Some Remarks on the "Book of Assumptions by Aqāṭun"

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Introduction and Conclusion

The treatise *Kitāb al-mafrūḍāt li Aqāṭun* (Book of Assumptions by Aqāṭun) has been treated in full in my doctoral thesis (Amsterdam, 1977). The present paper contains an abstract of this thesis. It consists of the contents of the treatise, i.e. propositions only without either proofs or drawings, along with some remarks describing the manuscript and pointing out several influences. In addition the questions of the original Greek title and putative Greek name of the author are extensively discussed. The offered solution is, however, too vague to have been included in the thesis.

My conclusion is that the treatise contains interesting propositions, but no sensational theorems. Some propositions deal with fundamental properties of triangles, some go in the direction of trigonometry, others could be connected with optics. The author, i.e. Aqāṭun seems to have been a man with a good knowledge of the mathematical literature, judging from the different influences on his treatise [Thesis, Chapter IV]. He may have lived around the same time as Pappus: on the one hand Aqāṭun gives two converses to a lemma by Pappus (prop. 41), on the other hand both Pappus and Aqāṭun prove a lemma related to Apollonius' *Conics* (prop. 27) and a lemma connected with Euclid's *Porisms* (prop. 22); these proofs by Aqāṭun and Pappus are similar but not identical. The treatise was still of value and interest to Arabic mathematicians in the thirteenth century, according to the extensive marginal notes. Yet the sphere of influence apparently was not very large.

Description of the Manuscript [Thesis, Chapter I]

The treatise "Book of Assumptions by Aqāṭun", *Kitāb al-mafrūḍāt li Aqāṭun*, is contained in the Istanbul manuscript Aya Sofia 4830,5 (fol. 89v - 102v) [Krause, p. 439]. It consists of 43 propositions dealing with plane geometry. Nineteen propositions, all from the first half, form a separate treatise entitled "Book by Archimedes on the Elements of Geometry", *Kitāb Arshimīdis fī'l-uṣūl al-handasīya.* This is contained in the Bankipore

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manuscript 2468,29 (fol. 141r – 144v) [Sezgin, p. 135]. An Arabic edition of the latter together with Bankipore 2468,28 (fol. 134v – 141r, *Kitāb Arshimidis fī'l-dawā'ir al-mutamāssa*, has been published by the Osmania Oriental Publications Bureau (Hyderabad-Dn., 1947).

In the Aya Sofia manuscript the date of copying is given as ca. A. D. 1230, mentioning as place of copying, sometimes Damascus, sometimes Marāgha. At the end of the Bankipore manuscript the date of its composition is indicated as A. D. 1027/1028. In the present copy some of the treatises are dated Mosul A. D. 1234/35. Thus the present copies of both treatises date from the same period. Both are written in Naskhī. The mathematical quality of the treatise is higher in the Aya Sofia manuscript than in the Bankipore manuscript.

The reasons for accepting the title *Kitāb al-mafrūḍāt* and Aqāṭun as its author are laid down in Chapter II of the thesis.

The Contents [*Thesis, Chapter III*]

Prop. 1: If the base BG of the circle-segment ABG be extended in either direction with pieces of equal length, and from the endpoints D and E the tangents EZ and DH be drawn to the segment, then the line ZH connecting the tangential points is parallel to the line ED.

Prop. 2: Assume the two lines DB and DG are tangents to a circle. Let the chord BG connecting the tangential points be extended to E, and let from E a third tangent be drawn, touching at A and meeting DG in Z and BD in T, then TE : EZ = TA : AZ.

Prop. 3: Assume the two lines EG and ED are tangents to a circle, while EB cuts it at H and B. Let DA be the chord through D parallel to EB, and let AG meet BH in Z. Then BZ = ZH.

Remark: There is no drawing in the Aya Sofia manuscript as the room left open for this purpose is too little. Later I discovered that the drawing had been made on a loose leaflet which lies now, on the microfilm, between fol. 87v and fol. 88r.

Prop. 4: Let ABG be an isosceles triangle and AD the perpendicular to the base BG. Assume on AB the points Z, E such that $BD^2 = BE \cdot BZ$. Let ZD be joined, ZH be drawn parallel to BG, and EH be joined, then $\sphericalangle EHG = 2 \sphericalangle AZD$.

