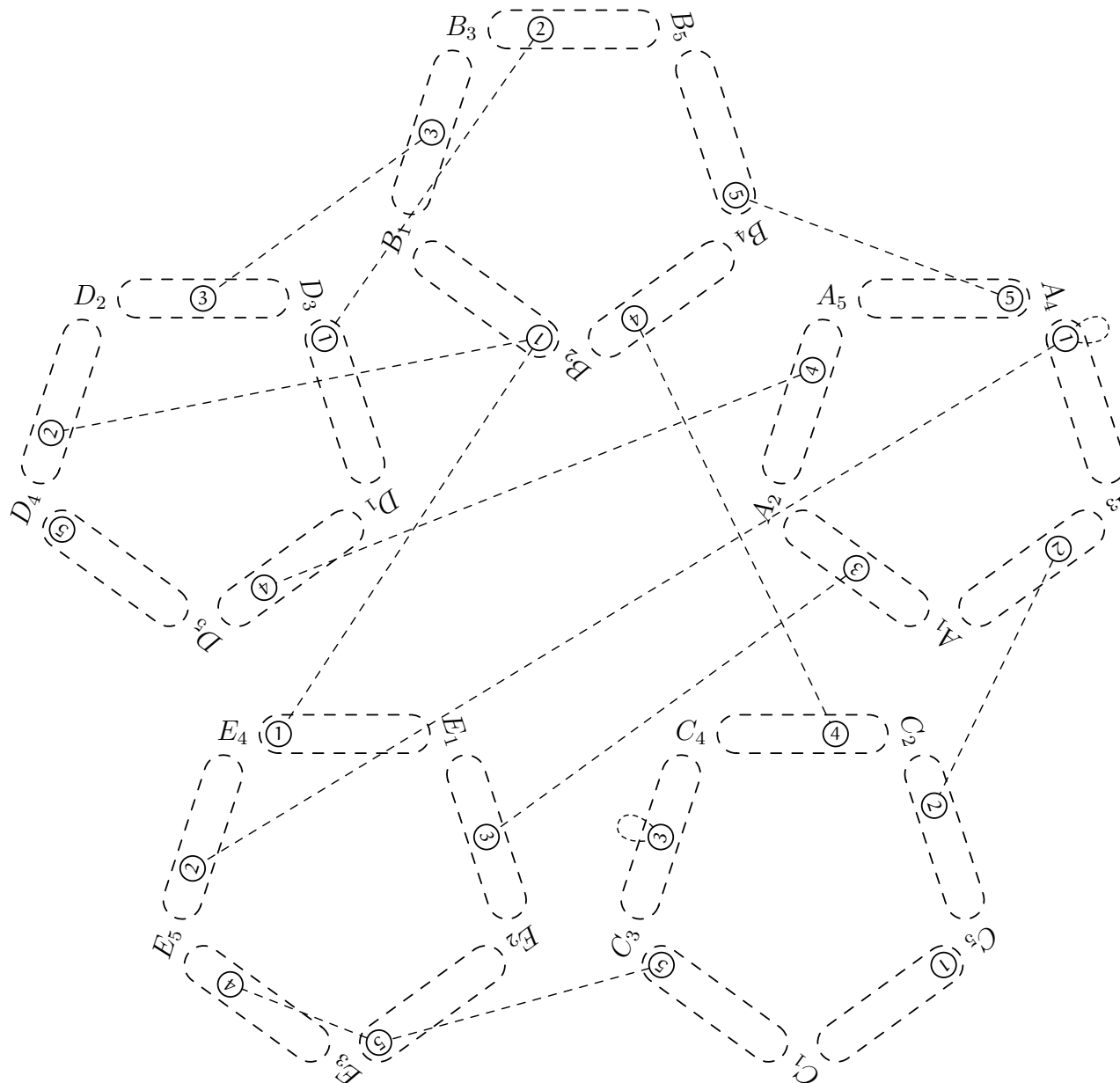
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

