SPICE Simulation of Transmission Lines by the Telegrapher's Method

Part 1: Putting the Telegrapher's Equations into a Circuit

Roy McCammon, Senior Engineer, 3M Communication Markets Division

Users of SPICE have used two methods to simulate transmission lines: use one of SPICE's built in transmission line models or use a series of lumped component sub-circuit approximations. Neither method has been completely satisfactory. Neither method directly addresses the fact that some of the line parameters are functions of frequency.

SPICE has two transmission-line components: T, the lossless transmission-line component; and O, the lossy transmission-line component. The lossless line simulates only delay and characteristic impedance. For the lossy line; you must enter the primary line parameters, but at this time it is not possible to have all four primary parameters non-zero at the same time. Furthermore, it is only good at a single frequency or at best a narrow band of frequencies because real line parameters are functions of frequency and the ones in the O component are constants.

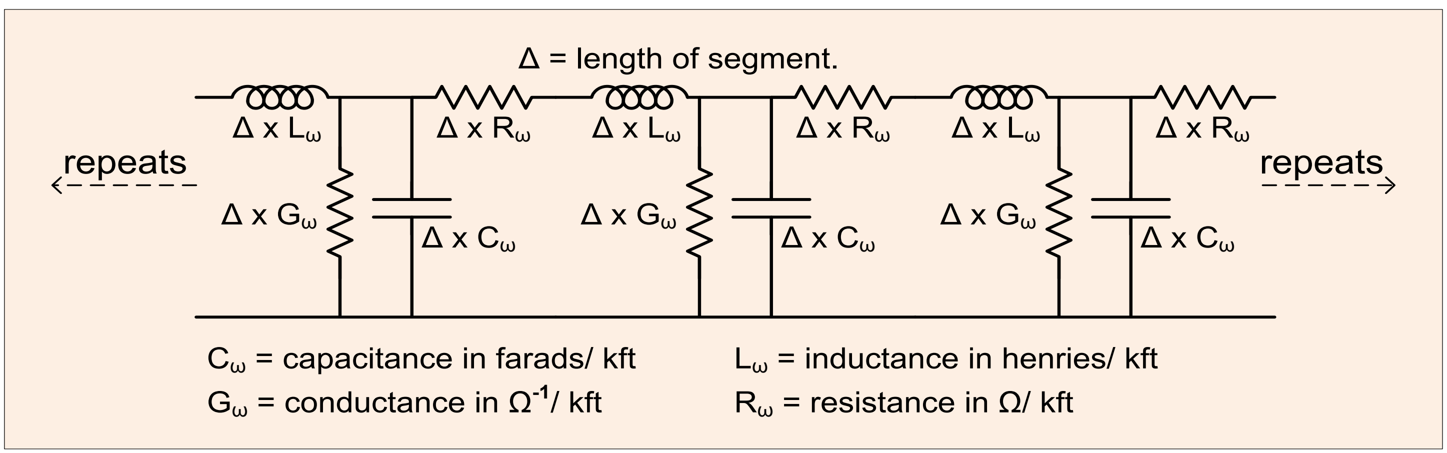

Figure 1. Primary line parameters.

Lumped-circuit modeling uses a series of lumped-element subsections as shown in **Figure 1**. Each line parameter is multiplied by the length of the section. The fact that the line parameters are dependent on frequency is emphasized by the subscript of ω. Frequency dependence has been modeled by replacing some components with a parallel ensemble of tuned components with staggered center frequencies (not shown in the figure). Usually, only the variation resistance is modeled.

Finally, the two-way delay (time from source to load and back again) of each sub-circuit must be short with respect to the rise times of the signal. For example, if the rise time was 50 ns, the delay of the lumped component sub-circuit should be less than about 10 ns, which for typical twisted pair cable is the delay of about three feet. A 1000-foot line would require approximately 300 lumped-component sub-circuits!

There is a better way, one which requires only about two-dozen components, regardless of the length or bandwidth of the transmission line. We will call it the *Telegrapher's Method* and the sub-circuit we create to implement it will be called the Telegrapher's Model or TM. By the end of this article, we will have a SPICE sub-circuit that will implement the Telegrapher’s method with full frequency dependence and will be as easy to use as the built-in components.

The TM two-port model

Figure 2 depicts a two-port equivalent circuit model of the solutions of the Telegrapher's Model. By using Thévenin and Norton equivalents, many different depictions are possible. This particular depiction will be called the canonical two-port.

In this model, P1, P2, Z1 and Z2 are frequency dependent and will be implemented with Laplace components. P1, P2 (propagation functions) account for loss, delay, dispersion and anything that happens to the signal in transit. The Z1, Z2 (the characteristic impedances) and the voltage doublers A1 and A2 account for the interaction of the transmission line with the external environment. As drawn, Z1 and Z2 would be implemented with a "G" component (a voltage dependent current source) and P1 and P2 would be implemented with an "E" component (a voltage dependent voltage source). The term "Laplace component" will be used to mean either the G or E components with the Laplace option.

