BAYESIAN MODEL AVERAGING IN R

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Abstract. Bayesian model averaging has increasingly witnessed applications across an array of empirical contexts. However, the dearth of available statistical software which allows one to engage in a model averaging exercise is limited. It is common for consumers of these methods to develop their own code, which has obvious appeal. However, canned statistical software can ameliorate one's own analysis if they are not intimately familiar with the nuances of computer coding. Moreover, many researchers would prefer user ready software to mitigate the inevitable time costs that arise when hard coding an econometric estimator. To that end, this paper describes the relative merits and attractiveness of several competing packages in the statistical environment R to implement a Bayesian model averaging exercise.

1. Introduction

Bayesian model averaging (BMA) is an empirical tool to deal with model uncertainty in various milieus of applied science. In general, BMA is employed when there exist a variety of models which may all be statistically reasonable but most likely result in different conclusions about the key questions of interest to the researcher. As Raftery (1995, pg. 113) notes "In this situation, the standard approach of selecting a single model and basing inference on it underestimates uncertainty about quantities of interest because it ignores uncertainty about model form." Typically, though not always, BMA focuses on which regressors to include in the analysis. The allure of BMA is that one can quickly determine models, or more specifically sets of explanatory variables, which possess high likelihoods. By averaging across a large set of models one can determine those variables which are relevant to the data generating process for a given set of priors used in the analysis. Each model (a set of variables) receives a weight and the final estimates are constructed as a weighted average of the parameter estimates from each of the models. BMA includes all of the variables within the analysis, but shrinks the impact of certain variables towards zero through the model weights. These weights are the key feature for estimation via BMA and will depend upon a number of key features of the averaging exercise including the choice of prior specified.

The implementation of BMA, which was first proposed by Leamer (1978, Sections 4.4-4.6), for linear regression models is as follows. Consider a linear regression model with a constant term, β_0,

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and k potential explanatory variables $x_1, x_2, \ldots, x_k$,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon. \tag{1}$$

Given the number of regressors, we will have 2^k different combinations of right hand side variables indexed by M_j for $j = 1, 2, 3, \ldots, 2^k$. Once the model space has been constructed, the posterior distribution for any coefficient of interest, say β_h, given the data D is

$$Pr(\beta_h|D) = \sum_{j:\beta_h \in M_j} Pr(\beta_h|M_j)Pr(M_j|D) \tag{2}$$

BMA uses each model's posterior probability, $Pr(M_j|D)$, as weights. The posterior model probability of M_j is the ratio of its marginal likelihood to the sum of marginal likelihoods over the entire model space and is given by

$$Pr(M_j|D) = Pr(D|M_j)\frac{Pr(M_j)}{Pr(D)} = Pr(D|M_j)\frac{Pr(M_j)}{\sum_{i=1}^{2^k} Pr(D|M_i)Pr(M_i)} \tag{3}$$

where

$$Pr(D|M_j) = \int Pr(D|\beta^j, M_j)Pr(\beta^j|M_j)d\beta^j \tag{4}$$

and β^j is the vector of parameters from model M_j, $Pr(\beta^j|M_j)$ is a prior probability distribution assigned to the parameters of model M_j, and $Pr(M_j)$ is the prior probability that M_j is the true model. The estimated posterior means and standard deviations of $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k)$ are then constructed as

$$E[\hat{\beta}|D] = \sum_{j=1}^{2^k} \hat{\beta} Pr(M_j|D), \tag{5}$$

$$V[\hat{\beta}|D] = \sum_{j=1}^{2^k} (Var[\beta|D, M_j] + \hat{\beta}^2)Pr(M_j|D) - E[\beta|D]^2. \tag{6}$$

For further discussions on BMA, including its limitations and implementation, we refer the reader to the comprehensive review of Bayesian model averaging by Hoeting, Madigan, Raftery & Volinsky (1999).

The following sections provide an overview of three currently available packages in the statistical computing language of R (R Development Core Team 2010) that can implement a BMA empirical exercise. The main features under the user's control for each of the packages, including the set of prior probabilities and model sampling algorithms as well as the plot diagnostics available to visualize the results, are described. Several detailed examples to compare the performance of these different packages are also provided along with functioning R code in an appendix.

To our knowledge, R is the only mainstream statistical platform which offers a suite of routines to conduct a BMA analysis. The availability of BMA routines in other statistical software is limited.

Neither Gauss nor Stata possess built-in packages which allow the user to implement a genuine, linear regression BMA.[1,2] Matlab, while lacking a comprehensive BMA toolbox,[3] supplies users with the core functionality of the `BMS` package (discussed below) via installation of the *BMS toolbox for Matlab*. Fortran users have access to a ready-to-use BMA toolbox stemming from Fernandez, Ley & Steel's (2001*b*) publicly available code. And finally, while SAS provides some functionality for implementing BMA it is incapable of handling a large-scale BMA analysis.

Beyond our review of the functionality of the three available packages, we contrast estimates and posterior inclusion probabilities across the three packages with a mock empirical example. This is done both with a set of covariates that allows for full enumeration of the model space as well as requiring the implementation of a model space search mechanism which is what truly distinguishes the three packages. The time performance of the three packages as both the sample size and the covariate space increase is also supplied. Finally, we examine whether these packages can replicate the results of recently published econometric research that employs BMA techniques. Overall, all three packages share relative advantages against their peers, yet we advocate for the `BMS` package given its versatility with user defined priors as well as the numerous options to customize one's BMA analysis.

2. Available Packages

2.1. **The BMS Package.** The `BMS` (an acronym for Bayesian Model Selection) package employs standard Bayesian normal-conjugate linear model as the base model and "Zellner's g prior" as the choice of prior structures for the regression coefficients (Feldkircher & Zeugner 2009). Since the form of the hyperparameter g is crucial in BMA analyses, the `BMS` package sets g equal to the sample size, usually known as the unit information prior (UIP). `BMS` also provides alternative formulations regarding the choice of g. The main function in the `BMS` package to implement a BMA regression analysis is `bms()`.

2.1.1. *Model Sampling.* Since enumerating all potential variable combinations becomes infeasible quickly for a large number of covariates, the `BMS` package uses a Markov Chain Monte Carlo (MCMC) samplers to gather results on the most important part of the posterior distribution when more than 14 covariates exist. The MCMC sampler walks through the model space using the Metropolis-Hastings algorithm[4], which works as follows: Suppose that the current model at step i is M_i with posterior model probability $p(M_i|y, X)$. The MCMC sampler for the `BMS` package

[1]Millar (2011) has recently published a Stata module that uses the Bayesian Information Criterion (BIC) for estimating the probability that a variable is a part of the final model. This module, available at `http://fmwww.bc.edu/repec/bocode/b/bic.ado`, calculates the BIC statistic for all possible combinations of the independent variables.

[2]Gauss users can find the code used in Sala-i-Martin, Doppelhofer & Miller (2004) to implement the BACE technique at `http://www.nhh.no/Default.aspx?ID=3075`.

[3]Matlab's Econometrics Toolbox comes with a function called `bma_g` that provides very basic BMA functionality.

[4]See Metropolis, Rosenbluth, Rosenbluth, Teller & Teller (1953), Hastings (1970), Chib & Greenberg (1995), and Liu (2008).

randomly draws a candidate model and then moves to this model if its marginal likelihood is superior to the marginal likelihood of the current model. In this algorithm, the number of times each model is kept will converge to the distribution of posterior model probabilities $p(M_i|y, X)$. The `BMS` package offers two different MCMC samplers to look at models within the model space. These two methods differ in the way they propose candidate models. The first method is called the *birth-death* sampler `(mcmc=bd)`. In this case, one of the potential regressors is randomly chosen; if the chosen variable is already in the current model M_i, then the candidate model M_j will have the same set of covariates as M_i but drop the chosen variable. If the chosen covariate is not contained in M_i, then the candidate model will contain all the variables from M_i plus the chosen covariate; hence the appearance (birth) or disappearance (death) of the chosen variable depends if it already appears in the model. The second approach is called the *reversible-jump* sampler `(mcmc=rev.jump)`. This sampler draws a candidate model by the birth-death method with 50% probability and with 50% probability the candidate model randomly drops one covariate with respect to M_i and randomly adds one random variable from the potential covariates that were not included in model M_i.

The precision of any MCMC sampling mechanism depends on the number of draws the procedure runs through. Given that the MCMC algorithms used in the `BMS` package may begin using models which might not necessarily be classified as 'good' models, the first set of iterations do not usually draw models with high posterior model probabilities (PMP). This indicates that the sampler will only converge to spheres of models with the largest marginal likelihoods after some initial set of draws (known as the burn-in) from the candidate space. Therefore, this first set of iterations will be omitted from the computation of results. In the `BMS` package the argument `(burn)` specifies the number of burn-ins (models omitted), and the argument `(iter)` the number of subsequent iterations to be retained. The default number of burn-in draws for either MCMC sampler is 1000 and the default number of iteration draws (excluding burn-ins) 3000.

2.1.2. *Model Priors.* The `BMS` package offers considerable freedom in the choice of model prior. One can employ the uniform model prior as the choice of prior model size `(mprior="uniform")`, the Binomial model priors where the prior probability of a model is the product of inclusion and exclusion probabilities `(mprior="fixed")`, the Beta-Binomial model prior `(mprior="random")` that puts a hyperprior on the inclusion probabilities drawn from a Beta distribution, or a custom model size and prior inclusion probabilities `(mprior="customk")`. Of the three packages currently available in R this is the only package that allows for custom priors.

