An extension of the labour theory of value

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Abstract

The known in classical political economy fundamental law of substitution of labourers' work by work of production equipment allows one to extend the labour theory of value, while one has to introduce and consider, in line with the conventional production factors: capital and labour, an energy production factor – the substitutive work of production equipment. It provides a consistent description of economic growth, as it is shown for the US economy as an example. In a thermodynamic interpretation, a flux of information and work eventually determine new organisation of matter, which acquires forms of different commodities (complexity), whereby the production processes can be considered as processes of materialisation of information, the cost of which is work of production system. Value appears to be a close relative to entropy with the reverse sign.

Key words: Energy in production; Labour theory of value; Law of substitution; Thermodynamics of production; The US economic growth.

JEL - classification: E11, O4.

Contents

1 Introduction

The concept of *economic value* is a well-established concept of our life. Every product has a price, which, one believes, reflects its value. The products are exchanging according to their values, which allows one to ascribe a certain quantity of value in arbitrary money units to a product and to estimate the value of a set of products, for example, the value of all products that are produced by a nation during a year, which one calls gross domestic product (GDP). The concept of value in economics seems to be as much important as the concepts of energy and entropy in physics, and economics itself could be defined as a science, which investigates processes of appearing, movement and disappearing of value, being hardly interested in its material carrier.[1] Recounting fluxes of value (in arbitrary money units) allows one to create general descriptive schemes of production and consumption (Blaug, 1997).

During the many centuries scholars have tried to understand how products acquire their value and what can be the meaning of value (Blaug, 1997). There is a strong believe that value of a set of products can be reduced to original sources of value, so-called production factors, among them one finds work of labourers, or labour consumption, as indisputable factor, and consumption of energy,[2] as a hypothetical production factor. The spectrum of opinions on the relationship of energy with value is very broad. The majority of economists, who believe in the productive force of capital, consider energy (or more correctly: energy carriers) to be an ordinary intermediate product that contributes to value of produced commodities by adding its cost to the price, which means that consumption of energy does not create value. However, many others are ardent proponents of a quite different point of view: energy must be considered as the only source and measure of value. The last conviction has a long and illustrious history, dating back to the 1860s

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[1] Surprisingly, I did not find the entry 'value' in the Cambridge Encyclopaedia, though, in the Oxford Dictionary of Current English, one can read, that value is 'worth of something in terms of money or other goods for which it can be exchanged'. In contrast to scholars of classical political economy, modern economists avoid the concept of value as much as possible – they prefer to speak about prices. But many researchers consider that economics as a science needs in the concept of value.

[2] It is a custom to speak about energy consumption, though, for the sake of precision, the word *consumption* should be replaced by the word *conversion.* Energy cannot be *used up* in production process, but it can only be converted into other forms: chemical energy into heat energy, heat energy into mechanical energy, mechanical energy into heat energy and so on. The measure of potential converted energy (work) is exergy.

(Mirowski, 1988), and is a foundation of some general schemes, aiming to valuate both natural and artificial things by description fluxes of energy (or exergy, or emergy), developed in recent years (Odum, 1996; Sciubba, 2001; Valero, 1998). In any case, energy is universally vital to the performance of both nature and economy and its special role in production is worthy to discuss once more.

In this paper, I attempted to describe the phenomenon of production of value. In the foundation of our understanding of performance of the production system, is laid a distinct thesis, which is discussed in Section 2, about the substitution of labourers' work by work of external sources, elaborated in classical political economy. In this case, in line with the conventional production factors of neo-classical economics (Cobb and Douglas, 1928; Solow, 1957): capital K and labour L, one has to introduce and consider a new production factor – the substitutive work of production equipment P. Section 3 discusses the main principles of the quantitative theory, which were originally formulated in previous publications of the author (Pokrovski, 1999, 2003), and demonstrates the ability of the theory to describe a real situation on an example of the US economy. In Section 4, possible connections of the concept of value with thermodynamic concepts are discussed. The Conclusion contains the discussion of the problem.

2 The law of substitution

The role of the production system of the human society is to transform 'wild' natural forms of substances into 'useful' forms (dwellings, food, clothes, buildings, machines, transport means, sanitation, home appliances, machinery and other commodities), which support the very existence of the human population. The artificial things can be characterised from different points of view, but it is very important that there is an unique and unifying measure of all services and commodities: value can be ascribed to every product, so that one can speak both about production of things and about production of value. The production system consists of many production units: enterprises, factories, plants, firms *et cetera*, which create all what the man needs, but, in this paper, we shall consider the production system as a whole, one says, in *macroeconomic* (as opposed to *microeconomic*) or phenomenological approach, which allows us to include some general characteristics of technology into the description and to formulate a phenomenological (macroeconomical, no fluctuations are discussed) theory of production.

