$$\Delta W = -0.153\left(0.99 \times 0 + 0.456\,\frac{0.250}{1} + 1 - 1\right) = -0.017$$

At $T = 0.2$ sec $W = 0 - 0.017 = -0.017$

During the first time interval mean $W = 1/2\,(0 - 0.017) = -0.0085$

$$\Delta s = -0.0085 \times 0.2 = -0.002$$

At $T = 0.2$ sec

$$s = 0.591 - 0.002 = 0.589$$

From Fig. 2 for $s = 0.589$ we have $p_1 = 0.748$; $q_1 = 0.767$

$\Delta q_1 = 0.767 - 0.769 = -0.002$,

$$\Delta h = -\frac{2 \times 1 \times (-0.002)}{0.273\sqrt{1} + 0.767} = 0.004$$

At $T = 0.2$ sec

$$h = 1 + 0.004 = 1.004$$

$$p = 0.748 \times 1.004 \times \sqrt{1.004} = 0.753$$

mean $p = 1/2\,(0.750 + 0.753) = 0.7515$

$$\Delta n = 0.0247\,\frac{0.7515 - 0.500}{1} = 0.0062$$

At $T = 0.2$ sec

$$n = 1 + 0.0062 = 1.0062$$

References

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6 L. M. Hovey, "Optimum Adjustment of Governors at the Hydro Generating Stations of the Manitoba Hydro-Electric Board," Paper No. 10, 1960 Annual General Meeting, Engineering Institute of Canada.

7 E. C. Koenig and W. C. Schultz, "How to Select Governor Parameters With Analog Computers," a Palimpsest on the Electric Analog Art, George A. Philbrick Researchers, Inc., Boston, Mass., 1955, pp. 237–238.

DISCUSSION

C. L. Avery[2]

The author is to be commended for an extensive mathematical analysis of the subject of his paper. As such the paper is of interest.

For design purposes, the procedures which he discusses have been supplanted by the techniques for designing servomechanisms and regulating systems, which have been developed over the last twenty years, in conjunction with the use of digital and analog computers. These methods permit the rapid determination of a complete spectrum of the operation of a given control scheme and thereby a determination of the effects of various factors by adjustment of significant computer constants. Within the boundaries of discontinuities inherent in the design of the control system, linearity can be assumed and consequent simplified methods of calculation which have been well established in the literature and practice of the industry are valid.

[2] Engineer, Woodward Governor Company, Rockford, Ill. Mem. ASME.

There are two significant characteristics of speed control for small step-load changes which are of importance:

1 The characteristic of the time history of the speed transient. This is determined by the physical characteristics of the installation T_w and T_m and the governor adjustments δ and T_r. The interrelation of these parameters has been discussed in numerous papers. As a matter of fact, the author has presumably used Paynter's optimum values of δ and T_r, although on this basis his values are somewhat low. Paynter's optimum values are based on relative output rather than relative gate, and the full length of water passages from head water to tail race should be used in determining T_w. The author's Fig. 3 corresponds qualitatively to Paynter's optimum transient.

2 The maximum relative deviation in speed. For stable control, this is the magnitude of the first peak ($n_{\max}$).

The dominant governor characteristic which determines this is the temporary droop. In the author's example, the temporary droop is assumed to be 0.18, considering the temporary droop only; for 17 per cent change in servomotor stroke, the maximum speed deviation would be 3.0 per cent. If a more realistic value of temporary droop of about 0.40 were used, its effect would be relatively greater.

We believe the author overemphasizes the importance of "the influence of relay valve stem position" on speed regulation, provided the system does comply with the standards of good design which any reputable manufacturer of governing equipment must observe. His argument would be stronger if he had demonstrated the relative importance of this factor and established limits for satisfactory speed control. Modern governors are provided with adjustment of this characteristic. It has been our experience that doubling the gain over the normal adjustment does not affect the speed control appreciably.

As a matter of interest we have recalculated the speed rise for the author's example, using a simplified speed rise formula for part gate step load change (load-off):

$$n_{\max}{}^2 = 1 + \frac{T_{1-2}}{T_{m1}}\left\{\left(1 + \frac{P_2}{P_1}\right)h_{\max}{}^{3/2} - 2\,\frac{P_2}{P_1}\right\} \quad (10)$$

Here P_1 and P_2 are the initial and final power levels and T_{m1} corresponds to the initial power level (75 per cent) or 10.8 seconds.

