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US Money Demand, Monetary Overhang, and Inflation Prediction

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Abstract

We analyze US money demand stability and the indicator properties of derived money overhang measures of various monetary aggregates for predicting inflation over a sample from 1987Q1 to 2008Q2. In contrast to a large part of the literature, we find evidence of a stable money demand function for M2 in the framework of the cointegrated VAR (CVAR) model without resorting to 'exotic' determinants or redefinitions of M2. Previous evidence suggesting instability of the M2 money demand function may have been related to two kinds of misspecification: First, with regard to the specification of the deterministic components, and secondly, with regard to the imposition of *theoretically plausible* but *empirically rejected* restrictions imposed on the model from the outset. Using formal stability tests, we find that stability of the long-run coefficients cannot be rejected, while stability of the short-run parameters is doubtable. Inference is not only based on asymptotics, but also on small-scale (parametric) bootstraps. We find some evidence that money overhang is a useful information variable for predicting changes in the inflation rate. First, our estimates obtained from the CVAR model suggest that money overhang Granger-causes inflation. Secondly, recursive out-of-sample forecasts which we conducted over a hold-back period show that taking account of derived money overhang measures significantly improves forecasts of the change in inflation over long horizons (about 3 years). Finally, we provide some evidence that the importance of money overhang for predicting (changes in) inflation may have increased over time.

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1 Introduction

Although the FED at least formally attributes a much less prominent role to the development of monetary aggregates and their potential impact on inflation than the ECB does, we believe that a careful reassessment of the properties of money demand in the US and the potential leading indicator qualities of derived money overhang measures for inflation are warranted. Previous empirical evidence has been skeptical of both, the stability of US money demand with respect to various monetary aggregates, as well as money's leading indicator qualities for inflation (see for instance Friedman and Kuttner, 1992, Estrella and Mishkin, 1997, and Stock and Watson, 1999). However, previous evidence suggesting instability of the M2 money demand function may have been partly due to two kinds of misspecification: First, with regard to the specification of the (long-run) deterministic components in the econometric model (see Swanson, 1998, and Ahking, 2002). And secondly, with regard to the imposition of *theoretically plausible* but *empirically rejected* restrictions imposed on the model from the outset. Following Juselius (2006), we think that it is time to 'let the data speak freely'.[1] To take this approach seriously, our analysis starts from a very general model specification and each imposed restriction is formally tested whether it is 'compatible with the data'. We thereby carefully employ the Cointgrated VAR (CVAR) *approach* from Juselius (2006). To address previous concerns regarding stability, our model specification is augmented by a rigorous stability analysis, examining both the long-run and the short-run stability of the final (overidentified) model. Our more favorable results in terms of M2 money demand stability may also partly be related to a different sample choice. In contrast to most previous studies we do not examine the full sample for which data is available (starting 1959Q1), but focus our analysis on a sample starting 1987Q1. While our sample-choice may superficially be criticized as 'data-mining', we believe that our approach to shorten the sample length by shifting the sample start is reasonable. First it is very likely that a sample spanning about 50 years of data contains (possibly several) regime shifts which would require adequate and often complex modeling. Generally, one would prefer a model to be as parsimonious as possible. Secondly, by picking a sample starting in the mid 80s we exclude the period preceding the 'Great Moderation', which may especially be advantageous with respect to our forecasting exercise. Thirdly, our sample choice is *ex post* rationalized by an (informal) recursive stability analysis which clearly points towards instability of the estimated long-run parameters during the period excluded from our previous analysis.

[1] See also Hoover et al. (2008).

If there is evidence of a reasonably stable money demand relation, derived measures of money overhang (which are defined as the deviation from the respective monetary aggregate from the money demand equilibrium) may well serve as an 'information variable' regarding the future state of the economy, and be - more specifically - helpful to predict future rates of inflation. Friedman's famous proposition that 'inflation is always and everywhere a monetary phenomenon' or at least Taylor's (1992) (cited in Nelson, 2003) 'softened' version that '*substantial* inflation is always and everywhere a monetary phenomenon' (emphasis added) remain intriguing.

To our knowledge no study has so far examined the leading indicator of *money overhang measures* for predicting (changes of) inflation in the US.[2] However, this is not surprising against the background of the above mentioned widespread skepticism regarding the stability of conventional money demand functions.

This article is structured as follows. In section 2 we will shortly review previous evidence on US M2 money demand (instability) and comment on potential shortcomings, which serve as a motivation for our article. In section 3 we will visually inspect the properties of the time series used in our analysis. In section 4 we will describe our estimation methodology and the model specification. Following Juselius (2006) we examine the cointegration space more closely and systematically test whether certain linear combinations suggested by economic theory are stationary in section 5. This will help us to find a sensible long-run identified structure of the model. In section 6 we will examine the stability of the long-run parameters, and how precisely they have been estimated. We also carefully analyze how our results are affected by the inclusion of a restricted smooth-shift dummy variably in the early 1990s, which has previously been often used in the literature on M2 money demand. We will formally test whether such a dummy is long-run excludable. In section 7 we examine the short-run structure of the overidentified model and examine the constancy of the short-run equations, which may be crucial for obtaining reliable inflation forecasts. A special focus will be on the analysis of the short-run 'inflation-equation' and the 'money-equation'. In section 8 we will conduct an extensive out-of-sample forecasting analysis (based on two distinct approaches) to assess whether our previously derived money overhang measures are useful to predict (changes in) inflation over various forecasting horizons (from 1 quarter up to 3 years). Finally, section

[2]Carlson et al. (2000) use derived money overhang measures to forecast nominal GDP, Dotsey et al. (2000) use an estimated money demand function to assess the 'nowcasting' properties of money for the other variables contained in the cointegration vector, Orphanides and Porter (2000) analyse whether a P^*model based on recursively estimated M2 money velocity is helpful to predict inflation.

9 concludes the article.

2 Previous Evidence and Sources of Misspecification

The following short overview about previous US money demand studies focuses on M2. A short review of previous empirical results and the presentation of our own results with respect to other monetary aggregates are moved to the appendix. We keep our literature review very short and instead refer to the comprehensive and excellent survey by Duca and Vanhoose (2004). Previous evidence on the relevance of monetary indicators for predicting inflation is summarized in section 1.8.1.

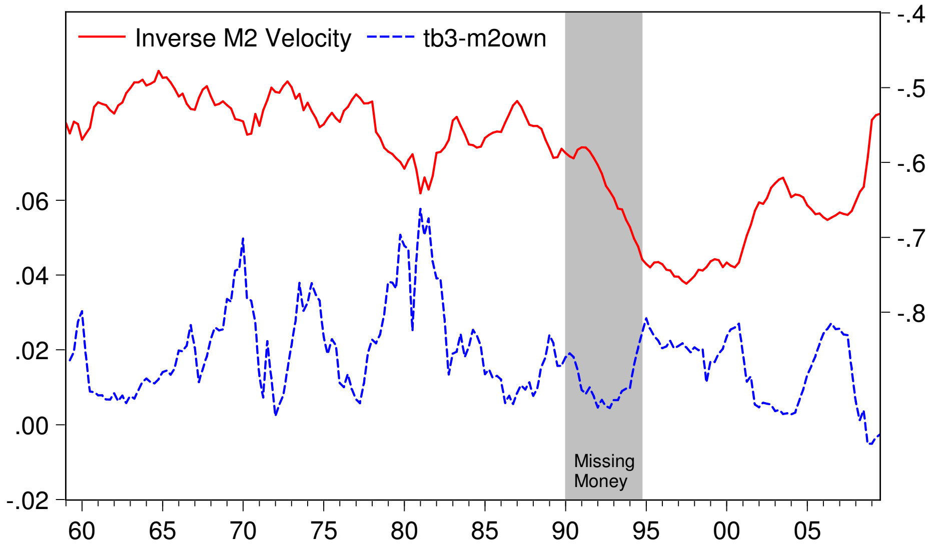
Until the early 1990s M2 money demand was deemed stable (see for instance Duca, 1995, Dotsey et al., 2000, and for the most extensive empirical study Carlson et al., 2000). If the sample end was shifted to include the early 1990s M2 money demand appeared to have become instable however. A notable decrease in the opportunity costs of holding money could not easily be reconciled with a slow-down of M2 growth (see Carlson et al., 2000). Those who acknowledged a break in that period basically found two ways to circumvent the problem and 're-establish' stability: Either by including dummy variables, smooth-shift variables (Carlson et al., 2000), or other variables such as the long-term bond rate over the respective period (see Koenig, 1996); or by redefining M2 in such a way to either exclude supposedly instable components or include alternatives to which shifts may have occurred.[3]

To illustrate this further, figure 1 depicts the inverse of US M2 velocity together with the commonly hypothesized opportunity costs of holding M2, that is the spread between the 3-month treasury bill rate ($tb3$) and the M2 own rate (own), which is the weighted average interest rate paid on the components included in M2.

The shaded area depicts the period which is often held responsible for instabilities in the US M2 money demand function, the so called 'period of missing money'. While this period looks peculiar if the end of the sample is in the midst of the 90s, it does not appear to be particularly special if the full sample is considered. Apart from this it should be noted that a contra-intuitive comovement of both series does not occur over five years, but is restricted to a very few quarters (most notably 1991Q4 and 1992Q1).

[3] Carlson and Keen (1996) for instance suggest that MZM is a more appropriate aggregate than M2, because it is per definitionem not affected by shifts to money market and bond funds (especially in the early 1990s) which are both included in MZM but not in

Figure 1: Inverse M2 Velocity vs. Interest Rate Spread



Note: Inverse M2 Velocity is calculated as $m2-y$. The shaded area depicts the period which is often held responsible for supposed instabilities of M2 money demand (see for instance Carlson et al. (1999) who include a dummy which is equal to 0 before and 1 after this period, and linearly increases in between.)

The robustness of previous results is questioned by Swanson (1998) and Ahking (2002). Swanson notes that estimation results fundamentally differ if the deterministic trend assumptions are changed. Building on this Ahking *systematically* examines the sensitivity of cointegration rank test results for various specifications including real M2, real GDP and different interest rates (which are however included separately) to changing the deterministic assumptions. He concludes that cointegration test results heavily depend on the deterministic components being correctly specified and claims that the deterministic components have often not been adequately dealt with in previous studies. Neither is formally tested for the appropriate deterministic components, nor is a clear cut made between restricted and unrestricted trends/constants. He therefore warns that 'there is a need for a more careful modeling of the deterministic components of long-run economic models than had been the case in the past'. In contrast to a large part of the literature, Ahking (2002) finds weak evidence of a long-run relationship for a

M2.

sample covering the early 90s without including any binary variables.[4] However, Ahking does not provide any estimated coefficient values (neither of the long-run parameters nor of the short-run parameters) which leaves us with a doubt whether the long-run relationship can really be interpreted as a money demand function. Evidence of cointegration is a necessary but not a sufficient condition for the existence of a (stable) long-run money demand function.

Beside a possible misspecification of the deterministic components in previous studies, there is another source of misspecification: the imposition of restrictions which are not 'accepted by the data'. To illustrate this let us consider equation 1.1, which is an econometric model for a quite general form of a money demand function that encompasses the most commonly tested specifications:

$$m2_t = \beta_0 + \beta_1 y_t + \beta_2 tb3_t + \beta_3 own_t + \beta_4 \Delta p_t + \varepsilon_t, \tag{1}$$

where $m2_t$ is the natural log of real M2 at time t, y_t is the log of real income, $tb3_t$ the three-month treasury bill rate, own_t the own rate of M2, and Δp_t the annualized quarterly inflation rate.

Almost all previous studies on US M2 demand (except Dotsey et al., 2000) define the opportunity costs of holding M2 as the difference between $tb3$ (or alternatively a long-term bond yield) and own, or alternatively only include *tb3* and disregard *own* completely. In terms of equation 1 this means that either the restriction $\beta_2 = -\beta_3$ or $\beta_3 = 0$ is imposed from the outset without testing its compatibility with the data. If either of these restrictions is mistakenly imposed the estimator will be biased, and closely related, cointegration tests may fail to find evidence of a long-run relationship among the variables.

Anticipating our results it turns out being crucial not to impose either of these restrictions. We find that both are incompatible with the data. Furthermore and closely related, restricting β_2 and β_3 in either way we do not find evidence of a long-run money demand relationship, but leaving both coefficients unrestricted we do.

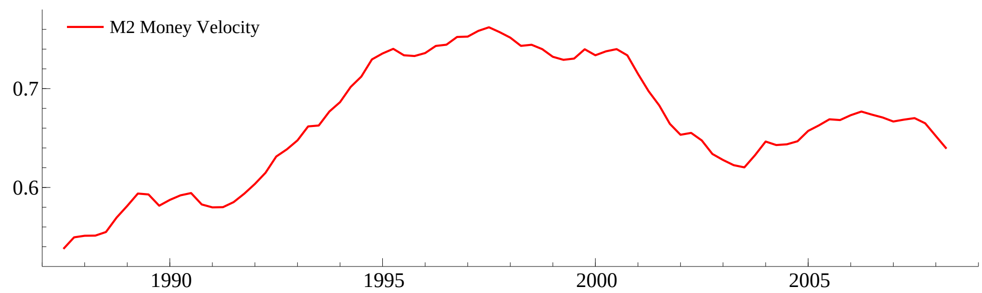
[4] He abstains from using dummy variables because the tabulated critical values underlying the cointegration rank tests are no longer valid. We instead choose to simulate the critical values in those cases where binary variables are included.

3 A Visual Inspection of the Time Series

Our benchmark model includes the five previously defined endogenous variables: $m2_t$, y_t, own_t, $tb3_t$, and Δp_t. Before we introduce the model more formally in the next section, we have a preliminary look at the involved series in this section, which also serves as a motivation for the choice of the estimation methodology.

