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LRS Bianchi Type-I Universe in Creation-Field Cosmology

K.S. Adhav, M.V. Dawande, R.B. Raut, M.S. Desale

Department of Mathematics, Sant Gadge Baba Amravati University, Amravati, India

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Abstract. We have studied Locally Rotationally Symmetry (LRS) Bianchi type-I space-time filled with perfect fluid in the Hoyle-Narlikar C-field cosmology. The solutions have been studied when the creation field C is a function of time t only. The geometrical and physical aspects for models are also studied.

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1 Introduction

The observation of the cosmic microwave background (CMB) radiation indicates that our Universe is globally isotropic to a very high degree of precision. Therefore, our Universe is usually assumed to be described by the Friedmann-Robertson-Walker (FRW) metric in most of the literatures. The Bianchi cosmologies which are spatially homogeneous and anisotropic play an important role in theoretical cosmology and have been studied since the 1960s. For simplification and description of the large scale behavior of the actual universe, LRS Bianchi models have great importance. Lidsey [1] showed that these models are equivalent to a flat Friedmann-Robertson-Walker (FRW) universe. We know that close to the big bang singularity, neither the assumption of spherical symmetry nor of isotropy can be strictly valid. In order to study problems like the formation of galaxies and the process of homogenization and isotropization of the universe, it is necessary to study problems relating to inhomogeneous and anisotropic space-time [2]. Hence, we consider LRS Bianchi type-I space-time which is less restrictive than the spherical symmetry and provide an opportunity for the study of inhomogeneity. LRS Bianchi type-I space-time has been widely studied by many researchers [3–16].

The phenomenon of expanding universe, primordial nucleon-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) were supposed to be successfully explained by big-bang cosmology based on Einstein's field equations. However, Smoot *et al*. [17] has revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly meet our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big bang type of the model. Also, CMBR discovery did not prove it to be an outcome of big bang theory. In fact, Narlikar *et al*. [18] have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle [19], Bondi and Gold [20] have proposed steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. Moreover, they have shown that the statistical properties of the large scale features of the universe do not change. Further, the constancy of the mass density has been accounted by continuous creation of matter going on in contrast to the one time infinite and explosive creation of matter at $t = 0$ as in the earlier standard model. But the principle of conservation of matter was violated in this formalism. To overcome this difficulty Hoyle and Narlikar [21] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to account for the matter creation. In the C-field theory introduced by Hoyle and Narlikar there is no big bang type of singularity as in the steady state theory of Bondi and Gold [20]. A solution of Einstein's field equations admitting radiation with negative energy massless scalar creation fields C was obtained by Narlikar and Padmanabhan [22]. The study of Hoyle and Narlikar theory [21, 23, 24] to the space-time of dimensions more than four was carried out by Chatterjee and Banerjee [25]. RajBali and Tikekar [26] studied C-field cosmology with variable G in the flat Friedmann-Robertson-Walker model. Whereas, C-field cosmological models with variable G in FRW space-time have been studied by RajBali and Kumawat [27]. The solutions of Einstein's field equations in the presence of creation field have been obtained for Bianchi type universes by Singh and Chaubey [28].

In the present paper, we have considered a spatially homogeneous and anisotropic LRS Bianchi type-I cosmological model in Hoyle and Narlikar C-field cosmology. We have assumed that the creation field C is a function of time t only, *i.e.* $C(x,t) = C(t)$.

2 Hoyle and Narlikar *C*-field Cosmology

Einstein's field equations are modified by introducing a mass less scalar field called as creation field *viz.* C-field [21, 23, 24]. (Here $G = 1$ and $c = 1$).

