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THE CONTACT BETWEEN CONFORMAL SURFACES ACCORDING TO STEUERMANN'S THEORY. COMPARISON BETWEEN STEUERMANN'S, HERTZ'S AND PANTON'S THEORIES

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ABSTRACT
The paper presents an analysis concerning the contact cases between a cylinder rolling wheel and a cylinder cavity as well between a circular cylinder and a cylinder cavity when the simplified hypothesis of the Hertz's contact theory can not be accepted. There may be observed a very good correspondence between Steuermann's and Hertz's theory in case of contact angles less than 20 ^{0}.
Key-words:
conformal surfaces, elastic half-spaces, contact pressure

1. INTRODUCTION

The classical contact theory between the elastic bodies, which was proposed by Hertz and Beleaev (1), allowed the approximation that the contact surface sizes have very small dimensions in comparison with the dimensions of the bodies in contact. The classical theory did not consider an important aspect which lead to difficulties in obtaining the solution. It represents the non-linear aspects of every contact problem, leading to a non-iterative solution.

According to Hertz`s theory, the initial contact takes place in a single point, resulting an ideal case which is not often found in practice. The contact between conformal surfaces didn`t allow to approximate the surface equations by second order polinomial functions, as Hertz considered for the non-conformal surfaces, neglecting the higher order terms.

In order to eliminate the approximations, new theories (Panton, Steuermann and others) have been advanced during the time in order to calculate the state of stress and deformation in contact problems.

2. THE STEUERMANN`S CONTACT THEORY

The initial distance h between the points placed on a common vertical line which will be in contact in the next moments is:

$$h = z_1 + z_2 = A_1 x^2 + A_2 x^4 + ... + A_n x^{2n} + ... \qquad (1)$$

The mathematical relation is valid for the two-dimensional contact when the surfaces in contact present a simmetry to the initial contact point. When the bodies in contact present a simmetry to an initial contact axys, the following mathematical relation is convenable to be used:

$$h = A_1 r^2 + A_2 r^4 + ... + A_n r^{2n} + ... \quad (2)$$

The surfaces in contact z_1 and z_2, which for Hertz approximated their equations, correspond to the case n = 1.

For an axys-simmetrical contact, Steuermann used sophisticated mathematical relations for pressure and displacement (compression):

$$p_n(r) = \frac{nA_n E^* a^{2n-2}}{\pi}\left[\frac{2\cdot 4...2n}{1\cdot 3...(2n-1)}\right]^2\left[\left(\frac{r}{a}\right)^{2n-2} + \frac{1}{2}\left(\frac{r}{a}\right)^{2n-4} + ... + \frac{1\cdot 3...(2n-3)}{2\cdot 4...(2n-2)}\right]\left(a^2 - r^2\right)^{\frac{1}{2}}$$

respectively:

$$\delta = \frac{2\cdot 4\cdot...2n}{1\cdot 3...(2n-1)} A_n a^{2n} \quad (3)$$

A particular problem, with an important practical application, and which has been analyzed by some authors (1),(2),(3) and (4), is represented by the contact between a cylinder (roller) and a cylinder cavity (Fig.1).According to Steuermann`s theory, the bodies in contact will not be considered as elastic half-spaces. Moreover the radius difference $\Delta R = R_2 - R_1$ present small values in comparison both with R_1 and R_2. The deformed contact zone is presented in Fig.2. The points belonging to surfaces S_1 and S_2, which will come in contact on a common surface S, will present both radial u_r, and tangential u_θ displacements.