The factor which relates to the governor is the time for the gates to travel from initial to final power level (T_{1-2}). For this we assume an empirical value of one second dead time plus $^1/_4 T_g$ (for 25 per cent load change) or 2.62 seconds.

For the change in head during the transient we use the rigid water column theory, which is valid for the closure time contemplated here. The head rise for full gate closure is determined from the equation:

$$\frac{\sqrt{h_{\max}}}{h_{\max} - 1} = \frac{T_g}{T_w} \text{ (using the nomenclature of the paper)}$$

whence $h_{\max} - 1 = 0.12$; for $^1/_4$ load rejection, $(h_{\max} - 1)$ is assumed to be $^1/_4$ of this or $h_{\max} = 1.03$ approximately. The author shows $h_{\max} = 1.035$.

Substituting in equation (10):

$$n_{\max}{}^2 = 1 + \frac{2.62}{10.8}\left\{\left(1 + \frac{0.50}{0.75}\right)(1.03)^{3/2} - \left(2 \times \frac{0.50}{0.75}\right)\right\} = 1.098$$

or

$$n_{max} = 1.048$$

This compares to the author's value of 1.0495.

For ordinary overspeed calculations, equation (10) appears to be quite adequate within the degree of accuracy usually required.

L. M. Hovey[3]

Mr. Swiecicki is to be congratulated on the preparation of this paper which takes into account the gate closing time factor and the beta characteristics of the hydraulic prime mover. This he does by plotting the speed transient curve for a given step load change using a step-by-step method. It is felt by the writer that where extreme accuracy is required this method will serve a useful purpose. However, as far as utility operation is concerned, the writer feels that, on an isolated system with zero net damping, the approximate figure of peak speed regulation following a load change is satisfactory enough.

If we use Dr. Paynter's formula $\Delta n/N \approx 2.5\ T_w/T_m \cdot \Delta p/p$ and substitute the author's parameters of T_w and T_m for a load change of 0.25 per unit, the calculated speed rise is 0.056 per unit which compares quite favorably with the 0.05 per unit speed rise as shown in Fig. 3 of the paper. Although Dr. Paynter's formula is based on relatively small load changes, it would seem to tie in very closely to the peak speed as calculated by the author.

On page 448 of the paper, the author calculates a T_w of 0.733 second using a flow which corresponds to rated power of 108,000 hp at rated head. Would it not have been more precise to calculate T_w based on the actual flow as based on the initial turbine loading which is assumed to be operating at 75 per cent of rated capacity just prior to the 25 per cent reduction in load? This would have given a water starting time approximately 25 per cent less which would have modified the calculations quite materially.

The organization with which the writer is associated just recently completed a hydro generating station of 150,000 hp in Northern Manitoba which supplies about 100 Mw to a mining company. This station is not tied in with the major transmission network and is thus an isolated plant supplying a load which consists of a composite of motors, electrolytic load, and a large proportion of load consisting of resistance type electric furnaces. It is the intention of our organization to perform tests in the near future on the system where blocks of electric furnace load will be applied and rejected during which speed transients will be measured to determine how the system behaves in relation to the calculated responses, and it is intended that this will form a basis of a paper to be presented at one of the Engineering Societies on this Continent in the next year or so.

George R. Rich[4]

The author has prepared an excellent and ingenious analysis of turbine speed regulation including the effect of all governor adjustments. Since the paper will probably be widely used for future reference, the discusser would like to submit a qualification of the statement that "the differential equations for governing stability given in reference [4] are complicated and cannot be used directly for calculation of speed changes."

The pertinent equations are given on page 74 (3-31) and (3-32). By substituting the appropriate physical constants, it will be demonstrated that they yield with comparative ease results practically identical with those obtained by the author.

