

The effect of Non-Darcy Flow on Induced Stresses Around a Wellbore in an anisotropic in-situ stress field

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Abstract After drilling a well, the stresses will be altered and the induced stresses present the new state of stress. These induced stresses which result in geomechanical problems. The studies indicate that the flow of hydrocarbon into the wellbore influences the induced stresses. Darcy equation has been used in the past; however, the laminar flow assumption embedded in this equation cannot correctly model the flow of fluid when a non-Darcy flow dominates near a wellbore. Example of such situation includes the gas wells.

In this study, analytical equations are developed to incorporate the effect of non-Darcy flow on the induced stresses around a wellbore. These equations developed based on Forchheimer flow equation. Then the simplified solutions were presented by considering the second term of Forchheimer flow equation. It was found that the difference between the results from Darcy and non-Darcy flow models is proportional to the drawdown pressure.

Further studies included numerical simulation of non-Darcy fluid flow in a typical reservoir. Comparison of the results with the analytical models indicated that the magnitude of stresses in non-Darcy flow is larger than of Darcy flow.

Finally, sensitivity of the reservoir properties on the induced stresses in the non-Darcy flow regime was investigated.

Keywords: Non-Darcy flow, Wellbore induced stresses, Analytical solution, Numerical models, Sensitivity analysis

1. Introduction

Drilling operation causes redistribute of stresses around the well. The redistributed

stresses, known as induced stresses are the tangential, radial and vertical stresses. As the wellbore wall is a principal stress plane, it is most severely affected by the presence of induced stresses as a result of drilling operation. This will result in different incidences; with wellbore instability and sand production being only two examples of its many. The financial loss resulted from these incidences are significant and reported to be over six billion dollars in the case of wellbore instability [1], and tens of billion dollars due to sand production annually [2].

Estimation of drilling induced stresses is fundamental to several applications in oil and gas. These stresses are the input to wellbore stability analysis, sanding prediction, hydraulic fracturing design and determining the optimum completion [3][4][5].

The impact of the flow of hydrocarbon when enters into the wellbore on the induced stresses around the wellbore have been reported by researchers[6][7][8][9]. For example the tangential stresses were increased due to stress concentration and fluid withdrawal that leads to compressive failure around a cavity of a well [6] or when the pore pressure gradient is very high, the effective radial stress may be tensile [8].

While the assumption of laminar flow near wellbore for oil reservoirs may be justified, this will introduce a significant error when applies to gas wells where turbulent flow near wellbore is dominant [10][11]. This implies that Darcy flow calculations cannot be used to modify the induced stresses around the wellbore in case of a gas well. This not only applies to wells producing gas, but also to the wells where the gas is injected for enhanced oil recovery (EOR) or storage purposes. The non-Darcy fluid flow also exists in hydraulically fractured formations [12][13]In petroleum engineering, the effect of non-

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Darcy flow on well performance has been studied widely and the concept of skin effect has been introduced to determine the pressure drop [14][15][16]. This is while most of the current solutions to estimate the stress around the wellbore are based on Darcy flow, i.e. laminar flow assumption near wellbore [3][7][14][17]. There are only few studies which assumed non-Darcy flow to developed sand production model for high rate gas wells [6] or derived modified equations for non-Darcy effect on isotropic stress field [9].

This indicates the need for developing solutions which includes the non-Darcy flow to correctly model the turbulent flow near wellbore in case of, for example, gas production or injection. This paper expands the application of the Forchheimer equation to non-Darcy flow. Numerical simulations also were conducted to support the applicability of the developed models.

2. Coupling Fluid Flow and Induced Stresses

Kirsch's equations are perhaps the most commonly used analytical solutions to estimate the stresses induced around a wellbore. Kirsch (1898) derived these equations for a circular hole in an infinite solid medium under a uniform loads applied far from the hole [18]. Kirsch's solution is based on assumptions such as a two-dimensional, elastic and non-porous medium. Further studies by Bradley [19] based on Fairhurst's works [20] included the effect of mud pressure in calculating the drilling induced stresses. In this work, however, plane strain condition and omission the flow of fluid near wellbore and its impact on induced stresses were the two major assumption. In 1986, Santarelli et al developed a nonlinear elastic relationship based on their studies on shale formations and concluded that maximum stress can take place near the wellbore not at the wellbore wall. Yet, this model also ignores the near wellbore fluid flow[21].

Poroelastic theory proposed by Yew and Liu (1992) addressed the issue of flow and deformation coupling in porous media [22]. They introduced poroelasticity theory in order to study the effects of fluid flow on wellbore

stability because this flow induces additional normal stresses. Bradford and Cook (1994) developed a semi-analytical elastoplastic model [19]. This model applies to vertical wells subjected to isotropic in-situ stress and pore pressure fields that predict the onset of sanding. Chen et al (2003) developed a model that included the poroelastic, chemical, and thermal effects [23]. Han and Dusseault (2003) derived the effective stress distributions around a wellbore producing oil with poro-inelastic and different boundary conditions[7]. All of the above models are based on Darcy's fluid flow assumption.