**All constraints are satisfied.**



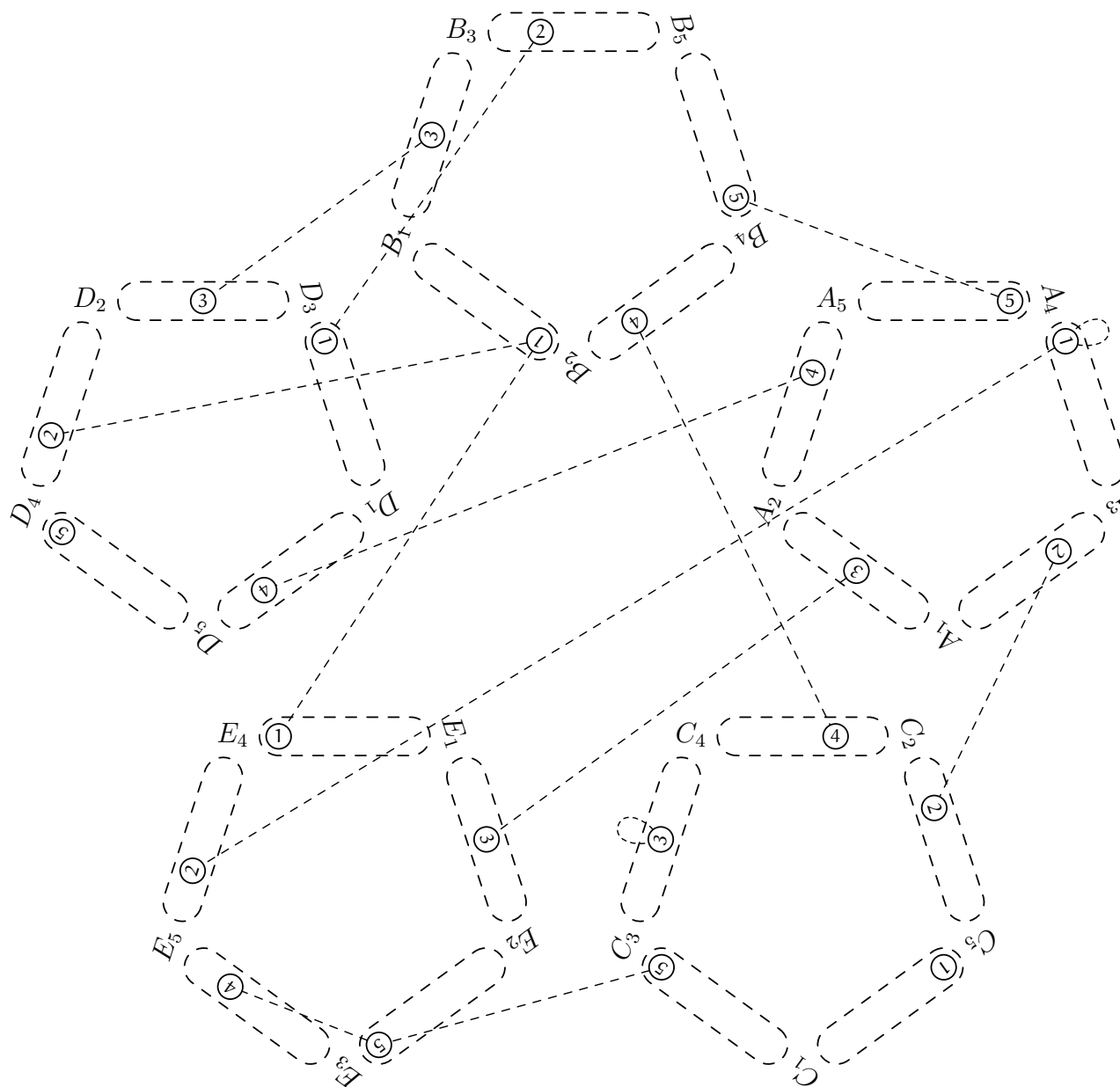
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