1st Interval.

$$\Delta t = 0.2 \text{ sec}$$

$$p_0 = \frac{P - P_0}{P_0} = \frac{(108{,}000 \times 0.75) - (108{,}000 \times 0.50)}{(108{,}000 \times 0.50)} = 0.500$$

$$j = 1.00\ \rho = \frac{aV_0}{2gH_0} = \frac{4100 \times 11.6}{64.4 \times 269} = 2.75$$

From Davis' "Handbook of Hydraulics," second edition, page 746, table 5.

For $\rho = 2.75 \quad r = 0.159 \quad s = 0.849$

$$\theta = \frac{LV_0}{gH_0} = \frac{411 \times 11.6}{32.2 \times 269} = 0.55$$

$$C = \frac{N_0^2(WR^2)}{Hp} = \frac{200^2(35.5 \times 10^6)}{81{,}000} = 17.5 \times 10^6$$

$$T = \frac{C}{16.2 \times 10^5} = \frac{17.5 \times 10^6}{1.62 \times 10^6} = 10.8 \text{ sec}$$

$$T_r = \frac{jT}{j - 3r} = \frac{1.00 \times 10.8}{1 - 3 \times 0.159} = 20.6 \text{ sec}$$

$$T_d = 3.7 \text{ sec} \qquad t_g = 6.5$$

$$T_g = k\beta t_g = 1.50 \times 0.05 \times 6.5 = 0.487$$

$$T'_g = T_g + (k_r\delta')T_d = 0.487 + 0.18 \times 3.7 = 1.153$$

Substituting the foregoing physical constants in Equations (3-31) and (3-32) we obtain:

$$\frac{\Delta\omega}{0.2} = \frac{0.500}{20.6} - \frac{3}{2}\,\frac{0.849}{1.00}\,\frac{0.55}{10.8}\,\frac{\Delta p_0}{0.2} \qquad (3\text{-}31)$$

$$\frac{\Delta p_0}{0.2} = -\frac{1}{1.153}\left(0.00 + 3.70\,\frac{\Delta\omega}{0.2}\right) \qquad (3\text{-}32)$$

Solving

$$\Delta\omega = 0.0061 \qquad \Delta p_0 = -0.0196$$

$$p_0 = 0.480$$

To obtain the power at the end of the first interval, see page 55:

$$(P - P_0)550(0.2) = \frac{4\pi^2(35.5 \times 10^6) \times 200 \times 1.22}{32.2 \times 3600}$$

$$P_0 = 54{,}000 \qquad P = 80{,}800 \text{ hp}$$

$$N = 1.0061(200) = 201.22 \text{ rpm}$$

2nd Interval. $\Delta t = 0.2$ sec. Same physical constants still accurate enough for second interval. These constants will be changed as necessary for later intervals.

$$p_0 = 0.500 - 0.0196 = 0.480$$

$$\frac{\Delta\omega}{0.2} = \frac{0.480}{20.6} - \frac{3}{2}\,\frac{0.849}{1.00}\,\frac{0.55}{10.8}\,\frac{\Delta p_0}{0.2} \qquad (3\text{-}31)$$

$$\frac{\Delta p_0}{0.2} = -\frac{1}{1.153}\left(0.0061 + 3.7\,\frac{\Delta\omega}{0.2}\right) \qquad (3\text{-}32)$$

Solving $\Delta\omega = 0.0062 \qquad \Delta p_0 = -0.0211 \qquad p_0 = 0.459$

$$\omega = 0.0061 + 0.0062 = 0.0123\,\frac{N - 200}{200} = 0.0123 \quad N = 202.46$$

$$(P - P_0)550(0.2) = \frac{4\pi^2(35.5 \times 10^6) \times 201.22 \times 1.24}{32.2 \times 3600}$$

[3] P. Engineer, The Manitoba Hydro-Electric Board, Winnipeg, Manitoba, Canada.

[4] Director, Chas. T. Main, Inc., Partner Uhl, Hall & Rich, Boston, Mass. Mem. ASME.

$$P_0 = 54{,}000 \qquad P = 81{,}400 \text{ hp}$$

As is well known, the character of the receiving load has an appreciable effect on governing stability.[5] This factor may be readily incorporated in the discusser's Equation (3-31)[6] giving for the second interval ($\omega = 0$ in the first interval) with $\alpha = 2.00$:

$$\frac{\Delta\omega}{0.2} = \frac{0.480}{20.6} - \frac{3}{2}\,\frac{0.849}{1.00}\,\frac{0.55}{10.8}\,\frac{\Delta p_0}{0.2} - \frac{2.00 \times 0.0061}{10.8} \qquad (3\text{-}36)$$

As before

$$\Delta p_0 = -0.0011 - 3.21\Delta\omega \qquad (3\text{-}32)$$

Solving

$$\Delta\omega = 0.0057$$

This effect becomes much more important for the later intervals when ω has become relatively large.