Prop. 5: If AG is the diameter of a semicircle and B the middle of the arc AG; let from D, on the extension of AG, DB be joined cutting the circle at Y. If YE = EB and from the center Z, ZE be drawn until it meets the extension of AB in H, then AH : HB = DZ : ZB.

Prop. 6: If BG is the diameter of a semicircle, and the chords AG, BD meet in Z, and if BA, GD be drawn meeting in E, then $BD \cdot DZ = GD \cdot DE$.

Prop. 7: Let BG be the diameter of a semicircle, and the chords AG, BD meet in Y. Take Z on BD and E on AG so that $BD \cdot DY = DZ^2$ and $GA \cdot AY = AE^2$, and let EB, ZG be joined meeting in H, then $ZH = HE$.

Prop. 8: In the equilateral triangle ABG the heights AZ, BD, GE are of equal length.

Prop. 9: Let ABG be an equilateral triangle, and AD its height. Let from a point E on BD the perpendiculars EZ and EH be drawn to the sides GA and AB, then $AD = EZ + EH$.

Prop. 10: Let ABG be an equilateral triangle, and AD its height. Let from an interior point E the perpendiculars to the sides EZ, EH, ET be drawn, then $AD = EZ + EH + ET$.

Prop. 11: If in triangle ABG the line AD be drawn meeting BG at D such that angle BAD be equal to angle AGD, then $GB \cdot BD = AB^2$.

Prop. 12: Let in triangle ABG, with AB equal to AG, AD be drawn perpendicular to AB meeting the extension of BG in D. If AB be bisected at E and ED be joined cutting AG at Z, and if through Z, HZ be drawn parallel to AB, then $DA \cdot AH = AG^2$.

Prop. 13: If in triangle ABG, with AB equal to AG, AD be drawn perpendicular to BG, then $2\,DG \cdot GB = AG^2$.

Prop. 14: If in triangle ABG the perpendicular AD to BG be drawn, then $BD^2 - DG^2 = BA^2 - AG^2$.

Prop. 15: Let line AB be equal to line AG and line BD equal to line DG, and let both angles BAG and BDG be right, then $\sphericalangle ABD = \sphericalangle AGD$.

Prop. 16: Let A be the right angle of the right-angled triangle ABG. If BG be bisected at D and AD be joined, then $BD = DG = DA$.

Prop. 17: Let in the right-angled triangle ABG, with angle BAG right, on the extension of AG a point D be taken, from which DE is drawn perpendicular to BG and cutting AB at Z. Let GZ be joined and on it H be taken such that $BH^2 = AB \cdot BZ$ and let DH be joined, then $DH^2 = DE \cdot ZD$.

Prop. 18: Let the lines AB and BG meet at B, and on AB point D be taken, such that $AB^2 = AD^2 + BG^2$. Let DG be joined and bisected at E, and AE be joined, then $\sphericalangle DAE = \sphericalangle DGB$.

Prop. 19: If in the isosceles triangle ABG, with AB equal to AG, an arbitrary line AD be drawn cutting BG at D, then $BD \cdot DG + DA^2 = AG^2$.

Prop. 20: If in the isosceles triangle ABG, with AB equal to AG, lines AE and AD be drawn, meeting BG at E and D such that $BD \cdot DG : DA^2 = GE \cdot EB : EA^2$, then DA = AE.

Prop. 21: Let in triangle ABG angle BAG be bisected by line AD, meeting BG at D, then $(BA + AG) : GB = AB : BD$.

Prop. 22: Let from triangle ABG AB be extended to D and AG to E, let DH be drawn parallel to EB and EZ parallel to DG, then ZH will be parallel to BG.

Prop. 23: Let in triangle ABG AD be equal to BE and AZ equal to GH, let GE, GD, BZ, BH be joined, GE and BZ intersecting in W and GD and BH in S. If AS and AW be joined and extended meeting BG in T and Y, then BT is equal to GY.

Prop. 24: Let angle AGB in the right-angled triangle ABG, with angle ABG right, be bisected by line GD, and angle DAE be equal either to angle AGD or angle BGD, then $GD > GE$.

Prop. 25: Let angle AGB in the right-angled triangle ABG, with angle ABG right, be divided by line GD such that angle BGD is twice angle DGA, then $BG \cdot GA > GD^2$.