# We have solved the problem.



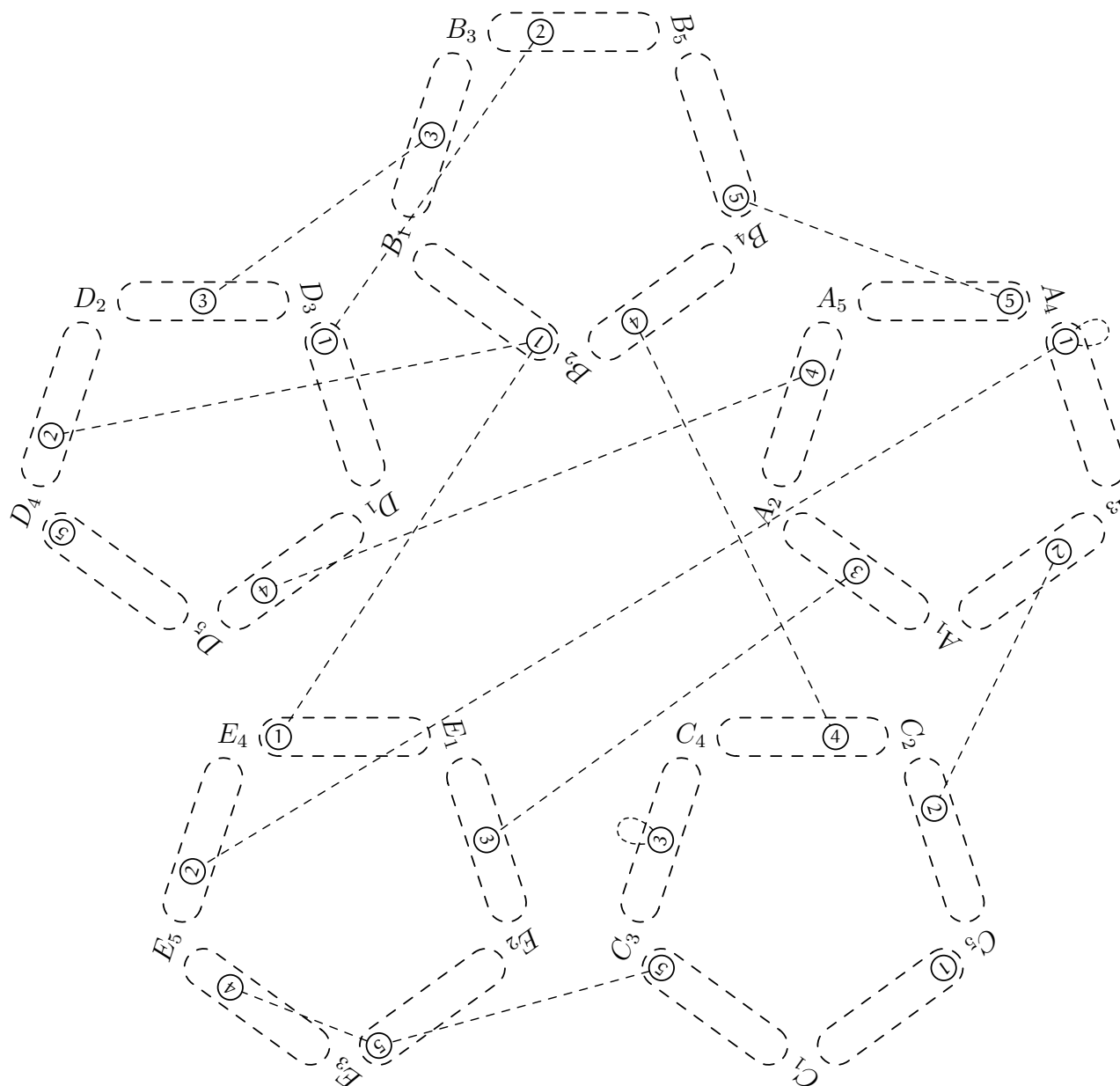
- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

$D_5 = 5$  (the zebra).



- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue

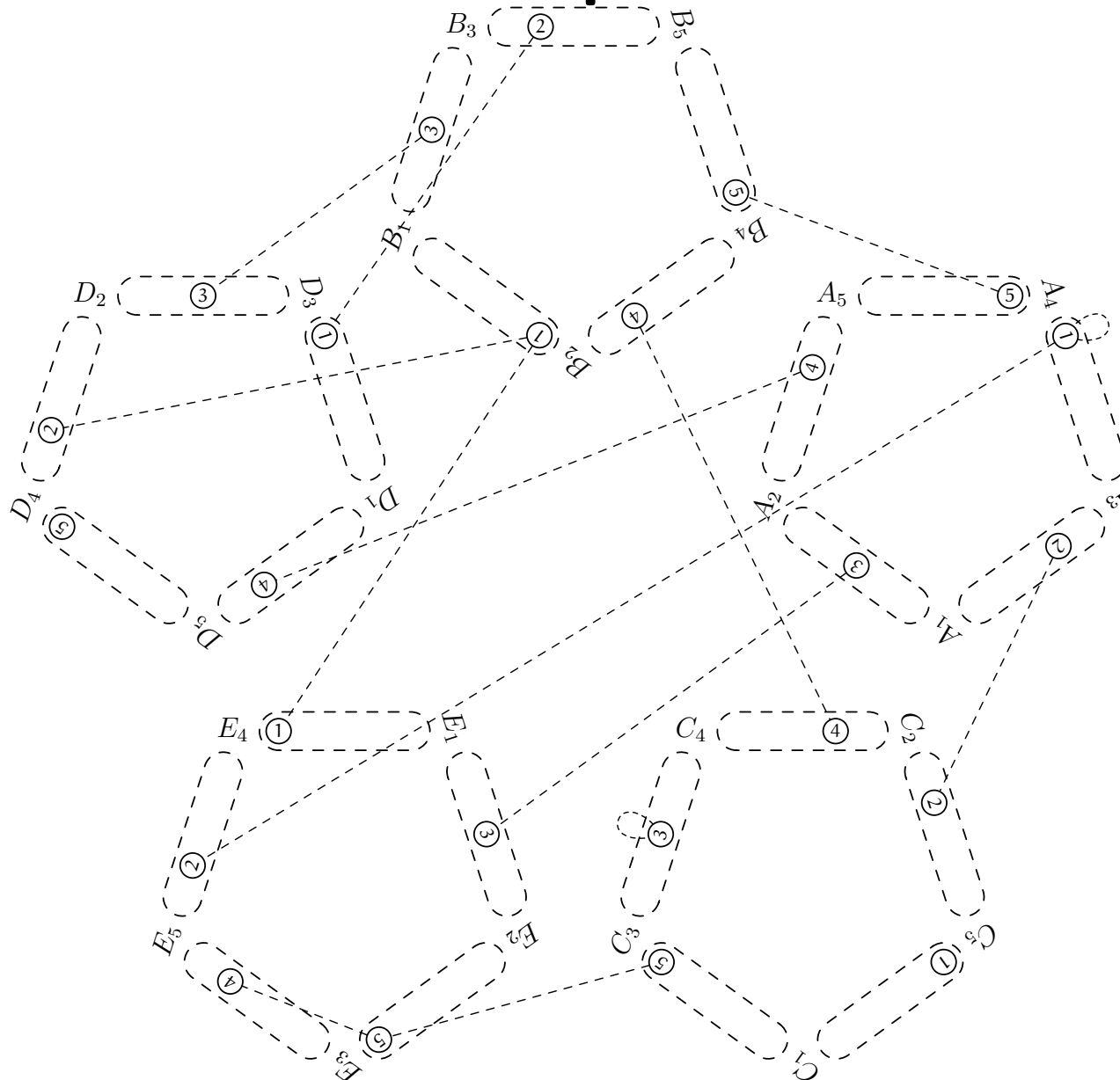
$D_5 = 5$  (the zebra).  $A_5 = 5$  (the Japanese).