It is also well known that the amortisseur winding of the generator rotor acts like the squirrel cage rotor of an induction motor during load changes and this tends to improve governing stability. Appropriate calculations to include this factor may be made by including as physical constants the subtransient reactances of the generator.

The discusser has not given the numerical values of the head changes in the foregoing illustrative computations; but they may of course be readily shown using Equation (3-21) page 65 or the vectorial components given on page 67.

C. G. Smallridge[7]

The author has presented an interesting analysis of the problem of speed regulation of a hydraulic turbine taking into consideration the effects of water hammer, mechanical inertia, and governor adjustments in such a way that a detailed step-by-step analysis can be made for any transient load change. The paper is very timely and is a valuable complement to the literature already existing on this subject.

The first comprehensive treatment of this subject was given in the classic Strowger-Kerr paper of 1926 (see author's bibliography) and it was there that the arithmetic integration method was first enumerated for speed regulation studies. This work, however, did not take into account the effect of initial nonlinear gate movement in the period before the governor relay valve ports are completely uncovered and the servomotors move at full speed. This effect is negligible when the load changes are large but can be of considerable significance for small load changes, particularly in cases where the water inertia is high and the governor time relatively long.

The writer was concerned recently with the study of the speed regulating capabilities of an isolated plant with a relatively long penstock which was the only source of power for a mining load including electric drag lines, locomotives, and large synchronous motors with "across the line" starting. The regulation requirements set out by the customer were unusually stringent, and it was necessary to evaluate the effect of additional WR^2 and varying governor adjustments on speed transients following specified step load changes. Using the equations developed by the author as a basis, the method was extended and programmed for a digital computer so that a number of different cases could be studied rapidly and the speed regulating capabilities determined. The problem was programmed for a "Stantec-Zebra" computer using simple coded instructions and it was found that a complete set of

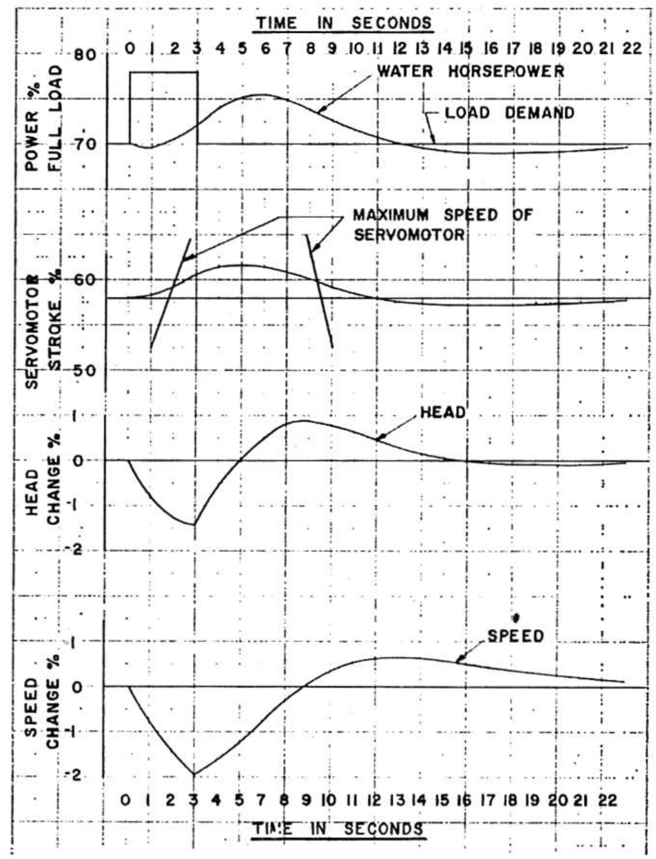

Fig. 4 Transient load acceptance, 3500 kw

transients for speed, head, flow, and power could be obtained in a matter of minutes. By this means, it was possible to assure the customer of reasonable regulation from the power plant and to give information to the designers of the electrical mining equipment comprising the load.