2.1 *The neo-classic concept of substitution*

The phenomenon of production, as production of value, has been investigated by prominent scholars of classical political economy and neo-classical economics during many centuries (Blaug, 1997). Some investigations have been aiming to uncover sources of value, so-called production factors, which are some general inputs of production processes, such as labour, capital and, perhaps, something else – the factors which one needs to create any of the products. Adam Smith, David Ricardo, Karl Marx and many others considered work of labourers L in production processes to be a sole source of value. According to Smith (1976), "value of any commodity... to the person who processes it and who means not to use or consume it himself, but to exchange it for other commodities, is equal to the quantity of labour which enables him to purchase or command". According to Marx (1952)], "all commodities are only definite masses of congealed labour time". However, some discrepancies had emerged later: the growth rate of the consumption of labour in production appeared to be less than the growth rate of output in developed economies, and, to explain the phenomenon of economic growth, other production factors ought to be added into consideration. The neo- classical approach to the theory of production has introduced stock of production equipment, measured by its value K (capital stock), as an important factor, which could substitute labour in production. The output, or production of value, Y (in money units), is assumed to be a function of outlay of labour L, measured, for example, in working hours per year and capital stock K measured by its value. For the interpretation of empirical data, different forms of production function were proposed, but the scholars often use a simple presentation – the Cobb-Douglas production function (Cobb and Douglas, 1928)

$$Y = Y_0 \frac{L}{L_0} \left(\frac{L_0}{L} \frac{K}{K_0} \right)^{\alpha} . \tag{1}$$

The index α is an internal characteristic of the production system. Though the hypothesis of substitution of labour by capital and even the very concept of capital (value of production equipment) was heavily criticised (Robinson, 1955a, 1955b, 1956), it survives up to now in the foundation of the neo-classical theory of production with some corrections: for the proper description of empirical situations, labour and capital services, which are some-what different from labour and capital, are considered real sources of growth (Solow, 1957; Brown, 1966; Ferguson, 1969; Jorgenson and Griliches, Jorgenson and Stiroh, 2000). In fact, this assumption implicitly incorporates some unknown production factors, which appear to be an object of investigation in the last decades. Remaining in a framework of the neo-classical approach, many candidates for new production factors, such as technology, human capital, stock of knowledge and others were tested (Barro and Sala-i-Martin, 1995; Aghion and Howitt, 1998). Some scholars (Berndt and Wood, 1979; Kümmel, 1982; Ayres et al, 2003; Beaudreau, 2005) insist that energy (or exergy) ought to be included in the list of production factors, and a question, whether consumption of energy carriers ought to be considered a source of value or, on the opposite, economic growth is responsible for increasing energy consumption, was debated for many decades.

2.2 *The role of production equipment*

To change forms of matter, that is, to transform, for example, ores of different chemical elements into an aircraft which can fly, some specific work[3] must be done. Modern technologies assume that this work can be done by a human being himself and/or by some external sources (water, wind, coal, oil, et cetera), one can say by energy, simultaneously. The same result can be obtained at different energy consumed and at different labourer's work. So, for example, to grind corn into flour a man can use a hand mill, or a water mill, a wind mill, or a steam mill. In the last cases, the labourer's work is substituted by the work of falling water, or wind, or heat. In these cases, as in many others, production equipment is some means of attracting of external sources to the production of things. No matter who or what does the work: the whole work must be done to obtain the result.

It is possible that the first, who wrote about the functional role of machinery in production, was Galileo Galilei. He realized that all machines transmitted and applied force as special cases of the lever and fulcrum principle. A prominent historian of science and technology Donald Cardwell (1972) wrote that Galileo in his notes *On Motion* (1590) and *On Mechanics* (1600) recognized that "the function of a machine is to deploy and use the powers that nature makes available in the best possible way for man's pur-

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[3] One can understand work as a process of conversion of energy in technological processes from one form to another, for example, from mechanical into thermic form.

poses... the criterion is the amount of work done – however that is evaluated – and not a subjective assessment of the effort put into accomplishing it" (pp. 38-39). The advantage of machines is to harness cheap sources of energy because "the fall of a river costs little or nothing."

The relevance of technology to economic performance was clearly recognized by Marx (1952). He described the functional role of machinery in production processes in chapter XV *Machinery and Modern Industry* of his famous book in such words as follow:

> On a closer examination of the working machine proper, we find in it, as a general rule, though often, no doubt, under very altered forms, the apparatus and tools used by the handicraftsmen or manufacturing workman: with this difference that instead of being human implements, they are the implements of a mechanism, or mechanical implements (pp. 181-182). The machine proper is therefore a mechanism that, after being set in motion performs with its tools the same operations that were formerly done by the workman with similar tools. Whether the motive power is derived from man or from some other machine, makes no difference in this respect (p. 182). The implements of labour, in the form of machinery, necessitate the substitution of natural forces for human force, and the conscious application of science instead of rule of thumb (p. 188). After making allowance, both in the case of the machine and of the tool, for their average daily cost, that is, for the value they transmit to the product by their average daily wear and tear, and for their consumption of auxiliary substances such as oil, coal and so on, they each do their work gratuitously, just like the forces furnished by nature without the help of man (p. 189).

Thus, the classics of physics and economics recognised the main functional role of machinery in production process, as a role connected with *substitution of labourer's work by work of machines moved by external sources of energy*, while the amount of substitution depends on employed technology embodied in the production equipment. Although one needs production equipment (capital) to attract an extra amount of labour and/or external energy, work can be replaced only by work not capital.