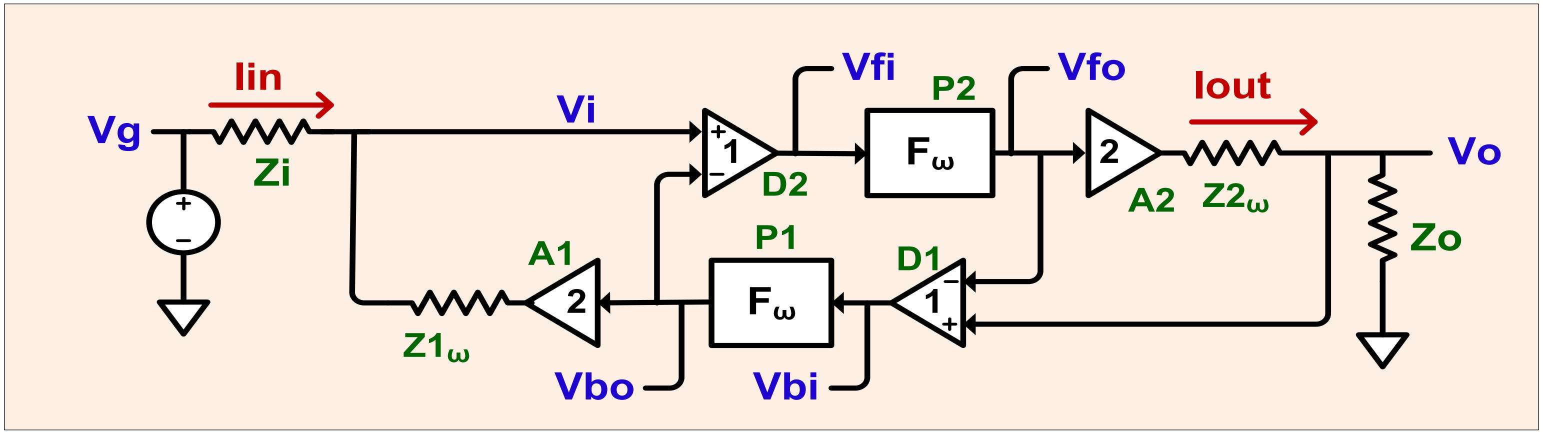

Figure 2. Canonical two-port representation of the solutions of the Telegrapher's Equations

Let's examine this model to if it at least qualitatively behaves as we would expect. We know that if a transmission line is perfectly matched at the load then there will be no reflections from the load. A perfect match means Zo = $Z2_\omega$. Zo and $Z2_\omega$ form a voltage divider which divides exactly by 2.

Let's start with the node Vfo. That voltage is doubled by A2 and then the voltage divider divides it by 2, so we know that Vo = Vfo. The inputs of difference amplifier D1 see Vfo on one input and Vo on the other. These signals exactly cancel so the output of D1 is zero and there is no reflection. Since the output of D1 is zero, the output of P1 and the output of A1 is also zero. Let's move our focus back to the node Vi. Since Vbo (the output of P1) is zero, the output of D2 is just Vi. The output of P2 (Vfo) is just F_ω x Vfi and we have already determined that Vo is exactly equal to Vfo so we know now that Vo/Vi = F_ω. F_ω is just the transfer function of a perfectly terminated transmission line. That works.

What if we now let Zi = $Z1_\omega$, let Zo become infinite (open), and let Vg be a 1-V step? This configuration, called series termination, is often used to drive logic when there is a single transmitter and a single load. Initially all nodes including the output of A1 are zero. Vg generates a one-volt step. Zi and $Z1_\omega$ form and exact divide by two divider so Vi is a ½-V step. D2 sees this ½-V step on its positive input and zero on its negative input, so its output it also a ½-V step. This is input to F_ω which accounts for delay, dispersion and attenuation.

Just for this case, let's suppose that F_ω is just a pure 150-ns delay without dispersion or attenuation. So, 150 ns after the input step occurs, a ½-V step comes out of P2 at node Vfo. It gets doubled to 1 V. There is no divider, so the load sees a 1-V step even as the input is a ½-V step. D1 sees Vo (a 1-V step) at its positive input and Vfo (½-V step) at its negative input, so its output is a ½-Vstep. P1 delays that by another 150 ns, at which time its output, Vbo, produces a ½-V step.

This is doubled to a 1-V step by A1. Vi rises from ½ V to 1 V. D2 now sees 1 V at its positive input and ½ V at its negative input, so it produces ½ V. But it was already producing ½ V, so nothing changes at the output of D2 so there are no more events. The reflection from the load was absorbed by the matched impedance at the source side. To recap, we see the input rise from zero to ½ V. 150 ns later we see the output rise to 1 V. Finally, 150 ns after that, the input rises to 1 V and then nothing else happens. This all agrees with experience.

But what about the doublers? Transmission lines are just wire and plastic. There are no amplifiers in there. Suppose there was an ideal, lossless 50-ohm transmission line with a 2-V, 50-ohm source and a 50-ohm load. Zi, Z1, Z2 and Zo all have 1 volt across them and they all dissipate 20 mW for a total of 80 mW. But, the source is only providing 40 mW. How can this two-port be accurate? It does not even conserve energy.