Prop. 26: Let in the semicircle with diameter AD arc AB be equal to arc BG, let GD be joined, and BE be drawn perpendicular to AD, then $GD < ED$.

Prop. 27: If in triangle ABG BG be bisected at D and AD be joined. If from B an arbitrary line be drawn cutting AD in Z and AG in E, and GZ be joined and extended to H on AB, then, if HE be joined, it is parallel to BG.

Prop. 28: If in the circle-segment standing on line BG arc BG be bisected at A, E be taken on the extension of BG, and AG and AE cutting the segment at D be joined, then $EA \cdot AD = AG^2$.

Prop. 29: If through two circles intersecting in A and D, an arbitrary line be drawn cutting one circle in Z and H and the other in B and E, and ZA, AB, ED, DH be joined, then $\sphericalangle ZAB = \sphericalangle EDH$.

Secondly, if HD and ZA be extended to K and T on circle ABED, and BK and ET be joined intersecting in L, then BL = LE.

Prop. 30: If from triangle ABG with angle BAG right, BA be extended to D, and from D, DE be drawn perpendicular to BG cutting AG in Z, then $BD \cdot DA = GZ \cdot ZA + ZD^2$.

Conversely: If $BD \cdot DA = GZ \cdot ZA + ZD^2$, then $\sphericalangle DEB = 90^\circ$.

Prop. 31: If the tangents ED and EA to circle ABTD be drawn, and if an arbitrary line EZ be drawn cutting the circle at Z and B; if AD be joined, if to EZ the perpendicular ZT be drawn cutting AD in Y and the circle in T, and if EY and ZD be joined, then $\sphericalangle$ DEY $= 2 \sphericalangle$ DZT.

Prop. 32: If to the semicircle with diameter AG the tangents EA and EB be drawn, EB and AG be extended until the intersection point D, if BZ be drawn parallel to AD, DZ and AB be joined intersecting in H, and HT be drawn perpendicular to AG, then T is the center of the circle.

Prop. 33: If line AG of triangle ABG be bisected at D, line BA be extended to E, and ED be joined and extended to Z on line BG, then BE : EA = BZ : ZG.

Conversely, if BE : EA = BZ : ZG, then AD = DG.

Prop. 34: If line BA of triangle ABG be extended from A to E, line BG bisected at Z, and line AG divided at D such that BE : EA = GD : DA, then E, D, Z are collinear.

Prop. 35: Through point D outside line AB let the lines ED and DZ be drawn both parallel to line AB, then EDZ is a straight line.

Prop. 36: From point E let the lines EZ and ED be drawn cutting the lines AB, AG, AD respectively in Z, T, L and in B, G, D, and let EZ : ZT = EL : LT, then EB : BG = ED : DG.

Prop. 37: In the right-angled triangle ABG, with angle BAG right, let the lines AD and AE be drawn such that angle DAE is equal to angle ABG, and from point B let the perpendicular BZ to AE [cutting AD in T] be drawn and extended to H on AG. Then DA·AT + GB·BD = HB·BT + GA·AH; and secondly GB·BD + GA·AH = AB^2· DA·AT + GB·BD = HB·BT + GA·AH.

Prop. 38 (1): Let the lines AB, AG, AD be of equal length and DG, GB, BD be joined, then $\sphericalangle$ GBA + $\sphericalangle$ GDB = 90°.

(2): Moreover, if AB = AG and $\sphericalangle$GBA + $\sphericalangle$GDB = 90°, then the lines AB, AG, AD are of equal length.

(3): Also, if AD = AB and $\sphericalangle$ GBA + $\sphericalangle$ GDB = 90°, then the lines AD, AG, AB are of equal length.

(4): Finally, if AD = AG and $\sphericalangle$ GBA + $\sphericalangle$ GDB = 90°, then the lines AD, AG, AB are of equal length.

Prop. 39: If in triangle ABG line AD is drawn meeting BG in D such that angle DAG is equal to angle ABG, then

$$BG : GD = BG^2 : GA^2 = BA^2 : AD^2.$$

Prop. 40: (= Prop. 43): If in the right-angled triangle ABG with angle ABG right, angle BAG is bisected by line AD, line AE drawn at random, from point E line EZ constructed parallel to line AD, ZB joined cutting AD in T, and ET joined, then AZ : ZT = AE : ET.