2.1.3. *Alternative Zellner's* g *Priors.* Different mechanisms have been proposed in the literature for specifying g priors. The options in the `BMS` package are as follows

(1) `g="UIP"`; Unit Information Prior (UIP), that corresponds to $g = N$, the sample size.[5]
(2) `g="RIC"`; Sets $g = K^2$ and conforms to the risk inflation criterion.[6]

[5]See Fernandez, Ley & Steel (2001*a*)
[6]See **?** for more details

(3) `g="BRIC"`; A mechanism that asymptotically converges to the unit information prior ($g = N$) or the risk inflation criterion ($g = K^2$). That is, the g prior is set to $g = max(N, K^2)$.[7]
(4) `g="HQ"`; Follows the Hannan-Quinn criterion asymptotically and sets $g = log(N^3)$.
(5) `g="EBL"`; Estimates a local empirical Bayes g-parameter.[8]
(6) `g="hyper"`; Takes the "hyper-g" prior distribution.[9]

2.1.4. *Outputs.* The main objects returned by `bms()` are the posterior inclusion probabilities (PIP) and posterior means and standard deviations. In addition, a call to this function returns a list of aggregate statistics including the number of draws, burn-ins, models visited, the top models, and the size of the model space. It also, returns the correlation between iteration counts and analytical PMPs for the best models[10], those with the highest posterior model probabilities (PMP).

2.1.5. *Plot Diagnostics.* `BMS` package users have access to plots of the prior and posterior model size distributions, a plot of posterior model probabilities based on the corresponding marginal likelihoods and MCMC frequencies for the best models that visualize how well the sampler has converged. A grid with signs and inclusion of coefficients vs. posterior model probabilities for the best models and plots of predictive densities for conditional forecasts are also produced.

2.2. **The BAS Package.** The `BAS` package, (an acronym for Bayesian Adaptive Sampling), performs BMA in linear models using stochastic or deterministic sampling without replacement from posterior distributions (Clyde 2010). Prior distributions on coefficients are from Zellner's g prior or mixtures of g priors corresponding to the Zellner-Siow Cauchy Priors or the Liang hyper g priors (see Liang et al. 2008). Other model selection criteria include AIC and BIC. The main function in the `BAS` package to implement a BMA regression analysis is `bas.lm()`.

2.2.1. *Model Sampling.* If the number of covariates is less than 25, the `BAS` package enumerates all models, otherwise it implements three different search algorithms to find the models with the highest posterior probability. The first algorithm is named the Bayesian Adaptive Sampling algorithm[11], (`method="BAS"`), that samples without replacement using random or deterministic sampling, (`random ="TRUE/FALSE"`). The Bayesian Adaptive Sampling algorithm samples models from the model space without replacement using the initial sampling probabilities, and will update the sampling probabilities using the estimated marginal inclusion probabilities. If the explanatory variables are orthogonal, then the deterministic sampler provides a list of the top models in order of their approximate posterior probability, and provides an effective search if the correlations of variables is small to modest. The second algorithm is the Adaptive MCMC sampler (`method="AMCMC"`). This sampling mechanism requires various parameters to be tuned appropriately for the algorithm to

[7]See Fernandez et al. (2001*a*)
[8]See George & Foster (2000) and Liang, Paulo, Molina, Clyde & Berger (2008)
[9]See Liang et al. (2008) and Feldkircher & Zeugner (2009)
[10]Note that here and throughout the remainder of the document, the terminology "best models" or "best" corresponds to best as judged by the analytical likelihood.
[11]See Clyde, Ghosh & Littman (2010)

converge reasonably well. The package currently handles these parameter tunings automatically by getting the computer to update tuning parameters and other choices during the course of sampling from the model space. The last method runs an initial MCMC to calculate marginal inclusion probabilities and then samples without replacement as in `method="BAS"` (`method="MCMC+BAS"`). The default number of models to sample is "NULL", meaning that a call to `bas.lm()` enumerates all combinations. The user can also indicate a model to initialize the sampling by passing a binary vector to the BAS functions (`bestmodel`). In default mode, sampling starts with the full model.

2.2.2. *Model Priors.* The family of prior distribution on the models is nearly identical to the `BMS` package allowing uniform, Bernoulli or Beta-Binomial distributions as priors (`modelprior="uniform", "Bernoulli", "beta.binomial"`).

2.3. **Alternative g Priors.** To determine the prior distribution for the regression coefficients, the user has the following choices:[12]

(1) `g="AIC"`; Akaike information criterion.
(2) `g="BIC"`; Bayesian information criterion.
(3) `g="g-prior"`; Takes $g = N$, the sample size, corresponding to the UIP.
(4) `g="ZS-null"`; Employs Zellner & Siow's (1980) suggestion. If the two models under comparison are nested, the Zellner-Siow strategy is to place a flat prior on common coefficients and a Cauchy prior on the remaining parameters. This option utilizes the null model as the base model and compares each model with the null model.
(5) `g="ZS-full"`; The distinction between this method and `ZS-null` is the choice of base model. `ZS-full` method utilizes the full model (i.e., all covariates included) as the base model and compares all other models with the full model.
(6) `g="hyper-g"`; This option uses a family of priors on g that provides improved mean square risk over ordinary maximum likelihood estimates in the normal means problem. Strawderman (1971) introduced this set of priors. An advantage of the hyper-g prior is that the posterior distribution of g given a model is available in closed form.
(7) `g="hyper-g-laplace"`; This method is same as `hyper-g` and the only difference is that it uses a Laplace approximation to calculate the priors on g. This avoids issues with modes on the boundary and leads to improved solutions.
(8) `g="EB-local"`; This procedure estimates a separate g for each model. The local empirical Bayes (EB) estimate of g is the maximum marginal likelihood estimate constrained to be nonnegative.
(9) `g="EB-global"`; The global EB technique sets one common g for all models and borrows strength from all models by estimating g from the marginal likelihood of the data, averaged over all models.

[12] See a thorough review of g priors used in `BAS` package in Liang et al. (2008).

2.3.1. *Outputs.* The main objects returned from a call to `bas.lm()` are posterior inclusion probabilities, posterior mean and standard deviations for each coefficient, as in a call to `bms()`. Moreover, `bas.lm()` also returns the prior inclusion probability for each variable along with the posterior probability that each variable is non-zero.

2.3.2. *Diagnostic Plots.* The main plotting command in this package provides a panel of four graphs. The first is a plot of the residuals versus fitted values using the Bayesian model averaged coefficient estimates. The second is a plot of the cumulative marginal likelihoods of all the visited models. The third is a plot of log marginal likelihood vs. model dimension and finally the fourth plot shows the posterior marginal inclusion probabilities.

2.4. **The BMA Package.** The `BMA` package (an acronym for Bayesian model averaging), performs BMA analysis assuming a uniform distribution on model priors and using a simple BIC (Bayesian Information Criterion) approximation to construct the prior probabilities on the regressions coefficients (Raftery, Hoeting, Volinsky, Painter & Yeung 2010). This package is built upon Raftery's (1995) algorithm. The main functions in the `BMA` package to implement a BMA regression analysis are `bicreg()` and `iBMA.bicreg()`.

2.4.1. *Model Sampling.* If the number of covariates is less than 30, the `BMA` package enumerates all models. `BMA` first uses a backward elimination procedure, in which variables are eliminated one at a time starting from the full model, to reduce the initial set of variables to 30. The package screens the t-statistics for the estimated parameters and removes the variables for which statistics are small. It then utilizes a specific BIC difference, according to Table 1, to compare model M_j to M_i to find models that are more likely a part of the final set of good models. The models that are left are said to belong to *Occam's window*, a generalization of the famous Occam's razor, or principle of parsimony in scientific explanation. If the initial set of models in Occam's window is large and all models are linear, `BMA` uses the leaps and bounds algorithm of Furnival & Wilson Jr (1974) to select a reduced set of good models. The user can set a number specifying the maximum ratio for excluding models in Occam's window (`OR`). The default value of this ratio is 20. The maximum number of columns in the design matrix (`maxCol`), including the intercept, to be kept defaults to 31.

[Table 1 about here.]

2.4.2. *Model Priors.* The `BMA` package is inflexible regarding model priors, assuming all models are on equal footing *a priori*. The model prior is set to $1/2^K$ where K is the total number of variables included in the analysis.

2.4.3. *Regression Coefficients Priors.* Unlike the `BMS` and `BAS` packages, `BMA` does not employ Zellner's g priors as its choice of the prior distributions for the regression coefficients. Instead, the package employs the BIC approximation, which corresponds fairly closely to the UIP. Specifically, for any coefficient, say β_1, `BMA` calculates the BIC for all models that include β_1 and then the posterior probability that β_1 is in the model would be

$$Pr(\beta_1 \neq 0|D) = \sum_{j:\beta_1 \in M_j} p(M_j|D), \text{ such that: } p(M_j|D) = exp(-BIC_j/2)/\sum_{i=1}^{K} exp(-BIC_i/2). \tag{7}$$

2.4.4. *Outputs.* The call to `bicreg()` returns the posterior inclusion probabilities, the posterior mean and standard deviation of each coefficient (as with `bms()` and `bas.lm()`), along with the values for the BIC of the models selected, the posterior mean of each coefficient conditional on the variable being included in the model, and the R^2 for the models deemed to be the most important.

2.4.5. *Diagnostic Plots.* `BMA` generates a plot of the posterior distribution for each of the estimated coefficients.

3. Comparison of the Packages

Having briefly described each of the `BMS`, `BAS` and `BMA` packages, we present the main function calls available to the user in Table 2 for ease of reference. The remainder of this section culls similarities of the key features and provides a detailed comparison of the available options for model sampling and search, construction of model priors, call outputs and diagnostics so that the reader is presented with enough information to judge the merits of each package.