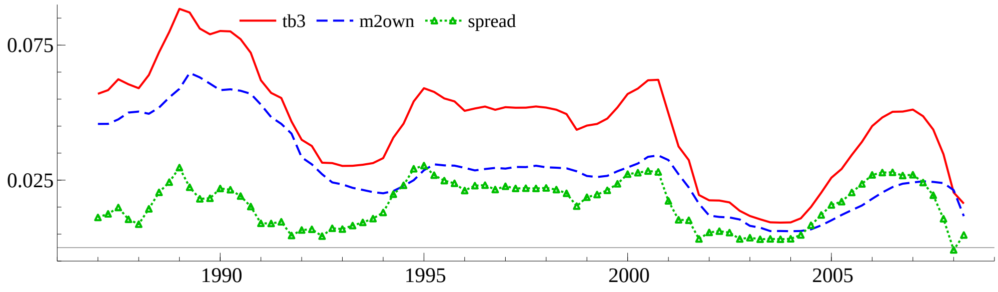
All data is taken from the Fed's FRED database. Real variables are obtained by dividing nominal values by the GDP deflator. Annualized quarterly inflation rates are obtained as four times the quarter-on-quarter percentage change of the GDP deflator. All rates are measured as fractions of 100. Graphs of the series over the estimation sample are depicted in figures 2 to 4. First of all, it can be clearly seen from figure 2 that US M2 money velocity is clearly non-stationary over the sample period. As an alternative to modeling the demand for money M2, we could have modeled M2 velocity. However, as described in the previous section, by doing so we would implicitly impose the non-tested long-run restriction that the money demand to income-elasticity is equal to unity and additionally impose restrictions on the short-run adjustment parameters. Wrongly imposing this restriction would again make the estimator biased.

Figure 2: M2 Money Velocity



It is also clearly visible in figure 3 that the assumption of a stationary interest rate spread would be quite hazardous. This visual assessment is formally supported in the succeeding analysis. It implies that the spread between *tb3* and *own* potentially qualifies as a determinant of the demand for $m2$, which itself is also clearly non-stationary. But again, instead of directly including the spread as a determinant, we include both interest rates individually. Figure 4 depicts annualized quarterly inflation rates, one based on the

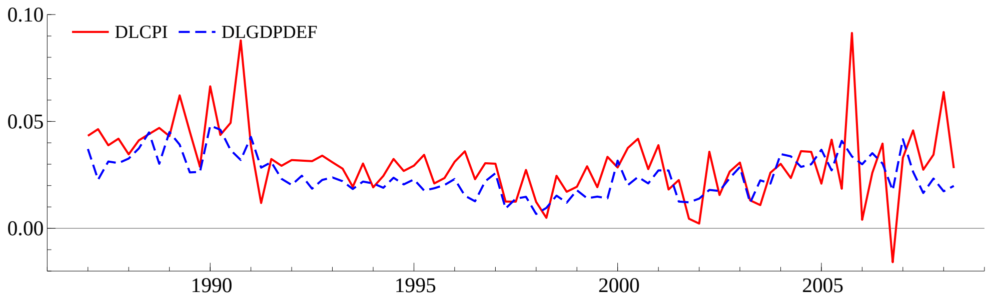
Figure 3: Interest Rates and their Spread



Note: The spread is calculated as *tb3-m2own*.

GDP deflator, the other on the CPI. Both series generally comove, however, CPI inflation is in some periods far more volatile than the one based on the GDP deflator and thereby causes, in contrast to the latter, unpleasant ARCH effects. The order of integration of inflation is less obvious than for the other series. Just by looking at the graphs it is hard to say whether inflation (however defined here) is I(0) or I(1) (and this is also unclear according to several univariate unit root tests we conducted). Its order of integration has crucial impacts for both, the model specification, as well as for our forecasting exercise. Our derived money overhang measure, which is defined as the difference between actual *m2* and the value predicted by the long-run money demand function (or more technically, the residual from the respective cointegration relationship), is per definitionem stationary and cannot plausibly be a useful predictor of a non-stationary variable. So whether inflation is I(0) or I(1) will determine whether money overhang may be a useful indicator for predicting the inflation rate or *changes* in the inflation rate.
Because all series (maybe except Δp) are nonstationary, cointegration analysis is the proper tool to analyse potential (long-run) relationships among the variables. Simple OLS regressions could lead to so-called 'spurious regressions' and estimating the model in first differences could cause omitted variable bias if there are cointegration relationships among the undifferenced series which are not taken into account in the model in first differences by including the error correction term(s).

Figure 4: US CPI Inflation Rate vs. GDP Deflator Inflation Rate



4 Estimation Methodology and Model Specification

It is unlikely that all variables except money are weakly exogenous so that a systems approach is generally warranted. In contrast to single equation approaches such as dynamic OLS (DOLS) or fully-modified OLS (FMOLS), which can also cope with endogenous regressors, a systems approach will furthermore give us insights into the adjustment dynamics. Additionally, it is straightforward to use the respective VECM for forecasting (as opposed to the DOLS estimator, which uses a two-sided filter to 'whiten' the residuals).

Following Johansen (1995) we write the CVAR model in vector error correction form as:

$$\Delta x_t = \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \alpha\beta' x_{t-1} + \Phi D_t + \varepsilon_t, t = 1, ..., T,$$

where $x_t = (m2_t, y_t, tb3_t, own_t, \Delta p_t)'$ is the vector of endogenous variables, Γ_i are matrices of short-run coefficients, D_t is a vector of deterministic components, and k the order of the VAR in levels. The number of cointegrating relationships is given by the rank of $\Pi = \alpha\beta'$ and will be determined after having set up a well-specified model.[5]

[5]If x_t is integrated of order 1, i.e. $I(1)$, and the variables contained in that vector were not cointegrated, it would imply that the equations would be unbalanced (unless $\Pi = 0$), because Δx_t which is $I(0)$ if x_t is $I(1)$ would have to equal x_{t-1} which is nonstationary plus some other *stationary* components. Because the sum of a stationary and a nonstationary series is itself nonstationary, the equation would be unbalanced and logically inconsistent (see Juselius, 2006).

We expect the demand for money to be positively related to real income, y_t, positively to the own rate of M2, own_t, and negatively to the 3-month treasury bill rate, $tb3_t$. The sign of the coefficient of Δp_t is ambiguous. On the one hand, Δp_t may be included to capture the opportunity costs of holding money compared to real assets. In this case we would expect a negative relationship. On the other hand, a positive sign may be rationalized in the presence of adjustment costs and nominal inertia (see Dregers and Wolters, 2008, Wolters et al., 1998, as well as the references therein). Another reason why Δp_t needs to be included is that by assuming that the I(2) trends in nominal money and prices cancel each other out, we impose long-run homogeneity. If we would not include Δp_t (or alternatively $\Delta m2_t$) we would impose the much more controversial restriction of short-run homogeneity as well (see Juselius, 2006). To restrict our analysis to the 'I(1)-world' seems appropriate based on the visual inspection of the series but is not formally tested.[6] Although the series have already been gathered in seasonally adjusted form, we include centered seasonal dummies to account for seasonalities in the data, which may have 'survived' the filtering process. *Centered* seasonal dummies are used in order not to create seasonal trends.

A money demand relationship does not have to be the only sensible long-run relationship we may find among the endogenous variables. Other plausible long-run relationships such as a long-run Fisher effect or a Phillips-curve relationship are equally possible and will be formally tested in the succeeding sections.

Our base model includes an unrestricted constant (to account for deterministic trends in the data) and a trend restricted to the cointegration relationship to allow for the possibility that the *deterministic* trends of the involved series do not cancel in the cointegration relationship(s).[7] We furthermore identified two extraordinary outliers in 1992Q1 and 2001Q1, which we first captured by including two restricted level shifts in these periods. The first coincides with the most peculiar quarter during the 'period of missing money', the latter is in the proximity of the burst of the new-economy stock market bubble and may be rationalized by a subsequent shift to liquidity. Later on we will formally test whether there was a shift in the equilibrium

[6] Juselius (2006) carefully explains how to formally test whether this so-called 'nominal-to-real transformation' is appropriate in the respective system. This however necessitates setting up an I(2)-model, which we abstain from doing. However, our analysis does not suggest 'severe I(2)-ness' in the transformed data - neither based on graphical evidence nor based on other criteria such as huge differences between uncorrected and Bartlett-corrected trace statistics.

[7] Such cointegration relations are called trend-stationary, which means that the relations are stationary around a *deterministic* trend.

mean in these periods, or whether a permanent impulse dummy is sufficient.

The choice of the correct rank is very important and crucially depends on the VAR model being well-specified. In table 1 we present misspecification test results for two similar competing models.[8] For both models we choose a lag-length of k=*2* as recommended by the Hannon-Quinn information criterion (HQC). Whereas model A contains a restricted trend and two restricted level shifts (*DS921* and *DS011*), model B includes an unrestricted constant, a permanent impulse dummy in 1992Q1 (*DUM921P*) as well as the restricted level shift in 2001Q1. Neither for model A nor model B do the residual diagnostics show clear evidence of a serious misspecification. However, in model A we find evidence of ARCH effects at the second and fourth lag, but only at the 5%-level. According to Rahbek et al. (2002), moderate ARCH effects will however not have a large impact on the results of the cointegration rank tests. So this should not be too concerning. In the more parsimonious model B the model diagnostics are even slightly better. Here we cannot reject the null hypothesis of no ARCH effects for the presented lags any longer. So, according to the misspecification tests, we would slightly favour model B over model A. However, we decide to base our choice of rank on both specifications. Additionally, after setting the rank we will have the opportunity to test the acceptability of the deterministic assumptions with respect to the trend assumptions as well as with respect to the dummy variables.

Because both of our models include level shifts and/or impulse dummies the critical values underlying the Johansen trace test are affected and *tabulated* critical values cannot be used. We therefore provide simulated critical values depending on the included deterministic components in each of the models.[9] Because the Johansen test is known to obey relatively poor small-sample properties and our sample is not that large, we furthermore provide empirical p-values based on parametric bootstrapping (with 9,999 bootstrap replications).[10] Cointegration rank test results are presented in table 2.

At the 5% level we would choose a cointegration rank of 2 for model A. For model B we would choose a rank of 2 if we follow the empirical p-value and a rank of 3 if we follow the asymptotic one. Since the choice of the rank for model B is not absolutely clear, we also estimated the model for

[8]We present diagnostics for both models, because model A is the starting point of our analysis (in which we impose testable restrictions), while model B turned out to be our preferred specification in the later part of the analysis. Because it is known that the cointegration rank may differ if the deterministic components are specified differently, it is sensible to show both specifications.

[9]Simulations are performed in CATS 2.0.

[10]Bootstrapping is conducted in S-VAR (version 0.43), which can be downloaded from http://www.texlips.net/svar/source.html.

Table 1: Misspecification Tests

	Model A	Model B
Deterministic Assumptions	Restricted Trend DS921 (restricted) DS011 (restricted)	Unrestricted Constant DP921 (unrestricted) DS011 (restricted)
No AC		
LM(1)	$\chi^2(25) = 24.07$ (0.52)	$\chi^2(25) = 27.32$ (0.34)
LM(2)	$\chi^2(25) = 35.29$ (0.08)	$\chi^2(25) = 26.51$ (0.38)
LM(4)	$\chi^2(25) = 34.69$ (0.09)	$\chi^2(25) = 31.00$ (0.19)
Normality		
	$\chi^2(10) = 14.19$ (0.17)	$\chi^2(10) = 11.07$ (0.35)
No ARCH		
LM(1)	$\chi^2(225) = 227.48$ (0.44)	$\chi^2(225) = 210.27$ (0.75)
LM(2)	$\chi^2(450) = 510.63$ (0.03)	$\chi^2(450) = 459.93$ (0.36)
LM(4)	$\chi^2(900) = 988.17$ (0.02)	$\chi^2(900) = 970.83$ (0.05)

Note: p-values in brackets. Null hypotheses for misspecification tests are: No autocorrelation (AC) at lag p, residuals are normally distributed, and no ARCH effects at lag q.

Table 2: Simulated and Bootstrapped Cointegration Tests

Model A	EV	Trace	Trace*	Frac95	p-value	p-value*	p-value**
$r = 0$	0.505	165.175	141.415	110.778	0.000	0.000	0.001
$r \leq 1$	0.421	106.182	89.132	82.413	0.000	0.013	0.022
$r \leq 2$	0.342	60.220	51.023	57.406	0.028	0.162	0.218
$r \leq 3$	0.202	25.055	20.109	36.438	0.451	0.755	0.779
$r \leq 4$	0.070	6.127	5.081	18.436	0.857	0.925	0.958
Model B	**EV**	**Trace**	**Trace***	**Frac95**	**p-value**	**p-value***	**p-value****
$r \leq 0$	0.503	136.625	116.972	65.768	0.000	0.000	0.000
$r \leq 1$	0.383	77.838	65.423	45.229	0.000	0.000	0.010
$r \leq 2$	0.291	37.301	31.362	28.170	0.003	0.021	0.159
$r \leq 3$	0.075	8.467	7.300	14.694	0.331	0.437	0.850
$r \leq 4$	0.023	1.916	1.128	3.720	0.162	0.284	0.551

Note: * refers to values obtained using simulated Bartlett-corrected test distributions, ** refers to values obtained using bootstrapped empirical distributions with 9,999 replications.

$r = 3$. However the system turned out to be much less stable than for $r = 2$, and we therefore stick to the more conservative choice of $r = 2$ also for this specification.

5 Tests for Long-run Exclusion and Stationarity of Linear Combinations

The long-run parameters of the model have neither been statistically nor economically identified so far. This requires the imposition of identifying restrictions. Not to impose arbitrary restrictions, but to narrow down two sensible long-run relationships, we systematically conduct a number of hypotheses tests on the stationarity of linear combinations of the variables based on economic priors.[11]

[11] This systematic approach to identify long-run relationships has previously been applied by Juselius and MacDonald (2004) to investigate international parity relationships between Germany and the US. For an application similar to ours, but with respect to the Danish monetary transmission mechanism see Juselius (2006). To our knowledge this approach has not yet been implemented to analyze US money demand and related long-run relationships – likely due to the direct modeling of the interest rate spread. Trivially, if the cointegration rank was set to one, a systematic approach was not necessary, because

Before we do so we formally check which specification of the deterministic components is acceptable/preferable.