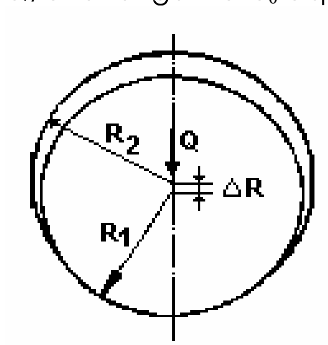

Fig.1. The contact between a cylinder and a cylinder cavity

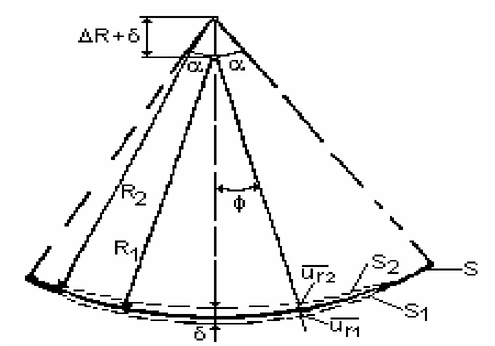

Fig.2. The deformed state in the contact zone

If $\delta << R_1, R_2$, the following geometrical relation is valid:

$$\left(R_2 + \overline{u}_{r2}\right) - \left(R_1 + \overline{u}_{r1}\right) = (\Delta R + \delta)\cos\Phi \quad (4)$$

resulting: $\overline{u}_{r2} - \overline{u}_{r1} = \delta\cos\Phi - \Delta R(1 - \cos\Phi)$

Because $-\alpha < \Phi < \alpha$, with α which can not be neglected (case of contact inside bearings), the above mentioned geometrical relation presents important differences in comparison with Hertz`s displacement equation for the contact between two cylinders with parallel axys:

$$w_1 + w_2 = \delta - \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)x^2 \quad (5)$$

For this contact case, the contact patch is a rectangle with the width 2b.

The real problem (a reversed problem) was to find a state of pressure which conduce to a radial relative displacement $\left(\overline{u}_{r2} - \overline{u}_{r1}\right)$ as in relation (4). The problem was solved by Persson who proposed special pressure dependences and functions. Persson`s pressure distribution is presented in Fig.3. It may be observed that the contact pressure increase when decreasing the Φ contact angle and when, of course, the sizes of the contact patch decrease too.

The variation of the contact angle limits (the arc) α (for the case $\Delta R > 0$ as well for the case $\Delta R < 0$) function of the normal surface loading Q is presented in Fig.4. In order to obtain a suggestive comparison, the variation proposed by Hertz and Steuermann are plotted on the same system of axys.

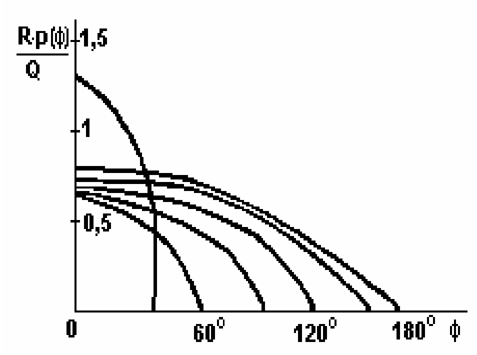

Fig.3. The Persson`s contact pressure distribution

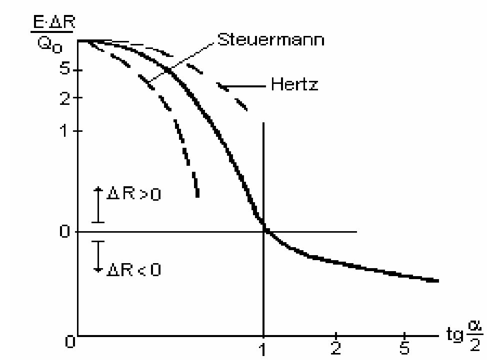

Fig.4. The variation of the contact angle limits (the arc) α function of the normal surface loading Q

It may be observed that the obtained results, according to the above mentioned contact theories, are in a convenable agreement only for small values of the normal load Q or for high values of the curvature radius difference ΔR. For high values of the normal load Q or for small values of the curvature radius difference ΔR, the results are far from that obtained with the most accurate Steuermann`s theory which consider the bodies in contact as elastic half-spaces.

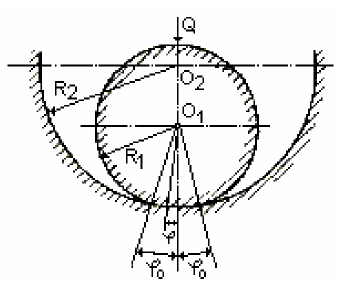

Fig.5. Contact between a cylinder and a cylinder cavity

A similar problem, namely the contact without friction between a sphere and a spherical cavity has been studied by Goodman and Kerr (5). They concluded that the displacement of the centers of the bodies in contact δ is 25% higher than the displacement predicted by Hertz`s theory.