2.3 *Effect of substitution on the estimate of value*

Every scholar of economics would agree that labour is the most important factor of production, but not the only one: the situation has appeared to be rather complicated: something else ought to be added into the theory. One can guess that the 'something', what is needed to be introduced, is Marx's phenomenon of 'the substitution of natural forces for human force'. Indeed, after having understood this phenomenon, Marx could suggest that the substitution affects the mechanism of production of value. To understand how gratuitous work influences the value of products, he could analyse the performance of two similar enterprises. He could consider that the first of the enterprises uses the technology, which require some amounts of labour L and substitution work P, and, to produce the same quantity of the same product, the second one uses the technology with the quantities $L-\Delta L$ and $P+\Delta P$ of production factors. So far as the products are considered to be identical, the exchange values of the products of either enterprises on the market are equal, despite of the difference in labour consumption. So, as Marx could continue to argue, value cannot be determined by labour only, but properly accounted work by natural forces ought to be taken into account. To produce the same quantity of value, the decrease in labourer's work ought to be compensated by increase in work of external sources, so that one can write the relation

$$-\beta\,\Delta L + \gamma\,\Delta P = 0,$$

where productivities β and γ of corresponding production factors are introduced. Thus, equally with human work, work of natural forces appears to be an important production factor. It is easy to see, that the quantity β/γ determines an amount of gratuitous work of external sources, which is needed to substitute unit of human work to get the equal effect in production of value.

In general case, the work performed by labour L and productive energy P has to be corresponded to a set of products, which has the exchange value Y, and one can write, assuming that the production system itself remains unchanged, the relation between differentials of the quantities

$$\mathrm{d}Y = \beta\,\mathrm{d}L + \gamma\,\mathrm{d}P. \tag{2}$$

The coefficients $\beta > 0$ and $\gamma > 0$ correspond to the value produced by the addition of unit of labour input at constant external energy consumption and by the addition of unit of work of production equipment at constant labour input, respectively; in line with the existing economic theories, these quantities can be labelled as marginal productivities of the corresponding production factors. The two production factors, work of labour and work of external sources of energy, can substitute for each other and, in this sense, be equivalent, so that labour remains eventually to be, using Adam Smith's words, 'the only universal, as well as the only accurate measure of value, or the only standard by which we can compare the values of different commodities at all times, and at all places'. Taking into account the effect of substitution, one can say that the only universal and accurate measure of value is the work of labourers or other agents used for production.

3 A quantitative theory of production

The law of substitution of labour by work of external sources allows one to develop a theory, principles of which were discussed earlier in the monograph (Pokrovski, 1999), where, unfortunately, the theory was not formulated in a complete form: in particular, the concept of productive energy was not clearly defined, and the important effect of changes of the production system itself on the production of value was not taken into account. The improved version of the theory (Pokrovski, 2003) allows one to present the consistent description of economic growth, which was illustrated on the data of the US economy.

3.1 *A concept of substitutive work*

Though there is no doubt that any consumption of energy carriers is productive, that is useful for production of things and services, the specific term *productive energy* was introduced (Pokrovski, 2003) allows to dub that part of consumed energy which is used to substitute work of labourers with work done by production equipment. Productive energy is a service provided by the production equipment - capital service. In economic terms, energy carriers are considered as intermediate products that contribute to value of produced products by adding its cost to the price quite similar to other intermediate products participating in production processes. However, the substitutive work or *productive energy* P has to be considered not only as an ordinary intermediate product but also as a value-creating factor which has to be considered equally with production factors of conventional neoclassical economics – capital K and labour L. The universal importance of labour and capital for economic performance is recognised: they are monitoring very thoroughly by special bodies. The usefulness of energy in production is indisputable: there are many data for consumption of primary carriers of energy – primary energy E, but necessity of separation and evaluation of substitutive work (genuine productive energy), which is used to make the production equipment to work, was not recognised until recently (Ayres et al, 2003; Pokrovski, 2003). In order to analyse economic performance in a proper way, methods of estimation of work of production equipment on the base of empirical data have been developing (Ayres et al, 2003; Pokrovski, 2007).

3.2 *Three-factor production function*

According to equation (2), output Y is a function of labour L and work of production equipment P. One has also to take into account the amount of production equipment measured universally by its value K (capital), so that the production function can be presented in two alternative forms

$$Y = \begin{cases} Y(K) \\ Y(L,P) \end{cases}, \qquad \mathrm{d}Y - \mathit{\Delta}\mathrm{d}t = \begin{cases} \xi(K)\,\mathrm{d}K \\ \beta(L,P)\,\mathrm{d}L + \gamma(L,P)\,\mathrm{d}P \end{cases} \tag{3}$$

where $\mathit{\Delta} dt$ is a part of an increment of production of value which is connected with change of characteristics of the production system (the technological and structural changes). In line with the existing economic practice, the quantities ξ, β and γ can be labelled as marginal productivities of the corresponding production factors. Considering the production system itself does not change, the marginal productivity ξ corresponds to value produced by addition of a unit of capital; the marginal productivities β and γ correspond to value produced by addition of a unit of labour input at constant external energy consumption and by the addition of a unit of energy at constant labour input, respectively. One has to consider that all marginal productivities are positive. One uses production factors to create useful commodities and an addition of any production factor must increase in production of things – this is known as the productivity principle.