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

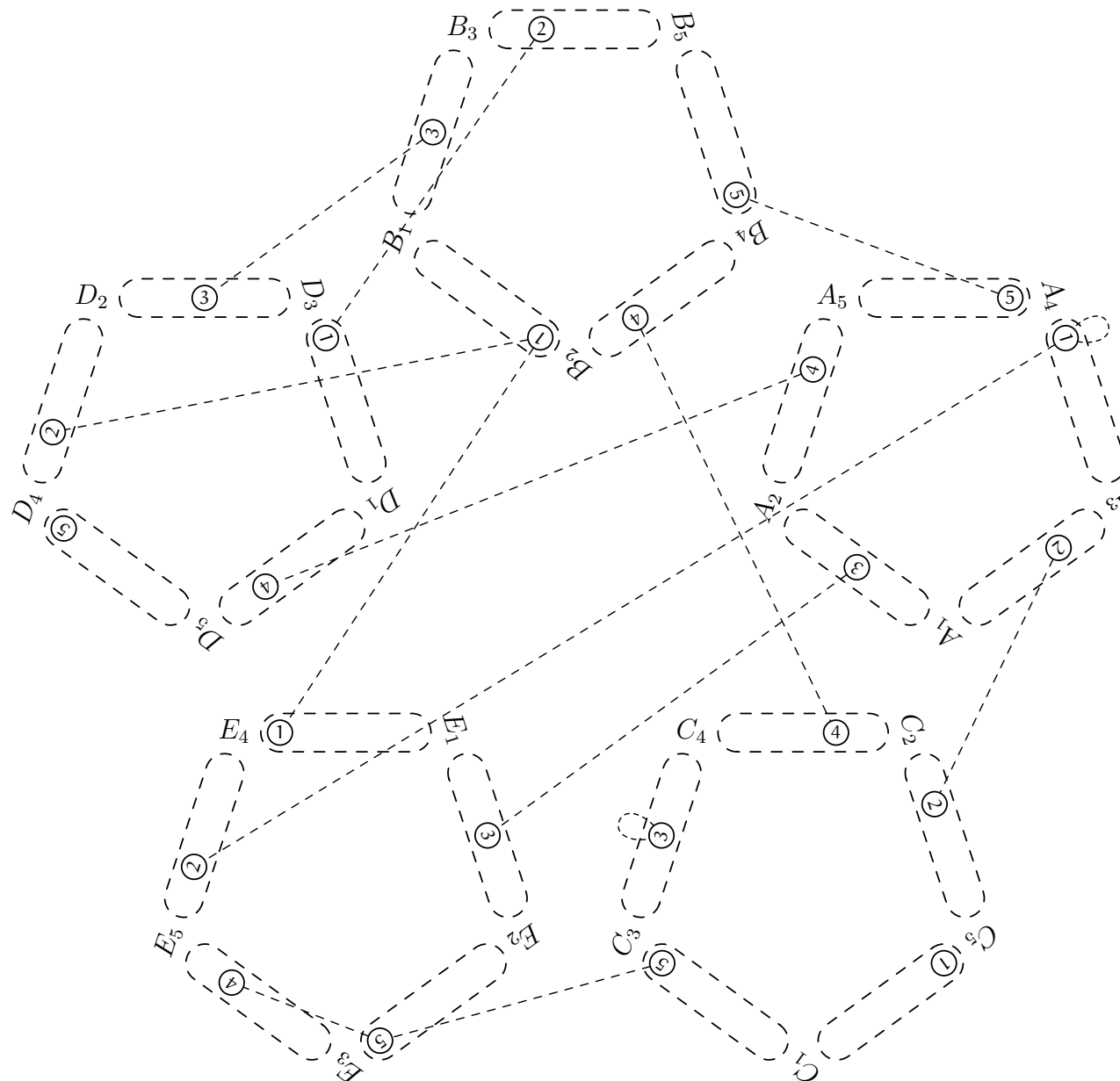


$D_5 = 5$  (the zebra).  $A_5 = 5$  (the Japanese).  
 Therefore, the Japanese owns the zebra.



