Kiyotaki and Moore [1997]

Econ 235, Spring 2013

- Heterogeneity: why else would you need markets!
- When assets serve as collateral, prices affect allocations
- Importance of who is pricing an asset
- Best users vs second-best users
- Forward-looking asset prices
- Asset trades as imperfect replacements for missing markets
- Amplification and persistence

1 The basic model

1.1 Technology

- K is the endowment of land
- There is a measure 1 of farmers and a measure m of gatherers, with access to different technologies
- If used by farmers, land produces fruit at a rate of

$$y_{t+1} = (a + c)\, k_t$$

- c output is "nontradeable" the farmer must consume it in the period: upper bound on savings rate
- If used by gatherers, land produces fruit a a rate of

$$y_{t+1} = G(k_t^G)$$

with $G' > 0$, $G'' < 0$

1.2 Preferences

- Both farmers and gatherers are risk-neutral and discount the future at rates

$$\beta^F < \beta^G$$

- This pins down the interest rate at $R = \frac{1}{\beta^G}$. At this interest rate, Farmers want to borrow as much as possible (there are infinite gains from trade!) but a borrowing constraint will prevent this

1.3 Markets

- Competitive market for land: q_t units of fruit per unit of land
- One-period borrowing at gross interest R
- Debt requires collateral

$$Rb_t \leq q_{t+1}k_t$$

  - Threat not to work
  - Ability to steal output

- Note the timing convention and the lack of uncertainty
  - Borrow b_t at t
  - Need to repay Rb_t at $t+1$
  - Own k_t at the end of period t
  - This land will yield $(a+c)\,k_t$ and be worth $q_{t+1}k_t$ in period $t+1$

  (sometimes people call the amount of capital you buy in period t to use in period $t+1$ "k_{t+1}". Here we are calling that "k_t")

- Key implicit assumption: lenders cannot foreclose on output, just on land
- Paper has a long discussion of this, the threat of renegotiation, etc.

2 Equilibrium

2.1 Farmer's problem

$$\max_{x_t,k_t,b_t} \sum \left(\beta^F\right)^t x_t$$

$$s.t. \quad x_t + q_t k_t + R b_{t-1} \leq (a+c)\, k_{t-1} + q_t k_{t-1} + b_t$$

$$R b_t \leq q_{t+1} k_t \tag{1}$$

$$x_t \geq c k_{t-1} \tag{2}$$

Assumption 1. $c > \left(\frac{1}{\beta} - 1\right) a$

Proposition 1. *Near a steady state, both (1) and (2) bind*

- Note that this is a statement about *both* the solution to the farmer's problem and about equilibrium prices in steady state
- The proof is by guess and verify
- Let
$$u_t \equiv q_t - \frac{q_{t+1}}{R} \tag{3}$$
- u_t is the user cost of capital: it costs q_t to buy it but you can resell it tomorrow for q_{t+1}
- u_t is also the downpayment requirement for a unit of land: it costs q_t to buy it but you can borrow $\frac{q_{t+1}}{R}$ against it.
- Why? Comes from the specific form of the borrowing constraint
  - Lender knows he can foreclose on land *tomorrow*
  - Lends *today* up to the PV of land tomorrow
  - In order to buy land the downpayment is the difference between the value of land today q_t and the PV of land tomorrow $\frac{q_{t+1}}{R}$
  - This is also the cost of using the land for one period
- Consider the following uses for the marginal unit of tradeable fruit:
  1. Consume it
  2. Invest it with maximum leverage and reinvest the net tradeable fruit you obtain because of this, also with maximum leverage (consuming the extra nontradeable fruit)

3. Use it to repay debt for one period, and then resume path 2

- The path of additional consumption you obtain:

1. $\{1, 0, 0, \ldots\}$
2. The following things happen:
   - k_t increases by $\frac{1}{u_t}$
   - Rb_t increases by $\frac{q_{t+1}}{u_t}$
   - Therefore $(a+c)\,k_t + q_{t+1}k_t - Rb_t$ (which is RHS of budget constraint at $t+1$) increases by $(a+c+q_{t+1})\frac{1}{u_t} - \frac{q_{t+1}}{u_t} = (a+c)\frac{1}{u_t}$.
   - Because he can pledge the land but not the fruit, investing with maximum leverage increases the farmer's net worth by the amount of additional fruit he will produce
   - By assumption he will consume the nontradeable fruit $\frac{c}{u_t}$ and buy land with the tradeable fruit, with maximum leverage, obtaining $\frac{a}{u_t}\frac{1}{u_{t+1}}$ units of additional land in period $t+1$, etc.
   - The path of additional consumption is $\left\{0, \frac{c}{u_t}, \frac{a}{u_t}\frac{c}{u_{t+1}}, \frac{a}{u_t}\frac{a}{u_{t+1}}\frac{c}{u_{t+2}}, \ldots\right\}$
3. The following things will happen:
   - $(a+c)\,k_t + q_{t+1}k_t - Rb_t$ will increase by R due to the reduced borrowing
   - By assumption, the farmer will use this extra net worth to increase land purchases with maximum leverage, obtaining $R\frac{1}{u_{t+1}}$ units of land in period $t+1$, etc.
   - The path of additional consumption will be $\left\{0, 0, R\frac{c}{u_{t+1}}, R\frac{a}{u_{t+1}}\frac{c}{u_{t+2}}, \ldots\right\}$

- To compare these we need to find out the equilibrium values of R and u
- Now guess that the proposition is true and solve for capital holdings for the farmer:

$$
\begin{aligned}
x_t + q_t k_t + Rb_{t-1} &= (a+c)\,k_{t-1} + q_t k_{t-1} + b_t \\
q_t k_t + Rb_{t-1} &= ak_{t-1} + q_t k_{t-1} + \frac{q_{t+1}k_t}{R} \\
k_t\left[q_t - \frac{q_{t+1}k_t}{R}\right] &= ak_{t-1} + q_t k_{t-1} - Rb_{t-1} \\
k_t &= \frac{1}{u_t}\left[ak_{t-1} + q_t k_{t-1} - Rb_{t-1}\right] \qquad (4)
\end{aligned}
$$

- Interpretation: tradeable net worth with maximum leverage

- Now impose that the proposition is true at $t-1$ as well:

$$\begin{aligned} k_t &= \frac{1}{u_t}\left[ak_{t-1} + q_t k_{t-1} - q_t k_{t-1}\right] \\ &= \frac{1}{u_t} a k_{t-1} \end{aligned} \tag{5}$$

- Impose steady state:

$$\begin{aligned} k &= \frac{1}{u} a k \\ u^{ss} &= a \end{aligned}$$

- User cost is equal to tradeable marginal product
  - Because you dedicate all the tradeable output to servicing debt
- Note than gatherers have no borrowing constraints, so the loan market will clear only if $R = \frac{1}{\beta^G}$
- Now we are in a position to compare the farmer's options.

1. NPV on consuming at the margin: 1
2. NPV of leveraged investment

$$0 + \beta^F \frac{c}{a} + \left(\beta^F\right)^2 \frac{c}{a} + \left(\beta^F\right)^3 \frac{c}{a} + ... = \beta^F \frac{\frac{c}{a}}{1-\beta^F}$$

   Assumption 1 exactly guarantees that this is better than immediate consumption

3. NPV of saving first and leveraged investment later:

$$0 + 0 + \left(\beta^F\right)^2 R\frac{c}{a} + \left(\beta^F\right)^3 R\frac{c}{a} + \ldots = \left(\beta^F\right)^2 R\frac{\frac{c}{a}}{1-\beta^F}$$

   which is inferior to leveraged investment right away as long as

$$\begin{aligned} \left(\beta^F\right)^2 R &< \beta^F \\ \Leftrightarrow \beta^F &< \beta^G \end{aligned}$$

   which we have assumed

- Note on modeling technique: highly nonstandard assumptions (linear utility, difference in discount rates, weird nontradeable technology, etc...) to highlight one single mechanism.