Prop. 41 (1) Let in the right-angled triangle ABG, with angle BAG right, from point A to line BG the lines AD and AE be drawn such that angle DAG is equal to angle GAE, then BE : EG = BD : DG.

(2): Conversely: If BE: EG = BD: DG [and angle BAG be right], then ∢ DAG = ∢ GAE.

(3): (margin) Let ∢ EAG = ∢ GAD, and BD: DG = BE :EG, then ∢ GAB = 90°.

Prop. 42 Let in triangle ABG angle B be bisected by line BD and angle G by line GD, then angle A is bisected by line AD.

Prop. 43 is identical with Prop. 40 but has been proved in a different way. Here Propositions 41 and 42 are used, whereas for the proof of Prop. 40 Propositions 38 and 39 are applied.

From these contents the impression is gained that Aqāṭun made an effort to understand plane geometry. Maybe this was done in a school where this treatise served as a textbook or exercise book. In this case, however, one would suppose more copies of the treatise to be extant. It seems therefore more probable to me that this was a one-man effort. Aqāṭun worked out new propositions and also, like Pappus, gave some proofs that had been left out in the existing mathematical literature, but no references are added. There may have been a mutual influence between Pappus and Aqāṭun. Other mathematicians exercising an influence on our author appear to be Archimedes, Euclid, and Apollonius. Also a connection with Menelaus exists: propositions 33 and 34 contain two special cases of Menelaus' theorem in the plane. Some of the propositions in the *Kitāb al-mafrūḍāt* may be later Arabic insertions [Thesis, Chapter I].

As the treatise is only moderately original, which may be the reason why not many copies seem to have existed, its impact on later mathematics was small. The only connection found is in Ibn al-Haytham. An interesting feature of our copy are the marginal notes. In these notes improvements on the text are made, i. e. some missing words are inserted and some different proofs added. This is done conscientiously and with great care. E.g. in the case of Prop. 6 the addition ends "I had written down this remark before looking at the construction of the proof. So I apologize." The marginal notes at the beginning and the end of the treatise inform us about the redactor.

His name is given as Muḥammad ibn Sartāq from Marāgha. It is remarked that he has studied this book by Aqāṭun, has verified and corrected it. The redactor is not otherwise known to us.

To the title of the Bankipore manuscript is added "Thābit ibn Qurra translated the treatise from the Greek language into the Arabic language". This leads to the question of what the original Greek title may have been, and the Greek spelling of Aqāṭun [On author and title see thesis, Chapter II]. The suppositions were excluded from the thesis, as no definitive answer can be given. The hypotheses are laid down in the following.

a. *The Title*

In the Arabic sources we find the titles *Kitāb al-ma'khūdhāt*, *Kitāb al-muᶜṭayāt*, *Kitāb al-mafrūḍāt*. The first of these is connected with the verb *akhadha* (أخذ) with basic meaning "to take". This corresponds with the Greek verb λαβεῖν. Also the original meaning of *ma'khūdhāt*, i. e. takings, receipts, returns (commerce) and of λημματα are equivalent. Thus *Kitāb al-ma'khūdhāt*, refers to the Greek title "Lemmata", e. g. Archimedes.

The second title, *Kitāb al-muᶜṭayāt* is related to the verb *aᶜṭaya* (عطى IV) which means "to present, offer". The corresponding Greek verb is διδόναι. Thus *Kitāb al-muᶜṭayāt* is the translation of the Greek title δεδόμενα, e. g. Euclid's Data.

As for the title *Kitāb al-mafrūḍāt*: in the case of Thābit ibn Qurra this title is sometimes translated as "Data". As I pointed out above, Data has usually a different Arabic equivalent. Therefore I tried to find another possible Greek title. *Mafrūḍāt* is a form of the verb farada (فرض) meaning something like "to decide, impose, assume, suppose, postulate". This could be rendered by the Greek verb υποτιθεσθαι (inf. med.), and thus *mafrūḍāt* could be a translation of υπόθεσεις. To this E. M. Bruins objects that nowadays the Arabic word for assumption, supposition, hypothesis is *iftirāḍ*. He therefore suggests starting from a more special meaning of *faraḍa*, namely "to take for granted". This could then correspond with the Greek verb συγχωρεω and we could assume *mafrūḍāt* to be a translation of συγχωρησεις.