One can specify function (3) further, requiring that the description ought to be universal, that is independent from the initial point (the principle of universality) and assuming also that production is homogeneous. Thus, under the simplest schematisation, when the production system is viewed as a collection of equipment (measured by its value K), getting its ability to act from labour (L) and capital services (P) inputs, the production function for output Y can be specified in the form

$$Y = \begin{cases} \xi K, & \xi > 0 \\ Y_0 \dfrac{L}{L_0} \left(\dfrac{L_0}{L} \dfrac{P}{P_0} \right)^{\alpha}, & 0 < \alpha < 1 \end{cases} \tag{4}$$

where L_0 and P_0 are values of labour and capital services in the base year. This formula presents two complementary descriptions of the process of production of value. The first line relates output to the amount of production equipment (capital stock), the second one describes the process of production through property of the same equipment to attract labour and energy (labour and capital services). The complementary descriptions of the process can be traced back. The first line in formula (4) reminds us about Harod-Domar approach (Harrod, 1939, 1948; Domar, 1946, 1947), while the function in the second line coincides with the Cobb-Douglas production function (1), in which substitutive work P stands in the place of capital stock K. The productivity of capital stock ξ and the index α in equation (4) are internal characteristics of the production system itself and connected with each other. One can note also that, in the conventional neo-classical approach, capital as variable plays two distinctive roles: capital stock as value of production equipment and capital service as a substitute for labour. These roles are ascribed to different variables in the discussed theory: equation (4) contains productive energy P as a capital service and capital stock K as a measure of amount of production equipment.

It is easy to see that the above relations provide the following expressions for marginal productivities

$$\xi = \frac{Y}{K}, \quad \beta = Y_0 \frac{1-\alpha}{L_0} \left(\frac{L_0}{L} \frac{P}{P_0} \right)^{\alpha}, \quad \gamma = Y_0 \frac{\alpha}{P_0} \left(\frac{L_0}{L} \frac{P}{P_0} \right)^{\alpha - 1}. \tag{5}$$

The index α in equation (4) and (5), as a characteristic of the production system, is connected, as shown earlier (Pokrovski, 2003), with characteristics of technology and can be called the technological index, which can be estimated on the base of all available information about the technological performance of the production system. Moreover, a condition regarding the optimal use of production factors enables us to establish a relation between the parameter α on one hand and the shared costs of production factors on the other one (Pokrovski, 2003). This provides the different means of estimating of the technological index.

3.3 *Application to the US economy*

3.3.1 *The consistent description of growth*

The relations (4) were tested for the US economy for years 1890 - 2000 (Pokrovski, 2003). The time series for output Y, capital K and labour L are easily available from the US governmental websites and were collected in Appendix of paper (Pokrovski, 2003). The empirical dependences are shown in Fig. 1 in line with total primary consumption of energy E, which is the amount (in energy units) of energy carriers, including primary productive consumption of energy. To test the theory, one needs in two quantities more: the productive energy P and technological index α, which are variables both. Note that the theory has no arbitrary parameters at all. Fortunately, methods for estimating of capital services – the third production factor P and the index α for given time series of Y, K and L can be developed, so that one can find (Pokrovski, 2003) such values of both capital services P and the technological index α, shown also in Fig. 1, that calculated values of output coincide with the empirical ones. It was show by a special analysis, that the calculated values of capital services, which are needed to obtain the correct values of output, correspond to estimates of real work of production equipment (Ayres et al, 2003). The index α represents also the share of expenses needed for utilisation of capital services in total expenses for production factors and can be evaluated due to estimates of cost of consumption of production factors. The different estimates of the technological index α are consistent (Pokrovski, 2003).

Thus, it was shown that production function (4) gives a consistent description of past empirical situation for years 1900 – 2000, and one can think that the theory can be helpful to forecast the future production. In the last case, one has to forecast the alteration of production system itself, which in this approximation is described by a change of α due to technological and structural modifications and the future values of production factors. The technological index α changes slowly and can be considered constant during