Hypotheses tests with regard to the exclusion of variables from the cointegration tests can be conducted despite the model not being identified, since the imposed restrictions underlying these hypotheses are not identifying but nevertheless binding (see Juselius, 2006). First we test whether the specified restricted level shifts can be excluded from the long-run relationships and whether it is necessary to include a restricted trend in the cointegration relationships.

It turns out that DS1992Q1 and the restricted linear trend are individually ($\chi^2(2) = 5.33[0.07]$, respectively $\chi^2(2) = 3.22[0.20]$) and jointly ($\chi^2(4) = 5.63[0.23]$)[12] long-run excludable, which supports our previous notion that model B is more appropriate than model A. Due to these considerations and to save space the focus of our succeeding analysis is on model B.

The unrestricted coefficient estimates for $r = 2$ are:

$$\beta' x_t = \begin{bmatrix} 8.24 & -11.73 & 96.46 & -277.40 & 226.30 & -3.31 \\ 51.24 & -54.11 & 340.28 & -523.52 & -105.85 & -2.13 \end{bmatrix} \begin{bmatrix} m2_t \\ y_t \\ tb3_t \\ own_t \\ \Delta p_t \\ DS011_t \end{bmatrix}$$

Normalizing on Δp, respectively $m2$ we obtain:

$$\beta' x_t = \begin{bmatrix} 0.04 & -0.05 & 0.43 & -1.23 & 1 & -0.02 \\ 1 & -1.06 & 6.64 & -10.22 & -2.07 & -0.04 \end{bmatrix} \begin{bmatrix} m2_t \\ y_t \\ tb3_t \\ own_t \\ \Delta p_t \\ DS011_t \end{bmatrix}$$

with the respective adjustment coefficients (t-statistics in brackets):

the long-run parameters were directly identified.

[12] p-values in square brackets.

$$\alpha = \begin{pmatrix} -0.42 & -0.08 \\ (3.61) & (-2.98) \\ -0.04 & 0.03 \\ (-0.37) & (1.20) \\ 0.23 & -0.01 \\ (3.27) & (-0.33) \\ 0.04 & 0.02 \\ (1.85) & (4.05) \\ -0.62 & 0.08 \\ (-4.56) & (2.51) \end{pmatrix}$$

Based on the reported t-statistics we observe significant error correction towards equilibrium of at least two variables towards both long-run relationships thereby supporting our previous choice of $r = 2$. It is furthermore noteworthy that y does neither significantly adjust to the first, nor to the second long-run relationship, implying that it may be weakly exogenous. Therefore we test whether y does not adjust significantly to *either* of the cointegration relationships by imposing two restrictions on α. These restrictions are clearly accepted ($\chi^2(2) = 0.80$ [0.67]).[13]

We now test the long-run excludability of any of the remaining included variables (including the restricted shift dummy $DS011$ to check whether the model can be further 'simplified'), long-run homogeneity between m and y, and a long-run homogeneous interest rate spread. The only one of these restrictions which cannot be rejected is a long-run homogeneous relationship between m and y in both cointegration relations (for details see table 3).

While the previous test results are helpful to narrow down possible long-run relationships and possibly adjusting variables, they alone do not statistically and economically identify the model. To find a sensibly identified long-run structure of the model, we test whether certain linear combinations of the included endogenous variables are stationary.

A number of combinations could be expected to be stationary based on economic priors; among others: Money velocity (although that seems unlikely based on our previous visual assessment of the respective graph), a classic money demand function (involving m, y, one or several opportunity cost variables and possibly *own*), a monetary policy reaction function (involving the own rate of M2, which may be regarded as the closest substitute to a

[13] Based on this result we also set up a partial model (conditional on y). However, results do not qualitatively change, so that we stick to this model specification. It furthermore allows us to check whether y may even be strongly exogenous at a later stage of our analysis.

Table 3: Tests of Same Restriction on all Cointegration Relations

	Restriction	Interpretation	$\chi^2(2)$	p-value
H1	$\beta_{1\cdot} = 0$	m is l-r excludable	11.41	0.00
H2	$\beta_{2\cdot} = 0$	y is l-r excludable	12.17	0.00
H3	$\beta_{3\cdot} = 0$	*tb3* is l-r excludable	12.54	0.00
H4	$\beta_{4\cdot} = 0$	*own* is l-r excludable	15.84	0.00
H5	$\beta_{5\cdot} = 0$	Δp is l-r excludable	26.74	0.00
H6	$\beta_{6\cdot} = 0$	*DS011* is l-r excludable	9.48	0.01
H7	$\beta_{1\cdot} = -\beta_{2\cdot}$	l-r homog. betw m and y	3.73	**0.16**
H8	$\beta_{3\cdot} = -\beta_{4\cdot}$	l-r homog. betw *tb3* and *own*	19.38	0.00

Note: l-r means long-run. All test statistics have alternatively been computed with a Bartlett-correction. Results only change marginally, not qualitatively, and are therefore not reported here.

monetary policy instrument in our set of variables, y and/or Δp), a stationary interest rate spread (revealing efficient financial markets) and stationary ex-post real interest rates (with respect to *tb3* and/or *own*) as implied by the (long-run) Fisher effect.

Table 4 shows the test results for the respective underlying hypotheses together with a number of additional hypotheses tests.[14]

In contrast to the usually applied univariate unit root tests the null hypothesis is reversed here. Non-rejection of the null therefore implies stationarity of the linear combination in this case. We test in a two-step procedure. First we test whether the respective linear combination is stationary when we include the level-shift in 2001Q1 (i.e. the coefficient of *DS011* is left unrestricted). If we cannot reject the null of stationarity we repeat the respective test without the level-shift included.

Hypotheses 1 to 5 (*H1* to *H5*) refer to the stationarity of each of the variables individually (with level-shift included). In all cases we clearly reject the null meaning that *all* variables are integrated of order 1, i.e. I(1). The stationarity of money velocity (*H6*) is also clearly rejected. So are the null on the ex-post real tb3 interest rate (*H7*) and the ex-post real own interest rate (*H8*), if we impose a homogeneous relationship between each of the interest rates and Δp. However, once we relax the latter restriction stationarity of a linear combination between *own* and Δp can no longer be rejected. This

[14] A similar set of hypotheses tests is examined in Juselius (2006) to analyze the monetary transmission mechanism in Denmark. For a careful examination of the acceptability of imposed restrictions in an analysis of Euro area money demand see Bruggeman et al. (2003).

Table 4: Hypotheses Tests on Stationarity of Linear Combinations for r=2

	$m2$	y	$tb3$	own	Δp	$DS011$	$\chi^2(v)$	p-value
H1	1.000					−1.124	35.37 (3)	0.000
H2		1.000				−0.976	35.32 (3)	0.000
H3			1.000			0.042	16.40 (3)	0.000
H4				1.000		0.036	24.94 (3)	0.000
H5					1.000	0.004	26.19 (3)	0.000
H6	1.000	−1.000				−0.033	29.64 (3)	0.000
H7			1.000		−1.000	0.039	11.31 (3)	0.010
H8				1.000	−1.000	0.029	19.25 (3)	0.000
H9			1.000	−1.000		0.011	16.79 (3)	0.001
H10	1.000	−1.000	−8.980			−0.384	12.28 (2)	0.002
H11	1.000	−1.000		−6.119		−0.204	11.55 (2)	0.003
H12	1.000	−1.000			−10.523	−0.025	10.01 (2)	0.007
H13	1.000	−1.000	6.056	−10.492		−0.089	2.37 (1)	0.124
H14	1.000	−1.000	10.102	−13.037			9.98 (2)	0.007
H15		1.000			29.336	−0.263	23.97 (2)	0.000
H16		1.000		18.316		0.311	22.94 (2)	0.000
H17		1.000	32.331			1.042	15.77 (2)	0.000
H18		1.000		110.689	−186.628	2.819	1.22 (1)	0.270
H19		1.000	−27.774		58.573	−1.355	9.47 (1)	0.002
H20		1.000	−44.652	39.499		−0.928	10.98 (1)	0.000
H21			1.000		−1.437	0.039	10.49 (2)	0.005
H22			1.000	−0.689		0.020	12.30 (2)	0.002
H23			1.000	−17.435	29.393	−0.456	1.60 (1)	0.207
H24				1.000	−1.781	0.028	1.70 (2)	0.428
H25				1.000	−5.114		25.86 (3)	0.000

Note: The null hypothesis underlying each of the conducted tests is that the respective linear combination is stationary. A p-value larger than 0.05 therefore indicates that the variables are cointegrated at the 5% level.

does not hold for $tb3$ and Δp. In line with the previous visual inspection we do not find the interest rate spread between $tb3$ and *own* to be stationary. *H10* to *H14* refer to a number of possible money demand functions.[15] The only 'candidate' which turns out to be stationary is a linear combination of (inverse) money velocity, *own* and $tb3$ (*H13*). The signs are in line with our theoretical priors. It remains to be tested however, whether this linear combination really qualifies as a money demand function, which also depends on the short-run adjustment parameters. In the next section we will therefore test which of the variables correct(s) a long-run disequilibrium. It is furthermore important to note that stationarity of this linear combination can no longer be accepted once the level-shift is excluded.[16]

In *H15* to *H20* we check whether several linear combinations of real income, interest rate(s) and the inflation rate are stationary. A long-run relation between inflation and real income is unsurprisingly clearly rejected (*H15*). Only *H18* cannot be rejected, i.e. a linear combination of y, *own* and Δp is stationary. One may be tempted to interpret this as evidence in favor of an IS-schedule relating real income to the real interest rate. But first of all, the additionally imposed coefficient restriction that the coefficient of *own* is equal to minus the coefficient of Δp, which would facilitate the interpretation in terms of a stationary relationship between real income and the real ex-post interest rate, is clearly rejected ($\chi^2(2) = 15.12[0.00]$). Secondly, the coefficient estimates of the interest rate semi-elasticities are so large that the inclusion of real income in the linear combination may not be necessary to 'achieve' stationarity. Indeed, we find the linear combination of only *own* and Δp to be stationary with an even higher p-value (*H24*). Whereas the linear combination tested in *H18* is a reducible cointegration relationship, the combination tested in *H24* is an irreducible cointegration relationship (Davidson, 1998), meaning that excluding either of the variables would make the relation non-stationary (because *H4* and *H5* have been rejected). We furthermore find a linear combination of both interest rates and Δp to be stationary (*H23*). However, this relation again is not irreducible, but only the subset tested in *H24*.

Summarizing, we find evidence of two irreducible cointegration relationships, one of which might be interpretable as a money demand function. The number of irreducible cointegration relationship matches with our previous choice of the cointegration rank. We will interpret the stationary relationships more carefully after having imposed the respective overidentifying

[15] More specifically, we test whether money velocity together with Δp and/or $tb3$ and/or *own* is stationary, because the homogeneity restriction between m and y is imposed here.

[16] We furthermore observe that the coefficient estimates of *own* and *tb3* increase if the level shift is excluded (disregarding the obvious misspecification of the model in that case).

restrictions on β.

By imposing these restrictions we obtain the following overidentified long-run relations:

$$\beta' x_t = \begin{bmatrix} 0 & 0 & 0 & -0.55 & 1 & -0.02 \\ & & & (-12.30) & & (-10.22) \\ 1 & -1 & 6.06 & -10.49 & 0 & -0.09 \\ & & & (13.55) & & (-10.59) \end{bmatrix} \begin{bmatrix} m2_t \\ y_t \\ tb3_t \\ own_t \\ \Delta p_t \\ DS011_t \end{bmatrix},$$

which are clearly accepted, both according to the asymptotic p-value ($\chi^2(3) = 3.90[0.27]$) as well as the empirical p-value [0.49].

Money adjusts significantly to a deviation from long-run equilibrium supporting our interpretation of the second cointegration relationship as a money demand function. About 11% of a disequilibrium are corrected in the next period (keeping the other variables constant). Furthermore, signs are as predicted by theory. m is positively related to y, positively to *own*, and negatively to *tb3*. However, some things are noteworthy: First, the estimated interest semi-elasticities are comparably high, secondly, money demand does not react symmetrically to changes in both interest rates (the model is clearly rejected if this restriction is additionally imposed), and thirdly, (presumably) the burst of the stock-market bubble around 2000/2001 caused an upward level shift in money demand, i.e. an increased preference for holding more liquid assets. Carstensen (2006b) as well as Dreger and Wolters (2010) provide similar interpretations for a dummy variable included in Euro area money demand specifications around that time.

The interpretation of the other cointegration relationship is less obvious. It lends support to a long-run Fisher effect, because a nominal interest rate and the inflation rate are cointegrated. It does however not support the 'full Fisher effect' (Miskin, 1992) because Δp and *own* do not move one for one in the long-run. We furthermore see that the inflation rate exclusively adjusts to long-run disequilbria in this relation. A tentative explanation is that an increase in the *expected* inflation rate will push up the nominal interest rate causing a long-run disequilibrium between the nominal interest rate and the inflation rate, which is then corrected by an increase in the *actual* inflation rate. In this sense, *own* may qualify as a predictor of inflation as hypothesized by Fama (1975) with respect to short-term nominal interest rates in general. However, in contrast to the predictions by Fama, we do not find a unit long-run coefficient. Crowder and Hoffman (1996) provide an explanation for this.