3.1. **Model Sampling.** When the covariate space is small all three packages enumerate the model space and estimate all models to conduct the averaging. However, when the sample space of covariates is large, each of the packages engages in totally different search behavior across the space of candidate models. Both the `BAS` and `BMS` packages use adaptive search algorithms to determine a feasible list of models to construct the posterior distribution of model probabilities. These gives added credibility to these packages relative to the `BMA` package which uses a much simpler mechanism to adjudicate across candidate models. As we will see in our empirical examples, when we enumerate the model space both `BMS` and `BAS` provide roughly identical results (posterior inclusion probabilities, means and standard deviations), whereas when we resort to model sampling the interpretations of the effects and inclusion of specific variables can be different. The `BMA` package produces roughly similar estimates of posterior means and standard deviations, but deviates substantially with respect to the inclusion probabilities.

[Table 2 about here.]

3.2. **Model Priors.** As one can see, `BAS` is comparable to `BMS` in alternative options for model priors except that the `BMS` package offers the opportunity to construct a prior outside of the standard options available, making it more versatile for specific applications when specification of the g-prior is paramount.

3.3. **Alternative Zellner's *g* Priors.** The BAS package provides additional options, including BIC and AIC, relative to the BMS package. Recall that the BMA package does not provide options for coefficient priors consistent with either of the other packages.

3.4. **Outputs.** All three packages return the posterior probability of inclusion as well as the posterior mean and standard deviations for each coefficient. However, only the BMS package provides a suite of diagnostic information to help the user gauge if further adjustment of the BMA setup is needed or to likely robustness of the results to various options within the model averaging exercise.

3.5. **Diagnostic Plots.** All three packages provide diagnostic output for the user to understand the results of the model averaging exercise. However, both the BMS and BAS packages provide plots of the dimensionality of the model whereas the BMA package does not.

4. Empirical Illustration

This section compares the results of a basic BMA exercise using the UScrime dataset available in the MASS package (Venables & Ripley 2002) in R. This is a dataset on per capita crime rates in 47 U.S. states in 1960 (see Ehrlich 1973). There are 15 potential independent variables, all perceived to be associated with crime rates. The last two, probability of imprisonment and average time spent in state prisons, were the predictor variables of interest in Ehrlich's (1973) original study, while the other 13 were control variables. This small sample will allow us to compare the results from the three packages when we can enumerate the model space. However, as a likely difference in the performance of the three packages, and the results gleaned from them, will depend upon a large model space, we also consider the same dataset with 35 fictitious variables (generated as independent standard normal random deviates) included so that each package must engage in model sampling.

4.1. **Enumeration of the Model Space.** We first compare the results of performing basic BMA with the original UScrime dataset, which is small enough to fully enumerate the model space for all three of the separate BMA packages discussed above. Table 4 shows the probabilities that each variable belongs to the final model (PIP). All three packages in this setup fully enumerate all $2^{15} = 32,768$ models. Column (i) shows the PIPs calculated by the BMS package. The call to this package uses a uniform distribution for model priors and unit information priors for the distribution of regressions coefficients. Column (ii) presents the PIPs computed by the BAS package. Model priors are set to have uniform distributions and the priors on the regressions coefficients come from Zellner-Siow's g priors. Column (iii) indicates the PIPs that the BMA package computes. The BMS and BAS packages produce nearly identical PIPs while those of BMA are markedly different. The different internal algorithm for calculating Bayes factors and other parameters is the main reason for this disparity. Table 5, on the other hand, shows the posterior means and standard errors of regressors calculated from the three packages. Similar to the posterior inclusion probabilities, the estimated coefficients and standard errors computed by the BMS and BAS packages are literally identical but the estimates obtained from the BMA package are relatively different from their counterparts.

[Table 3 about here.]

[Table 4 about here.]

4.2. **Model Sampling.** In this section we compare the results across the three packages when there is a large model space and model search must be undertaken to ascertain the best models. We create a large model space by adding 35 independent, fictitious variables obtained from a standard-normal distribution to the original `UScrime` dataset (for a total of 50 covariates). Tables 6 and 7 present the results. Using the available controls for each package we tried our best to set up the example so that the three packages are as identical as possible.

[Table 5 about here.]

Table 6 shows the PIPS. Column (i) presents the probabilities obtained from the `BMS` package. This package employs the *birth-death* sampler using an MCMC search algorithm to find the posterior probabilities with a burn-in of 2000 models. Its choices for model priors and the distribution of the model coefficients are uniform distribution and unit information priors, respectively. Column (ii) displays the results from the `BAS` package. We choose a uniform distribution for the model prior, Zellner-Siow's g prior for the distributions of the coefficients, and an adaptive MCMC method for model sampling. Column (iii) shows the PIPs that the `BMA` package assigns to each variable. As discussed earlier, the `BMA` package takes an entirely different model sampling approach than either the `BMS` or `BAS` packages. It first reduces the initial set of variables using a backward elimination algorithm and then implements the iterated BMA method for variable selection. The package calls repeatedly to a BMA procedure, iterating through the variables in a fixed order. After each call only those variables which have posterior probability greater than a specified threshold, controlled through `thresProbne0` in the call to `iBMA.bicreg()`, are kept and those variables whose posterior probabilities do not meet the threshold are replaced with the next set of variables. The call to the `BMA` package uses the BIC approximation as its choice for the distributions of model coefficients which is roughly identical to UIPs.

[Table 6 about here.]

Table 7 shows the estimated posterior means and standard deviations of the covariates. Moreover, how long each of the packages takes to run is shown in the last line of the table. The `BMS` and `BAS` packages that use similar internal search algorithms for model sampling generate roughly identical posterior means and standard deviations, and fairly identical posterior probability that each variable is non-zero. The `BMA` package, the fastest of the three, however, calculates significantly different posterior probabilities, means and standard deviations from what `BMS` and `BAS` produce. This difference lies in the fact that the `BMA` package relies on a hierarchical OLS t-statistic to obtain a handful of likely models, directly conflicting with the model sampling approaches in the `BMS` and `BAS` packages.

4.3. **Plot Diagnostics.** Plotting is an important tool that helps the user to visualize the shape of the posterior distributions of coefficients, assess the final model size, compare PIPs and look at model complexity. The `BMS`, `BMA`, and `BAS` packages all plot the marginal inclusion densities as well as images that show inclusion and exclusion of variables within models using separate colors. However, the `BMS` and `BAS` packages provide more graphical visualizations for the users. The following figures are some examples of the plots that these packages provide. These figures all correspond to the above example where we fictitiously incorporated 35 white noise covariates to force the packages to engage in searching over the model space.

[Figure 1 about here.]

Figures 1 and 2 are combined plots provided by the `BMS` package. The upper plot in each figure shows the prior and posterior distribution of model sizes and helps to illustrate to the user the impact of the model prior assumption on the estimation results. Consistent with the research of Ley & Steel (2009) and Eicher, Papageorgiou & Raftery (2011), who stress the importance of model priors in applied work, the plots produced by the `BMS` package allow for visual clarification of choice of model prior on posterior results. For example, the upper plot of Figure 1 assumes a uniform distribution for the model prior whereas the upper plot of Figure 2 assumes a "fixed" common prior inclusion probability for each regressor as an alternative to a uniform prior.[13] These plots allow the user to graphically compare across a range of model size priors to determine the impact on the posterior distribution. In the example here it seems that the model prior (either fixed or uniform) has little effect on the posterior distribution of model sizes.

[Figure 2 about here.]

The lower plot of both figures shows the analytical likelihood of the best 200 models and their MCMC frequencies/draws of these models from the MCMC sampler. If the sampler has converged, then the MCMC draws should conform to the analytical/exact likelihoods. This is expressed in the correlation of the two lines by the correlation reported in parentheses beneath the plot's title. In other words, these graphs are an indicator of how well the current 200 best models encountered by the search algorithm of the `BMS` package have converged. These plots are useful in determining the search mechanisms ability to find 'good' models throughout the model space. If the user deems that the best models are not acceptable then longer runs of the search algorithm or more burn-ins may be specified to help refine the search over the model space.

Figure 3(a) is a visualization of the mixture marginal posterior density for the coefficient of GDP produced by the `BMS` package. The bar above the density shows the PIP, and the dotted vertical lines show the corresponding standard deviation bounds in from the MCMC approach. Figure 3(b) shows the same visualization using the `BMA` package. The posterior probability that the coefficient is zero is represented by a solid line at zero, with height equal to the probability. The nonzero part of the distribution is scaled so that the maximum height is equal to the probability that the coefficient is nonzero. Figure 3(c) shows the same visualization using the `BAS` package. Similarly, the posterior probability that the coefficient is zero is represented by a solid line at zero, with height equal to the probability. The nonzero part of the distribution is scaled so that the maximum height

[13]See Sala-i-Martin et al. (2004) for more details.

is equal to the probability that the coefficient is nonzero. We mention here that Figure 3(b) is the only plot produced within the BMA package.

[Figure 3 about here.]

The plots of Figure 4 are created within the BAS package. The first panel, (a), is a plot of the residuals versus the fitted values from the model averaging exercise. This graph allows the user to visualize if the fitted values are heavily dependent on the residuals. If one can see a specific pattern or trend in this graph, then it could be a sign that an important variable is missing. Panel (b) is a plot of the log marginal likelihood versus the overall model dimension. Using this graph, the user can visually predict the dimensionality of models with the highest marginal likelihoods. For instance the majority of the best models is concentrated around 10 to 12. Panel (c) shows the posterior marginal inclusion probabilities for each variable.[14] This diagnostic plot makes it immediately obvious that the additional white noise variables we included almost uniformly have very low PIPs.

[Figure 4 about here.]