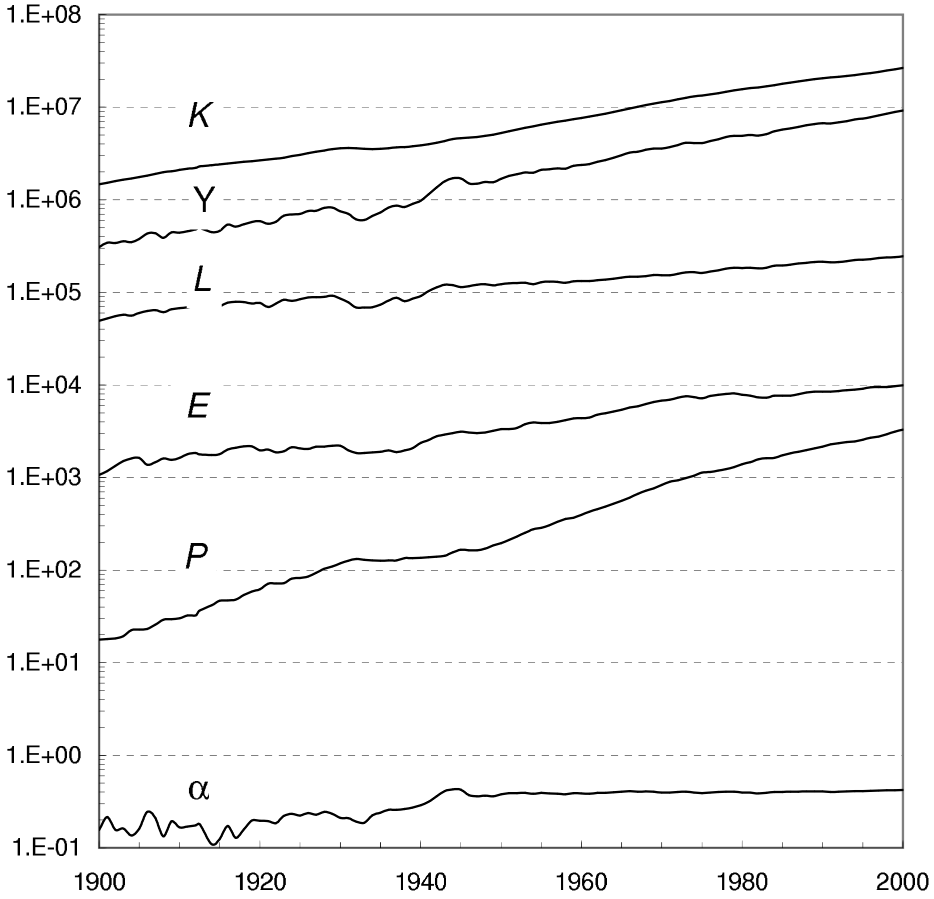

Figure 1. The consistent description of the US economy growth

The picture shows the empirical estimates of production of value (GDP) Y, million of 1996 dollars per year; production equipment (capital stock) K, million 1996 dollars; consumption of labour L, million working hours per year. The values of genuine substitutive work (productive energy) P, 10^{16} joules per year, and the technological index α are calculated (Pokrovski, 2003) to be corresponded to the above values of Y, K and L. Total consumption of primary energy carriers E, 10^{16} joules per year, is also shown.

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decades; as one can see in Fig. 1, the essential changes of the technological index are trigged by extraordinary events similar to the Second World War in years 1940 - 45. Methods of forecasting of the production factors ought to be developed.

3.3.2 *The 'stylised' economic growth*

There is some interest in deduction approximate relations to describe the 'stylised' facts of economic growth, that is, exponential growth of output, when the time dependencies of the production factors can be also approximated by exponential functions

$$K = K_0 e^{\delta t}, \qquad L = L_0 e^{\nu t}, \qquad P = P_0 e^{\eta t}. \tag{6}$$

Taking these equations into account, on the base of relation (4), the output can be written in the following form

$$Y = Y_0 e^{[\nu + \alpha(\eta - \nu)]t} = Y_0 e^{\delta \, t}. \tag{7}$$

In this approximation, the growth rate of output is equal to the growth rate of capital stock, as, indeed, one can see in Fig. 1 in the 'calm' period of years 1950 − 2000, and is connected with the growth rates of labour and capital services. The resulting expression implies that the growth rate of labour productivity is determined by the difference in the growth rates of energy and labour; it equals to $\alpha(\eta - \nu)$. In the 'calm' period of years 1950 − 2000 in the US economy (take a look at Fig. 1), $\delta = 0.0316$, $\nu = 0.0146$, $\eta = 0.0588$. The empirical averaged growth rate of output 0.0329 is approximately equal to the growth rate of capital $\delta = 0.0314$. One can directly estimate contributions of labour and capital services in the growth of output. Taking into account that, for this span of time, empirical value of α can be taken as 0.4, the contributions to the growth of output are $(1 - \alpha)\nu \approx 0.0088$ from the labour growth and $\alpha\eta \approx 0.0235$ from the capital services growth on average. Though capital stock is the means of attracting the production factors to production, increase in consumption of the production factors is connected with an increase in capital stock, and one can also separate the growth rate of capital stock δ in the growth rate of capital services η to get the breakdown of the growth rate of output in conventional terms: the contribution from the labour growth is $(1 - \alpha)\nu \approx 0.0088$, the contribution from the capital growth is $\alpha\delta \approx 0.0126$, and the contribution from the total factor productivity is $\alpha(\eta - \delta) \approx 0.0109$. In conventional interpretation, the latter is connected with changes of production system itself, but,in our interpretation the total factor productivity is connected with the growth of production factors. There is no contribution from alternation of the production system itself, when exponential growth (6) and (7) is assumed.

3.4 *What is the productivity of capital?*

In line with the productivity of capital stock ξ, one can also consider marginal productivities of labour and productive energy, β and γ, correspondingly, introduced by relations (2) and (3). The explicit forms of the marginal productivities are given by expressions (5) and allow one to evaluate the quantities on the base of empirical data. Note that these quantities, in virtue of equations (4) and (5), are always connected with each other by the relation

$$\xi = \beta\,\frac{L}{K} + \gamma\,\frac{P}{K}. \tag{8}$$

The empirical estimates of the quantities ξ, $\beta\,\frac{L}{K}$ and $\gamma\,\frac{P}{K}$ for the US economy (Pokrovski, 2003) are shown in Fig. 2. For the 'calm' years 1950 – 2000, the average value of the capital-stock marginal productivity is (0.309 ± 0.035) $year^{-1}$, whereas average value of the right hand side of Eq. (8) is (0.320 ± 0.041) $year^{-1}$. The values of the marginal productivity practically coincides with the averaged bulk productivity Y/K, which is (0.318 ± 0.010) $year^{-1}$; this is an evidence that the capital marginal productivity does not depend on argument K.

Thus, indeed, the marginal productivity of capital stock can be considered as the 'sum' of the marginal productivities of labour and capital services and appears to be a fundamental characteristic of the production system. The production equipment (capital stock) attracts labour and capital services to the production. Productivity of capital is, in fact, productivity of labour and energy. No other production factors are needed to be included into the theory.

The growth rate of capital marginal productivity cannot be reduced to any function of production factors. It is determined by the technological and/or structural evolution of the production system itself induced by abundance or lack of investment, labour and/or productive energy. Productivity of capital stock ξ can be calculated when more detailed approaches are applied. In the multi-sector approach (input-output model), this quantity is connected with the fundamental technological matrixes as will be shown elsewhere (Pokrovski, prepared to submission).