They attribute coefficient estimates larger than 1 to the taxation of nominal interest income. Against their hypothesized value of 1.3 to 1.5 our point estimate of 1.8 still seems large, however.[17] But since we cannot reject a model in which the additional restriction that the respective coefficient value is equal to 1.5 ($\chi^2(4) = 7.57[0.11]$), our results are still consistent with their hypothesized range of suitable coefficient values once we take into account estimation uncertainty.[18]

6 Tests for Constancy of the Long-run Coefficients and Estimation Precision

Even if model B is well-specified according to the misspecification test results presented in table 1, it does not imply that the estimated long-run and short-run parameters are stable. In this section we will conduct a number of stability (or constancy) tests with regard to different estimated parameters. We will conduct three kinds of tests. First we will check whether the eigenvalues are stable, we will then check the constancy of the estimated long-run parameters. The stability of the short-run coefficients is examined in the next section.

We approach the tests in the specified order, because it can help us to narrow down the reasons for instabilities in the model in general. If the long-run parameters are instable, we would generally be more concerned as if the short-run effects are instable.

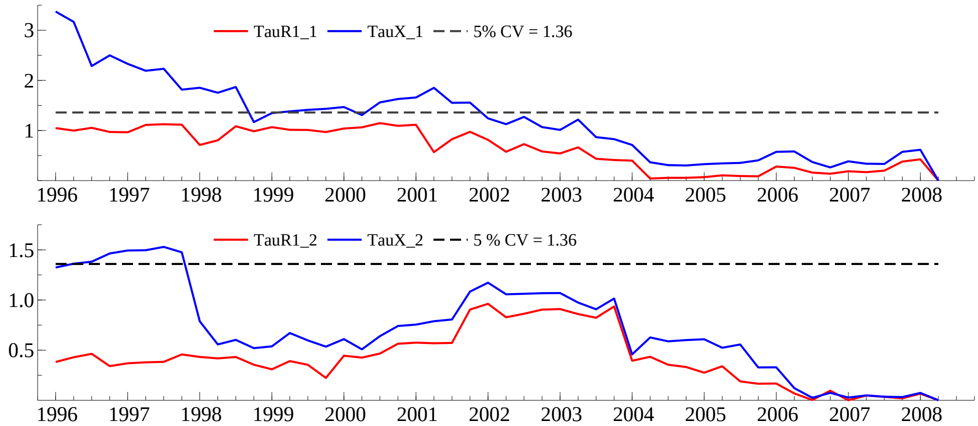
First we will conduct the eigenvalue fluctuation test. The eigenvalues are (quadratic) functions of both, α and β, so that a rejection of constant eigenvalues could (among others) imply instable α- and/or β- parameters. Figure 5 graphically depicts the recursively calculated (τ) test-statistics.

Whereas we observe non-constancies of the eigenvalues in the full model version (X-form), we do not so in the concentrated model version (R-form). This may either be due to a too short base sample, which leads to less precise estimates especially in the X-form (because there are more parameters to be estimated), or to non-constant short-run effects.

[17] In table 4 we have chosen another normalization, because Δp adjusts to disequilibria; normalizing the first cointegration relationship on *own* gives a point estimate of 1.8 for the coefficient of Δp.

[18] The lowest possible imposed coefficient value which does not lead to a rejection of the model (where all other previous restrictions remain imposed) at the 5% level is 1.43. This value would imply an average marginal tax rate of around 30% (since $\frac{1}{1-\tau} = 1.43$), where τ is the average marginal tax rate. For further details see Darby (1975).

Figure 5: Eigenvalue Fluctuation Test



Note: This figure depicts the recursively calculated τ-statistics and the 5% critical value, both for the X-form and the R-form. τ-statistics larger than the critical value imply rejection of the null hypothesis of constancy of the respective eigenvalue.

To check whether the estimated long-run parameters are stable, we apply the Nyblom-type test for parameter constancy by Hansen and Johansen (2002).[19] According to the test result, constancy of the long-run parameters cannot be rejected, neither based on the the asymptotic distribution (see figure 6 where the respective test-statistics are depicted), nor on the empirical distribution.

Figure 6: Nyblom-test of β-Constancy

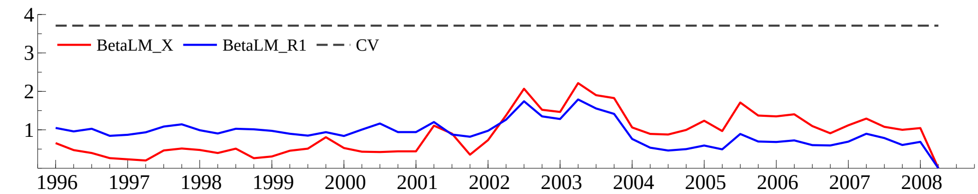
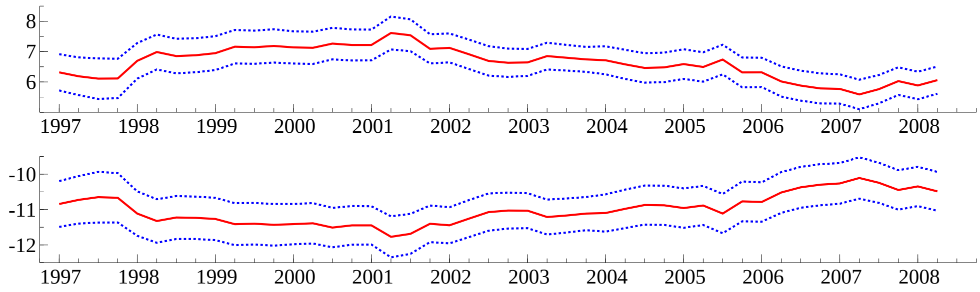


Figure 7 shows the recursively estimated money to interest semi-elasticities. In line with the results from the long-run stability test we observe that they are reasonably stable.

[19]See also Nyblom (1989).

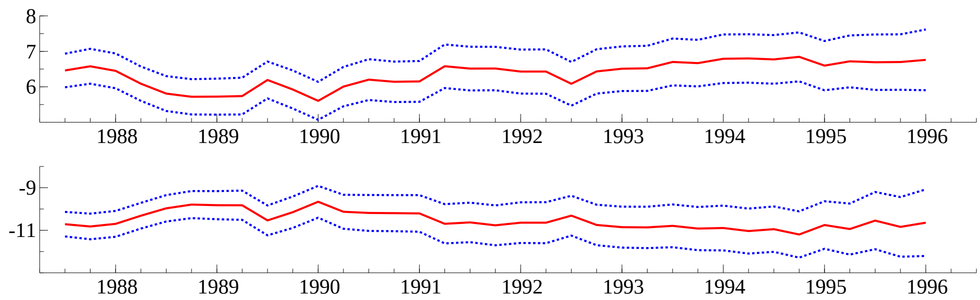
Figure 7: Recursively estimated Coefficient of *tb3* and *own* (based on concentrated form)



Note: The sample end for each recursive estimation is denoted on the horizontal axis, the respective estimated coefficient value on the vertical axis.

To assess the stability of the interest rate semi-elasticities over the period which is mostly held responsible for perceived instabilities of the money demand function (the early 1990s) we *backwards* recursively estimate the parameters over this period. Our base sample is 1996Q1 to 2008Q2. We then successively include an additional quarter at the beginning of the sample and re-estimate the model. The coefficient estimates again are reasonably stable (see figure 8).

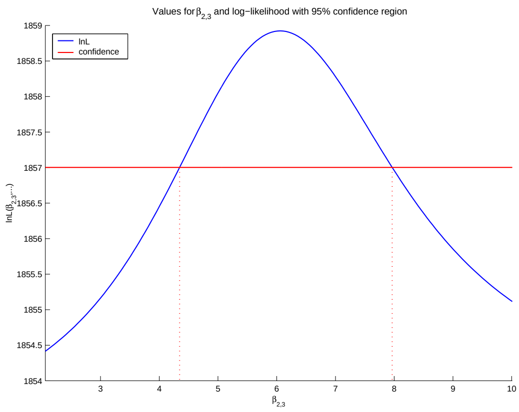
Figure 8: Backwards recursively estimated Coefficient of *tb3* and *own* (based on concentrated form)



Note: This figure should be read from right to left. Here we always estimate until 2008Q2 and subsequently add observations at the beginning of the sample.

While the long-run coefficients with respect to the interest rates seem to be quite stable over time, they are estimated very imprecisely - most likely due to the high collinearity among both rates.

Figure 9: Log-likelihood Values for Different Values of $\beta_{2,3}$ for Model B re-estimating all other Parameters



Note: A coefficient value outside the 95% confidence interval implies rejection of the null that the coefficient is equal to the hypothesised value at the 5% level.

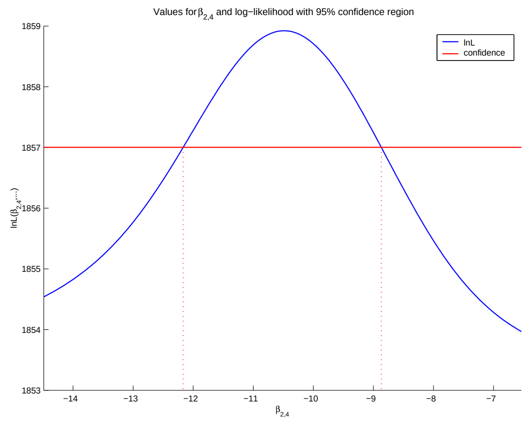
To illustrate, figure 9 and 10 depict log-likelihood values for specific values of interest rate semi-elasticities.[20] They are obtained by imposing the respective coefficient value and re-estimating all other coefficients. This gives a certain log-likelihood value, which is denoted on the vertical axis. All log-likelihood values outside the denoted 95%-confidence interval imply rejection of the imposed hypothesis at the 5% level according to a LR-test. We see clearly that the confidence intervals are huge.

Finally, figure 11 depicts various confidence ellipses for certain parameter values of $\beta_{2,3}$ (i.e. the long-run coefficient of *tb3*) and $\beta_{2,4}$ (the long-run coefficient of *own*). These confidence ellipses are based on values of the log-likelihood function for fixed values of the short rate *and* the own rate semi-elasticities re-estimating all other parameter values. Analogously to the interpretation for a single parameter restriction, the area outside the $X\%$ confidence *ellipse* (where $X = 80, 90, 95, 97.5, 99\%$) depicts value pairs of $\beta_{2,3}$ and $\beta_{2,4}$ for which a LR test would reject the null hypothesis that $\beta_{2,3}$ and $\beta_{2,4}$ have the hypothesised values at the $(100 - X)\%$ level of significance. We observe that the confidence intervals are quite large implying that the coefficients are estimated imprecisely. Additionally, the coefficient estimates are negatively correlated.

We now compare the stability of two competing models, model B and the

[20]These estimations have been conducted in S-VAR (version 0.43). Bruggeman et al. (2003) use this approach to investigate estimation uncertainty with respect to long-run coefficients for a Euro area money demand function.

Figure 10: Log-likelihood Values for Different Values of $\beta_{2,4}$ for Model B re-Estimating all other Parameters



Note: A coefficient value outside the 95% confidence interval implies rejection of the null that the coefficient is equal to the hypothesised value at the 5% level.

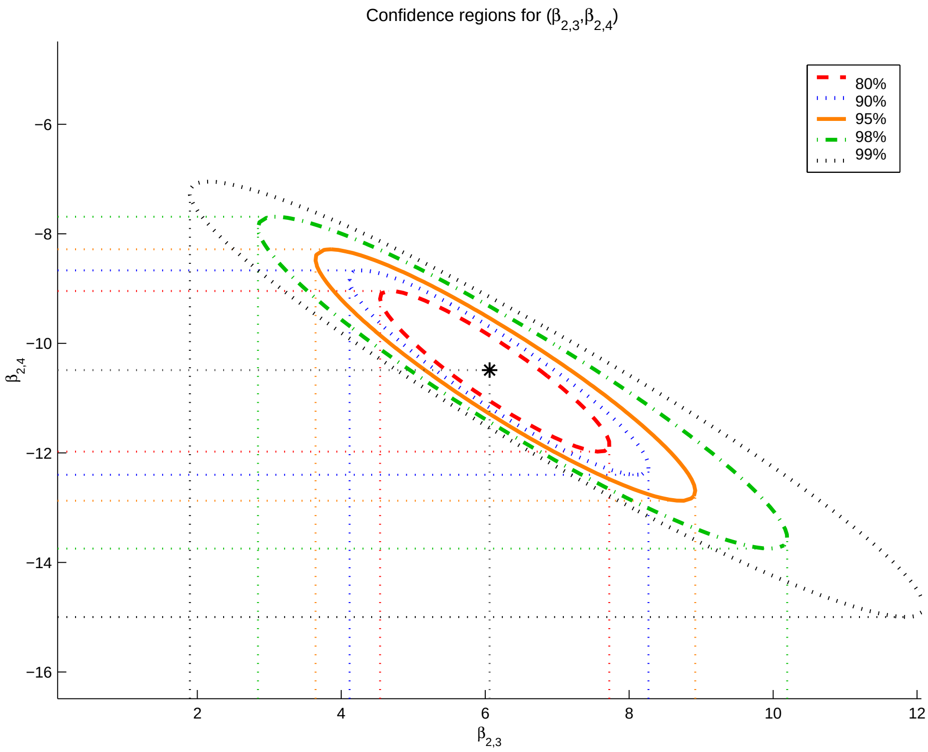
same model without the level shift in 2001Q1 (DS011) included (in figure 12 denoted as B1, respectively B2). The results are clear. While the recursively calculated likelihood ratios of our previous model are below the 5% critical value and therefore not rejected, the opposite is the case for the same model without the level-shift included (see figure 12) .[21]

Because most previous articles used some kind of a shift dummy for the period 1990Q1 to 1994Q4, we also considered such a specification. We re-specified the model to additionally include a restricted smooth-shift dummy as an exogenous variable. We define this dummy in the same way as Carlson et al. (2000), i.e. it takes the value 0 until 1989Q4, the value 1 from 1994Q4 onwards and linearly increases in between. It was designed in such a way to capture a period of financial innovation, which supposedly made M2 money demand instable in this period. Our results show that the inclusion of such a smooth-shift dummy is not necessary, at least not for our sample. For all suitable choices of the cointegration rank we find this restricted dummy to be clearly long-run excludable (for r=2: $\chi^2(2) = 2.65[0.27]$, for r=3: $\chi^2(3) = 4.65[0.20]$) and therefore prefer our more parsimonious specification.