5. Time Performance of the Three Packages

A key component of any statistical software is the amount of time it takes to produce results. To that end, this section reports the run times for the three packages across various sample sizes and dimensions of the covariate space. The size of the covariate space comes from $\{15, 30, 60, 120\}$ while the sample size is taken from $\{50, 500, 5000, 50000\}$. The dependent variable as well as the covariates are drawn from a standard normal distribution. We run our analysis on a 1.60 GHz Intel(R) Core(TM) 2 Duo processor with 3.00 GB of DDR3 memory. Table 8 presents the run times in seconds for each package. Within each set of parenthesis are the specific run times for a given covariate-sample size combination from baseline[15] calls to the BMS, BAS, and BMA packages, respectively.

The first thing to notice is that due to the fact that the BMA package uses a simple model selection algorithm, it is the fastest among the three packages when the number of covariates is less than 30. The BAS package that provides roughly the same options as the BMS package is slower than the BMA package but noticeably faster than the BMS package. Once the number of covariates is greater than 15 and the sample size is beyond 500, the BAS package is uniformly the fastest of the packages. One reason that the BAS package realizes faster computing times relative to the BMS package with similar controls is that the BAS package is an R wrapper for a C library. This enables the call to the BAS package to take advantage of the high speed compiled C code used in the package.

Note that all three packages display increases in computation time as the dimensionality of the covariates increases. The number of covariates (intuitively) leads to larger increases in computation time than does increases in the sample size. Note that there are at least two counter-acting effects

[14] The variables with PIPs>0.5 are drawn in red for ease of identification for the user.

[15] Baseline options include uniform model priors, 500,000 iterations of the chosen MCMC algorithm, and discarding the first 3000 models (burn-in).

as the sample size, n, increases. A larger n puts more posterior mass on the best models. Therefore we expect that the MCMC sampler will converge quicker. On the other hand, the larger n, the more 'large' models will be emphasized, which slows down the sampler. If these two effects offset each other then we expect little change in run time over n. This is apparent in the BMS package. Notice how the BMS package has run times which decrease for a fixed level of covariates as we increase n while both the BAS and BMA packages actually take longer for $n = 50,000$ relative to $n = 5,000$.

[Table 7 about here.]

6. Replicability

In this section we examine the ability of these three packages in replicating the results of published work research deploying BMA using handwritten code. Fernandez et al. (2001*b*) (FLS hereafter) use a cross section of 72 countries along with 41 potential growth determinants for the period 1960 to 1992.[16] FLS apply BMA to find the key determinants of economic growth given the numerous plausible models that have emerged on the topic. We use the same dataset and deploy all three BMA packages, BMS, BAS, and BMA, to attempt to replicate their results. In order to maximize our opportunity to replicate the FLS results, we set the available options within each of the three packages as close as possible to the specifications listed in FLS. The BMS package applies the MCMC algorithm to search over the model space, burns the first 100,000 models and the number of iteration draws to be sampled by its MCMC sampler is 200,000. It assigns the uniform distribution to the model priors. Similarly, the BAS package employs MCMC method to walk through the model space, discards the first 100,000 models, draws samples from the model space 200,000 times, and sets the models priors to the uniform distribution. The BMA package, on the other hand, does not have enough options to directly mimic the setup in FLS (see sections 2.4 and 3.1). Having said this, we set the number of iteration draws used by its search algorithm to 200,000, the maximum ratio for excluding models in Occam's window, OR, to 20 and keep the the maximum number of columns in the design matrix at the default of 31.

[Table 8 about here.]

Table 9 shows the PIPs for the variables of interest. We do not present posterior means or standard deviations since FLS only reported the PIPs in the body of their paper. To eschew making statements regarding results which FLS did not cover, we focus exclusively on the ability of the packages to reproduce the PIPs found in FLS. Column (i) shows the published PIPs that Fernandez et al. (2001*b*) have reported in their work and the remainder of table presents the PIPs computed via the BMS, BAS, and BMA packages.

As is apparent, only the BMS package is reasonably successful at matching the reported PIPs in FLS while the PIPs produced by the BAS package display significant differences (compare PIPs marked by *). The BMA package also fails achieve the same PIPs of FLS.[17] This most likely lies

[16]This dataset is taken from the larger dataset used by Sala-i-Martin (1997) for his study on robust determinants of growth. The exact FLS dataset is publicly available on the *Journal of Applied Econometrics* online data archive.
[17]Both BAS and BMA, however, are computationally much faster than the BMS package.

in the fact that the `BMA` package was not called using exactly the setup in FLS and the difference in searching the model space that was described earlier. Interestingly, the PIPs returned from the `BMS` package almost uniformly match FLS' PIPs greater than 0.5. A key distinction between the results is that both the `BAS` and `BMA` packages suggest a set of variables that belong in the final model (PIP>0.5) beyond those found in FLS. Specifically, the `BAS` package finds 13 variables with PIPs >0.5 beyond FLS (and one variable with PIP <0.5 from FLS) while the `BMA` package finds 10 variables with PIPs >0.5 (and one variable with PIP <0.5 from FLS).

To further test the limits of these packages to replicate published results on BMA we attempt to reproduce the estimates in Doppelhofer & Weeks's (2009) research, (hereafter DW), who focused on the use of model averaging when jointness of the covariates is considered. DW's application is identical to FLS, studying the determinants of economic growth. Appendix B of their paper provides the BMA PIPs which we try to replicate using the ensemble of BMA packages. The data used in DW comprises 88 countries and 67 candidate variables as a cross section for the period 1960 to 1996. The definition of all 67 variables used can be found in data appendix B in DW and the dataset is publicly available on the *Journal of Applied Econometrics* data archive. As before we have tried to preserve the setup in DW by setting the packages' options as identical as possible.

[Table 9 about here.]

Table 10 displays our findings. Column (i) shows the published PIPs>0.50 that DW report in Appendix B of their paper. The rest of table presents the PIPs, posterior means and standard deviations from each of the packages.[18] The results indicate that the `BMS` package is the only one that successfully reproduces the reported PIPs and posterior mean/standard deviation. Both the `BAS` and `BMA` packages reasonably reproduce the posterior means/standard deviations but the computed PIPs are significantly different from the published PIPs. For instance, the probability that "Investment Price" belongs to the final model is roughly 77% according to DW but the `BAS` packages reports this probability at nearly 7% and the `BMA` package reports it at exactly 100%!

The estimated coefficients are reasonably close for all packages yet there remain some anomalies. The estimated posterior mean on Fraction of Tropical Area in the `BMS` package is about a third as small as that reported from the `BAS` package and half the size of the reported posterior mean from the `BMA` package. Moreover, both the `BAS` and `BMA` packages are suggestive that Fraction of Tropical Area is relevant from a pure t-ratio perspective (see Masanjala & Papageorgiou 2008). Beyond differences in several of the posterior means across the packages the standard deviations show noticeable differences; compare the results for Investment Price where the standard deviation from the `BMS` package is nearly double that from the `BMA` package and almost five times as large from the reported standard deviation in the `BAS` package.

[18]DW do not report estimated posterior means or standard deviations.

7. Conclusions

This paper has outlined the currently available BMA packages (BMS, BAS, and BMA) in the statistical computing environment R. Our goal was to familiarize users with the different options that the current versions of the packages have to offer. We highlighted how each of the packages implements a BMA analysis as well as the options available to the user and the outputs that are returned. To further cement the operation of these packages and to determine how similar the packages are in practice, we presented a simple empirical example that first allowed all three packages to fully enumerate the model space. Beyond this we enhanced our empirical example to force all three packages to engage in search mechanisms throughout the model space. When the model space is relatively small, we see that all three packages are successful at matching the PIPs, posterior means, and posterior standard deviations. However, for the larger model space similarity of the PIPs broke down considerably.

To further buttress our investigation and comparison of these packages we also compared runtimes of generic calls to each package for a range of covariate and sample sizes. In most instances the BAS package was the fastest, especially for large problems (both in terms of the number of covariates and the number of observations). Additionally, we also sought to replicate two recent studies that deployed BMA to investigate the determinants of economic growth. Both of these studies used high level programming outside of R and as such represent the perfect opportunity to see how well these freely available packages compare to computer code specifically tailored to the problem at hand. Our results were striking. The BMS package almost exactly reproduced the results from both studies while the BAS and BMA packages were not able to match the reports PIPS in either study but were reasonably accurate at constructing the posterior means and standard deviations of our second study (compared with the same estimates from the BMS package).

In sum, it appears that while the BMS package is invariably slower than its peers, its numerous options and flexibility suggest that it should makes its way into the toolkit of applied researchers seeking to use BMA in their analysis. The results from the empirical examples from published studies suggest that while both the BMS and BAS packages offer a similar array of options, the BMS package is capable of replicating published studies deploying BMA at the cost of slightly longer run times. Our apparent advocacy of the BMS package does not hinge on its ability to reproduce the results of published studies however, as this presumably just means that the original authors used an implementation similar to that of the BMS package which the other packages were unable to match. This in no way is an indicator of superiority. Lastly, the relative rigidity of the BMA package to that of both BMS and BAS suggests that its use in applied work should be carefully scrutinized.

References

Chib, S. & Greenberg, E. (1995), ‘Understanding the Metropolis-Hastings algorithm’, *American Statistician* **49**(4), 327–335.

Clyde, M. (2010), *BAS: Bayesian Adaptive Sampling for Bayesian Model Averaging*. R package version 0.92.
URL: http://CRAN.R-project.org/package=BAS

Clyde, M., Ghosh, J. & Littman, M. (2010), ‘Bayesian adaptive sampling for variable selection and model averaging’, *Journal of Computational and Graphical Statistics, to appear* .

Doppelhofer, G. & Weeks, M. (2009), ‘Jointness of growth determinants’, *Journal of Applied Econometrics* **24**(2), 209–244.