As stated earlier, only a few studies considered the effect of non-Darcy flow in rock mechanics applications in particular stress estimations. Wang et al (1991) have studied the effect of non-Darcy flow on the stability of perforation of gas wells using elastoplastic theory. They have concluded that non-Darcy flow will significantly increases the instability of perforation tunnels [24]. Ong et al (2000) have combined equilibrium equations with the Mohr-Coulomb failure criterion and non-Darcy fluid flow to study the onset of sanding in spherical or cylindrical perforation of high rate gas wells. They have inferred from the result of specific reservoir that the non-Darcy flow reduces the critical drawdown pressure by half as compared to the Darcy case [6]. They also have proposed the method of predicting the onset of formation solid production [25].

In this study based on the poroelastic theory the effect of fluid flow (Darcy/non-Darcy) on induced stresses near the wellbore were investigated.

3. Developed Analytical Model

Forchheimer (1901) established differential equation (1) to estimate the flowing pressure near wellbore as a function of wellbore radius [26]:

$$\frac{dP_r}{dr} = \frac{\mu}{k} \frac{Q_0}{2\pi rh} + \beta \rho \frac{Q_0^2}{4\pi^2 h^2 r^2} \quad (1)$$

The first term in this equation is the Darcy flow with laminar flow assumption. The second term accounts for turbulent flow where the Non-Darcy coefficient, β , is determined empirically as a function of absolute permeability. In equation (1), μ , Q_0 , k , ρ , h and r , are viscosity, flow rate, permeability, fluid density, thickness of production layer, and distance from the wellbore, respectively.

After solving differential equation (1), the boundary conditions need to be applied to converge the results to the near wellbore fluid flow which is the objective of this study. Here the boundary conditions are established as:

$$P_f|_{r=R_w} = P_w \quad (2)$$

$$P_f|_{r=R_0} = P_{f0} \quad (3)$$

Which states that the fluid pressure at the wellbore wall is P_w and it becomes equal to the reservoir pressure, P_{f0} at a large distance away from the wellbore wall ($r = R_0$). Applying boundary conditions of equations (2) and (3) into equation (1) will lead to equation (4).

$$\begin{aligned} P_{f(Forchheimerflow)} &= a \ln\left(\frac{r}{R_0}\right) + b\left(\frac{1}{R_0} - \frac{1}{r}\right) + P_{f0} \\ &= a \ln\left(\frac{r}{R_w}\right) + b\left(\frac{1}{R_w} - \frac{1}{r}\right) + P_w \end{aligned} \quad (4)$$

In equation (4), coefficients a and b are defined as:

$$a = \frac{P_0 - P_w}{\ln\left(\frac{R_0}{R_w}\right)} - \frac{b(R_0 - R_w)}{R_0 \cdot R_w \cdot \ln\left(\frac{R_0}{R_w}\right)} \quad (5)$$

$$b = \frac{\beta \rho Q_0^2}{4\pi^2 h^2} \quad (6)$$

In order to determine the near wellbore induced stresses, we adopt the equations for a homogenous medium subjected to anisotropic horizontal stresses. These equations are presented as equation (A.1) in Appendix (A)[3].

Coupling equations (A.1) and (4) will result in equations (7) to (9) which allows estimation of drilling induced radial, tangential and vertical stresses around a wellbore drilled in a homogenous formation subjected to anisotropic horizontal stresses.

$$\begin{aligned} \sigma_{r(Forchheimerflow)} &= a_1 \left(1 - \frac{R_w^2}{r^2}\right) + a_2 \left(1 + 3\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right) \cos 2\theta \\ &+ \tau_{xy} \left(1 + 3\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right) \sin 2\theta \\ &+ 2\frac{\eta}{r^2} \left\{ \frac{\left[\begin{aligned} &\left(-R_w^2 + r^2\right) \left(\frac{R_0}{2} (0.5a + b) \right) + \frac{a \cdot R_w^2 (\ln(\frac{R_w}{R_0}) - 0.5)}{2R_0} + b \left(\frac{R_w^2}{2R_0} - R_w \right) \end{aligned} \right]}{R_0^2 - R_w^2} \right. \\ &\left. + \frac{\left[\begin{aligned} &a \left(r^2 \ln\left(\frac{r}{R_0}\right) - 0.5r^2 - R_w^2 \ln\left(\frac{R_w}{R_0}\right) + 0.5R_w^2 \right) \end{aligned} \right]}{2R_0} + b \left(\frac{-R_w^2 + r^2}{2R_0} + R_w - r \right) \right\} \\ &+ P_w \frac{R_w^2}{r^2} \end{aligned} \quad (7)$$