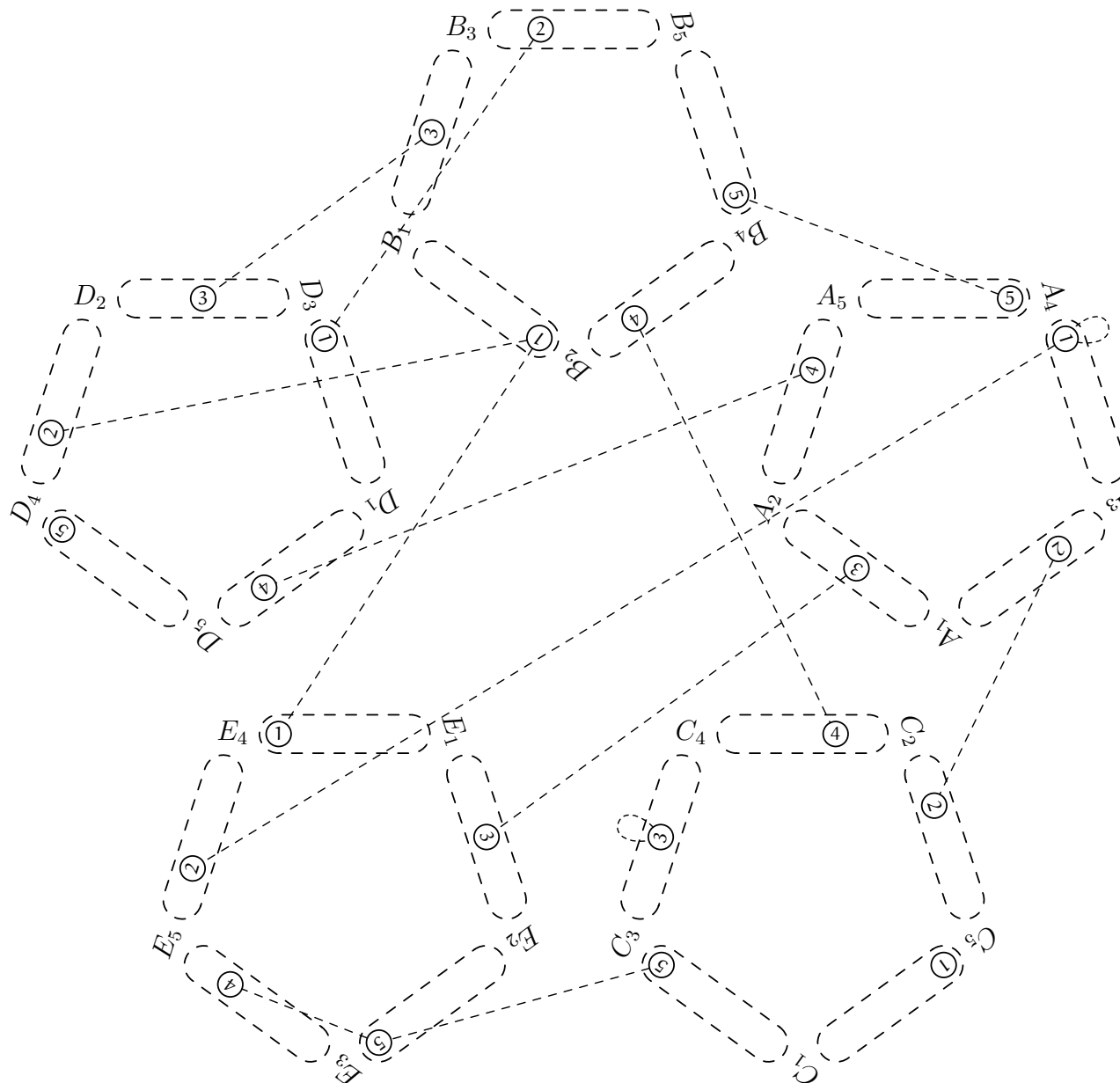
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

To some this may have come as a complete surprise.