Let us briefly summarize the main results of this section: First, we cannot reject the null that the long-run parameters are stable outside the base

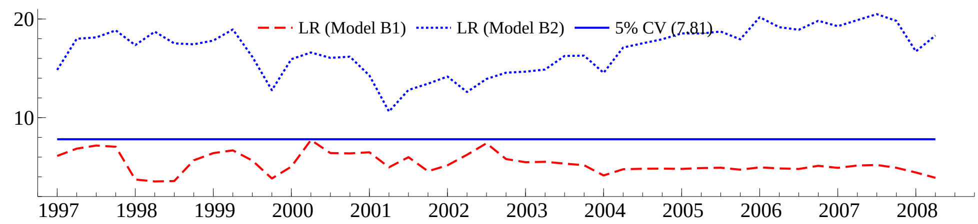
[21]That is a model where the coefficients of the level-shift are restricted to zero in both long-run relations. We also conducted backwards-recursive tests of the imposed restrictions. Results of the forward recursive exercise are matched by those of the backwards recursive exercise. Therefore we do not report the latter here, but they are available on request.

Figure 11: Various Confidence Intervals for $\beta_{2,3}$ and $\beta_{2,4}$ based on Log-Likelihood of Fixed Values of both Parameters for Model B re-estimating all other Parameters



Note: Confidence ellipses are based on values of the log-likelihood function for fixed values of the short rate and the own rate semi-elasticities re-estimating all other parameter values. The area outside the X% confidence ellipse (where $X = 80, 90, 95, 97.5, 99\%$) depicts value pairs of $\beta_{2,3}$ and $\beta_{2,4}$ for which a Likelihood ratio test would reject the null hypothesis that $\beta_{2,3}$ and $\beta_{2,4}$ have the hypothesised values at the $(100 - X)\%$ level of significance.

Figure 12: Recursive Likelihood Ratio Test of Restrictions: Model B1 vs. B2



Note: Based on concentrated form.

sample. Secondly, instable eigenvalues in the X-form as opposed to stable eigenvalues in the R-form suggest possible instabilities in the short-run effects. Thirdly, the estimated long-run coefficients are also reasonably stable in the period which has previously been made responsible for instabilities. Fourthly, a shift dummy such as the one considered by Carlson et al. (2000) is long-run excludable and does not significantly alter the results so that we abstain from including it into our model. Fifthly, an equilibrium shift in 2001Q1 on the other hand is necessary to obtain a stable model. At last, whereas the estimated interest semi-elasticities are reasonably stable, the estimation uncertainty is considerable, most likely due to the high collinearity among both series.

7 Short-run Stability and Identification

To check the stability of the short-run parameters we conduct the Ploberger-Krämer-Kronus fluctuation test.[22] Based on the asymptotic distribution we clearly have to reject stability of the short-run parameters in the 'inflation'- and in the 'own rate-equation'.[23] In this case, results obtained from bootstrapping are fundamentally different. The p-values from the respective empirical distribution are much more favorable in terms of constancy of the parameters. Whereas we have to reject constancy of the parameters in the 'own rate- and inflation-equation' at the 5, respectively 1% level based on the asymptotic distribution, the marginal significance levels increase to 32 (sup

[22]See Ploberger et al. (1989). Tests have been conducted in S-VAR (version 0.43). For careful stability analyses of Euro area money demand see Bruggeman et al. (2003) and Carstensen et al. (2009).

[23]More correctly, it is the 'change in inflation', respectively the 'change in the own rate'-equation, however our used terminology is less cumbersome.

S(13)=1.78) respectively 15% (sup S(13)=2.44) based on the bootstrapped distribution (using 999 parametric bootstraps) implying that we cannot reject constancy of either equation at the 5% level. All in all, we cannot rule out the possibility of non-constant short-run parameters, however. Additionally, the 'inflation-equation' seems to be the most likely candidate for having non-constant short-run parameters (with marginal significance levels of below 1, respectively 14.5%).

While the short-run dynamic structure of the model is formally identified, it is highly overparameterized, i.e. it contains many insignificant parameters. Using a general-to-specific approach we therefore subsequently delete insignificant variables from the simultaneous equations in a stepwise procedure. It is known that the sequence in which variables are excluded might affect which variables are retained in the final model. To increase transparency we use the same decision rule in each step. We always exclude the least significant variable (based on heteroscedasticity- and autocorrelation-consistent (HAC) standard errors) and check with a LR-test whether its exclusion is statistically acceptable. Before starting with this stepwise procedure, we first check whether the centered seasonal dummies can be jointly excluded from all equations. This restriction is not rejected at the 10 percent level, but at the 5 percent level ($\chi^2(15) = 23.39[0.08]$). In order not to be overly restrictive we decide not to exclude them. Table 5 and table 6 show two possible short-run representations for two distinct samples. Table 5 shows the most parsimonious short-run specification we obtain over the full sample (1987Q3 to 2008Q2) and over a restricted sample (from 1987Q3 to 2002Q4). The latter is presented here, because it will be used in the subsequent analysis. Table 6 shows another identified short-run structure, which is less restrictive. Here we left all variables included with a value of the t-statistics larger than 1.[24]

Because a detailed discussion of each of the components of the equations would consume too much space and would distract from the main message, we will only interpret the most notable findings or those with the closest impact on the aim of this study.

First of all, the lagged error correction terms ($EC1_{t-1}$ and $EC2_{t-1}$) matter for changes in all the variables except y irrespective of the sample period and the 'degree of restrictiveness'. This makes clear that modelling the system in first differences and disregarding the cointegrating relationships might have led to considerable bias due to omitted variables. Secondly, y is not only weakly, but strongly exogenous over the full sample. We do not want to over-

[24]Centered seasonal dummies are included in both specifications, but their coefficient estimates are not reported here.

Table 5: Parsimonious Short-Run Specifications

	1987Q3 - 2008Q2					1987Q3-2002Q4				
	$\Delta m2_t$	Δy_t	$\Delta tb3_t$	Δown_t	$\Delta^2 p_t$	$\Delta m2_t$	Δy_t	$\Delta tb3_t$	Δown_t	$\Delta^2 p_t$
$\Delta m2_{t-1}$	0.218	0	0.091	0.044	0	0.188	0	0.161	0.058	0
	(0.082)		(0.056)	(0.016)		(0.100)		(0.063)	(0.016)	
Δy_{t-1}	0	0.329	0.159	0.070	0	0	0.359	0.273	0.079	0
		(0.097)	(0.072)	(0.021)			(0.120)	(0.083)	(0.021)	
$\Delta tb3_{t-1}$	0	0	0.434	0.266	0	0	0	0.504	0.203	0
			(0.130)	(0.044)				(0.096)	(0.043)	
Δown_{t-1}	1.050	0	0	0.205	0	1.140	0	0	0.213	0
	(0.250)			(0.056)		(0.270)			(0.047)	
$\Delta^2 p_{t-1}$	0	0	0	0	0	0.190	0	-0.259	0	0
						(0.110)		(0.064)		
$ec1_{t-1}$	-0.157	0	0	0	-0.831	-0.419	0	0.328	0	-0.808
	(0.088)				(0.100)	(0.160)		(0.091)		(0.130)
$ec2_{t-1}$	-0.127	0	0.032	0.018	0.109	-0.132	0	0	0.025	0.088
	(0.018)		(0.017)	(0.005)	(0.021)	(0.022)			(0.005)	(0.021)
$const$	-0.087	0.005	0.021	0.012	0.082	-0.087	0.005	-0.005	0.016	0.066
	(0.013)	(0.001)	(0.013)	(0.004)	(0.016)	(0.015)	(0.001)	(0.001)	(0.004)	(0.015)
DUM921P$_t$	0	0.007	0	-0.004	-0.007	0	0.008	0	-0.004	-0.006
		(0.005)		(0.000)	(0.006)		0.005)		(0.001)	(0.005)
DIFC011$_t$	0.011	-0.008	-0.013	-0.002	0	0.012	-0.008	-0.014	-0.002	0
	(0.005)	(0.005)	(0.003)	(0.001)		(0.005)	(0.005)	(0.003)	(0.001)	
	$\chi^2(22) = 28.993(0.145)$					$\chi^2(20) = 17.645(0.6108)$				

Table 6: Less Restrictive Short-Run Specifications

	1987Q3 - 2008Q2					1987Q3-2002Q4				
	$\Delta m2_t$	Δy_t	$\Delta tb3_t$	Δown_t	$\Delta^2 p_t$	$\Delta m2_t$	Δy_t	$\Delta tb3_t$	Δown_t	$\Delta^2 p_t$
$\Delta m2_{t-1}$	0.426	0.065	0.052	0.036	-0.098	0.176	0	0.185	0.059	0
	(0.092)	(0.088)	(0.056)	(0.014)	(0.110)	(0.100)		(0.068)	(0.016)	
Δy_{t-1}	-0.289	0.231	0.172	0.081	0	0	0.279	0.251	0.081	0
	(0.120)	(0.120)	(0.076)	(0.020)			(0.120)	(0.083)	(0.021)	
$\Delta tb3_{t-1}$	0	0	0.557	0.244	0.390	0	0	0.322	0.179	0
			(0.150)	(0.042)	(0.310)			(0.160)	(0.046)	
Δown_{t-1}	1.260	0	0	0.177	-0.518	1.210	0	0	0.236	-0.644
	(0.290)			(0.054)	(0.390)	(0.280)			(0.048)	(0.310)
$\Delta^2 p_{t-1}$	0	0	-0.051	0	0	0.290	0.304	-0.162	0	0
			(0.045)			(0.120)	(0.120)	(0.073)		
$ec1_{t-1}$	-0.114	0	0	0	-0.809	-0.526	-0.322	0.217	0	-0.821
	(0.089)				(0.110)	(0.170)	(0.150)	(0.100)		(0.130)
$ec2_{t-1}$	-0.096	0.018	0.014	0.020	0.057	-0.126	0.034	0.037	0.027	0.114
	(0.021)	(0.017)	(0.018)	(0.005)	(0.036)	(0.023)	(0.022)	(0.023)	(0.006)	(0.024)
$const$	-0.061	0.018	0.008	0.013	0.045	-0.082	0.031	0.022	0.018	0.084
	(0.015)	(0.012)	(0.013)	(0.003)	(0.025)	(0.016)	(0.016)	(0.016)	(0.004)	(0.017)
$DUM921P_t$	0	0.008	0	-0.004	-0.008	0	0.010	0	-0.004	-0.008
		(0.005)		(0.001)	(0.006)		(0.005)		(0.001)	(0.005)
$DIFC011_t$	0.008	-0.009	-0.013	-0.002	0	0.012	-0.008	-0.014	-0.002	0
	(0.005)	(0.005)	(0.003)	(0.001)		(0.005)	(0.005)	(0.003)	(0.001)	
	$\chi^2(15) = 19.880(0.1766)$					$\chi^2(15) = 3.2463(0.9993)$				

interpret this finding, because our model is necessarily just a partial model such that relevant interdependencies that 'endogenize' y are probably simply not modeled.[25]

However, if we impose less restrictions on the model and retain all variables with t-statistics larger than 1, we find weak evidence for a short-run Phillips curve relationship, since Δp enters the y-equation positively and significantly at the 10% level.[26] This also implies that y is only weakly (and not strongly) exogenous over the restricted sample, but *only* in the case where less restrictions are imposed.[27]

Particularly interesting is the outcome that money overhang enters the inflation-equation significantly and with the predicted positive sign, implying that positive (negative) money overhang 'leads to' an increase (decreases) of the inflation rate (even) in the short-run.[28] Not surprisingly, inflation also adjusts to the other error correction term. It is furthermore noteworthy that in the parsimonious representation inflation only reacts significantly to both error correction terms and to none of the other variables included in the model. With regard to the 'money-equation' two things are noteworthy. First, the adjustment speed of money to long-run money demand disequilibria is very close to the adjustment speed in the initial model (it is only very slightly faster). Secondly, money also adjusts to disequilbria in the second long-run relationship. If we follow our previous tentative interpretation, short-run money demand increases if there is an increase in the expected inflation rate.

8 Forecasting (Changes in) US Inflation

8.1 Previous Empirical Evidence

A large number of studies examine the leading indicator properties of various macroeconomic indicators for forecasting inflation in the US (see Stock and

[25]Our model does not include the long-term bond yield which may play a crucial role in the monetary transmission mechanism. However, unsurprisingly, additionally including the long-term bond yield as a third interest rate made our estimates even less precise.

[26]We choose such a low t-value to make it more likely that all relevant variables remain included in the model. It clearly comes at the cost that also the number of irrelevant variables increases compared to our approach where a t-ratio of 1.96 is used as a selection criterion.

[27]Excluding y would however also be statistically acceptable according to the LR-test result in this case, albeit with a much smaller p-value than over the full sample.

[28]We are aware of the fact that temporal order does not imply causality, but nevertheless choose to use this 'sloppy language' instead of a technically more correct but often less intuitive language.

Watson, 1999, and the references therein). Stock and Watson themselves also provide the most extensive empirical study about forecasting US inflation. Among others, they examine the leading indicator properties of what they call a 'generalized version of the Phillips curve' in which an index of aggregate economic activity (composed of 61 real economic indicators) instead of unemployment is related to inflation. Howeverm they do only consider forecasts for the 12-month horizon. For this specific horizon their results point towards the superiority of their generalized Phillips-curve compared to other indicators derived from other frameworks such as monetary theories of inflation and the term structure.