These equations are based on the assumption that Forchheimer equation (4) represents the fluid flow near wellbore.

$$\sigma_{\theta(\text{Forchheimerflow})} = a_1 \left(I + \frac{R_w^2}{r^2} \right) - a_2 \left(I + 3 \frac{R_w^4}{r^4} \right) \cos 2\theta - \tau_{xy} \left(I + 3 \frac{R_w^4}{r^4} \right) \sin 2\theta - 2 \frac{\eta}{r^2} \left\{ \begin{aligned} & + \frac{1}{2} \frac{a \left(r^2 \ln \left(\frac{r}{R_0} \right) - 0.5 r^2 - R_w^2 \ln \left(\frac{R_w}{R_0} \right) + 0.5 R_w^2 \right)}{R_0} \\ & + \frac{1}{2} \frac{b \left(2 R_0 R_w - 2 R_0 r - R_w^2 + r^2 \right)}{R_0} - \frac{(R_w^2 + r^2)}{R_0^2 - R_w^2} \times \\ & \left[\frac{1}{4} R_0 (a + 2b) + \frac{1}{2} \frac{a R_w^2 \left(\ln \left(\frac{R_w}{R_0} \right) - 0.5 \right)}{R_0} \right. \\ & \left. - \frac{1}{2} \frac{b R_w (-R_w + 2 R_0)}{R_0} \right] \\ & - a r^2 \ln \left(\frac{r}{R_0} \right) - \frac{b r (R_0 - r)}{R_0} \end{aligned} \right\} - P_w \frac{R_w^2}{r^2} \quad (8)$$

$$\sigma_{z(\text{Forchheimerflow})} = \sigma_z^0 - 4 \nu \left(a_2 \times \frac{R_w^2}{r^2} \times \cos 2\theta + \tau_{xy} \times \frac{R_w^2}{r^2} \times \sin 2\theta \right) + 2\eta \left(a \ln \left(\frac{r}{R_0} \right) + b \left(\frac{1}{R_0} - \frac{1}{r} \right) \right) - \frac{4\eta \nu \left(\begin{aligned} & \frac{1}{2} R_0 \left(\frac{1}{2} a + b \right) + \frac{1}{2} \frac{a R_w^2 \left(\ln \left(\frac{R_w}{R_0} \right) - \frac{1}{2} \right)}{R_0} + b \left(\frac{1}{2} \frac{R_w^2}{R_0} - R_w \right) \end{aligned} \right)}{R_0^2 - R_w^2} \quad (9)$$

Coefficients a_1 and a_2 are defined as the equations (A.2) in Appendix A. It is to note that the non-Darcy parameter β is embedded into parameter b in equation 6. Determining parameter β is not straight forward and

several experimental correlations have been proposed to estimate this parameter [10].

In the next section, we discuss how parameter β can be discarded in the above equations considering the second term of equation (4) for specific case of fluid flow in high permeability gas wells [27].

4. Induced Stresses around a Gas Well

Here, equations (7) to (9) are used to estimate the drilling induced stresses in general form where turbulent flow is dominant. The past research studies have indicated that the flow rate in gas wells is proportional to the square root of the hydraulic gradient [27]. Therefore, the second term of the Forchheimer equation (equation (1)) was used to present the fluid flow in gas wells.

By solving the second term of the differential equation (1) and applying the boundary conditions (equations (2) and (3)) the pore pressure distribution as a function of radius from the well defined as equation (10):

$$P_{f(\text{Gas flow})} = \frac{(P_{f0} - P_w) R_0}{(R_0 - R_w)} \left(-\frac{R_w}{r} + 1 \right) + P_w = \frac{(P_{f0} - P_w) R_w}{(R_0 - R_w)} \left(-\frac{R_0}{r} + 1 \right) + P_{f0} \quad (10)$$

Equation (10) only requires pressure and radius parameters and contrary to equation (4) it does not include parameter β to calculate the pore pressure. On the other hand, applying the boundary conditions implies that equation (11) is valid:

$$\frac{(P_{f0} - P_w) R_0 R_w}{(R_0 - R_w)} = \frac{\beta \rho Q_0^2}{4\pi^2 h^2} \quad (11)$$

Thus the left side of equation (11) was used instead of the right side of this equation in equation (10).

