What Is Good for Mathematics? Thoughts on the Clay Millennium Prizes

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Around the year 2000, when information about the Clay Millenium Prizes for the solution of the seven specially selected mathematical problems became public, I met my old friend Arthur Jaffe, who was then president of the Clay Mathematics Institute. I asked him: "What is this being done for?" At the time I felt that the assignment of huge (million-dollar) prizes was more in keeping with the style of show business, aiming at drawing attention to something or somebody at any price, whereas scientific life should avoid cheap popularization.

Indeed, thought I, will "money tags" attached to the solution of scientific problems, and not their intrinsic interest, add to the mathematicians' enthusiasm?: if one of us is already working on the Riemann Hypothesis or the Poincaré Conjecture, no additional enticement is needed. And it will attract serious mathematicians to one these problems only if that person worked on the problem before or was an expert in the corresponding field.

Arthur answered me decidedly and professionally: "You understand nothing about the American way of life. If a politician, a businessman, a housewife will see that one can earn a million by doing mathematics, they will not discourage their children from choosing that profession, will not insist on their doing medicine, law, or going in for some other lucrative activity. And other rich philanthropists will be more likely to give money to mathematics, which is in such need of it."

At the time that answer partially convinced me. Nevertheless, in the years that followed, I have not learned to understand the American way of life any better, and seem to understand the logic of life in Russia less and less.

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Now that one of the "million dollar problems"—the Poincaré Conjecture—has been solved, I would like to come back to my original question: was the million-dollar venture useful for mathematics? Let me say at once that I have returned to my original position.

As I mentioned above, the number of people working on the Poincaré Conjecture and, probably, the other six problems, has not increased after the Clay millenium problems were announced. The person who solved it, G. Ya. Perelman, started working on it long before. The Clay Mathematics Institute has nothing to do with his solution. And the other mathematicians who still claim to have solved it, as far as I know, were also working on it before. And it is foolish to think that one of the nonspecialists (even a mathematician), having heard of the prize, has any chance of solving a problem of that level. Thus no rise in the progress of mathematics due to the new financial stimulus has occurred.

Intrinsically, the solution of the problem, as well as the method used to solve it, is a remarkable success of mathematics, an outstanding scientific achievement. And the Clay Institute has played no role in this.

I would also like to note that the stir created around the seven "millenium problems" creates the wrong impression in society about the work of mathematicians, supporting the hackneyed notion that it consists only in solving concrete problems.[1] You don't have to be an expert to understand how misleading that notion is. The discovery of new domains and relationships between different branches of mathematics, the setting of new problems, the development and perfection of the mathematical apparatus, and so on, are no less

[1] *A journalist from Russia once asked me: "In order to get the million, do you have to solve all seven problems, or will one do?"!*

important and difficult parts of our science, without which it cannot exist.

More generally, will the approach of the Clay Institute increase interest in mathematics and increase the influx of young people into the field? I am not sure. One must understand that somebody fascinated by mathematics as a teenager needs no additional stimuli, while those who, in their choice of profession, are primarily interested in ensuring a normal comfortable life do not need a million-dollar prize for solving an inaccessible problem, but need something completely different.

As to the interest of the general public in mathematics, it certainly did flare up for a while. No newspaper or TV channel passed by the sensational news. From August 20 and 21, when articles in the *New York Times* and the *New Yorker* appeared, to mid-September, passions did not die down. How many journalists inquired at our institute, seeking out mathematicians they knew or didn't know, asking for interviews, asking questions about the nature of the problem, demanding what its impact on everyday life is! Now at least everyone knows the name of Henri Poincaré and, of course, that of Perelman, and people interested in science did learn something about the problem. This was the case, and it was a good thing.

But what interests nonmathematicians above all? What questions are heard most often in this clamor? Such was the fate of the first test of the Clay Institute's initiative that it involved unexpected tragicomic circumstances. One can say that these circumstances were apparently unrelated to the expectations of the organizers, although who knows what is related, what isn't. The main question that interests the mass media and the general public is not the problem whose solution mathematicians were eagerly waiting for so many years, not what is going on in mathematics—all that was too difficult to understand, too inaccessible. And not even the personality of G. Ya. Perelman ("Completely nuts, he's a mathematician, they're all like that"). No. The main question was: "Why did he refuse the million?" Actually, after numerous explanations, some journalists (not all), and through them part of the public, finally understood that so far only the Fields Medal has been refused, the million has not been awarded yet. Nevertheless, the overwhelming part of the commentaries, at least in the Russian media, concerned that question only. Unfortunately, most of these commentaries were unintelligent, tactless, or even obnoxious.