Superficially, their results suggest that monetary indicators are not useful for predicting inflation. However, first it is important to note that Stock and Watson only consider a 12-month horizon which may not be the forecast horizon in which monetary indicators are useful, and secondly, they only consider two simple monetary indicators: money growth and the change in money growth. They do not consider more elaborate monetary indicators such as money overhang. With regard to the Euro area Nicoletti-Altimari (2001) and Carstensen et al. (2009) find evidence that money overhang significantly improves inflation forecasts over longer horizons of about one to two years.

In a more recent article, Stock and Watson (2007) emphasize that '*inflation has become harder to forecast, at least, it has become much more difficult for an inflation forecaster to provide value added beyond a univariate model*'.[29] They also acknowledge that the relative performance of the Phillips curve forecasts deteriorated sharply from their first to their second sample (1970Q1 to 1983Q4, respectively 1984Q1 to 2004Q4). Similarly, Orphanides and van Norden (2002) as well as Atkeson and Ohanian (2001) conclude that it has become difficult to beat inflation forecasts obtained from univariate models since the mid-80s.

However, there is also some evidence suggesting that money can be used as an information variable with regard to predicting inflation. Using standard as well as Bayesian VAR models Berger et al. (2008) find that the inclusion of money growth to their model significantly improves out-of-sample inflation forecasts, albeit the benefits are quantitatively small. Estrella and Mishkin (1997) are more skeptical. They set up a trivariate VAR in first differences containing nominal income growth, inflation and growth in M2. Over a sample from 1979 to 1995 inflation and nominal income growth Granger-cause money growth, while the opposite does not hold. Because the former model may be misspecified if the variables are cointegrated, they additionally set up

[29] Their univariate model is an MA(1)-model with time-varying parameters.

a bivariate CVAR model composed of M2 velocity and $tb3$ and augment the previous nominal growth equation with the obtained error correction term.[30] Since the joint significance level of the model does not increase, they conclude that there is no role for money as an information variable. Using predictive regressions D'Agostino and Surico (2009) find that US money growth does not provide marginal information content for predicting inflation over various horizons compared to univariate inflation forecasts.[31] This is consistent with our previous findings, because $\Delta m2_{t-1}$ was clearly insignificant in the $\Delta^2 p_t$-equation. However, we found evidence that *money overhang* Granger-causes inflation, which may be interpreted as preliminary evidence for the hypothesis that a more sophisticated monetary indicator such as money overhang is a more suitable predictor of (changes in) inflation than the growth rate of a monetary aggregate.

The two analyses which are methodologically most closely related to our forecasting exercise are Carlson et al. (2000) and Orphanides and Porter (2000). Carlson et al. isolate nominal income from their estimated long-run relation for $m2$ and calculate the difference between actual nominal income and the derived measure of equilibrium nominal income.[32] This term and its lags are added to a separate nominal income growth equation together with 6 or 9 lags of opportunity cost changes, nominal money growth, and (in some cases) the inflation rate. To check whether nominal money growth and the error correction term help to predict changes in nominal income growth they test whether they are jointly significant with a simple F-test. Their results are supportive.[33] However, in-sample significance (or non-excludability) does not imply that the variables also have predictive content out-of-sample. By only considering the in-sample properties the risk of 'overfitting' is not negligible. We will therefore assess both the in-sample and the (pseudo) out-of-sample predictive content of our derived money overhang measures.

Based on Hallman et al. (1991) Orphanides and Porter (2000) provide evidence that a P*model[34] based on recursively estimated (mean shifting)

[30]The existence of an error correction term implies evidence in favor of cointegration. This result is notable against the background of other published results covering this period, but it is unfortunately not further commented on by the authors.

[31]Only at the 4-quarter horizon there is some minor improvement. They however find that their measure of global liquidity helps to provide significantly better inflation forecasts at forecast horizons beyond one year.

[32]The cointegration vector includes real income. They calculate the natural log of nominal income by adding the natural log of the price index to the natural log of real income after having estimated the model with real values.

[33]As they do neither present any model diagnostics nor provide HAC standard errors it is not sure whether their test results are reliable.

[34]In this model $P^*(= MV^*/Q^*)$ is the equilibrium level of prices supported by the

equilibrium M2 money velocity helps to predict inflation in a real-time setting.

In summary, previous evidence in favor of monetary indicators providing useful (additional) information for predicting inflation is weak. Bringing to mind the results from Stock and Watson (2007) this is however not restricted to monetary indicators, but valid for nearly all macroeconomic indicators. Also the performance of previously successful multivariate models based on the Phillips curve deteriorated a lot. Since the mid-80s it has become hard for 'theory-augmented' multivariate models to improve on the performance of 'atheoretical' univariate models.

8.2 Forecast Methodology and Preliminary Considerations

In this section we will address several questions related to the usefulness of money overhang and our estimated model in general to predict changes in inflation. Against the background of the results by Stock and Watson (2007) outlined above we should not expect any 'miracles' in the sense of vast improvements in forecasting performance compared to univariate approaches. However, in contrast to the univariate forecasts, our forecasts are not purely atheroretical, but are based on (in most cases) sensible short- and long-run relationships among the variables, which have been identified in the previous sections.

We will address two fundamental questions in this section. First, we check how various VECM specifications perform in a (pseudo) out-of-sample forecasting exercise - compared to one another and compared to a univariate model (keeping in mind that the univariate benchmark is a difficult one).[35] Secondly, we will examine whether adding money overhang measures to a univariate model improves its out-of-sample forecasting performance over various horizons.

All results presented have in common that only information was used for the forecast, which would have been available to a forecaster at the respective forecast origin. Because it is sensible to assume that a forecaster would make use of as recent information as possible, many of the models are recursively (re-)estimated.

quantity of money in circulation M (Q^* is an estimate of real potential output, V^* an estimate of equilibrium velocity). If the actual price level P is lower than P^* it is expected that prices will increase. This is the idea underlying the inflation forecasts.

[35]It is called 'pseudo', because we created a hold-back period, for which we know the realized values of inflation, but not the forecaster at the forecast origin.

Before we address the question of how money overhang performs as an information variable, we will check the out-of-sample performance of various VECM specifications. Surely, these questions are not isolated from each other, because one of the long-run relationships we identified has been interpreted as a money demand function, and money overhang shown to Granger-cause inflation - both, over the full sample and over a restricted sample until 2002Q4. However, it is hard to assess how much of the model's predictive power can be attributed to the respective error correction term. Theoretically, one could compare the forecasting performance of the VECM with and without money overhang in the 'inflation equation', but since the exclusion of money overhang from the 'inflation-equation' is clearly rejected according to the LR-test result, we would essentially compare the forecasting performance of a well-specified to the performance of a clearly misspecified model. We abstain from doing so. Instead, the question of 'how much' money overhang may contribute to inflation forecasts will be addressed in a single-equation framework, which is outlined in section 8.4.

Before presenting the results of the forecasting exercises, we will shortly explain how the VECM can be used for (multi-step) forecasting.

8.3 VECM Forecasts of Changes in Inflation

After estimation of the CVAR model until the respective forecast origin and having imposed the long-run restrictions the I(1) system is 'mapped' to a stationary simultaneous equations system, on which possibly short-run restrictions are imposed. This model can be easily used to forecast changes in the inflation rate (and thereby also the inflation rate itself) *one* period ahead. All information we need for this forecast is directly observable at time t. We only need to 'plug in' the respective observed values into the 'change in inflation'-equation. *Dynamic* forecasts more than one period ahead are more difficult to obtain however, because this also requires forecasts of future long-run disequilibria. Put simply, the error correction terms need to be 'endogenized' to obtain fully dynamic multi-step ahead forecasts. This is achieved by defining the error correction terms as identities.[36] If we recursively re-estimate the model this procedure is further aggravated, because the identities have to be be redefined after each recursion.

To make this more clear, consider the case of a two-step ahead forecast of the change in inflation conducted in 2002Q4 as an example.

The first step in obtaining a two-step forecast is the estimation of the CVAR model until 2002Q4, followed by the imposition of sensible long-run

[36] See Hendry (1995).

restrictions. A sensible long-run identified structure which could have been obtained after estimation until 2002Q4 is:

$$\beta' x_t = \begin{bmatrix} 0 & 0 & 0 & -0.53 & 1 & -0.01 \\ & & & (-15.58) & & (-5.60) \\ 1 & -1 & 6.92 & -11.26 & 0 & -0.07 \\ & & (16.13) & (24.29) & & (-6.84) \end{bmatrix} \begin{bmatrix} m2_t \\ y_t \\ tb3_t \\ own_t \\ \Delta p_t \\ DS011_t \end{bmatrix}, \chi^2(3) = 0.38$$

The long-run estimates are close to the ones obtained over the full sample, and the same long-run restrictions are easily accepted according to the LR-test.[37]

The parsimonious short-run specification which has been presented in table 5 (columns: 1987Q3 to 2002Q4) has been obtained with regard to these long-run estimates and can now easily be used to forecast the change in inflation over the next quarter:

$$\Delta^2 p_{t+1} = -0.81 EC1_t + 0.09 EC2_t + deterministics \tag{2}$$

Forecasting the change in inflation two periods ahead is more difficult. A two-step ahead forecast of the change in the inflation ratio is obtained from

$$\Delta^2 p_{t+2} = -0.81 EC1_{t+1} + 0.09 EC2_{t+1} + deterministics \tag{3}$$

To get a forecast of $\Delta^2 p_{t+2}$ requires forecasts of $EC1_{t+1}$ and $EC2_{t+1}$. Future values of the error correction terms are obtained from the following two identities:

$$EC_{1,t+1} \equiv EC_{1,t} - 0.53 \Delta own_{t+1} + \Delta\left(\Delta p_{t+1}\right) \tag{4}$$

$$EC2_{t+1} \equiv EC2_t + \Delta m_{t+1} - \Delta y_{t+1} + 6.92 \Delta tb3_{t+1} - 11.27 \Delta own_{t+1} \tag{5}$$

All we need to forecast future long-run disequilbria are the current values of the long-run disequilibria and the forecasts of the changes in the other endogenous variables.

We can iterate forward until we obtain forecasts for the designated fore-

[37] Apart from the statistical acceptability of these restrictions and the long-run structure being the 'most sensible' one, we would have arrived at the same long-run restrictions if we repeated the steps of the analysis over the full-sample above over the restricted sample.

cast horizon. We have also shown these equations to point towards a likely reason for poor multi-step ahead forecasts: the instability of short-run parameters. Multi-step forecasts are obtained via iteration so that forecast errors can be propagated through the system, because 'everything depends on everything'. We saw in the previous section that short-run stability is doubtable in at least one of the equations, which just turns out to be the 'inflation-equation'. To deal with the problem of possibly instable parameters, we forecast 1-period ahead changes in inflation with a number of different approaches, which will be further explained below. Some of them may be better suited to cope with supposed instabilities and thereby improve the forecast accuracy of the model.

We use five different approaches to obtain forecasts of the change in the inflation rate over the next quarter. The period for which forecasts are obtained is 2003Q1 to 2008Q2, so that we get 22 forecasted values with each approach. These forecasted values can be compared to the realized values to calculate the forecast errors, which can be then used to calculate measures of forecast accuracy. We will report the relative mean squared forecast error (MSFE) of each of the models with respect to each other model. We now describe the five approaches we use to obtain the forecasts. They differ with regard to whether the long-run coefficients are recursively re-estimated, whether the short-run coefficients are recursively re-estimated, and whether short-run restrictions are (re-)imposed. In all of the models we impose sensible long-run restrictions.[38]

The five approaches we use are:

- *VECM1*: The CVAR model is estimated once until 2002Q4, no short-run restrictions are imposed. Coefficients are fixed for all forecast origins, the model is not re-estimated.

- *VECM2*: As VECM1, but short-run restrictions are imposed.

- *VECM3*: As VECM2, but the short-run coefficients are recursively re-estimated.

- *VECM4*: Long-run coefficients and short-run coefficients are recursively re-estimated, no short-run restrictions are imposed.

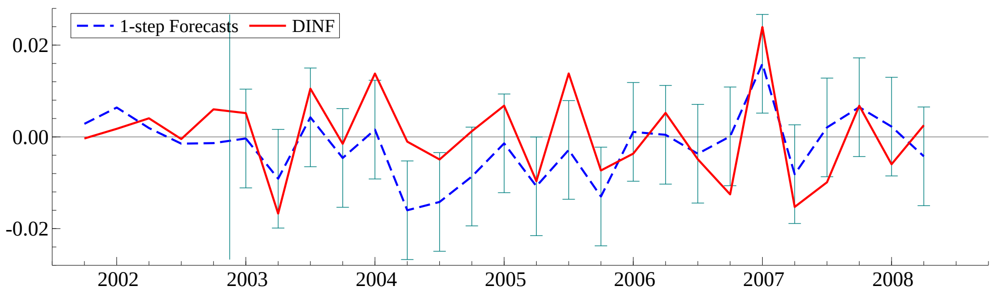
[38]While forecasting based on recursively estimated single-equation models and of *stationary* VARs is commonly applied in the literature (see for instance Stock and Watson, 1999, Nicoletti-Altimari, 2001, and Hubrich, 2005), to our knowledge such an exhaustive comparison of the forecast accuracy of models which differ with regard to the re-estimation of long-and/or short-run parameters and the (re-)imposition of short-run restrictions is unprecedented.

- *VECM5*: As VECM4, but short-run restrictions are imposed.

We compare the forecast performance of *VECM1* and *VECM2* to check whether a more parsimonious model (where short-run restrictions have been imposed) leads to better out-of-sample forecasts, which seems likely, since VECM1 is severely overparameterized. *VECM2* and *VECM3* are compared to evaulate whether recursive updating of the short-run coefficients can improve the forecast performance. This may be the case if the short-run coefficients are instable, which is less likely to negatively impact on the forecast accuracy if the coefficients are continuously updated (but surely, it may still occur). The relative forecasting performance of VECM4 compared to the other models will help us to assess whether it is worth the effort to recursively re-estimate the model in each forecast origin. Finally, the relative performance of VECM5 may give us hints whether the complete re-estimation of the model in each period together with the imposition of sensible short-run restrictions further improves the forecast performance.