Ehrlich, I. (1973), ‘Participation in illegitimate activities: A theoretical and empirical investigation’, *The Journal of Political Economy* **81**(3), 521–565.

Eicher, T. S., Papageorgiou, C. & Raftery, A. E. (2011), ‘Default priors and predictive performance in Bayesian model averaging with application to growth determinants’, *Journal of Applied Econometrics* **26**, 30–55.

Feldkircher, M. & Zeugner, S. (2009), Benchmark Priors Revisited: On Adaptive Shrinkage and the Supermodel Effect in Bayesian Model Averaging, IMF Working Papers 09/202, International Monetary Fund.
URL: http://ideas.repec.org/p/imf/imfwpa/09-202.html

Fernandez, C., Ley, E. & Steel, M. (2001*a*), ‘Benchmark priors for Bayesian model averaging’, *Journal of Econometrics* **100**(2), 381–427.

Fernandez, C., Ley, E. & Steel, M. (2001*b*), ‘Model uncertainty in cross-country growth regressions’, *Journal of Applied Econometrics* **16**(5), 563–576.

Furnival, G. & Wilson Jr, R. (1974), ‘Regressions by Leaps and Bounds’, *Technometrics* **16**(4), 499–511.

George, E. & Foster, D. (2000), ‘Calibration and empirical Bayes variable selection’, *Biometrika* **87**(4), 731–747.

Hastings, W. (1970), ‘Monte Carlo sampling methods using Markov chains and their applications’, *Biometrika* **57**(1), 97–109.

Hoeting, J., Madigan, D., Raftery, A. & Volinsky, C. (1999), ‘Bayesian model averaging: A tutorial’, *Statistical science* **14**(4), 382–401.

Leamer, E. (1978), *Specification searches: Ad hoc inference with nonexperimental data*, Wiley New York.

Ley, E. & Steel, M. (2009), ‘On the effect of prior assumptions in Bayesian model averaging with applications to growth regression’, *Journal of Applied Econometrics* **24**(4), 651–674.

Liang, F., Paulo, R., Molina, G., Clyde, M. & Berger, J. (2008), ‘Mixtures of g priors for Bayesian variable selection’, *Journal of the American Statistical Association* **103**(481), 410–423.

Liu, J. (2008), *Monte Carlo strategies in scientific computing*, Springer Verlag.

Masanjala, W. & Papageorgiou, C. (2008), ‘Rough and Lonely Road to Prosperity: A reexamination of the sources of growth in Africa using Bayesian Model Averaging’, *Journal of Applied Econometrics* **23**(5), 671–682.

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A. & Teller, E. (1953), ‘Equation of state calculations by fast computing machines’, *The Journal of Chemical Physics* **21**(6), 1087–1092.

Millar, P. (2011), ‘BIC: Stata module to evaluate the statistical significance of variables in a model’, Statistical Software Components, Boston College Department of Economics.
URL: http://econpapers.repec.org/RePEc:boc:bocode:s449507

R Development Core Team (2010), *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0.
URL: http://www.R-project.org

Raftery, A. E. (1995), ‘Bayesian model selection in social research’, *Sociological Methodology* **25**, 111–163.

Raftery, A., Hoeting, J., Volinsky, C., Painter, I. & Yeung, K. Y. (2010), *BMA: Bayesian Model Averaging*. R package version 3.13.
URL: http://CRAN.R-project.org/package=BMA

Sala-i-Martin, X. (1997), 'I just ran two million regressions', *The American Economic Review* **87**(2), 178–183.

Sala-i-Martin, X., Doppelhofer, G. & Miller, R. (2004), 'Determinants of long-term growth: A Bayesian averaging of classical estimates (BACE) approach', *American Economic Review* **94**(4), 813–835.

Strawderman, W. (1971), 'Proper Bayes minimax estimators of the multivariate normal mean', *The Annals of Mathematical Statistics* **42**(1), 385–388.

Venables, W. N. & Ripley, B. D. (2002), *Modern Applied Statistics with S*, fourth edn, Springer, New York. ISBN 0-387-95457-0.
URL: http://www.stats.ox.ac.uk/pub/MASS4

Zellner, A. & Siow, A. (1980), 'Posterior odds ratios for selected regression hypotheses', *Trabajos de Estadística y de Investigación Operativa* **31**(1), 585–603.

Appendix A. R codes to produce tables results

We used the following codes to generate the results shown in the tables.

```
# Loading datasets and required libraries
#########################################

set.seed(2011)
library(MASS)
data(UScrime); attach(UScrime)
UScrime1 <- (cbind(log(UScrime[,c(16,1,3:15)]), So))
noise<- matrix(rnorm(35*nrow(UScrime)), ncol=35)
colnames(noise)<- paste('noise', 1:35, sep='')
UScrime.log <- cbind(UScrime1,noise)
X <- UScrime1[,-1]; Y <- UScrime1[,1]
x <- UScrime.log[,-1]; y <- UScrime.log[,1]




# Full enumaration of model space (Tables 4 and 5)
##################################################

# BMS Package:
library(BMS)
bms_enu <- bms(UScrime1, mcmc="enumerate", g="UIP", mprior="uniform",
user.int=FALSE)
coef(bms_enu)
detach("package:BMS")

# BMA Package:
library(BMA)
bma_enu <- iBMA.bicreg(X, Y, thresProbne0 = 5, verbose = TRUE, maxNvar = 30)
summary(bma_enu)
detach("package:BMA")

# BAS Package:
library(BAS)
bas_enu<- bas.lm(y~., data=UScrime1, n.models=NULL, prior="ZS-null",
modelprior=uniform(), initprobs="Uniform")
```

```
coef(bas_enu)
detach("package:BAS")




# Using model space search mechanisms (Tables 6 and 7)
######################################################

# BMS Package:
library(BMS)
bms.time <- system.time(bms_sam <- bms(UScrime.log, burn=2000,
iter = 100000, mcmc="bd", g="UIP", mprior="uniform", nmodel=2000,
user.int=FALSE))
cat("Elapsed Time=", bms.time[3], "Seconds")
# Figure 1
plot(bms_sam[1:200])
# Figure 2
bms_sam2 <- bms(UScrime.log, burn=2000, iter = 100000, mcmc="bd",
g="UIP", mprior="fixed", nmodel=2000, user.int=FALSE)
plot(bms_sam2[1:200])
# Figure 3(a)
density.bma(bms_sam, reg = "GDP", addons="esEpl")
detach("package:BMS")

# BMA Package:
library(BMA)
bma.time <- system.time(bma_sam <- iBMA.bicreg(x, y, thresProbne0 = 5,
verbose = TRUE, maxNvar = 30, nIter = 100000))
cat("Elapsed Time=", bma.time[3], "Seconds")
#Figure 3(b)
bma_fig <- bicreg(x, y)
plot(bma_fig, include=5, include.intercept=FALSE)
detach("package:BMA")

# BAS Package:
library(BAS)
bas.time <- system.time(bas_sam <- bas.lm(y~., data=UScrime.log,
n.models=2^15, prior="ZS-null", modelprior=uniform(), initprobs="Uniform",
```

```
method="AMCMC", Burnin.iterations=2000, MCMC.iterations=100000))
cat("Elapsed Time=", bas.time[3],"Seconds")
# Figure 3(c)
plot(coefficients(bas_sam), subset=12, ask=TRUE)
# Figures 4(a), 4(b), and 4(c)
plot(bas_sam, add.smooth=TRUE, panel="add.smooth")
detach("package:BAS")




# Evaluating time performance of the packages (Table 8)
#######################################################

UScrime.time <- as.data.frame(matrix(rnorm(6050000),nclo=121))

# BMS Package:
library(BMS)
bms.mat <- matrix(0,4,4)
m <- 1; n <- 1
for(i in c(15,30,60,120)){
   for(j in c(50,500,5000,50000)){
       bms.run.time <- system.time(bms.run <- bms(UScrime.time[1:j,1:(i+1)],
       burn=2000, iter = 100000, mcmc="bd", g="UIP", mprior="uniform",
       nmodel=2000, user.int=FALSE))
       bms.mat[m,n] <- bms.run.time[3]
       m <- m+1
    }
  m <- 1
  n <- n+1
}
detach("package:BMS")

# BMA Package:
library(BMA)
bma.mat <- matrix(0,4,4)
m <- 1; n <- 1
x <- UScrime.time[,-1]
y <- as.data.frame(UScrime.time[,1])
```

```
for(i in c(15,30,60,120)){
   for(j in c(50,500,5000,50000)){
       bma.run.time <- system.time(bma.run <- iBMA.bicreg(x[1:j,1:i], y[1:j,],
       thresProbne0 = 5, verbose = TRUE, maxNvar = 30, nIter = 100000))
       bma.mat[m,n] <- bma.run.time[3]
       m <- m+1
    }
  m <- 1
  n <- n+1
}
detach("package:BMA")

# BAS Package:
library(BAS)
bas.mat <- matrix(0,4,4)
m <- 1; n <- 1
for(i in c(15,30,60,120)){
   for(j in c(50,500,5000,50000)){
       bas.run.time <- system.time(bas.run <- bas.lm(V1~.,
       data=UScrime.time[1:j,1:(i+1)],
       n.models=2^15, prior="ZS-null", modelprior=uniform(),
       initprobs="Uniform", method="AMCMC", Burnin.iterations=2000,
       MCMC.iterations=100000))
       bas.mat[m,n] <- bas.run.time[3]
       m <- m+1
    }
  m <- 1
  n <- n+1
}
detach("package:BAS")




# Evaluating the replicability of three packages (Table 9)
##########################################################

mydata <- read.dta("fls21.dta", convert.dates=TRUE, convert.factors=TRUE,
           missing.type=TRUE, convert.underscore=TRUE,
```

```
           arn.missing.labels=TRUE)

bmadata <- mydata[,-2]
y <- bmadata[,1] ; x <- bmadata[,-1]
attach(bmadata)

library(BMS)
bms.time <- system.time(fls.bms <- bms(bmadata, burn=10000,
iter=500000, g="UIP", mprior="uniform", nmodel=3000, mcmc="bd",
ser.int=FALSE))
coef(fls.bms)
detach("package:BMS")

library(BAS)
bas.time <- system.time(fls.bas <- bas.lm(gr56092~., data=bmadata,
n.models=2^15, prior="ZS-null", modelprior=uniform(), initprobs="Uniform",
method="AMCMC", Burnin.iterations=10000, MCMC.iterations=500000))
coef(fls.bas)
detach("package:BAS")

library(BMA)
bma.time <- system.time(fls.bma<- iBMA.bicreg(x, y, thresProbne0 = 5,
 verbose = TRUE, maxNvar = 30, nIter = 500000))
summary(fls.bma)
detach("package:BMA")

################################## END ##################################
```