The installation studied consists of two 60,000 hp units operating under a net head of 290 feet at a speed of 225 rpm. The units are fed from individual penstocks 1600 feet long and 13 feet 6 inches in diameter with governor times of 14 seconds opening and 9 seconds closing. The elastic water-hammer theory [Equation (2) of the author's paper] was used in the computer program since the rigid column approximation is not warranted and, in fact, may be quite inaccurate for certain high head layouts. In the installation under study, a wave velocity of 3200 feet per second was used.

Other approximations suggested by the author for the values of h in Equation (2) and p and n in Equation (9) give accurate values for the speed change, but may introduce an apparent oscillation in the governor action. The approximations are not necessary in the computer solution and mean values for each interval can be obtained by introducing an additional iteration loop in the program.

Figs. 4 and 5 show the speed, power, and head variations following a transient load increase of 3500 kw for a duration of three seconds and a sustained load rejection of 6000 kw. These curves were plotted directly from the tabulated computer results and illustrate the ease with which complete transients can be obtained for any imposed conditions.

The influence of temporary speed droop and dashpot time settings on the speed of the servomotor piston is considerable for small load changes, and the amount of step load changes necessary to cause the governor dashpot to bypass the produce full

[5] Reference [4], pp. 76–78.

[6] Reference [4], Equation (3-36), p. 78.

[7] Hydraulic Engineer, The Shawinigan Engineering Company Limited, Montreal, Canada. Assoc. Mem. ASME.

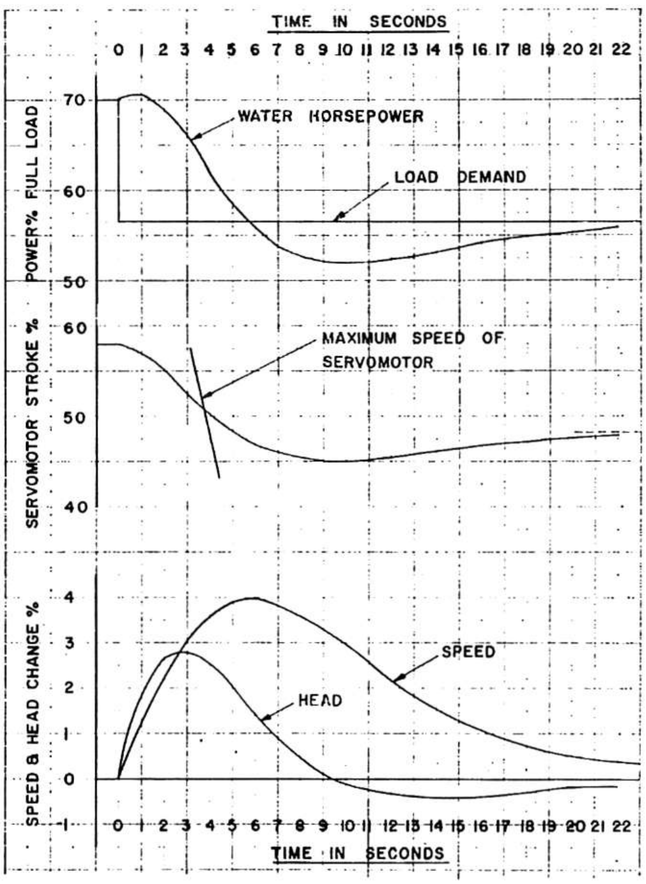

Fig. 5 Sustained load rejection, 6000 kw

rate of movement can be a large percentage of full load. The full rate of servomotor movement has been superimposed on the curves of servomotor stroke in Figs. 4 and 5, and it can be seen that a much larger load change than 6000 kw would be needed to allow the gates to reach full speed during the transient. In this example, a load change of approximately 50 per cent would be necessary for this condition.

The computer program derived from the author's equations takes these factors into account and permits an accurate determination of speed transients in cases where the load does not decrease to or start from zero. It can be readily applied to any layout, and it can be used to rapidly compare the regulating characteristics of alternative installations at an early stage in design.