Figure 13 exemplarily shows the one-step ahead forecasts of the change in the annualized quarterly inflation rate from 2003Q1 to 2008Q2 obtained from VECM2. We observe that the size of the forecast errors is quite large, but the direction of change of the inflation rate is predicted quite well.

Figure 13: 1-step Ahead Forecasts of the Change of the Inflation Rate



Note: 1-period ahead forecasts for the change of the inflation rate based on the CVAR model estimated until 2002Q4. The same long-run restrictions have been imposed as over the full sample. It should be noted that this long-run *structure* could have equally been obtained over this restricted sample (of course, coefficient estimates (slightly) differ). Short-run restrictions are imposed as in table 5 (restricted sample).

Generally, the results from this forecasting exercise (see table 7) are in line with our previous expectations. First, the imposition of short-run restrictions improves the forecast performance compared to a model where such restrictions have not been imposed (see the relative MSFE of VECM2 vs. VECM1,

Table 7: Relative MSFE for Various VECM vs. MA(1)

	VECM1	VECM2	VECM3	VECM4	VECM5	MA(1)
VECM1	1.00	0.95	0.96	0.95	0.91	0.89
VECM2	1.05	1.00	1.01	1.00	0.96	0.94
VECM3	1.04	0.99	1.00	0.99	0.95	0.93
VECM4	1.05	1.00	1.01	1.00	0.96	0.94
VECM5	1.10	1.04	1.06	1.04	1.00	0.98
MA(1)	1.12	1.07	1.08	1.07	1.02	1.00

Note: The VECM differ in three aspects. 1. whether the long-run coefficients are recursively re-estimated (Y or N), 2. whether the short-run-coefficients are recursively re-estimated (Y or N), and 3. whether short-run-restrictions are imposed (Y or N). VECM1: N,N,N; VECM2: N,N,Y; VECM3: N,Y,Y; VECM4: Y,Y,N; VECM5: Y,Y,Y.

respectively VECM5 vs. VECM4), most likely due to overparameterisation of the latter model. Secondly, the recursive re-estimation of the short-run parameters does not have a large impact on the forecast performance (VECM3 vs. VECM2). Thirdly, the recursive re-estimation of the complete model improves the forecasts, but only if short-run restrictions are imposed after each recursive estimation. Lastly, none of the models can beat a simple MA(1) process, but VECM5 comes very close and achieves about the same forecast performance (with a relative MSFE of 1.02 compared to an MA(1)). Overall, we could say that the benefits of the tedious updating procedure are small, but notable (with a relative MSFE of 1.10 of the 'nothing-updated' model, VECM1, compared to the 'everything-updated'-model, VECM5). One might ask what the benefit is of estimating a complex CVAR model, recursively re-estimating it every period, and recursively re-imposing restrictions every period, to finally obtain forecasts which are about as good as those obtained from one of the simplest possible univariate models. We think the benefit is clearly that the model is not purely atheroretical, but based on two sensible economic long-run relationships (a money demand function and a long-run Fisher equation), towards which variables (among others the inflation rate) adjust in case of long-run disequilibria. This means we do not just provide a forecast of the change in the inflation rate (put simply: a number), but also some economic rationale for it.

We also considered using the VECM for performing multi-period forecasts. How such forecasts can be obtained from the VECM was carefully explained above. However, multi-period forecasts turned out to be relatively bad, most likely due to the propagation of forecast errors. We furthermore think that the forecast errors are most likely due to the relatively bad forecasts of y, whose change is solely explained by its own lagged change. Re-estimation of the model conditional on y, i.e. setting up a partial model, and separately forecasting y with the help of another partial model would certainly be worth trying, but is left for further research.[39]

8.4 Does Money Overhang Improve Inflation Forecasts? Evidence from Predictive Regressions

We now turn to the question whether our derived measures of money overhang help forecast (changes in) inflation over various forecast horizons. We have already presented some supporting evidence (money overhang Granger-causing inflation). To assess whether money overhang improves forecasts of (changes in) inflation over various horizons we use predictive regressions as proposed by Stock and Watson (1999). Because it is still an ongoing discussion whether US inflation is $I(1)$ or $I(0)$ (more specifically, trend-stationary), we do not only assess money overhang's information content with regard to predicting changes in inflation, but also levels in inflation. In doing so we follow Nicoletti-Altimari (2001) who faces similar uncertainties with regard to the order of integration of Euro area inflation and opts for conducting both kinds of forecasts as well.

If inflation is really I(1) (as it has been tested inside the CVAR model), the stationary money overhang measures can plausibly only explain changes in inflation, because the equation would be unbalanced otherwise. If the inflation rate is best characterized as being stationary on the other hand, money overhang may be a useful indicator to predict the level of the inflation rate.

Following Stock and Watson (1999) the forecasting equations are specified as

$$\pi_{t+h}^{h} - \pi_t = a + b\,(L)\,\Delta\pi_t + c\,(L)\,ov_t + \varepsilon_{t+h}, \tag{6}$$

respectively

$$\pi_{t+h}^{h} = a + b\,(L)\,\pi_t + c\,(L)\,ov_t + \varepsilon_{t+h}, \tag{7}$$

[39] We also set up the respective partial model, which turned out to be even slightly more stable than our presented model, but as the long-run parameters almost did not change, the estimation results are not reported here (but can be obtained from the author upon request). However, we did not check how the muli-period forecasts are affected if y is forecasted separately.

where $\pi_t^h = \frac{400}{h} \ln\left(\frac{P_t}{P_{t-h}}\right)$ is the h-period annualized inflation rate, $\pi_t \equiv \pi_t^1 = 400 \ln\left(\frac{P_t}{P_{t-1}}\right)$ is the annualized quarterly inflation rate (i.e. $\pi_t = 100\Delta p_t$), ov_t is money overhang in period t, and L the lag operator which is set to 4, because data is quarterly.[40]

To assess whether money overhang improves the forecast accuracy compared to a model where money overhang is not considered, we estimate both regressions in two different ways. First we estimate both regressions without restrictions so that all money overhang measures are included. Then we estimate both regressions with the money overhang measures excluded (i.e. we impose the restrictions that all c-coefficients are equal to zero. If money overhang helps predict (changes in) inflation over the respective horizon, the former specification should provide more accurate forecasts.

We will provide $h = 1, 2, 4, 8$ and 12 quarter ahead forecasts of the respective dependent variable. The predictive regressions are recursively estimated over the sample 2002Q4$-h$ to 2008Q2$-2h$ and based on the estimated coefficients we obtain forecasts for the period 2002Q4+h to 2008Q2. For h=1, 2, 4, 8, 12 we therefore obtain 22, 21, 19, 15, respectively 11 forecasted values of the annualized (change in the) inflation rate for each of the models. These in turn can be used to calculate the respective mean squared forecast error (MSFE) for each of the models.

To illustrate the forecasting procedure, let us consider forecasts for the 8-period ahead annualized change in the inflation rate as an example. In order to conduct forecasts we need to estimate the regressors' coefficients first. We therefore estimate equation 1.6 over the sample 1987Q3 to 2000Q4. This regression equation contains the 8-period lead of the 8-quarter annualized change in the inflation rate as the dependent variable so that we implicitly use data from 2002Q4, which is only observable at that time. After having obtained the coefficient estimates we 'move to' 2002Q4 and calculate the fitted value of the 8-period lead of the 8-period annualized change in the inflation rate based on the previously estimated regression coefficients. The fitted value of the 8-period lead of the 8-quarter annualized change in the inflation rate in 2002Q4 is our forecast of the 8-quarter annualized change in the inflation rate from 2002Q4 to 2004Q4. Then we add one quarter to the estimation sample and re-estimate equation 1.6 from 1987Q3 to 2001Q1. This gives us the forecast for 2005Q1, and so on. The last parameter estimation sample is 1987Q3 to 2004Q2 (=2008Q2-2h), which gives the forecast for 2008Q2. Any later base sample would produce forecasts outside our hold-back period.

[40] In the specification of Stock and Watson the unemployment rate is used instead of money overhang.

One more question we need to deal with is which money overhang measures should be used when estimating equation 1.6. In contrast to the VECM approach where money overhang measures were endogenously determined, they are exogenous here. The long-run coefficients of the CVAR model estimated over the full sample (ending 2008Q2) were not known to a forecaster in 2002Q4, where all other information needed to run the first predictive regression from 1987Q3 to 2000Q4 is observable. Because the forecaster uses information from 2002Q4 to calculate the 8-period lead of the change in the inflation rate, it is sensible to assume that he also estimates the CVAR model from 1987Q3 to 2002Q4 to obtain estimates of money overhang in 2000Q4 (and previous quarters). He thereby uses all available information.

The next question concerns whether the coefficient estimates of the CVAR to derive the money overhang measures are recursively updated or not. For h=8, 12 we performed both, predictive regressions with fixed coefficient values to calculate the money overhang measures (those obtained from estimating the CVAR until 2002Q4), as well as with recursively updated ones for which the CVAR has been re-estimated until the respective forecast origin. In line with the results obtained from the long-run stability analysis, the influence of this updating procedure turned out to be negligible. We therefore only report the forecast results based on money overhang measures derived from CVAR estimates obtained until 2002Q4.

In table 8 we present the results from our forecasting exercise. The results are in line with our expectations and consistent with our 'classification' of inflation as I(1). First, we observe that adding money overhang measures to the univariate model does not improve forecasts of the annualized inflation rate. This was expected because a stationary variable (money overhang) should not help forecast a nonstationary variable (inflation rate). Secondly, at short horizons money does not seem to provide useful information for predicting changes in the inflation rate. However, at long horizons the money-overhang-augmented models have smaller MSFE than the univariate models. When predicting the change of the annualized quarterly inflation rate over the next three years, including money overhang measures to the univariate model reduces the mean squared forecast error by about 20%. To examine whether the forecasting accuracy of the augmented model is *significantly* better, we calculate the Clark and McCracken (2001) forecast encompassing statistic, which is obtained as:

$$ENC - NEW = P\frac{P^{-1}\sum_{t=R}^{T}\left(fe_{1,t+1}^2 - fe_{1,t+1}\cdot fe_{2,t+1}\right)}{P^{-1}\sum_{t=R}^{T} fe_{2,t+1}^2},$$

where P is the number of forecasts, R is the number of observations in the

Table 8: Relative Forecasting Performance Augmented vs. Univariate Model

h-period change of inflation rate					
Forecast horizon (h=)	1	2	4	8	12
Number of Forecasts	22	21	19	15	11
MSFE1 (w/ OV)	0.12	0.16	0.33	0.61	0.64
MSFE2 (w/o OV)	0.12	0.16	0.33	0.65	0.77
MSFE1/MSFE2	0.95	1.00	1.00	0.94	0.83
h-period inflation rate					
Forecast horizon (h=)	1	2	4	8	12
Number of Forecasts	22	21	19	15	11
MSFE1 (w/ OV)	0.85	0.63	0.64	0.75	0.78
MSFE2 (w/o OV)	0.84	0.62	0.58	0.68	0.74
MSFE1/MSFE2	1.02	1.02	1.10	1.10	1.05

Note: The table shows the MSFE for predicting inflation, respectively changes in inflation of the (recursively estimated) model that includes money overhang measures (MSFE (w/OV)) and the model that excludes money overhang measures (MSFE (w/o OV)) for five different forecast horizons. MSFE1/MSFE2 gives the relative MSFE of model 1 (w/ OV) vs. model 2 (w/o OV). A ratio smaller than 1 implies a higher forecast accuracy of the augmented model.

initial estimation sample, T the number of observations in the full sample, and $fe_{i,t}$ the forecast error of model i in period t. Model 2 (the augmented model) encompasses model 1 (the univariate model). The null hypothesis is that the forecast accuracy of both models is equal. Rejection of the null hypothesis implies that model 2 provides significantly better forecasts than model 1. We obtain an $ENC - NEW$ statistics of 1.26. This value has to be compared to the non-standard critical values tabulated in Clark and McCracken (1999).[41] The appropriate critical value depends on the proportion (Π) of the number of forecasts (P) to the number of observations in the initial sample (R), and the number of excess parameters (k_2) in model 2 compared to model 1. In our case $\Pi = 11/48 = 0.229$ and $k_2 = 5$. Because critical values are only tabulated for $\Pi = 0.2$ (1.198) and $\Pi = 0.4$ (1.639) we linearly interpolate the critical value for $\Pi = 0.229$. As 0.029 is equal to 14.58% of

[41] Critical values for $k_2 = 5$ are only provided in this working paper version of the above-cited journal article. The critical value we use takes into account that the models are nested and that forecasts are obtained after recursive re-estimation of the models.

the difference between $\Pi = 0.4$ and $\Pi = 0.2$ we add 14.58% of the difference between the critical value for $\Pi = 0.4$ and $\Pi = 0.2$ to the critical value for $\Pi = 0.2$. Using this procedure we get an interpolated 10% critical value of 1.25, which is smaller than our calculated $ENC - NEW$ statistics (albeit very slightly). We therefore reject the null hypothesis of equal forecasting accuracy and conclude that the 'money-overhang-augmented' model 2 produces significantly better forecasts of the 3-year change of the annualized quarterly inflation rate than the univariate model.

Finally, we examine whether the importance of money overhang to predict (changes in) inflation might have increased over time. To do so, we test whether the money overhang measures can be jointly excluded from the predictive regressions over two distinct samples, the first from 1987Q3 to 2002Q4-h, the second from 1987Q3 to 2008Q2-h. Table 9 shows the test results for h=4, 8, and 12.