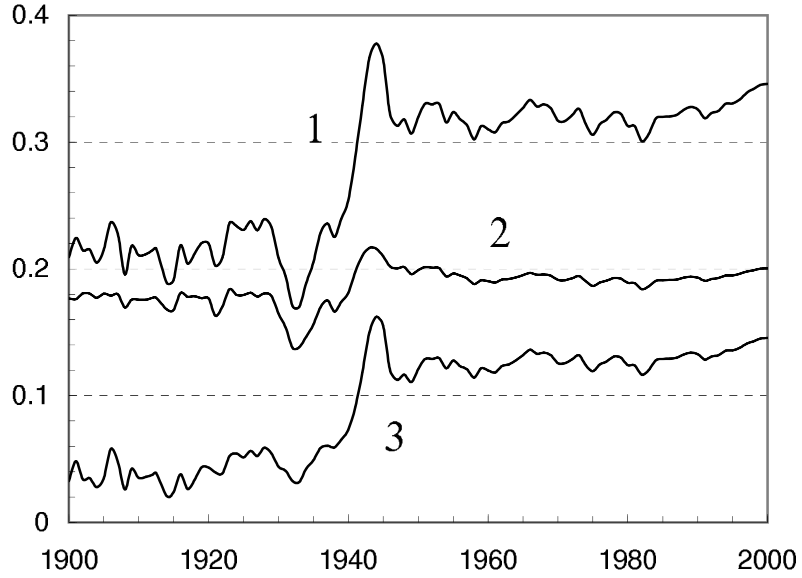

Figure 2. Marginal productivities in the US Economy

The solid lines 1 - 3 represent empirical estimates of the quantities: ξ, $\beta L/K$ and $\gamma P/K$, correspondingly, in $year^{-1}$.

4 Thermodynamics of production

The previous consideration demonstrates that production of value for unit of time Y (in money units, for year, for example), as a market estimation of the results of the work of labourers L and the work of external forces P, can be represented by an empirical non-linear function, which, in virtue of relations (4) and (5), can be written as

$$Y = \beta\, L + \gamma\, P. \tag{9}$$

From the other side, the results of the production are the changes in our environment (in the form of commodities and services) due to the work of production system, which, per unit of time, can be estimated as

$$dA = P + \frac{\beta}{\gamma}\, L. \tag{10}$$

The quantity dA/Y is genuine work needed to produce a thing or service

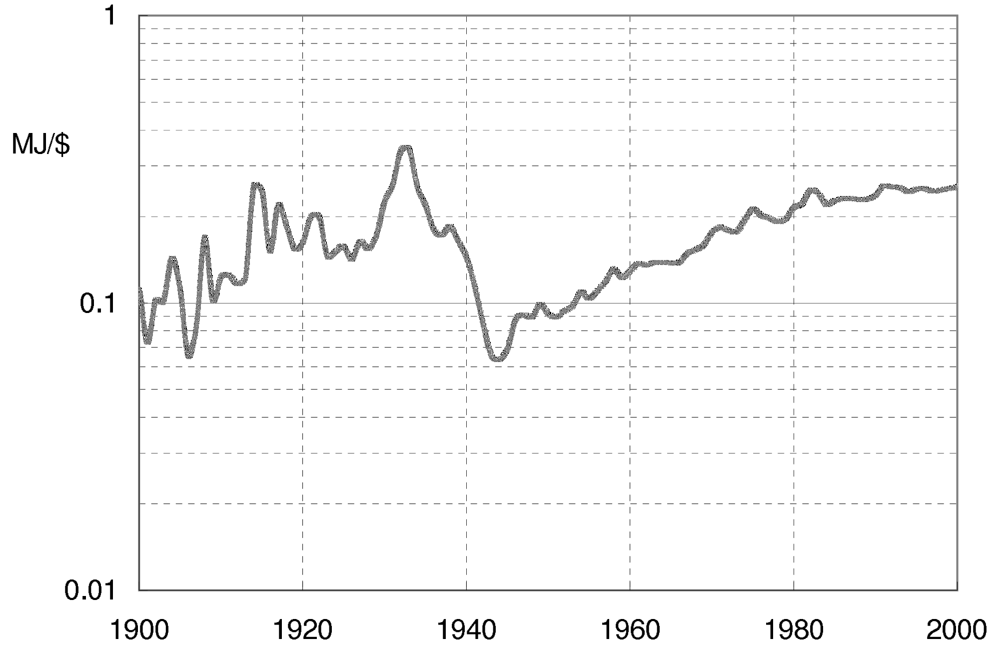

Figure 3. The 'energy content' of the US (1996) dollar

valued by a unit of money or, in other words, an 'energy content' of money unit. Figure 3 shows the 'energy content' of dollar for the US economy during the century. One can see that calculated in this way 'energy content' of dollar is $(1-3)\times 10^5$ joules per dollar of 1996; this quantity is naturally much less than 'total exergy or emergy content' (Odum, 1996; Sciubba, 2001), which is calculated 'from the very beginning', when all previous contributions of energy are accounted.