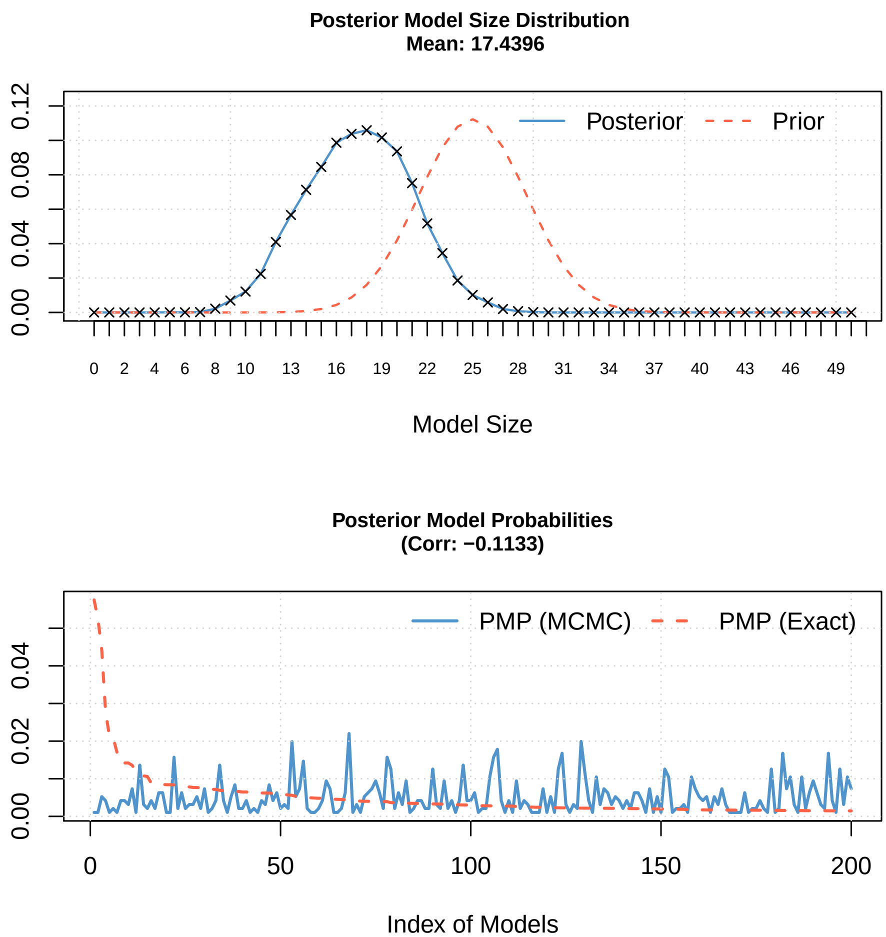


FIGURE 1. Posterior Model Size Distribution and Model Probabilities Produced from the `BMS` package with "uniform" model priors.

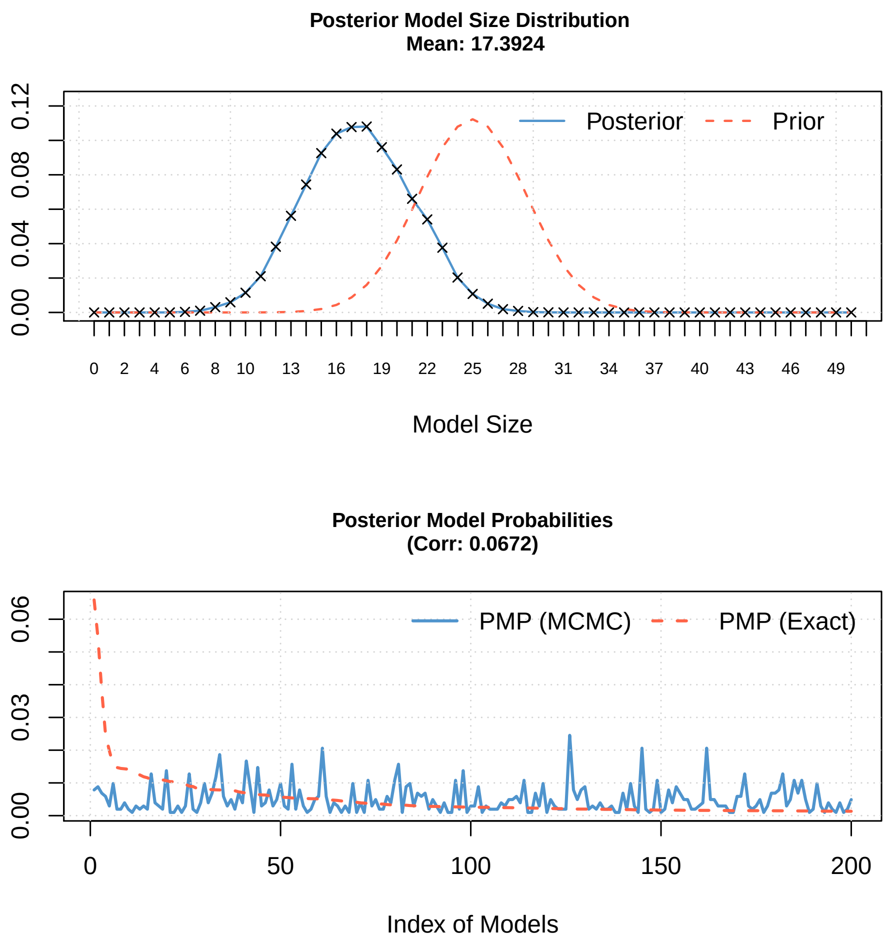
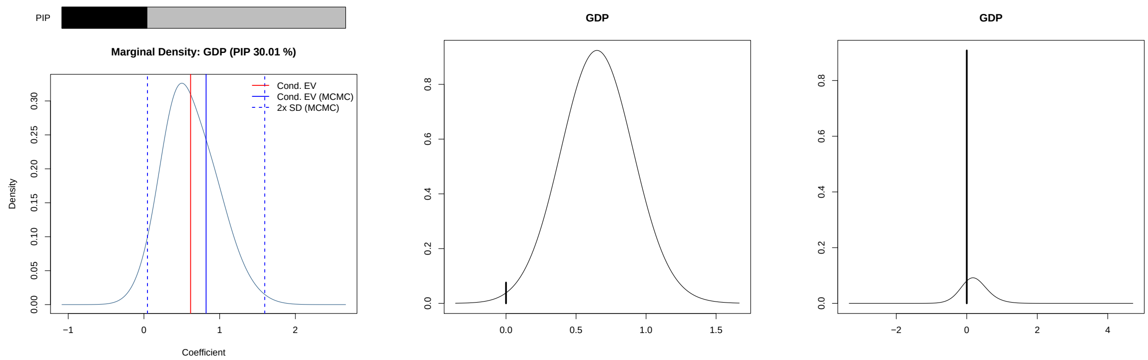
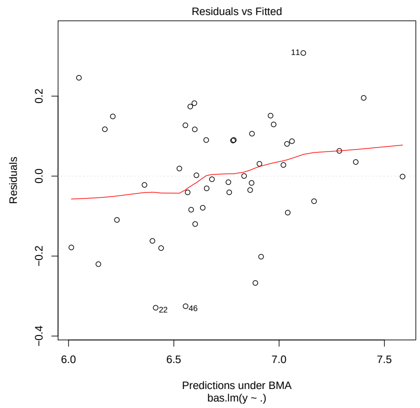


FIGURE 2. Posterior Model Size Distribution and Model Probabilities Produced from the `BMS` package with "fixed" model priors.