E. B. Strowger[8]

Mr. Swiecicki has analyzed the problem of regulation of a hydroelectric unit for a partial load change taking account of the governor compensation in altering the rate of gate travel during the transient. He not only has derived the differential equations showing the relation between increments of speed, head, flow, and power output, but has developed the working equations used in a typical example and, further, has applied arithmetic integration to the solution. His results in the form of curves showing speed and power against time (Fig. 3) are useful in determining the inherent stability of the unit.

He assumes that the power output P and the demand are equal at $T = 0$ and that instantly the demand becomes R, i.e., a load change of P-R with the demand remaining constant at the value R. Since he uses the value of the flywheel effect of the one unit it is obvious he is assuming an isolated system.

When a unit drops a sizeable load and at the same time becomes disconnected from the system, then the pilot valve moves over its full travel and ds/dT may be considered constant and the governor compensation has no influence on the problem. For this condition the author refers to methods of computation already described in the technical press. For such cases the writer has lately developed some approximate formulas that may be useful for computations for preliminary investigations where great accuracy is not required.[9]

It would be interesting to extend the analysis to include the kinematics of a double floating lever governor.

As pointed out by the author, in using the elastic water-column theory of water hammer, the value of $\Sigma\Delta h$ must be calculated, taking time intervals of $2L/a$ seconds of length and taking account of the negative waves as well as the positive ones.

Author's Closure

The author wishes to thank the discussers for their comments which greatly enhance this paper.

He has to apologize because, after the preprints of the paper had been circulated and before the paper was read, he introduced changes to the chapter "Turbine Head" and corrected some figures in Table 1. This resulted in numerical disagreement between a few values quoted in the discussions of Mr. Avery and Mr. Rich and the final appearance of Table 1.

Summarizing the discussion, the author shares the general contention that, as Mr. Hovey puts it, "as far as utility operation is concerned . . . . the approximate figure of peak speed regulation following a load change is satisfactory enough." This figure can be obtained, for instance, utilizing Mr. Strowger's formula.[9] However, sometimes, as in Mr. Smallridge's problem an accurate analysis as presented in this paper is required. The same is true when studying two significant characteristics of speed control for the small step-load changes mentioned by Mr. Avery: the time history of the speed transient and the maximum relative deviation in speed.

Mr. Avery apparently believes that "extensive mathematical analysis," for design purposes, can be supplanted by the techniques in conjunction with the use of electronic computers. He does not elaborate how this technique can be developed without mathematical analysis. On the other hand, Mr. Smallridge found that in programming the mathematics of this paper for a digital computer the transients for speed, head, flow, and power could be obtained in a matter of minutes.

Discussing the role of β, Mr. Avery refers, in broad terms, to experience saying "it has been our experience that doubling the gain over the normal adjustment does not affect the speed control appreciably." Unfortunately, he does not spell out what he considers to be the normal adjustment and its relation to other parameters because, for the set of conditions discussed as the example in this paper, the mathematical analysis indicates a considerable influence of β when other parameters remain unchanged.

Equation (6) of this paper gives the solution for a governor having $\beta = 0$. The author has called such a governor "ideal," having been inclined to believe that the smallest possible β is the best. Taking $\beta = 0$ for the example of this paper when $T = 0$, equation (6) gives:

$$W = -\frac{1}{0.18}\left(\frac{0.75 - 0.50}{8.1 \times 1} + \frac{1 - 1}{3.7}\right) = -0.171$$

An absolute value of W greater than $\dfrac{1}{T_g} = \dfrac{1}{6.5} = 0.154$ means

[8] Niagara Mohawk Power Corporation, Buffalo, N. Y. Mem. ASME.

[9] "American Civil Engineering Practice," vol. II, section 16, John Wiley & Sons, Inc., New York, N. Y., p. 29.