All this moves to the background the meaningful part of the event and impedes the reader's real understanding of it. And, of course, the most improbable gossip is picked up with enthusiasm, such as that Perelman was supposedly slighted, thrown out of the institute, his results were plagiarized and so on. For example, see the article by N. Lobastova and M. Hirst in the *Sunday Telegraph*

of August 28, or in the Wikipedia article about G. Ya. Perelman (which, most unfortunately, are referenced by the site of the International Mathematical Union). There are other, obviously unforeseen, consequences of the million-dollar undertaking.

Here is one of them. As the present example shows, some serious mathematicians have succumbed to the temptation of engaging in a discussion of the question of priority, involving real and imaginary complications. This is being done quite professionally, but it is difficult to avoid the thought that the aim is not only to share in the honor of solving an outstanding problem, but also to share in the spoils. Let us hope that the mathematical community will reject these claims, but where is the guarantee that in less obvious situations such attempts will not be successful?

In my opinion, all this clamor and fuss show that this method of promoting mathematics is warped and unacceptable, it does not popularize mathematics as a science, on the contrary, it only bewilders the public and leads to unhealthy interest. I don't think that these passions are only explained by the peculiarities in the behavior of today's hero, which of course tend to aggravate the emotions; things are deeper. The question is, does mathematics need such an indecent interest? Would such a reaction have occurred if not for the conspicuous presentation of the Clay prizes? Probably not. The proof of Fermat's Last Theorem in 1996 by Andrew Wiles did not lead to such a tumult, although the problem solved was no less important than Poincaré's.

The explanation is simple: two poorly compatible things became too closely connected: a serious scientific result and, out there in the forefront, the "million".

Within their own community, certain mathematicians, e.g., Paul Erdős, when setting a problem, liked to estimate their value by a number of bottles of beer, or glasses of martini, or a small number of dollars, but that was done in fun and was harmless. The French Academy at one time also proposed prizes for the solution of some mathematical problems, but the prizes were rather modest and were never presented with such pomp. The prestigious Fields Medal is above all a medal; the monetary prize given in parallel is modest, remains in the background, and is hardly ever talked about. The Nobel Prize and the Abel Prize, despite discussions about the equity of the choice of prizewinner, bring to mind, above all, the idea of outstanding scientific achievements. Rare refusals to accept the prizes have occurred in the past, but always had concrete motivations; convincing or not—but that is another question.

Certainly, after an important mathematical problem is solved, and many were solved in the twentieth century, the author should be

significantly encouraged (provided he/she does not refuse), and means for this are usually found. There is one thing wrong about mathematical prizes—there just aren't enough of them. Unfortunately, as things stand now, the few prizes that exist are distributed among the same small group of people. But again, this is another question.

In our case we are dealing with an a priori, an excessive monetary estimate of the solution of one of several scientific problems. Is it really necessary to estimate their cash equivalent (with a long line of zeros), and if so, how are we to go about it? The Hilbert problems were not evaluated in millions, but their popularity among active mathematicians did not suffer from this. To transform serious research problems into something like a million-dollar lottery is a totalistic means to indulge the bad taste of the mob. In response, we get a social effect in keeping with the underlying scale of "values". Science should be promoted by more sophisticated means, while the funds that far-seeing business people are willing to bestow on mathematics, and which we need so badly, should be used more efficiently. Popularization of math for the general public is indeed necessary, but not of the kind that is characteristic of the worst manifestations of present-day mass culture.

Undoubtedly, mathematics is in dire need of serious support, including financial support, as well as in the need of a general public much better informed about what goes on in our field. In contrast to this, the newspapers, especially in Russia, are presently "discussing" a completely different question: Is mathematical education, and mathematics itself, really necessary in contemporary society (see the series of articles on the subject in http://www.gazeta.ru and other sources)? However strange it may seem, the topic of the "millions" only inflames such passions and guides them in the wrong direction, while the situation of Russian scientists, especially the younger ones, remains as difficult as ever.

The Clay Institute conducts a very useful and successful program for supporting young mathematicians, helps organize scientific conferences and seminars, and so on, and this work can serve as an example for other foundations or private individuals.

But I firmly believe that the mathematical community can and must find new reasonable means of propaganda and promotion (and I don't mean popularization only). New means (including monetary ones, of course) are needed to attract attention to mathematics and to the outstanding events in our science, as are dignified ways of expressing recognition to its most outstanding representatives.