In order to develop the stress equations coupled with the non-Darcy flow equation, we substituted equation (10) into equations (A.1). This results in equations (12) to (14):

$$\begin{aligned}
 \sigma_{r(Gas\ flow)} = & a_1 \left(1 - \frac{R_w^2}{r^2} \right) + a_2 \left(1 + 3 \frac{R_w^4}{r^4} - 4 \frac{R_w^2}{r^2} \right) \cos 2\theta \\
 & + \tau_{xy} \left(1 + 3 \frac{R_w^4}{r^4} - 4 \frac{R_w^2}{r^2} \right) \sin 2\theta \\
 & + 2 \frac{\eta}{r^2} (P_{f_0} - P_w) \left(\frac{R_w R_0}{R_0^2 - R_w^2} \right) (r - R_w)(r - R_0) \\
 & + P_w \frac{R_w^2}{r^2} \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{\theta(Gas\ flow)} = & a_1 \left(1 + \frac{R_w^2}{r^2} \right) - a_2 \left(1 + 3 \frac{R_w^4}{r^4} \right) \cos 2\theta \\
 & - \tau_{xy} \left(1 + 3 \frac{R_w^4}{r^4} \right) \sin 2\theta \\
 & - 2 \frac{\eta}{r^2} (P_{f_0} - P_w) R_0 R_w \left[\frac{R_0 R_w - r^2}{R_0^2 - R_w^2} \right] \\
 & - P_w \frac{R_w^2}{r^2} \quad (13)
 \end{aligned}$$

$$\begin{aligned}
 \sigma_z(Gas\ flow) = & \sigma_z^0 \\
 & - 4 \nu \times \left(a_2 \times \frac{R_w^2}{r^2} \times \cos 2\theta + \tau_{xy} \times \frac{R_w^2}{r^2} \times \sin 2\theta \right) \\
 & + 2\eta (P_{f_0} - P_w) \left(\frac{R_w R_0}{R_0 - R_w} \right) \left(\frac{1}{R_0} - \frac{1}{r} \right) \\
 & + \frac{2 \eta \nu}{R_0 + R_w} (P_{f_0} - P_w) (R_w) \quad (14)
 \end{aligned}$$

In order to highlight the effect of non-Darcy assumption, the above equations are compared with the existing solutions based on Darcy assumption. Here the comparison was performed with the Darcy model presented in reference [3]. The results show that the difference between stresses in Darcy and non-Darcy flow regime is proportional to η ($P_{f_0} - P_w$). This means that in reservoirs with high

drawdown pressure ($P_{f_0} - P_w$), the difference between Darcy and Non-Darcy stresses is more significant.

5. Numerical Model

In this section, the applicability of the above proposed analytical solutions is examined numerically. The data corresponding to a typical reservoir were assumed for this purpose as shown in Table 1.

Abaqus software was used for numerical simulations. In numerical modeling three steps are followed: defining the geometry and the boundary conditions; assigning material characteristics as well as the loading conditions; and mesh generation and the model analysis. In this study, axisymmetric simulation of a vertical wellbore was carried out. The pore pressure at the top surface was set to zero, at the well bore wall was assumed to be P_w and at distances far from the well was considered equal to the reservoir pressure (P_{f_0}) across the production layer. The porous elastic material model was used in this simulation. At the bottom of the reservoir, displacements in the vertical directions were restrained and similarly the radial displacements at distances far from the well were restrained.

Finer mesh was used near the wellbore because of the rapid stress changes that are expected near the wellbore; and coarser mesh was used in the regions far from the wellbore.

Axisymmetric simulation was used in modeling with 78400 CAX8RP type (eight-node biquadratic displacement, bilinear pore pressure, reduce integration) elements as depicted in Figure 1. In addition, Non-Darcy flow was modeled by adding the velocity-dependent term to permeability and defining β in Forchheimer equation [28].

Figures 2 to 5 show the pore pressure, radial, tangential and vertical stresses corresponding to both Darcy and non-Darcy flows. In all figures, the distance from the well is presented for a distance of $5R_w$ (i.e. 0.6 m in this model) away from the wellbore wall where the disturbed zone due to drilling is expected to occur [29].

Figures 2 to 5 illustrate the results for analytical and numerical solutions in two regimes of Darcy and non-Darcy flows. As is observed there is the same trend between analytical and numerical results in the wellbore vicinity. The average difference between analytical and numerical results is less than 12.5 % near the wellbore wall in all cases. The difference may result from estimation of elastic input parameters for numerical simulation or variation of permeability with velocity in numerical modeling.

In Figure 2, the pore pressure at the wellbore wall is equal to P_w and it approaches P_{fo} as moving well far from the wellbore. The results indicate that the pore pressure calculated for non-Darcy Forchheimer flow (Eq 4) are 72% larger than that of the Darcy flow (Heidarian et al). This number from numerical simulations is 67% which is comparable with analytical results.

In analytical solution presented by Fjaer et al (1992) (Appendix A, Figure A.2) when P_f is constant, the radial stress increases from P_w to σ_H , which is a similar trend observed in Figure 3. In addition, the results of Figure 3 shows that the average radial stresses in non-Darcy Forchheimer flow regime (Eq 7) are 7.3% larger than the Darcy's flow regime, respectively. Correspondingly, the radial stress in non-Darcy flow is 7% larger than Darcy flow from the results of the numerical modeling. It should be mentioned that the amount of stress increase depends on the reservoir characteristics and it may vary from case to case.