Table 2 Results assuming $\beta = 0$

T	n	s	q_1	h	p_1	W
0.0	1.0000	0.591	0.769	1.000	0.750	−0.171
0.2	1.0066	0.560	0.731	1.076	0.700	−0.203
0.4	1.0134	0.529	0.690	1.121	0.649	−0.213
0.6	1.0196	0.498	0.648	1.144	0.601	−0.191
0.8	1.0248	0.467	0.605	1.160	0.557	−0.172
1.0	1.0289	0.436	0.562	1.166	0.512	−0.143
1.2	1.0317	0.407	0.522	1.161	0.470	−0.108
1.4	1.0331	0.385	0.491	1.137	0.437	−0.070
1.6	1.0331	0.371	0.472	1.091	0.415	−0.031
1.8	1.0320	0.365	0.464	1.046	0.407	−0.003
2.0	1.0302	0.364	0.463	1.014	0.406	+0.013
2.2	1.0281	0.367	0.467	0.993	0.410	+0.022

that, instantaneously, at the time $T = 0$ the servomotor begins to close at its full speed. The calculation for subsequent time intervals (results are presented in Table 2) shows that, not earlier than about one second after the beginning of the transient, W becomes smaller than 0.154 and, therefore, the average servomotor speed for time intervals is less.

Table 2 for the ideal governor shows the maximum relative turbine speed $n = 1.0331$ as contrasted with $n = 1.0602$ obtained for a governor having $\beta = 0.05$. The ideal governor transient time is considerably shorter.

However, the ideal governor has the unpleasant feature that an infinite displacement of the relay valve stem from its neutral position results in full positive or negative speed of the servomotor. The maintenance of any smaller average speed takes place by continuous oscillation between the two extremes. Theoretically, this oscillation is of a high frequency and, as such, should have no ill effects upon turbine regulation.

Theoretically again, it takes a very small value of β (such that would not affect materially the turbine regulation) to prevent this oscillation. To achieve the same result in practice, much greater values of β are required. The lost linkage motion and the governor piping oil starting time delay the response of the servomotor speed while, unfortunately, the relay valve stem overtravels its pertinent balance position. These factors lower the frequency of oscillation which might become resonant with the frequency of the turbine water passage. The above, as well as the problem of oil hammering, dictates the necessity of avoiding oscillations by means of utilizing sufficient β which become greater for smaller degree in excellency of governor design and for smaller size of oil pipes.

Governors, popular in America, having a pilot valve, control valve, and compensating mechanism between point A (Fig. 1) and the relay valve, in spite of all additional devices, produce the same regulating effect as the schematic arrangement represented in Fig. 1. For such a governor, when $\beta \neq 0$, at the beginning of a transient ($T = 0$) we have $ds/dT = 0$ and $dp_1/dT = 0$; because a finite displacement of the flyball collar must occur to produce a finite opening of the relay valve. Consequently, some time elapses before we can observe any finite amount of governor speed (see curve p_1, Fig. 3). Using approximate, fast methods of speed calculation this time lap sometimes is called the "dead time." It is related to the magnitude of β and it does not exist in an "ideal governor" having $\beta = 0$.

It is understandable that any guess as to the magnitude of the dead time would not affect the final results very much when it is known in advance that the dead time constitutes only a small fraction of the total time. But, for the solution of his equation (10) Mr. Avery guesses a dead time of one second which is 62 per cent of the given pertinent time $^1/_4\, T_g$. It is difficult to see how anyone can make such a guess without prior knowledge of the reasonable answer for n. Mr. Avery had this answer looking at Table 1. However, he was less successful with his simplified and empirical calculations when he arrived at the maximum pressure rise of approximately 3 per cent while the detailed calculation indicated 11.2 per cent instead (see Table 1).

Mr. Rich's equations (3–31) and (3–32) used in his discussion have been developed for a governor such that "the displacement of the flyball collar will depend upon the difference between uplift force of the flyball and the dashpot reaction force" (page 61, reference [4]).

Therefore, these equations are not applicable to the governor considered in this paper. Equation (3-32) does not even fulfill the requirement $dp_1/dT = 0$ when $T = 0$. The "results practically identical with those obtained by the author" have been calculated by Mr. Rich accidentally, due to a mistake.

His p_0, Θ, and C should have been calculated on the basis of the same load, while he used $P_0 = (108000 \times 0.5)$hp for calculation of his p_0 and 81000 hp for calculation of C and Θ. Correcting this, equations (3-31) and (3-32) yield relative speed rise 0.00404 during the first time interval, a result which is incorrect not only for the type of governor considered in this paper but for any conceivable design.