The modified field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi \left({}^{m}T_{ij} + {}^{c}T_{ij} \right), \tag{1}$$

where ${}^{m}T_{ij}$ is the matter tensor of Einstein theory and ${}^{c}T_{ij}$ is the matter tensor due to the C-field which is given by

$$^{c}T_{ij} = -f\left(C_iC_j - \frac{1}{2}g_{ij}C^kC_k\right), \tag{2}$$

where $f > 0$ is a coupling constant and $C_i = \dfrac{\partial C}{\partial x^i}$.

Because of the negative value of T^{00} $\left(T^{00} < 0\right)$, the C-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$$^{m}T^{ij}_{;j} = -^{c}T^{ij}_{;j} = fC^iC^j_{;j}. \tag{3}$$

Here the semicolon (;) denotes covariant differentiation, *i.e.* the matter creation through non-zero left hand side is possible while conserving the over all energy and momentum.

The above equation is similar to

$$mg_{ij}\frac{dx^i}{ds} - C_j = 0. \tag{4}$$

which implies that the 4-momentum of the created particle is compensated by the 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy. Further, the C-field satisfies the source equation

$$fC^i_{;i} = J^i_{;i} \qquad \text{and} \qquad J^i = \rho\frac{dx^i}{ds} = \rho v^i,$$

where ρ is homogeneous mass density.

3 Metric and Field Equations

The spatially homogeneous and anisotropic LRS Bianchi-type-I space-time is described by the line element

$$ds^2 = dt^2 - A^2dx^2 - B^2\left(dy^2 + dz^2\right), \tag{5}$$

where $A(t)$and $B(t)$ are the cosmic scale factors and the functions of the cosmic time t only (non-static case).

The matter tensor for perfect fluid is

$$^{m}T^i_j = diag\left(\rho, -p, -p, -p\right), \tag{6}$$

where ρ is the homogeneous mass density and p is the isotropic pressure.

We have assumed that the creation field C is function of time t only, *i.e.* $C(x,t) = C(t)$.

Now, the Einstein's field equations (1) modified by Hoyle-Narlikar for metric (5) with the help of Eqs. (2), (3), and (6) can be written as

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi\left(\rho - \frac{1}{2}f\dot{C}^2\right), \tag{7}$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = 8\pi\left(-p + \frac{1}{2}f\dot{C}^2\right), \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 8\pi\left(-p + \frac{1}{2}f\dot{C}^2\right), \tag{9}$$

$$\dot{\rho} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(\rho + p) = f\dot{C}\left[\ddot{C} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{C}\right], \tag{10}$$

where dot $(\cdot)$ indicates the derivative with respect to t.

The spatial volume is given by

$$V = a^3 = AB^2, \tag{11}$$

where a is the mean scale factor.

The above equation (10) can be written in the form

$$\frac{d}{dV}(V\rho) + p = f\dot{C}(V)\frac{d}{dV}\left[V\dot{C}(V)\right] \quad . \tag{12}$$

In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the C-field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing C-field energy-density, *i.e.* the rate of creation of matter energy density per unit proper-volume is given by

$$\frac{d}{dV}(V\rho) + p = \alpha^2\dot{C}^2 \equiv \alpha^2 g^2(V) \quad , \tag{13}$$

where α is proportionality constant and we have defined $\dot{C}(V) \equiv g(V)$.

Substituting it in Eq. (12), we get

$$\frac{d}{dV}(V\rho) + p = fg(V)\frac{d}{dV}(Vg). \tag{14}$$

Comparing right hand sides of equations (13) and (14), we get

$$g(V)\frac{d}{dV}(gV) = \frac{\alpha^2}{f}g^2(V). \tag{15}$$

Integrating, we obtain

$$g(V) = c_1 V^{\left(\frac{\alpha^2}{f} - 1\right)} \quad , \tag{16}$$

where c_1 is the arbitrary constant of integration.

We consider the equation of state of matter as

$$p = \gamma\rho\,. \tag{17}$$

Here γ varies between the interval $0 \leqslant \gamma \leqslant 1$, whereas $\gamma = 0$ describes the dust universe, $\gamma = 1/3$ presents the radiation universe, $1/3 < \gamma < 1$ ascribes the hard universe and $\gamma = 1$ corresponds to the stiff matter.