Considering Fjaer et al (1992) analytical solutions [3] when P_f is constant (see Appendix A, Figure A.2), the tangential stress around the wellbore is at maximum, however, it reduces rapidly moving away from the wellbore wall and reaches the in-situ stress σ_H . Similar trend is observed in Figure 4 for the case of current study. Here, the average tangential stress in the non-Darcy Forchheimer flow regime are 10.5% greater than the Darcy flow regime based on the results of analytical models. The results of non-Darcy flow in numerical simulations

appear to be 0.16% larger than that of the Darcy flow.

In Figure 5, the average vertical stress obtained using the analytical approach in Non-Darcy Forchheimer flow regime 5% greater than Darcy's flow regime, respectively. The results of numerical simulations show 2.5% larger value for the vertical stress in non-Darcy flow comparing to Darcy regime.

In interpreting the above results it is important to note the two implicit assumptions that are embedded in the above presented equations: these are radial flow into the wellbore and homogeneity of the formation as considered in developing equations in Appendix A.1. In addition, the sensitivity of the reservoir properties on the results needs to be investigated as a continuation of this work.

6. Sensitivity analysis of reservoir properties

In this section the sensitivity analysis of reservoir properties on induced stresses around a wellbore in non-Darcy flow regime was performed.

For this purpose, the effect of these parameters include drawdown pressure, thickness of the pay zone and drainage radius on induced stresses were investigated.

Parametric analysis according to table 2 was carried out to analytical equations (Eq 4 and Eqs 7 to 10) in the way that one parameter was changed and others as in table 1 were kept unchanged.

In figures 6 to 9 show the effect of drawdown pressure on radial, tangential and vertical stresses.

As shown in numerical value and figure 6, the radial stresses near the wellbore in non-Darcy flow regime very slightly decreased with increasing draw down pressure that can be ignored in practical application.

considering figure 7, by increasing the draw down pressure, the tangential stress near the wellbore increases. Also in figure 8 the vertical stress decreases by increasing the draw down pressure. This means that when draw down pressure increases the tangential stress increases and vertical stress decreases leading to more geomechanical problems like sand production.

Similar procedure like above was done to find out the effect of thickness of the pay zone (h) on induced stresses around the wellbore. thickness of the pay zone has a significant effect on radial stresses around a wellbore. So that by increasing the pay zone thickness from 5m to 200 m, radial stresses were increased rapidly from negative to positive value. The effect of the pay zone thickness on radial stress was shown in figure 9.

According to figure 10, when thickness increases from 5 m to 50 m the tangential stress increases rapidly and then decreases.

In figure 11 unlike the horizontal stresses, by increasing the pay zone thickness the vertical stress was decreased, so the potential of mechanical failure was increased dominantly in thin pay zone.

The drainage radius on stresses around a bore hole has a minor effect. By increasing the drainage radius, horizontal stresses (radial and tangential) were increased and vertical stress was decreased. However, tangential stress is more sensitive to the change of drainage radius than radial and vertical stresses.

7. Conclusions

According to this paper Darcy flow assumption was not appropriate in gas reservoir and in a situation like hydraulic fracture, so two sets of stress equations are developed that will be useful in geomechanical problem e.g. wellbore stability, sand production and completion design with a non-Darcy fluid flow assumption.

Analytical solutions were developed and compared to numerical modelling for a typical reservoir and the changes in pore pressure, radial, tangential and vertical stresses in non-Darcy flow regime were investigated.

The results indicated that the pore pressure and stresses in non-Darcy flow regime have greater values than Darcy flow regime, where the difference is a function of several parameters including the drawdown pressure.

From the presented results of this work, it is concluded that using Darcy flow assumption for stress calculations near wellbore for gas wells may result in significant error, providing incorrect stress data for design purposes including wellbore stability analysis. For example, the induced stresses extracted from equations 7 to 14 compared with appropriate failure criteria can be used in wellbore stability analysis.

Also the sensitivity analysis of reservoir properties on induced stresses around a well bore in non-Darcy flow regime was done and conclude that effect of the thickness of the pay zone has a more significant effect on induced stresses than draw down pressure and drainage radius.

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Table 1- Reservoir characteristics for laminar and turbulent flows

σ_v	57 MPa
$\sigma_H = \sigma_h$	28.5 MPa
P_w	5 MPa
P_{f_0}	20 MPa
R_0	1000 m
R_w	0.1 m
η	0.25
ϑ	0.46
β	0.005
ρ	$2000 \frac{\text{kg}}{\text{m}^3}$
Q_0	$570.5 \times 10^3 \frac{\text{m}^3}{\text{s}}$
h	200 m