In the nomenclature of this paper the author attempted to separate constant parameters from the values changing during a transient. His T_w and T_m are constant parameters, related to rated horsepower and rated discharge and, if Mr. Hovey examines the pertinent equations carefully, he will agree that they are constructed in such a manner that T_w based on any other flow than rated would produce results that were erroneous, not more precise, as Mr. Hovey suggested.

Mr. Avery's statement, "the full length of water passage from headwater to tailrace should be used in determining T_w" needs a qualification. For load-on it is absolutely correct. For load-off, air admission, and cavitation limit the possibility of pressure drop below the runner to a value in many cases, small enough to justify the complete negligence of the draft tube water starting time.

Mr. Smallridge presented an interesting problem. He is concerned that a small inaccuracy in equation (2), arising from the introduction of a small variation Δ, in place of the differential d and inaccuracy in equation (9) arising from insertion in it of constant values of p and n pertinent to the beginning of the time interval may result in the apparent oscillation of the governor action. If Mr. Smallridge felt that his time interval, of one second, was too big for accuracy he could have carried calculations on the basis of one half or one quarter of a second without any difficulties. Instead, he used iteration loops in the computer to obtain mean values for one second intervals. Something must have gone wrong because his head changes are too low.

Mr. Smallridge's turbine is rated $P_0 = 60000$ hp at $H_0 = 290$ ft. Therefore, the rated discharge is about $Q_0 = 2000$ cfs. The penstock has $L = 1600$ ft and cross-sectional area $A = 143$ ft.^{2} On this basis

$$T_w = \frac{2000}{32.2 \times 296} \times \frac{1600}{143} = 2.4 \text{ sec}$$

His curves for rejection of 6000 kw show that after 9.2 sec the head comes back to normal while the power comes down from 70 per cent to 52 per cent in the same time. Assuming constant efficiency, the change of the relative discharge is $\Delta q = 0.52–0.70 = -0.18$. On the above basis, for the first 9.2 seconds the average head rise is

$$h - 1 = -T_w \frac{\Delta q}{dT} = -2.4 \frac{-0.18}{9.2} = 0.047$$

while Mr. Smallridge's curve shows a maximum of only $h - 1 = 0.028$.

The author feels responsible for the difficulty which Mr. Smallridge encountered because the preprint of the paper did not contain equation (2a) which facilitates the calculation of head at the end of the second and the following time intervals. The direct application of equation (2) to any other than the first interval was too difficult. After the wave reflection, the selection of mean q_1 and h values, properly satisfying the equation and yielding the correct answer for h, is quite complicated.

The influence of the primary and the reflected waves is shown in Fig. 6 representing graphical solution of the water hammer problem. It helps to check the correctness of equations (2) and (2a).

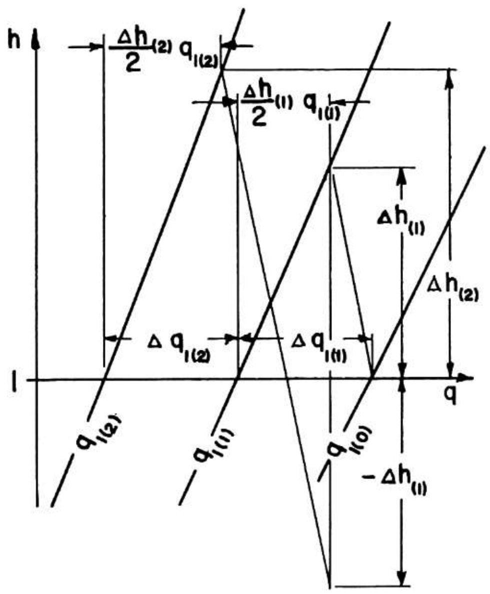

Fig. 6 Graphical solution of water hammer

To present the simplest calculation a step load change for an isolated unit (R = constant) was selected as an example. But all equations have been so constructed that they do not impose any more restrictions on the changes of R than on P. Therefore, they are readily applicable to the variable receiving load and, through the variation of R, they leave the door open for adaptation to studies of the interconnected systems mentioned by Mr. Strowger.