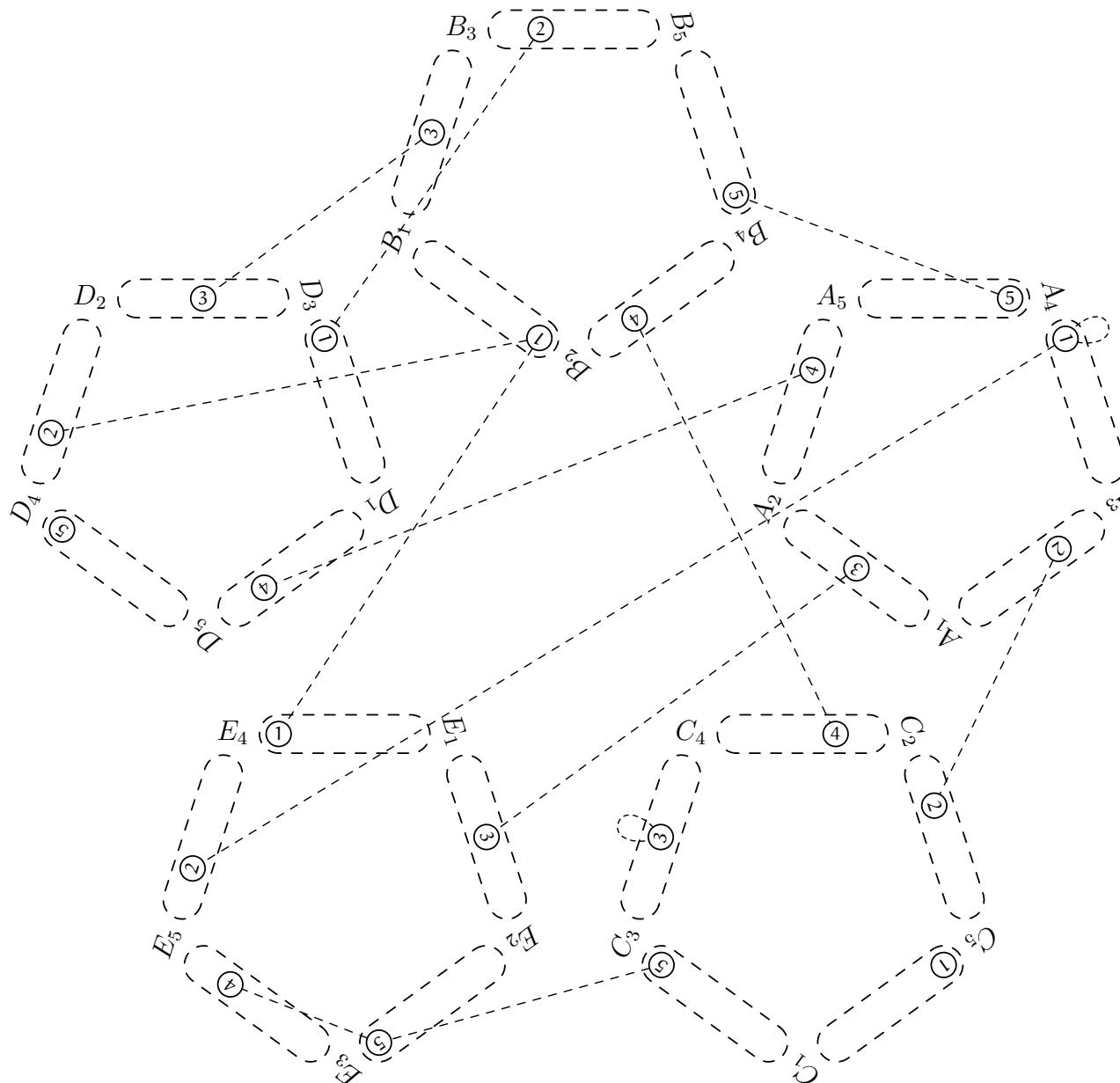
- A*<sub>1</sub> Englishman
- A*<sub>2</sub> Spaniard
- A*<sub>3</sub> Irishman
- A*<sub>4</sub> Nigerian
- A*<sub>5</sub> Japanese
- B*<sub>1</sub> go
- B*<sub>2</sub> cricket
- B*<sub>3</sub> judo
- B*<sub>4</sub> poker
- B*<sub>5</sub> polo
- C*<sub>1</sub> coffee
- C*<sub>2</sub> tea
- C*<sub>3</sub> milk
- C*<sub>4</sub> orange juice
- C*<sub>5</sub> Guinness
- D*<sub>1</sub> dog
- D*<sub>2</sub> snails
- D*<sub>3</sub> fox
- D*<sub>4</sub> horse
- D*<sub>5</sub> zebra
- E*<sub>1</sub> red
- E*<sub>2</sub> green
- E*<sub>3</sub> ivory
- E*<sub>4</sub> yellow
- E*<sub>5</sub> blue

$C_5 = 1$  (Guinness).