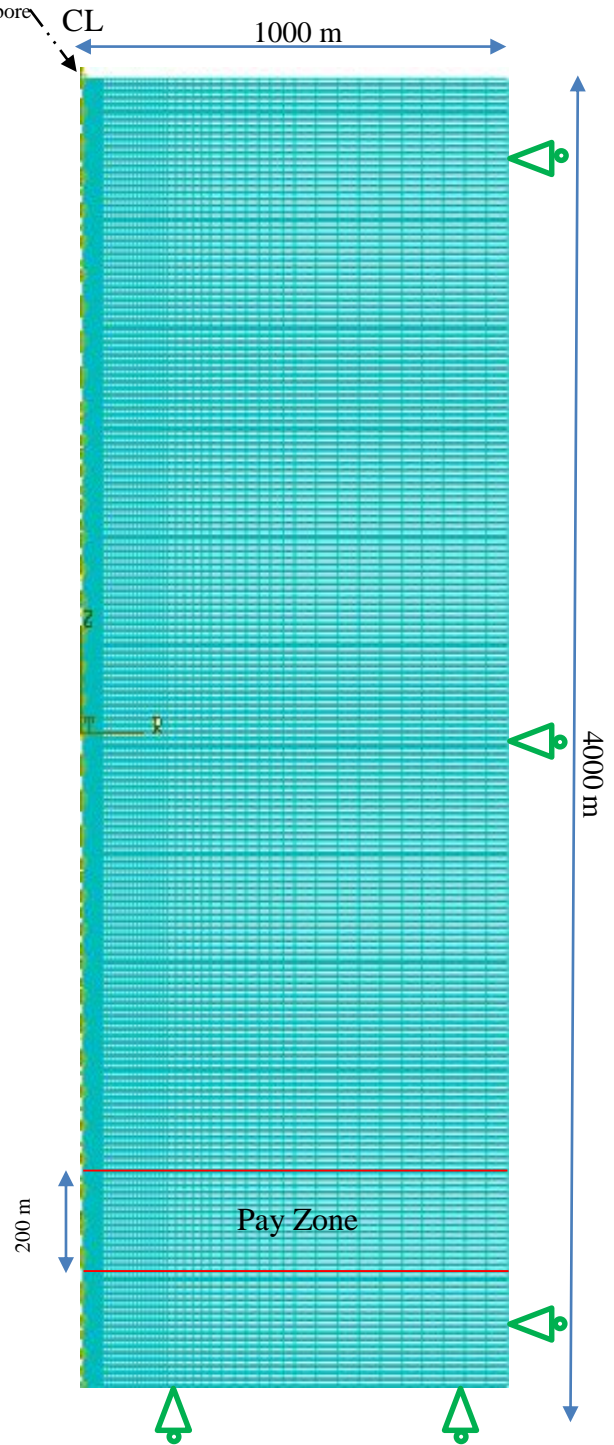


Fig. 1 Axisymmetric mesh used for the model

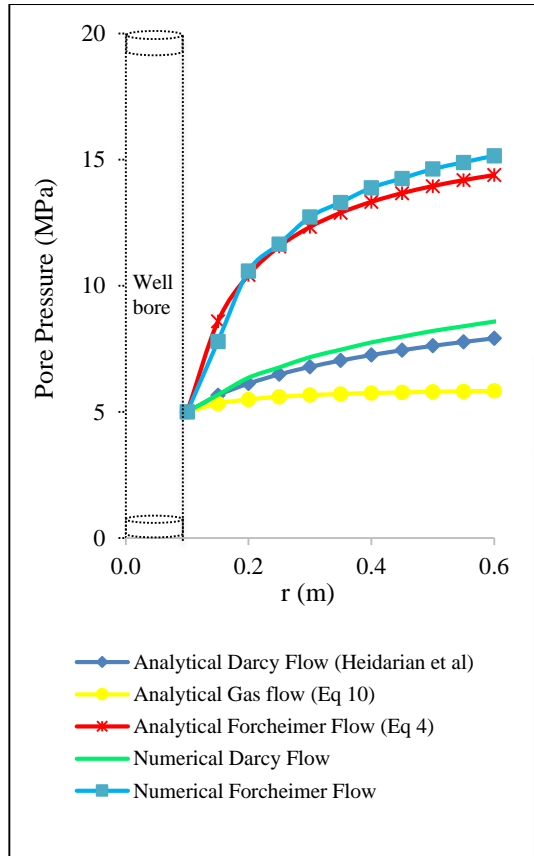


Fig. 2 Pore Pressure variation around the wellbore in Darcy and non-Darcy flow regimes

