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Intuitionistic fuzzy implication $\rightarrow_{190}$

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Abstract: A new – with current number 190 – intuitionistic fuzzy implication is introduced and some of its basic properties are studied.
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1 Introduction

During last 10 years, 180 new intuitionistic fuzzy implications are defined and their basic properties are studied. Here, a new one will be discussed.

Initially, following [2], we give some necessary definitions and notations.

In the intuitionistic fuzzy logic (see [1, 2]), each proposition, variable or formula is evaluated with two degrees – "truth degree" or "degree of validity" and 'falsity degree" or "degree of non-validity". Thus, to each one of these objects, e.g., p, two real numbers, $\mu(p)$ and $\nu(p)$, are assigned with the following constraint:

$$\mu(p), \nu(p) \in [0, 1] \text{ and } \mu(p) + \nu(p) \leq 1.$$

Let an evaluation function V be defined over a set of propositions $\mathcal{S}$, in such a way that for $p \in \mathcal{S}$:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

Hence the function $V : \mathcal{S} \to [0,1] \times [0,1]$ gives the truth and falsity degrees of all elements of $\mathcal{S}$ – the set of logical objects that we use (in general case – formulas).

In [5], we called the object $\langle \mu(p), \nu(p) \rangle$ an Intitionistic Fuzzy Pair (IFP).

We assume that the evaluation function V assigns to the logical truth T

$$V(T) = \langle 1, 0 \rangle,$$

and to the logical falsity F

$$V(F) = \langle 0, 1 \rangle.$$

As it was discussed [2], the first (classical) intitionistic fuzzy negation is $V(\neg_1 p) = \langle \nu(p), \mu(p) \rangle$. Below, for simplicity, we write $\neg$ instead of $\neg_1$.

Here, we define only the operations "disjunction" and "conjunction", originally introduced in [1], that have classical logic analogues, as follows:

$$V(p \vee q) = \langle \max(\mu(p), \mu(q)), \min(\nu(p), \nu(q)) \rangle,$$

$$V(p \wedge q) = \langle \min(\mu(p), \mu(q)), \max(\nu(p), \nu(q)) \rangle.$$

For the needs of the discussion below, we define the notions of Intuitionistic Fuzzy Tautology (IFT, see, e.g. [1, 2]) and tautology.

Formula A is an IFT if and only if (iff) for every evaluation function V, if $V(A) = \langle a, b \rangle$, then, $a \geq b$, while it is a (classical) tautology if and only if for every evaluation function V, if $V(A) = \langle a, b \rangle$, then, $a = 1,\ b = 0$.

When a given IFP is an IFT, we call it Intitionistic Fuzzy Tautological Pair (IFTP) and when it is a tautology – Tautological Pair (TP).

Below, when it is clear, we will omit notation "$V(A)$", using directly "A" of the intuitionistic fuzzy evaluation of A. Also, for brevity, in a lot of places, instead of the IFP $\langle \mu(A), \nu(A) \rangle$ we will use the IFP $\langle a, b \rangle$, where $a, b \in [0,1]$ and $a + b \leq 1$.

It is also suitable, if $\langle a, b \rangle$ and $\langle c, d \rangle$ are IFPs, to have

$$\langle a, b \rangle \leq \langle c, d \rangle \text{ iff } a \leq c \text{ and } b \geq d$$

and

$$\langle a, b \rangle \geq \langle c, d \rangle \text{ iff } a \geq c \text{ and } b \leq d.$$

2 Intuitionistic fuzzy implication $\to_{190}$ and some of its properties

Let everywhete below, the two IFPs x and y have the forms $x = \langle a, b \rangle$ and $y = \langle c, d \rangle$, where $a, b, c, d \in [0,1]$, $a + b \leq 1$ and $c + d \leq 1$.

Let

$$\text{sg}(x) = \begin{cases} 1, & \text{if } x > 0 \\ \\ 0, & \text{if } x \leq 0 \end{cases},$$

and

$$\overline{\text{sg}}(x) = \begin{cases} 0, & \text{if } x > 0 \\ \\ 1, & \text{if } x \leq 0 \end{cases}.$$

The new intuitionistic fuzzy implication has the form

$$x \to_{190} y = \langle a, b \rangle \to_{190} \langle c, d \rangle = \langle \frac{\overline{\text{sg}}(a-c) + \overline{\text{sg}}(d-b)}{2}, \frac{\text{sg}(a-c) + \text{sg}(d-b)}{2} \rangle.$$

First, the new operation is defined correctly. Really, we see directly, that

$$\overline{\text{sg}}(a-c) + \overline{\text{sg}}(d-b) \geq 0$$

and

$$\text{sg}(a-c) + \text{sg}(d-b) \geq 0.$$

On the other hand, obviously,

$$\frac{\overline{\text{sg}}(a-c) + \overline{\text{sg}}(d-b)}{2} \leq 1,$$

$$\frac{\text{sg}(a-c) + \text{sg}(d-b)}{2} \leq 1$$

and

$$\frac{\overline{\text{sg}}(a-c) + \overline{\text{sg}}(d-b)}{2} + \frac{\text{sg}(a-c) + \text{sg}(d-b)}{2}$$

$$= \frac{\overline{\text{sg}}(a-c) + \text{sg}(a-c)}{2} + \frac{\overline{\text{sg}}(d-b) + \text{sg}(d-b)}{2} = 1.$$