Table 9: Tests of Exclusion Restrictions on Money Overhang

h	h-period change of inflation			h-period inflation rate		
4	F(5,69)=	2.36	**[0.05]**	F(5,69)=	1.18	[0.33]
	F(5,47)=	1.41	[0.24]	F(5.47)=	0.95	[0.46]
8	F(5,65)=	2.74	**[0.03]**	F(5,65)=	1.37	[0.25]
	F(5,43)=	0.87	[0.51]	F(5,43)=	0.71	[0.62]
12	F(5,61)=	2.46	**[0.04]**	F(5,61)=	0.87	[0.51]
	F(5,39)=	0.89	[0.50]	F(5,39)=	0.54	[0.75]

Note: This table reports the F-statistics and the marginal p-values for the null-hypothesis of exclusion of the money overhang measures from the respective model. The statistics reported in the first row refer to the full sample, while the statistics reported in the second row refer to the restricted sample (for each of the reported forecast horizons).

One result is especially noteworthy. While we can exclude the money overhang measures for predicting changes in inflation over the restricted sample, we cannot exclude them over the full sample over all forecast horizons considered. While this may partially be related to the larger number of degrees of freedom over the full compared to the restricted sample, the large drop in the p-values suggests that the importance of money overhang measures for predicting inflation actually has increased over the period excluded from the restricted sample (especially for $h = 8, 12$). It is also striking that the money overhang measures are excludable in the models predicting the inflation rate, both over the restricted, as well as over the full sample. This supports our previous notion that the stationary money overhang measures can only be helpful for predicting the stationary change in the inflation rate, not the rate itself.

9 Conclusions

In this paper we have analyzed US money demand stability and the leading indicator properties of derived money overhang measures of various monetary aggregates for predicting inflation over a sample from 1987Q1 to 2008Q2. In contrast to a large part of the literature, we find evidence of a stable money demand function for M2 in the framework of the cointegrated VAR (CVAR) model. In addition to a long-run money demand function, we also find evidence of a long-run Fisher effect. As predicted by theory, the demand for real M2 is positively related to real income, and a unit coefficient of real income cannot be statistically rejected. Money demand is furthermore positively related to the own rate and negatively to the 3-month treasury bill rate. We find a homogeneous long-run relationship among both interest rates to be clearly rejected. Evidence in favor of a long-run money demand function collapses once we impose a homogeneous interest rate spread from the beginning. This could be regarded as a possible explanation for contrary previous results. However, not imposing this restriction from the outset comes at the cost of imprecisely estimated interest rate coefficients. This has been illustrated by depicting the log-likelihood values for different hypothesized coefficient values.

Formal tests show that the long-run parameters of the model are stable, while the stability of the short-run parameters is doubtable.

In the first part of our extensive forecasting exercise we have analyzed how various VECM models perform in predicting one-step ahead changes of the inflation rate, depending on whether long-run and/or short-run parameters are recursively re-estimated and whether short-run restrictions are (re-)imposed or not. The statistical acceptability of the model has been checked in each single recursion. The results from this exercise suggest that by recursively re-estimating the complete CVAR model in each period, and by (re-)imposing sensible and statistically accepted long-run and short-run restrictions, the forecast accuracy is about the same as the one from an MA(1) model, whose forecast performance is known to be hard to beat for this forecast horizon. While the price of this tedious procedure does not seem to be worth paying at first glance, we believe it is for the following reason:

Whereas the univariate model is atheroretical and, basically, only provides a number, our model provides an economic rationale for each forecasted value. More specifically, the predictions are based on the expected adjustment of the inflation rate in the case of disequilibria in the two identified long-run relations (i.e. money overhang and deviations from the long-run Fisher equation).

We find some evidence that money overhang is a useful information variable for predicting *changes* in the inflation rate. First, in contrast to money growth, money overhang Granger-causes inflation. Secondly, evidence based on the second part of our (recursive) forecasting exercise suggests that taking into account derived measures of money overhang significantly improves forecasts of the change in inflation over the 3-year horizon. Finally, some tentative evidence suggests that the importance of money overhang for predicting changes in inflation may have increased in recent years. Based on these results we believe that money overhang measures (albeit estimated imprecisely) can be considered a useful *supplementary* information variable for predicting long-run changes in the US inflation rate.

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10 Appendix

10.1 Rolling DOLS Estimation of Interest Rate Semi-Elasticities

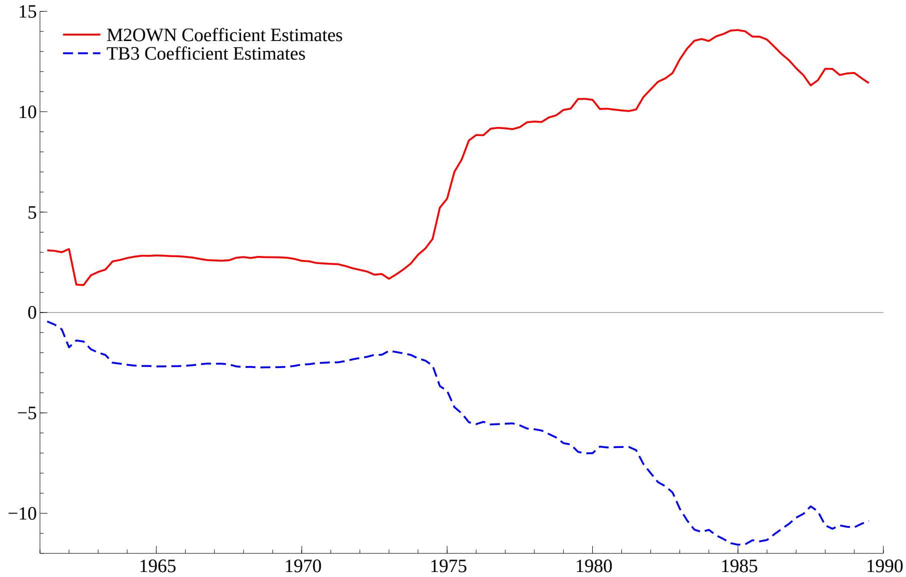
To further examine our comparably high interest rate semi-elasticities, we estimate moving window dynamic OLS regressions for the full sample, where data is available (so this sample also covers the period which has been excluded from our previous analysis). We set a *fixed* window size of 76 quarters, which seems high at first glance, but choosing smaller samples would yield to even less precise coefficient estimates than documented before, given the high collinearity among the interest rates. Swanson (1998) uses a 15-year-rolling window approach because he finds 10-year rolling windows to yield too imprecise estimates using *monthly* data. Against this background our chosen window size seems rather small than large.

We decide to fix the window size (as opposed to an approach where the window is growing) because we are primarily interested in the potentially time-varying parameter values. These would be much harder to detect in a growing window setting, where the picture is more blurred.

Figure 14 shows the point estimates of the interest semi-elasticities with respect to all 114 sample starts, which are depicted on the horizontal axis. In each of the regressions the lag order is chosen that minimizes the Hannan-Quinn criterion.[42]

[42]In order not to 'lose' too many degrees of freedom, we restrict the maximum lead-lag-length to 5.

Figure 14: Moving Window D-OLS Estimates of Interest Semi-Elasticities



Note: Window size is set to 76 observations. On the horizontal axis the starting period of each of the estimation samples is denoted (so the first (adjusted) sample is 1961Q2 to 1980Q3, the last of the 114 samples is 1989Q3 to 2008Q2). At each iteration the lead/lag length is chosen that minimizes the HQC; maximum lead/lag length for lead/lag length selection is set to 5.

We observe that after a period of relative stability, interest semi-elasticities increase a lot once the beginning of the sample is moved to the 1980s. While the relatively large window-size prevents us to assess parameter stability during our chosen sample within this more illustrative approach, our (backwards recursive) formal stability tests in the preceding analysis point towards an at least reasonable degree of parameter stability within our sample.

10.2 Results for other Monetary Aggregates

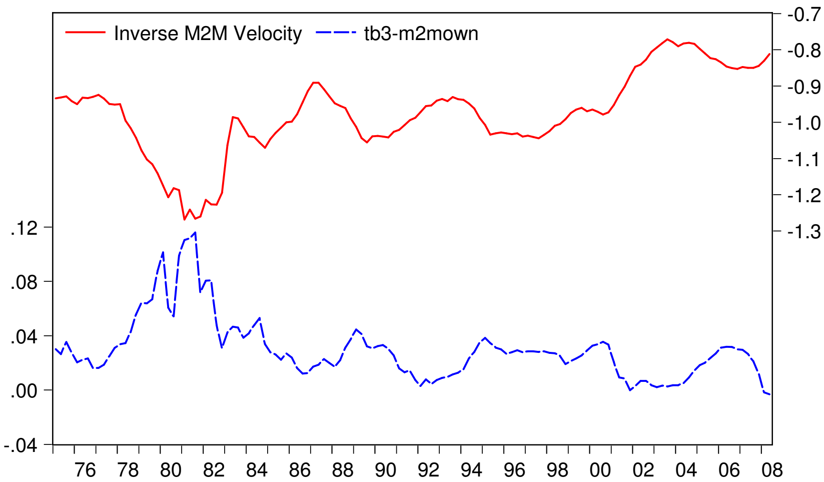
While the results for the monetary aggregate M2 have been favorable in terms of the presence of a long-run stable money demand function, this is not the case for the other monetary aggregates, which we examined, i.e. M2M (M2 minus short-term deposits), MZM (money at zero maturity), and M1. While we find evidence of cointegration for the former two aggregates, the estimates either do not point towards a money demand function (with reasonable coefficient values) or the money demand functions are highly instable, both in the long- and in the short-run. Below we present some more specific remarks on our results with respect to each of these aggregates as well as present graphs of the inverse velocity of the respective aggregates together with the most commonly hypothesized opportunity costs for that respective aggregate.

10.2.1 M2M

Figure 15 shows the inverse M2M velocity and the spread between the 3-month treasury bill rate (*tb3*) and the own rate of M2M (*m2mown*). In the graph we thereby imposed the 'restriction' that M2M money demand is affected symmetrically by both interest rates. We did not impose this restriction when estimating the various models for M2M.

We find evidence of a cointegration relationship resembling a money demand function. However, both the short-run and the long-run parameters are highly instable (indicated by the results obtained from the eigenvalue fluctuation test, the Nyblom test, as well as the Ploberger-Krämer-Kontrus test). This finding is robust against various trend assumptions, definitions of shift- and impulse dummies, and the (in this case) difficult choice of the cointegration rank (both, a rank of 2 and 3 seemed suitable). Due to the large number of tested specifications, we do not report the results here, but they can be obtained from the author upon request. Interestingly, our preferred (but still unacceptable) specification for M2M includes a restricted level-shift in 1992Q2, which could not be excluded according to the LR-test result.

Figure 15: Inverse M2M Velocity vs. Opportunity Costs



10.2.2 MZM

In figure 16 the inverse MZM velocity is depicted together with the spread between *tb3* and the own rate of MZM (*mzmown*).

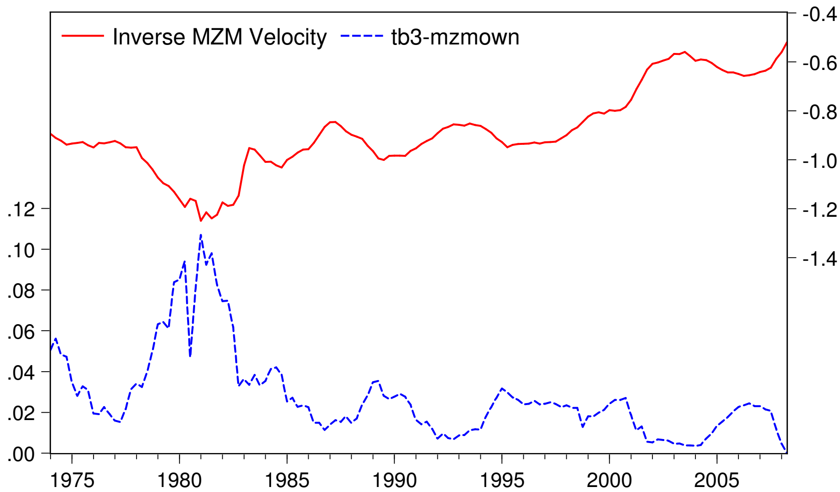
In contrast to the specifications for M2 and M2M there is no need for including a dummy variable in the beginning of the 1990s. Since we only find money adjusting towards the first of two cointegrating relationships, we estimate the 'money demand function' in a single-equation framework. The estimated interest semi-elasticities are in the proximity of those for M2 (6.75 for *tb*3, -11.50 for *mzmown*), but the estimated money-to-income elasticity appears unreasonably high (with a point estimate of 2.00). Furthermore, the recursively estimated eigenvalues are again highly instable.

10.2.3 M1

Lastly, figure 17 shows the inverse M1 velocity and two interest rates, *tb3* and the 10-year long-term government bond yield, *lt*. Previously identified periods of breakdown of the M1 money demand function (period of 'missing money' in the early 70s, the 'great velocity decline' in the early 80s , and the subsequent M1 explosion in the mid 80s where real M1 grew by more than 20% from 1985Q1 to 1986Q4) are shaded in grey.

In contrast to all other presented monetary aggregates previously analyzed, an own rate for the M1 components is not publicly available. Since we

Figure 16: Inverse MZM Velocity vs. Opportunity Costs



regard interest payments on the M1 components not to be of considerable importance, we disregard the own rate here, and present two interest rates possibly representing the opportunity costs of holding M1.

While we find a well-specified VAR model for M1 in terms of misspecification tests, there is clearly no evidence for a cointegration relationship among $m1$, y, $tb3$, and Δp so that only a short-run money demand equation could be specified. Results do not improve if we either augment the system with lt, or if we separately include lt instead of $tb3$.

Figure 17: Inverse M1 Velocity vs. Opportunity Costs