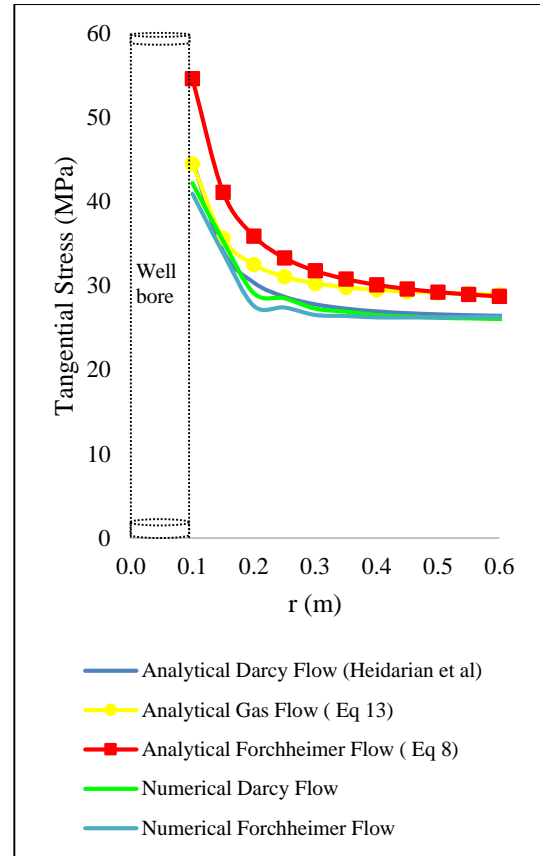


Fig. 4 Tangential stresses around the wellbore in Darcy and non-Darcy flow regimes

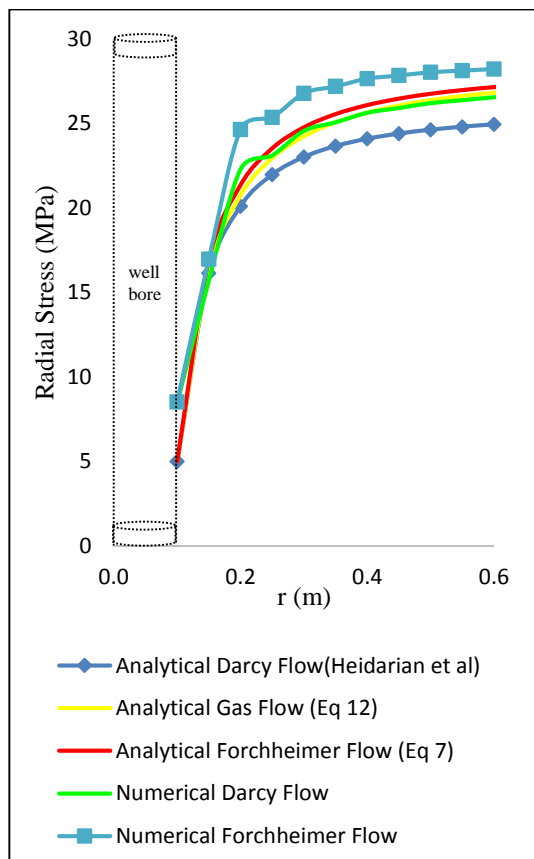


Fig. 3. Radial stresses around the wellbore in Darcy and non-Darcy flow regimes

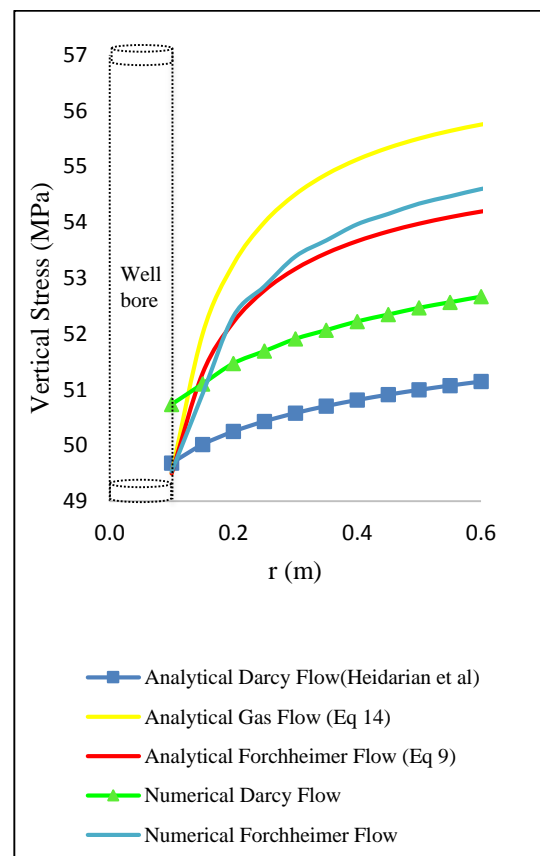


Fig. 5 Vertical stresses around the wellbore in Darcy and non-Darcy flow regimes

Table 2-Parametric analysis of reservoir properties

parameters	
Draw down pressure ($P_{f0}-P_w$)	2 MPa
	5 MPa
	10 MPa
	15 MPa
Thickness of The pay zone (h)	5 m
	25 m
	50 m
	200 m
Drainage Radius (R_0)	250 m
	500 m
	750 m
	1000 m