Is the two port accurate? The answer is yes and no. The two-port only promises to model the transmission line's effect on the external circuit. It makes no promise that there is anything like the model inside the transmission line. The model absorbs and generates energy from nowhere, but that is OK because energy is conserved in the external circuit. Yes, it is an accurate model of the transmission line's effect on the rest of the circuit, by which we mean the voltages and currents at the two ends of the model are the same as an actual transmission line.

The telegrapher's equations

In the circuit of Figure 2, the P1 and P2 are just voltage-dependent voltage sources and could be replaced with a SPICE E component. Impedance, like Z1 or Z2 can be thought of as a component that adjusts its pass current as a function of the voltage across it. In other words, impedance can be thought of as a voltage-dependent current source. Z1 and Z2 can be implemented with a G component that has its output connected to its input. This circuit, as drawn could be directly simulated, if we only knew Z(s) and F(s).

Fortunately, that problem was formulated and solved in the late 19^{th} century by Oliver Heaviside. The equations are derived from the incremental model in Figure 1 by mathematically letting the length of each segment approach zero. That produces a second order differential equation that can be solved. The solutions for Z and F are:

$$Z(s) = \sqrt{\frac{(R_\omega + sL_\omega)}{(G_\omega + sC_\omega)}}$$

1a.

$$\gamma(s) = \sqrt{(R_\omega + sL_\omega)(G_\omega + sC_\omega)}$$

1b.

$$F(s) = e^{-x\gamma(s)}$$

1c.

1d. **x = length of the transmission line.**

(Refer to **Reference 1 or 2** to see how these functions are derived.)

R_ω, L_ω, G_ω, and C_ω are the previously defined primary parameters of the transmission line. We would now be finished, except that the primary parameters are (possibly) functions of frequency.

Understanding the solution to the telegrapher's equations

Z(s) is called the characteristic impedance. It has an explicit frequency dependence caused by the "s" term and an implicit frequency dependence indicated by the **ω** subscripts on R_ω, L_ω, C_ω and G_ω which means each of the parameters is (potentially) also a function of frequency. There are several definitions of characteristic impedance that are all equivalent:

1. It is the impedance seen looking into an infinite length of transmission line.

2. It is the impedance that must be used to terminate the line to avoid reflections.

3. It is the impedance seen looking into a finite length of transmission line that is terminated at the other end by its characteristic impedance. So if I have 1000 feet of 50-ohm coax and terminate it with 50 ohms at the load and measure the impedance at the source end, it will be 50 ohms.

F_ω is called the transmission or propagation function. If the transmission line is terminated by its characteristic impendence, then the ratio of its output voltage to its input voltage is F(s). i.e. (Vo/Vi) = F(s). If F(s) and Vi are known, then Vo = F x Vi. The form of F may not be familiar. Clearly, it is a function of length (x), but it may look unusual. γ(s) (also known as the "propagation constant" even though it is not a constant) is a square root of a complex number, and so, in general, γ(s) is also a complex number. For convenience let α(s) and β(s) be the real and imaginary parts of γ(s), i.e.

2a. $$\gamma(s) \equiv \alpha + j\beta$$

where α and β are purely real. Now we can rewrite:

2b. $$F(s) = e^{-x\gamma(s)} = (e^{-x\alpha(s)})(e^{-xj\beta(s)})$$

The $e^{-x\alpha}$ term is just the exponential of a real number. As x gets larger, $e^{-x\alpha}$ gets smaller, which means as the transmission line gets longer, the attenuation increases. $e^{-x\alpha}$ accounts for the loss in the transmission line. The loss is x$\boldsymbol{\alpha}$ with $\boldsymbol{\alpha}$ being the loss in nepers per unit length (1 neper = 8.67 dBs).

The $e^{-xj\beta}$ term is the exponential of a purely imaginary number and is equal to cos(-xβ) + j sin(-xβ). This simply means that the phase shift between input and output is xβ in radians per unit length. Obviously, the longer the transmission line the greater the phase shift. A phase shift of xβ is equivalent to a delay at a particular frequency of d = xβ/$\boldsymbol{\omega}$ which increases proportionately to length as the transmission line gets longer (just what we would expect).

In the next part of this article we will show how to put frequency dependence into the solutions of the Telegrapher's Equations and then in the last part we will pull it all together into an easy to use sub-circuit.

References

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Resources

1. Linear Technology Corporation LT Spice IV download available with a very generous license.
   http://www.linear.com/designtools/software/ltspice.jsp
2. Yahoo user's group
   http://tech.groups.yahoo.com/group/LTspice/

About the author

Roy McCammon is a senior engineer with 3M's Communication Markets Division, and a graduate of the University of Texas Department of Electrical Engineering. He has operated a satellite tracking station in Antarctica, designed astronomical instruments and telescope servos, and has spent the last 28 years designing test equipment used by telecommunications providers and thinking about transmission lines. His interests include dancing the two-step, pushing miniature battleships around a table top, and simulations.