This small exposition may have made clear how difficult it is, if not impossible, to establish the original Greek title.

b. *The Author*

The author's Greek name, which was arabized into Aqāṭun can only be guessed at. Some want to explain Aqāṭun as a misreading of Aflāṭūn (= Plato). According to J. van Ess this cannot be correct as Aqāṭun is written with an article in the last two marginal notes (fol. 102v). Aflāṭūn is never written with an article. Van Ess adds that in general an article with the name of an ancient author is somewhat curious. As the addition "translated from the Greek language" has been omitted in the Istanbul manuscript, its redactor may not have recognized Aqāṭun as an ancient Greek author. As the name Aqāṭun occurs, according to Lippert in this same form in all codices of the *Ta'rīkh al-ḥukamā'* (p.195), I assume its writing to be correct. This would point to a Greek name like ʿΕκατον. A more probable Greek name would be ʾΑγαθων. However, this means three exceptions in one word from the normal rules of transliteration. M. Ullmann has shown that the Greek word λυκανθρωπος (werewolf) became *quṭrub* in Arabic by means of a Syrian intermediary, in which θ was already rendered by *ṭ*. In our case he also suggested an intermediary, namely Pahlavi (= the Iranian language of Sassanid Persia). D. N. MacKenzie and W. Sundermann agreed with this eventual possibility, and gave more details:

Although one expects *g* in Pahlavi for γ, *k* sometimes occurs *(hykmwn* for ηγεμόν*)*. Then ʾΑγαθων could have been rendered by **'k'twn*, which would have to be written in Arabic as **'q'ṭwn*. Because of the ambiguity of some Pahlavi letters, the ending *–wn* could have been misread as *-n'*. In this way one could arrive at *'q'ṭn*, Aqāṭun. Already C. A. Nallino pointed out that scientific works were translated from Greek into Pahlavi into Arabic. He notes in this context that the extreme ambiguity of Pahlavi writing makes it impossible to read foreign names with certainty. A. Schall comments that the rules in everyday life are not so strict: the correlation γ ~ ق (*q*) still exists in dialects, especially in names, e.g. (Jordan:) قُسُّس (priest) is rendered by Goussous, and (Sudan:) عبد القادر by Abdel Gadir. ʾΑγαθων was a rather common Greek name, but no mention of a mathematician of this name has been transmitted to us.

Appendix

Among the pages of the treatise one, fol. 95, does not belong to the text, which after fol. 94v continues on fol. 96r. This page may belong to another treatise of the manuscript, as it contains on one side, fol. 95v, a referential proposition which has no apparent connection with the contents of the *Kitāb al-mafrūḍāt*. It is written in the same hand as our treatise. Fol. 95r consists of two three-dimensional drawings not belonging to the proposition on fol. 95v. Seemingly they are not of an astronomical nature.

The proposition on fol. 95v reads: Proposition by Muḥammad ibn Mūsā from the book "On the Sphere", which refers to it:

The intersection of any circular cone or cylinder with a plane parallel to the base is also a circle. And the line drawn from the vertex to the center of the base passes through the center of the intersection.

The proposition is followed by an example with proof, but no drawing.

References

Yvonne Dold-Samplonius, *Book of Assumptions by Aqāṭun* (Doctoral Thesis, Amsterdam, 1977).

Max Krause, "Stambuler Handschriften islamischer Mathematiker", *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B, vol. 3 (Berlin, 1934), 437-532.

Carlo A. Nallino, "Tracce di opere greche giunte agli Arabi per trafila pehlevica", *A Volume of Oriental Studies Presented to Edward G. Browne*, T. W. Arnold and R. A. Nicholson eds. (Cambridge, 1922), 345-363.

Ibn al-Qifṭī, *Ta'rikh al-ḥukamā'*, Julius Lippert ed. (Leipzig, 1903).

Fuat Sezgin, *Geschichte des arabischen Schrifttums.* Band V, Mathematik. Bis ca. 430 H. (Leiden, 1974).

Manfred Ullmann, "Der Werwolf", *Wiener Zeitschrift für die Kunde des Morgenlandes*, 68 (1976), 171-184