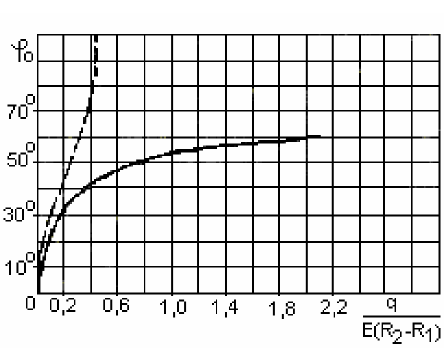

Fig.6. Comparison between Steuermann`s and Hertz`s results

In case of a linear contact on the common generatrice between a cylinder and a cylinder cavity with parallel axys (Fig.5) the value of the ratio q = Q/l represents the uniformly distributed pressure on the initial common contact generatrice.

3. CONCLUSIONS

A comparison between the results obtained by Steuerman and Hertz (Fig.6) leads to the following conclusion: when the ratio $\frac{q}{E\left(R_2 - R_1\right)}$

increase, the difference between the results (according to the above mentioned theories) also increase. Anyway, there is a relative good agreement between the results for contact angles less than 20^0.

The most important conclusion is that for a significant contact patch, the contact problem between a cylinder and a cylinder cavity may be solved only by Steuermann`s theory instead of it`s mathematical sophisticated apparatus.

The normal contact force Q function of the contact angle φ is calculated according to Steuermann`s, with the formula:

$$\begin{aligned} Q(\varphi) &= -\sec^2\varphi F(tg\varphi) - \frac{k}{\pi} R(tg\varphi) - \\ &- \frac{k}{\pi^2}\int_0^{tg\varphi}\left[\pi^2 F(t) + \frac{\pi k R(t)}{1+t^2}\right]\sin k(\varphi - arctg\, t)dt + \frac{k}{\pi}\int_0^{tg\varphi}\left[F(t) - \frac{qt}{a^2-t^2}\right]\cos k(\varphi - arctg\, t)dt + k\cos k\varphi \end{aligned} \tag{6}$$

The relevance of notations in relation (6) is presented in (2), page 154.

For the particular case when the two cylinders are manufactured from the same material, E. Panton proposed the following approximate formula:

$$Q(\varphi) = \frac{Q}{R_1(\sin\varphi_0\cos\varphi_0 + \varphi_0)}\cos\varphi \tag{7}$$

There is presented in Fig.7 the distribution Q(φ) for three values of the contact angle: φ_0 = 30^0, 50^0 and 60^0 according to Panton`s (non-continuous line) and Steuermann`s theories (continuous line).

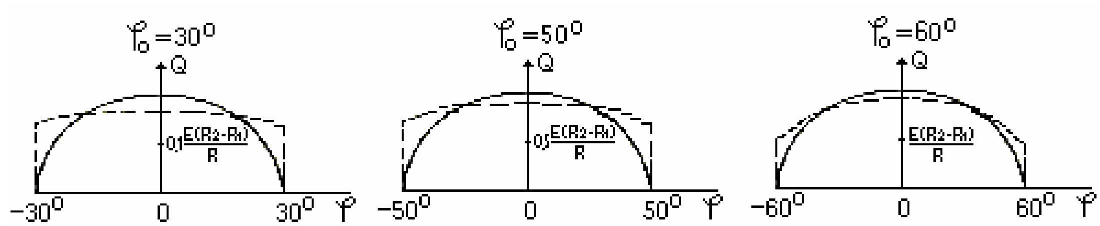

Fig.7. The distribution Q(φ) according to Panton`s (non-continuous line) and Steuermann`s theories (continuous line)

It may be observed in Fig.7 that the results according to the above mentioned two theories are in a perfect agreement when increasing the contact angle φ_0.

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