2.2 Gatherer's problem

$$\max_{x_t,k_t,b_t} \sum_{t=0}^{\infty} \left(\beta^G\right)^t x_t$$
$$s.t. \quad x_t + q_t k_t^G + Rb_{t-1} \leq G\left(k_{t-1}^G\right) + q_t k_{t-1}^G + b_t$$

- FOC:

$$\left(\beta^G\right)^t - \lambda_t = 0 \tag{6}$$
$$-q_t\lambda_t + \left[G^{'}\left(k_t\right) + q_{t+1}\right]\lambda_{t+1} = 0 \tag{7}$$
$$\lambda_t - R\lambda_{t+1} = 0 \tag{8}$$

- From (6) and (8), we confirm that
$$R = \frac{1}{\beta^G}$$

- From (7) and (8),

$$\begin{aligned} G^{'}\left(k_t^G\right) &= q_t \frac{\lambda_t}{\lambda_{t+1}} - q_{t+1} \\ &= q_t R - q_{t+1} \\ &= Ru_t \end{aligned} \tag{9}$$

2.3 Equilibrium

- In equilibrium, the land market has to clear, so

$$k_t + mk_t^G = K$$

where k_t satisfies (4) and k_t^G satisfies (9)

Assumption 2. $G'(0) > Ra > G'(\frac{K}{m})$

- This assumption ensures that there will always be an interior solution for how much land the gatherers hold

- Define
$$u(k) \equiv \frac{1}{R}G'\left(\frac{K-k}{m}\right) \tag{10}$$

- This is the value that the user-cost of land must have in order for the land market to clear if the farmers have k units of land

- i.e. the user cost of land that will persuade gatherers to hold the rest of the land
- An equilibrium can be characterized by equations:
  - (4): evolution of farmers' land holdings
  - (1): borrowing constraint, which always binds
  - (10): user cost of capital for the market to clear
  - (3): relation between land price and user cost

  starting from any initial b_{t-1}, k_{t-1}

3 Dynamics

3.1 Steady state

- We have seen that in steady state $u = a$. Using (3)

$$\frac{R-1}{R} q^{ss} = u^{ss} = a \tag{11}$$

- Also, using (10):

$$\frac{1}{R} G' \left(\frac{K - k^{ss}}{m} \right) = u^{ss}$$

- And using (1):

$$b^{ss} = \frac{a}{R-1} k^{ss}$$

3.2 Impulse response

- Consider a completely unexpected shock at $t = 0$, which multiplies output by $1 + \Delta$ for one period only, assuming that the economy was in steady state
- (Notice that what will matter about its shock is not its effect on output but its effect on wealth.
- Start from (4):

$$k_0 u_0 = a(1 + \Delta) k_{-1} + q_0 k_{-1} - R b_{-1}$$

  and use:

  - $k_{-1} = k^{ss}$
  - $b_{-1} = b^{ss} = \frac{1}{R} q^{ss} k^{ss}$

- $u_0 = u\left(k_0^F\right)$

so that

$$k_0 u(k_0) = \left[a\left(1+\Delta\right) + q_0 - q^{ss}\right] k^{ss} \tag{12}$$

- For periods after 0, where we know that (1) binds, we can simply use (5), so

$$k_t u\left(k_t\right) = a k_{t-1} \tag{13}$$

- Equations (12) and (13) determine the dynamic path of how much land is used by farmers
- Log-linearize. Let

$$\hat{x} \equiv \log\left(\frac{x_t}{x^{ss}}\right)$$

and define

$$\frac{1}{\eta} \equiv \frac{d\log u(k_t)}{d\log k_t}$$

- η is the elasticity of land supplied to farmers w.r.t. the user cost for gatherers
- Related to the curvature of G. If G' is almost flat, a small increase in u releases a lot of land for farmers
- Log-linearize the LHS of equations (12) and (13):

$$k_t u(k_t) \simeq k^{ss} u\left(k^{ss}\right)\left[1 + \hat{k}_t + \frac{1}{\eta}\hat{k}_t\right]$$