As far as the environment ought to be considered as *a thermodynamic system*, the production system, by performing work fulfilled by labour and external energy sources, shifts the environment from one non-equilibrium state to another. To estimate the changes in the thermodynamic terms, consider the process of production of useful things as a sequence of cycles – production cycles: raw materials are transformed into finished and semi-finished goods, semi-finished goods – into other semi-finished and finished goods and so on, until the finished commodities, which can be used by man, are made. The production cycle is performed by unit of production equipment, which is able to execute special operations, remaining (to say nothing about tear and wear) unchanged after the cycle, on some bodies, the forms of which (one can assume that change of internal energy can be neglected) are being modified. A production cycle can be considered as a sequence of elementary operations $j_1, j_2, \ldots$, while a set of elementary operations is given. The j_l is an index of elementary operation which is fulfilled as number l in the sequence of operations. The unique choice of indexes determines where, when and how forces are allowed to act to perform work, while the total work can be considered as a sum of work at elementary operations

$$\Delta A = A_{j_1} + A_{j_2} + \ldots \tag{11}$$

Not to be too abstract, we shall demonstrate on a simple example what can be the result of a cycle.

4.1 *A simple production cycle*

Let us consider a system of $2N$ particles (ideal gas) in a container consisting of two compartments of volume V each, as shown in Fig. 4. There are some devices, which allow the compartments to be connected or isolated (let us call this operation A) and the volume of the second department to be diminished or restored to previous volume (operation B).

Let us assume that in an initial state each compartment has volume V and the compartments are connected with each other, while gas is in an equilibrium state, so that each compartment has on average N particles. We consider isothermal processes consisting of several elementary operations, while every operation is fulfilled in reversible manner. One can imagine that a deliberate sequence of operations can be apply to the system. After one has performed the sequence: B – decreasing of volume 2 in ΔV, A – isolating of the compartments, B – increasing of volume 2 in ΔV, and A – connecting the compartments, the number of particles in each compartment can be found to be

$$N_1 = N(1+\xi), \quad N_2 = N(1-\xi), \quad \xi = \frac{(\Delta V/2V)}{1-(\Delta V/2V)}. \tag{12}$$

After the cycle, the configuration of the outer devices is initial, but the gas appears to be in non-equilibrium state. Entropy of the system can be directly estimated according to Boltzmann formulae applied to this case

$$S = k \ln W, \quad W = \frac{(2N)!}{N_1!\, N_2!},$$

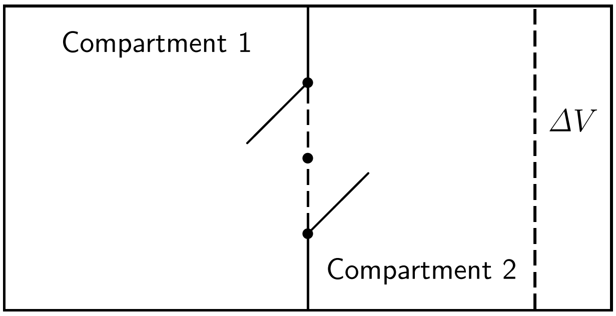

Figure 4 The scheme of the production container

The production devices can perform two operations: operation A allows the compartments to be connected or isolated and operation B allows the volume of the second department to be diminished or restored. Starting from the state of joined compartments, the sequence of operations BABA leads to creation of a non-equilibrium state of the working body.

so that the difference of entropy of the system in equilibrium and non- equilibrium states is

$$\Delta S = -kN\xi^2. \tag{13}$$

The terms of the third and higher orders are neglected here and further on.

The work ΔA which is needed to pass the system through the cycle can be calculated as a work of/on ideal gas in every of four steps of the cycle. One finds eventually that external forces have to produce extra work during the cycle

$$\Delta A = kTN\xi^2. \tag{14}$$

The internal energy of the system

$$E = 6NkT,$$

as internal energy of ideal gas, does not change in the process, so that the first law of thermodynamics can be written, considering the every step of the process to be reversible, in the form

$$T\,\Delta S = -\Delta A,$$

and the change of entropy of the system in the process can be estimated as

$$\Delta S = -kN\xi^2. \tag{15}$$

Thus, as a result of this production cycle, one has a working body in a particular non-equilibrium state, which cannot be created without work of external forces and without the deliberate choice of sequence of elementary operations. Somebody possesses certain sources of energy and has an aim to create an unique non-equilibrium form of matter. To achieve the goal, the creator sends the message in codes of elementary operations: BABA.[4] No other messages can be helpful. The information content of the deliberate message can be estimated, if one takes into account that this message is one in 8 possibilities. So, the message carries the information entropy in the amount

$$\Delta I = -\log_2 \frac{1}{8} = 3. \tag{16}$$

The information content of the message can be considered to be materialised in non-equilibrium form (complexity) of matter. The cost of materialisation is the work of production equipment ΔA.

4.2 *Entropy, value and utility*

Returning to the general case, one can say that each production cycle is designed to diminish entropy in our environment. The input of unique combination of information $\mathrm{d}I$ and work of production system $dA = P + (\beta/\gamma)\,L$ eventually determines new organisation of matter, which acquires forms of different commodities, whereby the production process is considered as a process of materialisation of information.

If changes in internal energy of the environment can be neglected, decrease in entropy of the entire environment is proportional to work of production system

$$-\mathrm{d}S = \frac{1}{T}\,dA. \tag{17}$$

To create and maintain the special complexity (far-from-equilibrium objects or dissipative structures), which has form of buildings, machinery and other

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[4] One can note that in similar situation Maxwell's Demon had a similar aim to create a non-equilibrium state of matter. In contrast to the creator, Demon can distinguish separate molecules and act without doing any work.

products, as in any thermodynamic system (Morowitz, 1968; Nicolis and Prigogine, 1977; Prigogine, 1980), there is a need for energy fluxes moving through the system.