Substituting Eqs. (16) and (17) in Eq. (14), we get

$$\frac{d}{dV}\left(V\rho\right) + \gamma\rho = \alpha^2 c_1^2 V^{2\left(\frac{\alpha^2}{f}-1\right)}. \tag{18}$$

Further which yields

$$\rho = \frac{\alpha^2 c_1^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} V^{2\left(\frac{\alpha^2}{f}-1\right)}. \tag{19}$$

Subtracting Eq. (8) from Eq. (9), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0. \tag{20}$$

Now, from Eqs. (11) and (20), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 \quad .$$

Integrating, this gives

$$\frac{A}{B} = d_1 \exp\left(x_1 \int \frac{dt}{V}\right), \quad d_1 = \text{const}, \quad x_1 = \text{const}. \tag{21}$$

From Eqs. (11) and (21), we obtain the scale factors as

$$A(t) = d_1^{2/3} V^{1/3} \exp\left[2\frac{x_1}{3}\int \frac{dt}{V}\right], \tag{22}$$

$$B(t) = d_1^{-1/3} V^{1/3} \exp\left[-\frac{x_1}{3}\int \frac{dt}{V}\right]. \tag{23}$$

Adding two times Eqs. (9), (8) and 3 times Eq. (7), we get

$$\left(\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 4\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}}{B}\right) = \frac{32}{2}\pi\left(\rho - p\right). \tag{24}$$

From Eq. (11), we have

$$\frac{\ddot{V}}{V} = \left(\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 4\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}}{B} \right). \tag{25}$$

From Eqs. (24), (25) and (17), we get

$$\frac{\ddot{V}}{V} = 12\pi \left(1-\gamma\right) \rho. \tag{26}$$

Substituting Eq. (19) in Eq. (26), we get

$$\frac{\ddot{V}}{V} = \frac{12\pi \left(1-\gamma\right) \alpha^2 c_1^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} V^{2\left(\frac{\alpha^2}{f}-1\right)}. \tag{27}$$

This further gives

$$V = \left\{ c_1(f-\alpha^2) \left[\frac{12(1-\gamma)}{(2\alpha^2 - f + \gamma f)} \right]^{1/2} \right\}^{\frac{f}{f-\alpha^2}} t^{\frac{f}{f-\alpha^2}}. \tag{28}$$

Substituting Eq. (28) in Eq. (16), we get

$$g = \frac{1}{(f-\alpha^2)} \left[\frac{12\pi \left(1-\gamma\right)}{(2\alpha^2 - f + \gamma f)} \right]^{-1/2} \frac{1}{t}. \tag{29}$$

Also, from equation $\dot{C}\left(V\right) = g\left(V\right)$, we get

$$C = \frac{1}{(f-\alpha^2)} \left[\frac{12\pi \left(1-\gamma\right)}{(2\alpha^2 - f + \gamma f)} \right]^{-1/2} \log t. \tag{30}$$

Substituting Eq. (28) in Eq. (19), the homogeneous mass density becomes

$$\rho = \frac{\alpha^2 f}{12\pi \left(1-\gamma\right) \left(f-\alpha^2\right)^2} \frac{1}{t^2}. \tag{31}$$

Using Eq. (17), the pressure becomes

$$p = \frac{\alpha^2 \gamma f}{12\pi(1-\gamma)(f-\alpha^2)^2} \frac{1}{t^2}. \tag{32}$$

From Eqs. (31) and (32), it is observed that

(i) when time $t \to \infty$, we get, density and pressure tending to zero, *i.e.*, the model reduces to vacuum;

(ii) when $f = \alpha^2$, there is singularity in density and pressure;

(iii) there is also singularity in density and pressure for $\gamma = 1$ (stiff fluid).