- Now the RHS of (12), using that, by (11), $q^{ss} = a\frac{R}{R-1}$

$$\begin{aligned}\left[a\left(1+\Delta\right) + q_0 - q^{ss}\right] k^{ss} &\simeq \left[a\left(1+\Delta\right) + q^{ss}\left(1+\hat{q}_0\right) - q^{ss}\right] k^{ss} \\ &= a\left[\left(1+\Delta\right) + \frac{R}{R-1}\hat{q}_0\right] k^{ss}\end{aligned}$$

- Now the RHS of (13):

$$a k_{t-1} = a k^{ss}\left(1 + \hat{k}_{t-1}\right)$$

- Equating LHS and RHS and using $u\left(k^{ss}\right) = a$: (12) can be approximated by

$$\begin{aligned}k^{ss} u\left(k^{ss}\right)\left[1 + \hat{k}_0 + \frac{1}{\eta}\hat{k}_0\right] &= a\left[\left(1+\Delta\right) + \frac{R}{R-1}\hat{q}_0\right] k^{ss} \\ 1 + \hat{k}_0 + \frac{1}{\eta}\hat{k}_0 &= 1 + \Delta + \frac{R}{R-1}\hat{q}_0 \\ \left(1 + \frac{1}{\eta}\right)\hat{k}_0 &\simeq \Delta + \frac{R}{R-1}\hat{q}_0 \end{aligned} \tag{14}$$

and (13) can be approximated by

$$k^{ss}u\left(k^{ss}\right)\left[1+\hat{k}_t+\frac{1}{\eta}\hat{k}_t\right]=ak^{ss}\left(1+\hat{k}_{t-1}\right)$$
$$\left(1+\frac{1}{\eta}\right)\hat{k}_t=\hat{k}_{t-1} \tag{15}$$

- Equations (14) and (15) give us a linearized dynamic system in k and q
- RHS of (14): impact of shock on net worth (in percentage terms)
  - Direct
  - Indirect via price of land. Scaled up by $\frac{R}{R-1}$ due to initial leverage
- LHS: increase in land holdings
  - They increase less than one-for-one with net worth because of increasing u (higher user cost/downpayment requirement)
- (15) shows that there will be persistence
  - With maximum leverage, net worth $= ak_t$
  - More land in the past ⇒ more net worth today ⇒ more land today

3.3 Initial effects of the shock

- (3) is a standard asset-pricing equation. Iterating forward and assuming there are no bubbles:

$$q_0=\sum_{t=0}^{\infty}R^{-t}u_t$$

- Log-linearize LHS:

$$q_o\simeq q^{ss}\left(1+\hat{q}_0\right)=u^{ss}\frac{R}{R-1}\left(1+\hat{q}_0\right)$$

- Now RHS:

$$\sum_{t=0}^{\infty}R^{-t}u_t\simeq\sum_{t=0}^{\infty}R^{-t}u^{ss}\left(1+\hat{u}_t\right)=\sum_{t=0}^{\infty}R^{-t}u^{ss}\left(1+\frac{1}{\eta}\hat{k}_t\right)$$