The comparison of relations (9) - (10) and (17) allows us to state that value is a close relative of negative entropy, though the correspondence is not accurate, because entropy is a function of state in contrast to value, which is not a function of state. The last statement means that, though one can write an expression for the increment of value of the stock of commodities $Q_1, Q_2, ..., Q_n$ as

$$\mathrm{d}W = \sum_{j=1}^{n} p_j \,\mathrm{d}Q_j = Y, \tag{18}$$

where p_i is value (price) of unit of product, depending on the amounts of the products $p_i = p_i(Q_1, Q_2, ..., Q_n)$, one can hardly expect that the form (18) is a total differential of any function. The fact, that value is not a function of state of the system, is well known in economics and a function of a state – utility function (subjective) has been introduced, as manifestation of sensation of preference of one aggregate of products as against another, to replace non-existing value functions in theoretical considerations (Blaug, 1997). A function, which is closely related to value can be introduced in a different way. Indeed, linear form (18) can be multiplied by an integrating factor, and, in the case, when a positive integrating multiplier can be found, instead of (17), one has a total differential of a monotonically increasing function of each variable

$$\mathrm{d}U = \sum_{j=1}^{n} \phi(Q_1, Q_2, ..., Q_n) p_j(Q_1, Q_2, ..., Q_n) \,\mathrm{d}Q_j \tag{19}$$

One can also call the introduced function a *utility function (objective)*, taking into account that the properties of function U coincide with those of the conventional *utility function (subjective).* Any two utility functions connected with a monotonic transformation are considered to be identical, so that the utility function U, introduced by relation (19) is also an utility function in conventional interpretation.

The existence of the conventional utility function is justified by the fact that there is a preference relation on the set of products. Similar to that, the existence of entropy can be justified by an acceptability relation on the space of thermodynamic variables. The similarity between the utility representation problem in economics and the entropy representation problem in thermodynamics was demonstrated by Candeal *et al* (2001). Astonishingly, it seems to be not just a formal analogy: the two functions appear to be equivalent estimates of a set of products.

So, the useful properties of the artificial environment can be connected with decreasing of entropy of our environment, which can be characterized by utility function U. The situation is being complicated by the fact that, simultaneously with useful products, the production system creates also useless and harmful products (waste and pollution), while all real processes are irreversible. Production of useful things stimulates processes of dissipation of energy and matter. One can estimate (Pokrovski, 2007) that the genuine substitutive work of production equipment P presents only a few percents of total energy directed to produce this work. The larger part converts into heat, which is coming eventually out of the environment, but production of useful things stimulates also the processes of mixing, dispersion and diffusion, so that one can think that the matter necessary for production would become progressively unavailable (Georgescu-Roegen, 1971). But given the availability of energy, the materials could be recovered from waste as from an ore pile (Ayres, 1997), so that the Earth is not waiting for the diffusion death: despite of some processes of degradation of matter, the essence of production processes is the creation of useful complexity in the environment. In other words, the total entropy of the environment is decreasing during the production of commodities despite of processes of diffusion and degradation.

5 Conclusion

The paper proposes some reconciliation of contrasting points of view on the role of energy in production of value. The developed extension of the labour theory of value closely relates to the conventional theory, which considers capital and labour as the main sources of production of value. At the cost of introduction of the third production factor – substitutive work or productive energy, the formulated theory allows one to unravel a proper role of energy in production of value, from one side, and to get rid from the contradictions of conventional neo-classical theory, from the other side. The simplest schematisation of production process allows us to formulate the consistent mathematical model, which allows one to separate influence of changes of the production factors and changes, which are connected, with the structural and/or technological alternations of the production system itself.

One can think that the new production factor – substitutive work or productive energy is, perhaps, the same one, which the scholars of modern endogenous theory of economic growth (Barro and Sala-i-Martin, 1995; Aghion and Howitt, 1998) are seeking. Indeed, the introduction of the stock of knowledge or human capital, as a significant production factor and genuine source of economic growth (Barro and Sala-i-Martin, 1995; Aghion and Howitt, 1998), corresponds to the introduction of productive energy as a production factor. To use external energy in the production, one ought to have available sources of energy and appliances, which utilise energy for production. Some devices have to be invented, made and installed for work, so that the supply of productive energy is determined by fundamental results of science, by research, by project works, and by materialisation of all human imagination about how to use energy for production. There is no doubt that developing machine technologies appears to give increase, via effect of substitution, in labour productivity. A progressively greater amount of energy is utilised by human societies via improvements in technology.

As a specific concept of economics, the concept of value does not need to be reduced to any scientific concepts, but, as far as the production process can be considered as a process of transformation matter from nature into forms useful for humans (mainly without change of internal energy), one can look for analogies in thermodynamics. All our environment can be considered as a thermodynamic system of course, and by performing work, the production system reduces entropy of the environment, so that value can be related to entropy with the reverse sign. The properly organised work of production system is needed to transform the natural environment into artificial environment. One can estimate the total amount of work, including properly accounted labour work, needed to produce a thing or service to correspond it to market value. This gives an absolute measure of value, though, due to difficulties and uncertainties of energy accounting, this valuation cannot apparently get any immediate advantages against the market valuation.

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