Now, substituting Eq. (28) in Eqs. (22) and (23), we get

$$A(t) = d_1^{2/3} K^{1/3} t^{\frac{f}{3(f-\alpha^2)}} \exp\left[\frac{2x_1}{3K}\left(1 - \frac{f}{\alpha^2}\right) t^{\frac{\alpha^2}{\alpha^2 - f}}\right], \tag{33}$$

$$B(t) = d_1^{-1/3} K^{1/3} t^{\frac{f}{3(f-\alpha^2)}} \exp\left[\frac{-x_1}{3K}\left(1 - \frac{f}{\alpha^2}\right) t^{\frac{\alpha^2}{\alpha^2 - f}}\right], \tag{34}$$

where

$$K = \left\{c_1\left(f - \alpha^2\right)\left[\frac{12\pi(1-\gamma)}{(2\alpha^2 - f + \gamma f)}\right]^{1/2}\right\}^{\frac{f}{f-\alpha^2}}.$$

4 Physical Properties

The expansion scalar θ is defined by $\theta = 3H$ and is found as

$$\theta = \left(\frac{f}{f - \alpha^2}\right)\frac{1}{t}. \tag{35}$$

The mean anisotropy parameter is defined by $\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)$ and is found as

$$\Delta = \frac{2x_1^2}{K^2}\left(\frac{f - \alpha^2}{f}\right)^2 t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)}. \tag{36}$$

The shear scalar σ^2 is defined by $\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 4H^2\right) = \frac{1}{2}AH^2$ and is found as

$$\sigma^2 = \frac{x_1^2}{9K^2} t^{2\left(\frac{f}{\alpha^2 - f}\right)}. \tag{37}$$

The deceleration parameter q is defined by $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$ and is found as

$$q = 2 - \frac{3\alpha^2}{f}, \tag{38}$$

where $\Delta H_i = H_i - H$.

Here H is the Hubble parameter and H_i are the directional Hubble parameter.

If $f > \alpha^2$ then for large t, the model tends to isotropic case.

Case I: $\gamma = \mathbf{0}$ (Dust Universe)

In this case, we obtain the values of various parameters as

$$
\begin{aligned}
g &= \frac{1}{f-\alpha^2}\left[\frac{\left(2\alpha^2-f\right)}{12\pi}\right]^{j12}\frac{1}{t}\,,\\
C &= \frac{1}{f-\alpha^2}\left[\frac{\left(2\alpha^2-f\right)}{12\pi}\right]^{j12}\log t,\\
\rho &= \frac{\alpha^2 f}{12\pi\left(f-\alpha^2\right)^2}\frac{1}{t^2}\,,\\
A(t) &= d_1^{2/3}K_1^{1/3}t^{\frac{f}{3(f-\alpha^2)}}\exp\left[\frac{2x_1}{3K_1}\left(1-\frac{f}{\alpha^2}\right)t^{\frac{\alpha^2}{\alpha^2-f}}\right],\\
B(t) &= d_1^{-1/3}K_1^{1/3}t^{\frac{f}{3(f-\alpha^2)}}\exp\left[\frac{-x_1}{3K_1}\left(1-\frac{f}{\alpha^2}\right)t^{\frac{\alpha^2}{\alpha^2-f}}\right],
\end{aligned}
$$

where

$$
K_1 = \left\{c_1(f-\alpha^2)\left[\frac{12\pi\left(1-\gamma\right)}{\left(2\alpha^2-f\right)}\right]^{1/2}\right\}^{\frac{f}{f-\alpha^2}}.
$$

In this case, the expansion scalar θ is given by

$$
\theta = \left(\frac{f}{f-\alpha^2}\right)\frac{1}{t}\,.
$$

The mean anisotropy parameter is given by

$$
\Delta = \frac{2x_1^2}{K_1^2}\left(\frac{f-\alpha^2}{f}\right)^2 t^{2\left(\frac{\alpha^2}{\alpha^2-f}\right)}.
$$

The shear scalar σ^2 is given by

$$
\sigma^2 = \frac{X^2}{9K_1^2}t^{2\left(\frac{f}{\alpha^2-f}\right)}.
$$

The deceleration parameter q is given by

$$
q = 3-\frac{4\alpha^2}{f},
$$

If $f > \alpha^2$, this model tends to isotropy for large t.