Second,

$$\langle 0, 1 \rangle \to_{190} \langle 0, 1 \rangle = \langle 1, 0 \rangle,$$

$$\langle 0, 1 \rangle \to_{190} \langle 0, 0 \rangle = \langle 1, 0 \rangle,$$

$$\langle 0, 1 \rangle \to_{190} \langle 1, 0 \rangle = \langle 1, 0 \rangle,$$

$$\langle 0, 0 \rangle \to_{190} \langle 0, 1 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0, 0 \rangle \to_{190} \langle 0, 0 \rangle = \langle 1, 0 \rangle,$$

$$\langle 0, 0 \rangle \to_{190} \langle 1, 0 \rangle = \langle 1, 0 \rangle,$$

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$$\langle 1, 0 \rangle \to_{190} \langle 0, 0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 1, 0 \rangle \to_{190} \langle 1, 0 \rangle = \langle 1, 0 \rangle.$$

More general, if $\langle a,b\rangle \leq \langle c,d\rangle$, i.e., $a \leq c$ and $b \geq d$, then

$$x \to_{190} y = \langle a,b\rangle \to_{190} \langle c,d\rangle = \langle \frac{\overline{\text{sg}}(a-c)+\overline{\text{sg}}(d-b)}{2}, \frac{\text{sg}(a-c)+\text{sg}(d-b)}{2}\rangle = \langle 1,0\rangle.$$

Third, the implication $\to_{190}$ generates the following negation

$$\langle a,b\rangle \to_{190} \langle 0,1\rangle = \langle \frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}, \frac{\text{sg}(a)+\text{sg}(1-b)}{2}\rangle,$$

that is a new – 55-th – intuitionistic fuzzy negation.

Now, we see that

$$\neg_{55}\langle 0,1\rangle = \langle 1,0\rangle,$$
$$\neg_{55}\langle 0,0\rangle = \langle \frac{1}{2}, \frac{1}{2}\rangle,$$
$$\neg_{55}\langle \frac{1}{2}, \frac{1}{2}\rangle = \langle 0,1\rangle,$$
$$\neg_{55}\langle 1,0\rangle = \langle 0,1\rangle.$$

Fourth, we see that

$$\neg_{55}\neg_{55}\langle a,b\rangle = \neg_{55}\langle \frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}, \frac{\text{sg}(a)+\text{sg}(1-b)}{2}\rangle$$

$$= \langle \frac{\overline{\text{sg}}(\frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}) + \overline{\text{sg}}(1-\frac{\text{sg}(a)+\text{sg}(1-b)}{2})}{2}, \frac{\text{sg}(\frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}) + \text{sg}(1-\frac{\text{sg}(a)+\text{sg}(1-b)}{2})}{2}\rangle$$

$$= \langle \frac{\overline{\text{sg}}(\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)) + \overline{\text{sg}}(2-\text{sg}(a)-\text{sg}(1-b))}{2},$$
$$\frac{\text{sg}(\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)) + \text{sg}(2-\text{sg}(a)-\text{sg}(1-b))}{2}\rangle.$$

Therefore, for each IFP x, so that $V(x) \neq \langle 1,0\rangle, \langle 0,1\rangle$,

$$\neg_{55}\neg_{55}\, x \neq x.$$

Following idea from [6], we can construct the following new disjunction:

$$x \vee_{190,1} y = \neg_{55} x \to_{190} y = \langle \frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}, \frac{\text{sg}(a)+\text{sg}(1-b)}{2}\rangle \to_{190} \langle c,d\rangle$$

$$\langle \frac{\overline{\text{sg}}(\frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}-c) + \overline{\text{sg}}(d-\frac{\text{sg}(a)+\text{sg}(1-b)}{2})}{2}, \frac{\text{sg}(\frac{\overline{\text{sg}}(a)+\overline{\text{sg}}(1-b)}{2}-c) + \text{sg}(d-\frac{\text{sg}(a)+\text{sg}(1-b)}{2})}{2}\rangle.$$

Finally, following the idea from [3], we see that for every two IFPs x and y, so that $x \neq y$,

$x \to_{190} y$ is a tautology if and only if $y \to_{190} x$ is not a tautology.

Really, let $x \to_{190} y$ be a tautology. Therefore,

$$\frac{\overline{\text{sg}}(a-c)+\overline{\text{sg}}(d-b)}{2} = 1$$

and

$$\frac{\mathrm{sg}(a-c)+\mathrm{sg}(d-b)}{2}=0,$$

i.e., $a \leq c$ and $b \geq d$. Let us assume that $y \to_{190} x$ is a tautology. Then

$$\frac{\overline{\mathrm{sg}}(c-a)+\overline{\mathrm{sg}}(b-d)}{2}=1$$

and

$$\frac{\mathrm{sg}(c-a)+\mathrm{sg}(b-d)}{2}=0,$$

i.e., $a \geq c$ and $b \leq d$. But this is possible only when $a = c$ and $b = d$, that is a contradiction.

3 Conclusion

The new implication can find application in different procedures, e.g., in the algorithm of the Intercriteria analysis (see, e.g., [2, 3]), in intuitionistic fuzzy decision making procedures (see, e.g. [4]) and others.

In next research other properties of the implication $\to_{190}$ will be introduced and studied. New operations (conjunctions and disjunctions) will be generated by the implication $\to_{190}$.

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