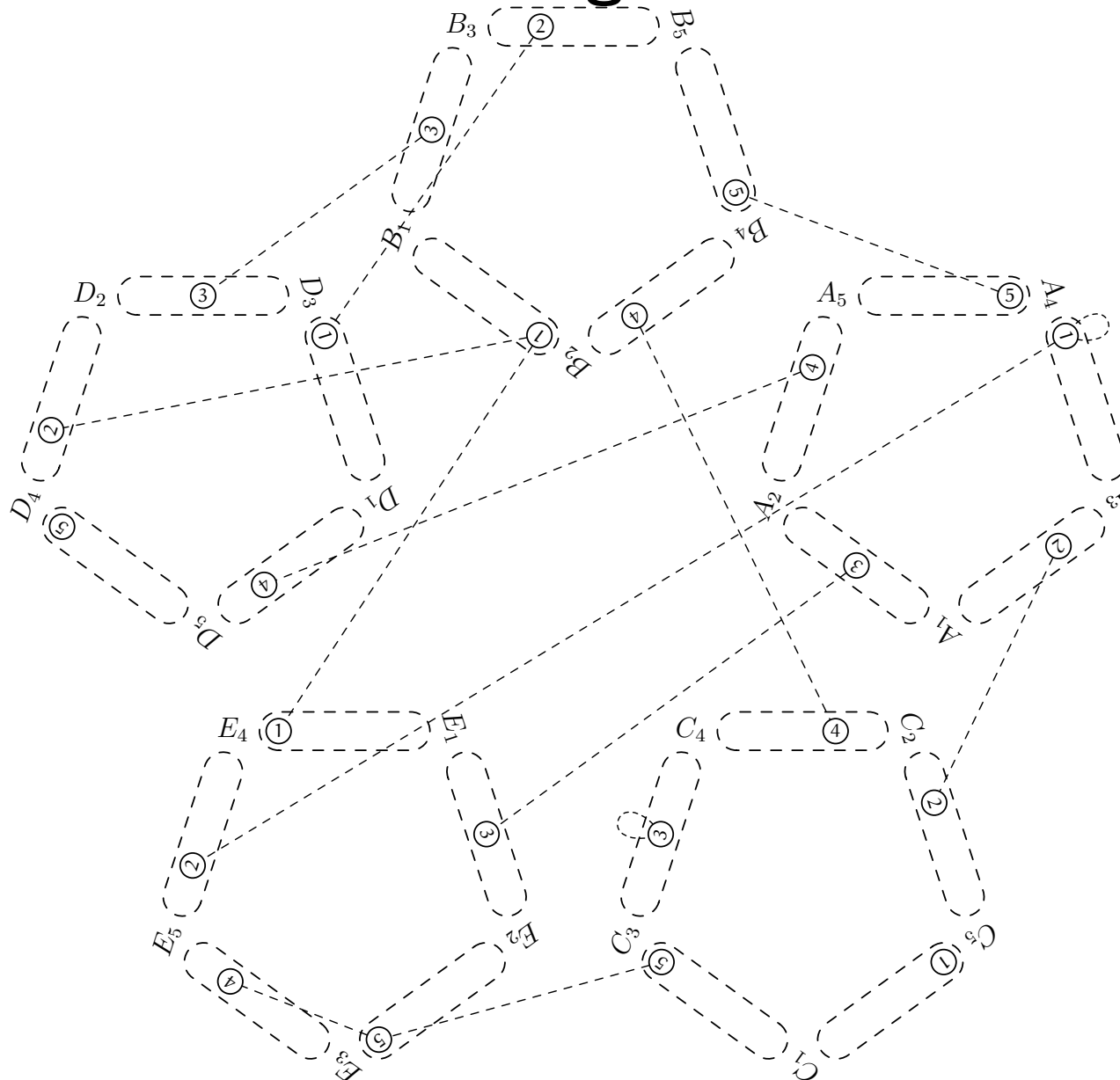
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

$C_5 = 1$  (Guinness).  $A_4 = 1$  (the Nigerian).