Case II: $\gamma = {}^1\!/_3$ (Disordered Radiation Universe)

In this case, we obtain the values of various parameters as

$$g = \frac{1}{f-\alpha^2}\left[\frac{3\left(\alpha^2-f\right)}{32\pi}\right]^{1/2}\frac{1}{t},$$

$$C = \frac{1}{f-\alpha^2}\left[\frac{\left(3\alpha^2-f\right)}{12\pi}\right]^{1/2}\log t,$$

$$\rho = \frac{3\alpha^2 f}{4\pi\left(f-\alpha^2\right)^2}\frac{1}{t^2},$$

$$p = \frac{\alpha^2 f}{4\pi\left(f-\alpha^2\right)^2}\frac{1}{t^2},$$

$$A(t) = d_1^{2/3}K_2^{1/3}t^{\frac{f}{3(f-\alpha^2)}}\exp\left[\frac{2x_1}{3K_2}\left(1-\frac{f}{\alpha^2}\right)t^{\frac{\alpha^2}{\alpha^2-f}}\right],$$

$$B(t) = d_1^{-1/3}K_2^{1/3}t^{\frac{f}{3(f-\alpha^2)}}\exp\left[\frac{-x_1}{3K_2}\left(1-\frac{f}{\alpha^2}\right)t^{\frac{\alpha^2}{\alpha^2-f}}\right],$$

where

$$K_2 = \left\{c_1\left(f-\alpha^2\right)\left[\frac{12\pi\left(1-\gamma\right)}{\left(3\alpha^2-f\right)}\right]^{1/2}\right\}^{\frac{f}{f-\alpha^2}}.$$

Here D_1, D_2, D_3, D_4 and X_1, X_2, X_3, X_4 are constants of integration, satisfying the relations $D_1D_2D_3D_4 = 1$ and $X_1 + X_2 + X_3 + X_4 = 0$.

In this case, the expansion scalar θ is given by

$$\theta = \left(\frac{f}{f-\alpha^2}\right)\frac{1}{t}.$$

The mean anisotropy parameter is given by

$$\Delta = \frac{4X^2}{K_2^2}\left(\frac{f-\alpha^2}{f}\right)^2 t^{2\left(\frac{\alpha^2}{\alpha^2-f}\right)}.$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{X^2}{2K_2^2}t^{2\left(\frac{f}{\alpha^2-f}\right)}.$$

The deceleration parameter q is given by

$$q = 2 - \frac{3\alpha^2}{f}.$$

For $f > \alpha^2$, this model also tends to isotropy for large t.

5 Conclusion

In this paper, we have considered the space-time geometry corresponding to LRS Bianchi type-I in Hoyle-Narlikar [21, 23, 24] creation field theory of gravitation. Here, we have observed that the ratio $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right)^2 = 0$, [for $\alpha^2 < f$ or $2f < 3\alpha^2$] hence, the model approaches to isotropy for a large value of t.

The deceleration parameter $q = 2 - \dfrac{3\alpha^2}{f} < 0$ [for $2f < 3\alpha^2$] and we get the accelerating universe. Also in this case we get negative deceleration parameter indicating that the universe is accelerating which is consistent with the present day observation. Pertmutter *et al.* [29, 30] and Riess *et al.* [31] have shown that the decelerating parameter of the universe is in the range $-1 \leqslant q \leqslant 0$ and the present day universe is undergoing accelerated expansion.

All results obtained by us are similar to the results obtained by Singh and Chaubey [28].

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