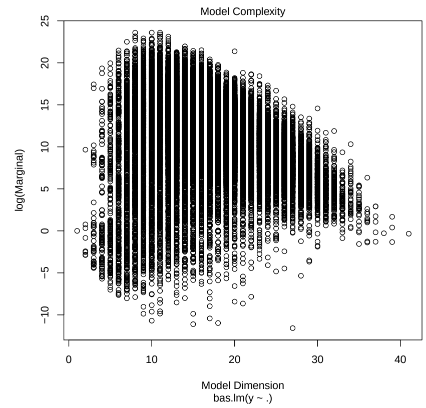


(a) Marginal Density of GDP: `BMS` Package (b) Marginal Density of GDP: `BMA` Package (c) Marginal Density of GDP: `BAS` Package

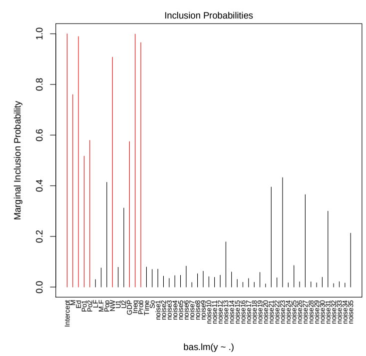
FIGURE 3. Marginal Posterior Density Plots Produced Across the Three Packages



(a) Plot of Residuals vs. Fitted Values



(b) Plot of Marginal Likelihoods vs. Model Dimension



(c) Marginal Posterior Inclusion Probabilities

FIGURE 4. Diagnostic Plots Produced With the `BAS` Package

TABLE 1. Grades of Evidence Corresponding to Values of the BIC difference for M_j against M_i

BIC Difference	Evidence
0–2	Weak
2–6	Positive
6–10	Strong
>10	Very strong

Table 2. A comparison of BMS, BAS, and BMA options

Package	Option	Description	Default
BMS:	mcmc	Indicates the model sampler to be used. If bd, a birth/death MCMC algorithm will be used. If rev.jump, a reversible jump algorithm is used that adds a swap step to the birth/death algorithm. If enumerate, the entire model space will be enumerated	bd
	burn	The number of burn-in draws for the MCMC sampler. Not taken into account if mcmc=enumerate	1000
	iter	Shows the number of iteration draws to be sampled excluding burn-ins	3000
	nmodel	The number of best models for which information is stored	500
	g	Indicates the hyperparameters on Zellner's g-prior for the regression coefficients. If UIP, the number of observations, $g = N$, is used as g-prior. If BRIC, $g = max(N, K^2)$, where K is the number of covariates, will be used. If RIC, $g = K^2$ is considered. If HQ, the package sets $g = log(N)^3$. If EBL, a local empirical Bayes g-parameter will be used. If hyper, the package takes the hyper-g prior distribution	N
	mprior	Indicates the choice of model prior. If fixed, then fixed common prior inclusion probabilities for each regressor is employed. The random option employs the random theta prior. If uniform, the uniform model prior will be utilized. The customk option, allows for cusom model size priors. Alternatively, the pip option allows for custom prior inclusion probabilities	random
	start.value	Specifies the starting model of the iteration algorithm	NA
	g.stats	If TRUE, statistics on the shrinkage factor $g/(1 + g)$ will be collected. Set to FALSE for faster iteration	TRUE
	force.full.ols	If TRUE, the OLS estimation part of the sampling algorithm relies on slower matrix inversion procedure. Setting this option to TRUE, slows down sampling but deals better with highly collinear data	FALSE
	fixed.reg	Includes the explanatory variables that the user wants to be always included in every sampled model	numeric(0)
BAS:	n.models	Specifies the number of models to sample. If NULL, this package enumerates the entire model space unless $K > 25$	NULL
	prior	Indicates the choice of prior distribution for regression coefficients. Choices include AIC, BIC, g-prior, ZS-null, ZS-full, hyper-g, hyper-g-laplace, EB-local, and EB-global	ZS-null
	modelprior	Shows the family of prior distribution on the models. Choices include uniform, Bernoulli, and beta.binomial	uniform
	method	Indicates the sampling method. If BAS, a Bayesian Adaptive Sampling without replacement will be used. The MCMC+BAS option runs an initial MCMC algorithm to calculate marginal inclusion probabilities and then samples without replacement. If AMCMC, an Adaptive, experimental MCMC is considered.	BAS
	initprobs	Introduces a vector of length K with the initial inclusion probabilities used for sampling without replacement	Uniform
	prob.rw	Indicates the probability of using the random-walk proposal	0.5
	Burnin.iterations	Specifies the number of iterations to discard when using any of the MCMC options	NULL
	MCMC.iterations	Provides the number of iterations to run MCMC algorithm	NULL
	lambda	A parameter used in the AMCMC algorithm	NULL
	delta	This is the truncation parameter that prevents sampling probabilities to degenerate to 0 or 1	0.025
BMA:	wt	A vector of weights for regression	rep(1,length(y))
	OR	Indicates the maximum ratio for excluding models in Occam's window	20
	strict	If FALSE, all models whose posterior model probability is within a facto of 1/OR will be returned. If TRUE, a more parsimonious set of models where any model with a more likely submodel is eliminated	FALSE
	maxCol	Specifies the maximum number of columns in the design matrix to be kept	31
	thresProbne0	A number that indicates the probability threshold for including variables as a percent	5
	nIter	Determining the maximum number of iterations that should be run	100

Notes: This table presents the available options and their default settings for the current Bayesian model averaging packages, BMS, BAS, and BMA within the statistical software environment R. In addition, a brief description of each option is also provided. For more details, see sections 2 and 3.

Table 4. A comparison of Posterior Inclusion Probabilities (PIPs) across the BMS, BAS, and BMA packages. (Full Model Enumeration)

	Packages		
	BMS	BAS	BMA
	(i)	(ii)	(iii)
PIP:			
Income Inequality	0.99	0.99	1.00
Education	0.97	0.97	1.00
Imprisonment Probability	0.89	0.89	0.98
Males	0.85	0.85	0.97
Non-white	0.67	0.68	0.89
Police Expenditure 1960	0.66	0.66	0.74
Unemployment 35-39	0.59	0.60	0.83
Police Expenditure 1959	0.42	0.43	0.25
Time	0.33	0.36	0.44
State Population	0.33	0.35	0.28
GDP	0.31	0.35	0.33
Southern State Dummy	0.23	0.26	0.06
Unemployment 14-24	0.20	0.24	0.11
Labor Force	0.15	0.19	dropped

Notes: This table reports the posterior inclusion probabilities (PIP) obtained from applying Bayesian model averaging to a dataset on per capita crime rate in 47 U.S. states, originally published in Ehrlich (1973), using the BMS, BAS, and BMA packages. The dependent variable is the rate of crimes in a particular category per head of population across 47 states for the year 1960. All models estimated contain an intercept. Column (i) shows the Bayesian posterior inclusion probabilities computed by the BMS package. Column (ii) shows the same probabilities reported by the BAS package. The posterior inclusion probabilities reported by the BMA package are presented in column (iii). Note since the number of explanatory variables in this case is 15, the model space is small enough that allows for full enumeration of model space.

Table 5. Comparison of Posterior Means and Standard Deviations across the `BMS`, `BAS`, and `BMA` packages. (Full Model Enumeration)

	Packages		
	`BMS`	`BAS`	`BMA`
	(i)	(ii)	(iii)
Posterior Mean & Std. Dev.			
Income Inequality	1.41	1.40	1.38
	(0.35)	(0.36)	(0.33)
Education	1.90	1.86	2.12
	(0.61)	(0.62)	(0.51)
Imprisonment Probability	-0.21	-0.21	-0.24
	(0.11)	(0.11)	(0.10)
Males	1.16	1.14	1.40
	(0.67)	(0.66)	(0.53)
Non-white	0.06	0.06	0.09
	(0.05)	(0.05)	(0.04)
Police Expenditure 1960	0.62	0.61	0.66
	(0.52)	(0.53)	(0.43)
Unemployment 35-39	0.20	0.20	0.27
	(0.21)	(0.21)	(0.19)
Police Expenditure 1959	0.32	0.32	0.22
	(0.51)	(0.51)	(0.40)
Time	-0.07	-0.08	-0.12
	(0.15)	(0.16)	(0.17)
State Population	-0.02	-0.02	-0.01
	(0.03)	(0.03)	(0.03)
GDP	0.18	0.19	0.19
	(0.35)	(0.36)	(0.35)
Southern State Dummy	0.03	0.03	0.006
	(0.08)	(0.09)	(0.03)
Unemployment 14-24	-0.02	-0.02	-0.03
	(0.15)	(0.17)	(0.13)
Labor Force	0.04	0.05	dropped
	(0.27)	(0.30)	—

Notes: This table reports the posterior means and standard errors of regressors calculated from the `BMS`, `BAS`, and `BMA` packages. The Bayesian model averaging is applied to a dataset on per capita crime rate in 47 U.S. states, originally published in Ehrlich (1973). The dependent variable is the rate of crimes in a particular category per head of population across 47 states for the year 1960. All models estimated contain an intercept. Column (i) shows the posterior means and standard deviations computed by the `BMS` package. Column (ii) shows the same statistics reported by the `BAS` package. The posterior means and standard errors of regressors reported by the `BMA` package are presented in column (iii). Standard errors are in the parentheses. Note since the number of explanatory variables in this case is 15, the model space is small enough that allows for full enumeration of model space.

Table 6. Posterior Inclusion Probabilities across the `BMS`, `BMA`, and `BAS` packages. (Model Sampling Required)

	Packages		
	`BMS` (i)	`BAS` (ii)	`BMA` (iii)
Post Probabilities:			
Income Inequality	0.99	1.00	1.00
Males	0.99	1.00	0.19
Imprisonment Probability	0.97	0.99	1.00
Education	0.97	0.99	1.00
Unemployment 35-39	0.74	0.93	1.00
Time	0.73	0.97	dropped
Police Expenditure 1960	0.57	0.89	dropped
Police Expenditure 1959	0.54	0.22	1.00
Southern State Dummy	0.51	0.79	0.42
Unemployment 14-24	0.33	0.12	0.91
GDP	0.31	0.17	0.97
Males per Females	0.25	0.08	0.86
Non-white	0.25	0.04	0.99
Labor Force	0.23	0.04	dropped

Notes: This table reports the posterior inclusion probabilities (PIP) obtained from applying Bayesian model averaging to a dataset on per capita crime rate in 47 U.S. states, originally published in Ehrlich (1973), using the `BMS`, `BAS`, and `BMA` packages. The dependent variable is the rate of crimes in a particular category per head of population across 47 states for the year 1960. All models estimated contain an intercept. Column (i) shows the Bayesian posterior inclusion probabilities computed by the `BMS` package. Column (ii) shows the same probabilities reported by the `BAS` package. The posterior inclusion probabilities reported by the `BMA` package are presented in column (iii). In this exercise, 35 independent, fictitious variables generated from a standard-normal distribution are added to the original dataset. Therefore, the model space is large enough that forces the packages to use a model space search mechanism.