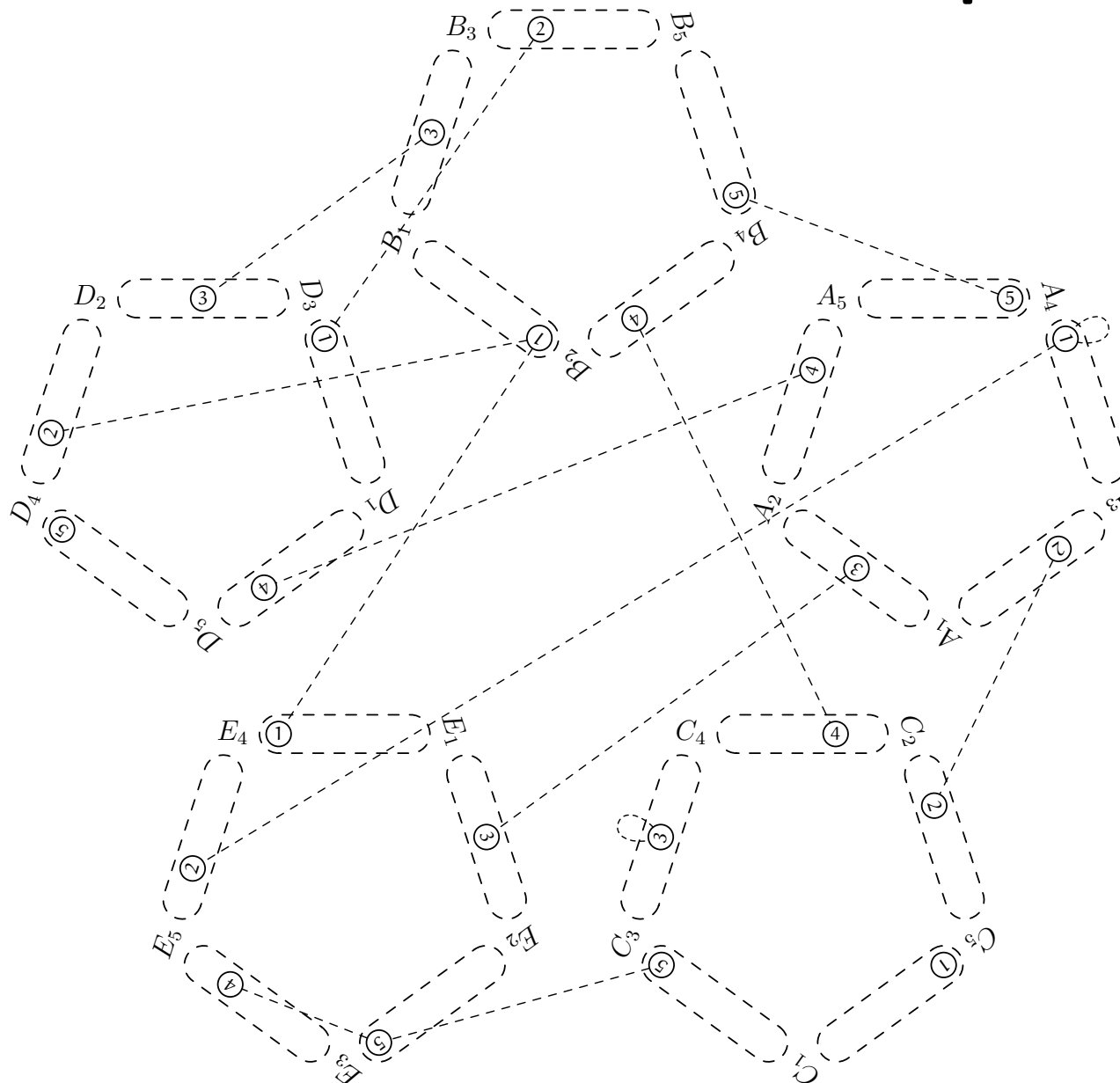
- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

$C_5 = 1$  (Guinness).  $A_4 = 1$  (the Nigerian).  
 Therefore, the Nigerian drinks Guinness.



- $A_1$  Englishman
- $A_2$  Spaniard
- $A_3$  Irishman
- $A_4$  Nigerian
- $A_5$  Japanese
- $B_1$  go
- $B_2$  cricket
- $B_3$  judo
- $B_4$  poker
- $B_5$  polo
- $C_1$  coffee
- $C_2$  tea
- $C_3$  milk
- $C_4$  orange juice
- $C_5$  Guinness
- $D_1$  dog
- $D_2$  snails
- $D_3$  fox
- $D_4$  horse
- $D_5$  zebra
- $E_1$  red
- $E_2$  green
- $E_3$  ivory
- $E_4$  yellow
- $E_5$  blue

Given that Lagos has a large Guinness brewery, this should not have come as a complete surprise.



- A<sub>1</sub> Englishman
- A<sub>2</sub> Spaniard
- A<sub>3</sub> Irishman
- A<sub>4</sub> Nigerian
- A<sub>5</sub> Japanese
- B<sub>1</sub> go
- B<sub>2</sub> cricket
- B<sub>3</sub> judo
- B<sub>4</sub> poker
- B<sub>5</sub> polo
- C<sub>1</sub> coffee
- C<sub>2</sub> tea
- C<sub>3</sub> milk
- C<sub>4</sub> orange juice
- C<sub>5</sub> Guinness
- D<sub>1</sub> dog
- D<sub>2</sub> snails
- D<sub>3</sub> fox
- D<sub>4</sub> horse
- D<sub>5</sub> zebra
- E<sub>1</sub> red
- E<sub>2</sub> green
- E<sub>3</sub> ivory
- E<sub>4</sub> yellow
- E<sub>5</sub> blue