- Therefore

$$u^{ss}\frac{R}{R-1}\left(1+\hat{q}_0\right) \simeq \sum_{t=0}^{\infty} R^{-t}u^{ss}\left(1+\frac{1}{\eta}\hat{k}_t\right)$$
$$\hat{q}_0 \simeq \frac{R-1}{R}\frac{1}{\eta}\sum_{t=0}^{\infty} R^{-t}\hat{k}_t$$

and using (15):

$$\begin{aligned}\hat{q}_0 &\simeq \frac{R-1}{R}\frac{1}{\eta}\sum_{t=0}^{\infty} R^{-t}\left(1+\frac{1}{\eta}\right)^{-t}\hat{k}_0 \\ &= \frac{R-1}{R}\frac{1}{\eta}\frac{1}{1-\frac{1}{R}\frac{\eta}{\eta+1}}\hat{k}_0 \end{aligned} \tag{16}$$

- Now (14) and (16) are a pair of equations that we can solve for $\hat{q}_0$ and $\hat{k}_0$. Solving:

$$\hat{q}_0 = \frac{1}{\eta}\Delta$$
$$\hat{k}_0 = \frac{\eta}{\eta+1}\left[1+\frac{R}{R-1}\frac{1}{\eta}\right]\Delta$$

- Effect on price is of same order of magnitude as shock! (even though the shock is very short lived).
- The term $\frac{R}{R-1}$ can be very large! So effect on k_0 could be much more than one-for-one
- Persistence is the reason why there is amplification. Suppose *future* user costs did not change. Then the change in the value of land would just be:

$$\hat{q}_0 = \frac{R-1}{R}\frac{1}{\eta}\hat{k}_t$$

(just the present effect, i.e. the first term of (16)). Solving simultaneously with (14), we would get:

$$\hat{q}_0 = \frac{1}{\eta}\frac{R-1}{R}\Delta$$
$$\hat{k}_0 = \Delta$$

Contrast these *static* multipliers with the *dynamic* multipliers above.

- The effect on total output at any point in time is given by:

$$\hat{y}_t = \frac{dY}{dk}\frac{k^{ss}}{Y^{ss}}\hat{k}_t$$

- Use that

$$\begin{aligned} Y_t &= (a+c)\,k_t + mG\left(\frac{K-k_t}{m}\right) \\ \frac{dY}{dK} &= a+c-G'(\cdot) \end{aligned}$$

and that in steady state

$$G'(\cdot) = Ra$$

to obtain

$$\begin{aligned} \hat{y}_t &= [a+c-Ra]\,\frac{k^{ss}}{Y^{ss}}\hat{k}_t \\ &= \frac{a+c-Ra}{a+c}\frac{(a+c)\,k^{ss}}{Y^{ss}}\hat{k}_t \end{aligned}$$

- $\frac{a+c-Ra}{a+c}$: productivity advantage of farmers
- $\frac{(a+c)k^{ss}}{Y^{ss}}$: share of farmers in output

4 Remarks

- The paper has a second section that most people don't read, which extends to capital accumulation (rather than fixed land)
- There is also an appendix where some of the most nonstandard assumptions are relaxed
- Some of the ideas in these less-known sections reappear in Kiyotaki and Moore [2012] and Kiyotaki and Moore [2005], which we will come to later
- The fact that shocks are unanticipated is quite important. Firms would want to insure against them if that was possible.
  - Krishnamurthy [2003]: they would want to get insurance, but maybe the collateral constraint of insurance providers are the key.
  - DiTella [2012]: firms would get insurance against certain types of shocks but not others.

References

Sebastian DiTella. Uncertainty shocks and balance sheet recessions. MIT Working Paper, 2012.

Nobuhiro Kiyotaki and John Moore. Credit cycles. *Journal of Political Economy*, 105(2):211–48, April 1997.

Nobuhiro Kiyotaki and John Moore. 2002 Lawrence R. Klein lecture: Liquidity and asset prices. *International Economic Review*, 46(2):317–349, 05 2005.

Nobuhiro Kiyotaki and John Moore. Liquidity, business cycles and monetary policy. http://www.princeton.edu/ kiyotaki/papers/km6-120215.pdf, 2012.

Arvind Krishnamurthy. Collateral constraints and the amplification mechanism. *Journal of Economic Theory*, 111(2):277 – 292, 2003.