Table 7. Comparison of Posterior Means and Standard Deviations across the BMS, BMA, and BAS Packages. (Model Sampling Required)

	Packages		
	BMS	BAS	BMA
	(i)	(ii)	(iii)
Posterior Mean & Std. Dev.			
Income Inequality	1.58	1.49	2.55
	(0.31)	(0.22)	(0.31)
Males	1.65	1.55	0.08
	(0.41)	(0.32)	(0.23)
Imprisonment Probability	-0.30	-0.34	-0.26
	(0.10)	(0.06)	(0.05)
Education	1.72	1.64	2.97
	(0.59)	(0.38)	(0.48)
Unemployment 35-39	0.23	0.26	0.67
	(0.17)	(0.11)	(0.20)
Time	-0.29	-0.36	dropped
	(0.22)	(0.13)	—
Police Expenditure 1960	0.51	0.81	dropped
	(0.53)	(0.37)	—
Police Expenditure 1959	0.49	0.14	0.83
	(0.55)	(0.35)	(0.14)
Southern State Dummy	0.08	0.14	0.08
	(0.11)	(0.09)	(0.12)
Unemployment 14-24	0.07	0.02	-0.54
	(0.20)	(0.11)	(0.28)
GDP	0.13	0.06	0.90
	(0.29)	(0.19)	(0.38)
Males per Females	-0.32	-0.10	-2.54
	(1.03)	(0.44)	(1.58)
Non-white	0.004	0.001	0.10
	(0.029)	(0.009)	(0.03)
Labor Force	0.03	0.01	dropped
	(0.44)	(0.11)	—
Computation Time			
(Seconds)	64	16	12

Notes: This table reports the posterior means and standard errors of regressors calculated from the BMS, BAS, and BMA packages. The Bayesian model averaging is applied to a dataset on per capita crime rate in 47 U.S. states, originally published in Ehrlich (1973). The dependent variable is the rate of crimes in a particular category per head of population across 47 states for the year 1960. All models estimated contain an intercept. Column (i) shows the posterior means and standard deviations computed by the BMS package. Column (ii) shows the same statistics reported by the BAS package. The posterior means and standard errors of regressors reported by the BMA package are presented in column (iii). Standard errors are in the parentheses. In this exercise, 35 independent, fictitious variables generated from a standard-normal distribution are added to the original dataset. Therefore, the model space is large enough that forces the packages to use a model space search mechanism. The bottom part of table shows the computation time in seconds.

TABLE 8. A comparison of BMS, BAS, and BMA run times. Tabular values are total run time in seconds for the BMS, BAS and BMA packages respectively.

Number of Covariates:	15	30	60	120
Sample Size:				
50	(44.86, 1.90, 0.74)	(42.30, 4.43, 2.17)	(42.33, 12.26, 8.15)	(58.42, 105.83, 101.69)
500	(39.03, 1.65, 0.50)	(40.44, 3.64, 2.78)	(39.36, 8.08, 24.67)	(42.46, 18.31, 75.21)
5,000	(34.97, 1.66, 1.00)	(35.70, 2.43, 4.78)	(38.89, 5.32, 38.09)	(39.78, 9.92, 81.70)
50,000	(34.05, 2.36, 6.95)	(34.40, 4.11, 13.02)	(38.89, 8.25, 34.47)	(44.45, 22.12, 195.68)

Notes: This table presents the run times in seconds for the three packages, BMS, BAS, and BMA, across various sample sizes and dimensions of the covariate space. The size of the covariate space comes from $\{15, 30, 60, 120\}$ while the sample size is taken from $\{50, 500, 5000, 50000\}$. The dependent variable as well as the covariates are drawn from a standard normal distribution. The analysis is run on a 1.60 GHz Intel(R) Core(TM) 2 Duo processor with 3.00 GB of DDR3 memory. Within each set of parenthesis are the specific run times for a given covariate-sample size combination from baseline calls to the BMS, BAS, and BMA packages, respectively. Baseline options include uniform model priors, 500,000 iterations of the chosen MCMC algorithm, and discarding the first 3000 models (burn-in).

TABLE 9. Performance of BMS, BAS, and BMA Packages for the Fernandez et al. (2001*b*) study.

	Published Results	BMS Package	BAS Package	BMA Package
	PIP	PIP	PIP	PIP
	(i)	(ii)	(iii)	(iv)
Variable:				
GDP 1960	1.000	1.000	1.000	1.000
Fraction Confucian	0.995	0.990	1.000	1.000
Life Expectancy	0.946	0.926	0.998	1.000
Equipment Investment	0.942	0.925	0.997	0.972
Sub-Saharan Dummy	0.757	0.747	*1.000	*1.000
Fraction Muslim	0.656	0.634	0.487	0.519
Rule of Law	0.516	0.510	*0.976	*1.000
Years Open Economy	0.502	0.502	*0.048	*0.055
Degree of Capitalism	0.471	0.485	0.460	*1.000
Fraction Protestant	0.461	0.450	0.355	*0.945
Fraction GDP in Mining	0.441	0.450	*0.978	*0.966
Non-Equipment Investment	0.431	0.441	*0.746	*0.987
Latin American Dummy	0.190	0.200	*0.884	*0.637
Primary School Enrollment, 1960	0.184	0.192	0.818	dropped
Fraction Buddhist	0.167	0.190	0.222	*0.551
Black Market Premium	0.157	0.177	*0.673	*0.841
Fraction Catholic	0.111	0.124	0.083	dropped
Civil Liberties	0.100	0.112	*0.733	dropped
Fraction Hindu	0.097	0.111	*0.991	*0.737
Primary exports, 1970	0.071	0.093	0.039	dropped
Political Rights	0.069	0.092	0.184	*0.419
Exchange Rate Distortions	0.060	0.077	0.087	dropped
Age	0.058	0.075	0.154	0.210
War Dummy	0.052	0.074	0.108	dropped
Size Labor Force	0.047	0.052	*0.972	*0.567
Fraction Speaking Foreign Language	0.047	0.065	0.042	dropped
Fraction of State Population. Speaking English	0.047	0.067	*0.496	0.241
Ethnolinguistic Fractionalization	0.035	0.057	*0.933	dropped
Spanish Colony Dummy	0.034	0.047	*0.785	0.339
S.D. of Black-Market Premium	0.031	0.047	0.033	dropped
French Colony Dummy	0.031	0.042	*0.745	0.207
Absolute Latitude	0.024	0.039	0.045	dropped
Ratio Workers to Population	0.024	0.040	0.032	dropped
Higher Education Enrollment	0.024	0.039	*0.936	dropped
Population Growth	0.022	0.040	0.049	dropped
British Colony Dummy	0.022	0.040	*0.651	0.053
Outward Orientation	0.021	0.036	*0.554	dropped
Fraction Jewish	0.019	0.030	0.034	dropped
Revolutions and Coups	0.017	0.027	0.032	dropped
Public Education Share	0.016	0.028	0.196	dropped
Area	0.016	0.020	0.032	dropped
Computation Time:				
(Seconds)	–	140	46	10

Notes: This table presents the results of applying Bayesian model averaging to the data used in Fernandez et al. (2001*b*) using BMS, BAS, and BMA packages. The dependent variable is the growth rate from 1960-1996 across 72 countries. All models estimated contain an intercept. Column (i) shows the Bayesian posterior inclusion probabilities reported by Fernandez et al. (2001*b*, Table I). Columns (ii), (iii), and (iv) report the posterior inclusion probabilities (PIP) using BMS, BAS, and BMA packages, respectively.

Table 10. Performance of BMS, BAS, and BMA Packages for the Doppelhofer & Weeks (2009) study.

	Published Results	BMS Package		BAS Package		BMA Package	
	PIP	PIP	Post Mean & SD	PIP	Post Mean & SD	PIP	Post Mean & SD
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Variable:							
East Asian Dummy	0.828	0.811	0.0219	0.999	0.0227	1.00	0.0297
			(0.0126)		(0.0071)		(0.0065)
Primary Schooling Enrollment	0.796	0.799	-0.00006	1.00	-0.00009	1.00	-0.00008
			(0.00004)		(0.00002)		(0.00002)
Investment Price	0.774	0.780	0.0167	0.071	0.0003	1.00	0.0168
			(0.0101)		(0.0022)		(0.0049)
Initial GDP 1960	0.685	0.701	-0.0065	0.999	-0.0085	1.00	-0.0090
			(0.0045)		(0.0026)		(0.0025)
Fraction of Tropical Area	0.570	0.565	-0.0082	0.236	-0.0211	1.00	-0.0155
			(0.0079)		(0.0033)		(0.0030)
Computation Time:							
(Seconds)			160		44		20

Notes: This table presents the results of applying Bayesian Model Averaging to the data used in Doppelhofer & Weeks (2009) using BMS, BAS, and BMA packages. The dependent variable is the growth rate from 1960-1996 across 88 countries. Standard errors are in parentheses. All models estimated contain an intercept. Column (i) shows the Bayesian posterior inclusion probabilities reported by Doppelhofer & Weeks (2009, Data Appendix B). Columns (ii), (iv), and (vi) report the posterior inclusion probabilities (PIP) using BMS, BAS, and BMA packages, respectively. Columns (iii), (v), and (vii) show the posterior estimates of model coefficients and their standard errors using the three packages. The bottom part of table shows the computation time in seconds.