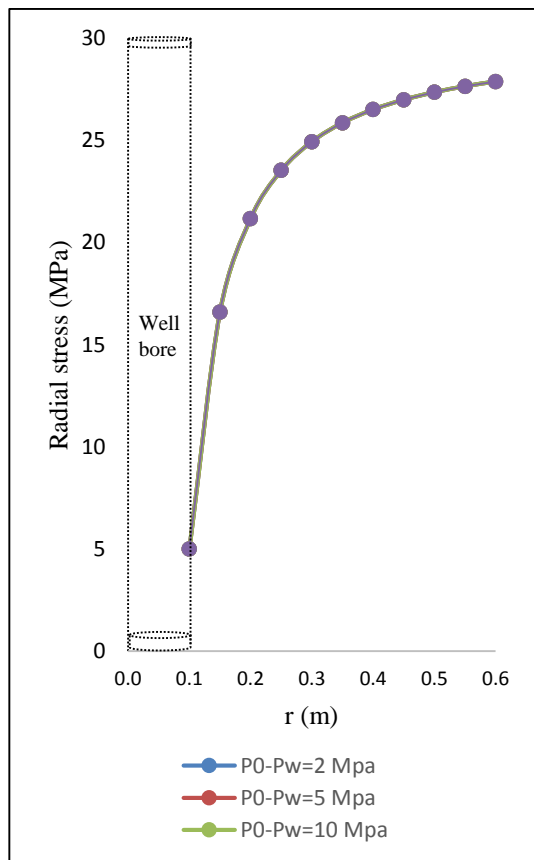


Fig. 6 The effect of draw down pressure on radial stresses in non-Darcy flow regime

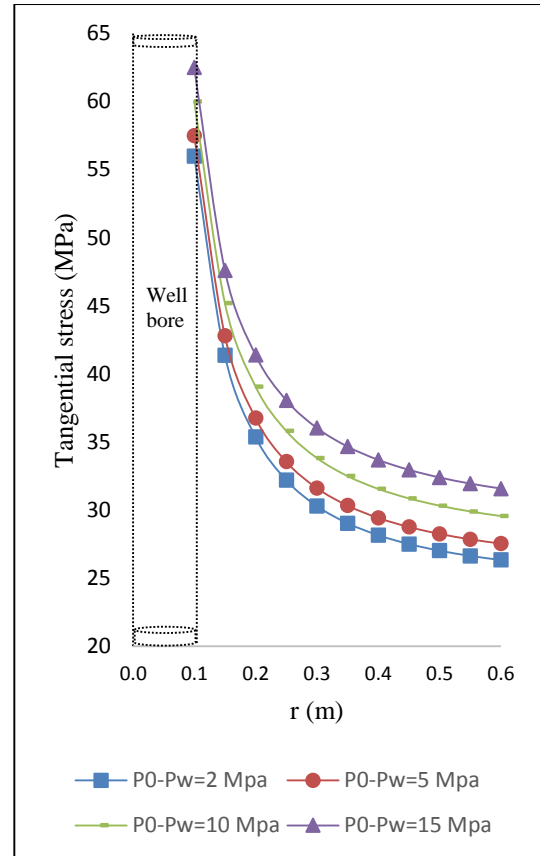


Fig. 7 The effect of draw down pressure on tangential stresses in non-Darcy flow regime

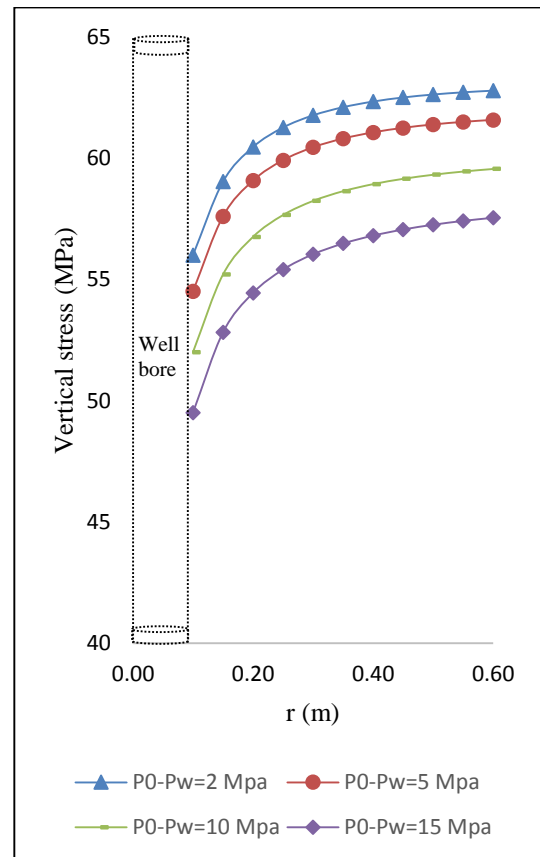


Fig. 8 The effect of draw down pressure on vertical stresses in non-Darcy flow regime

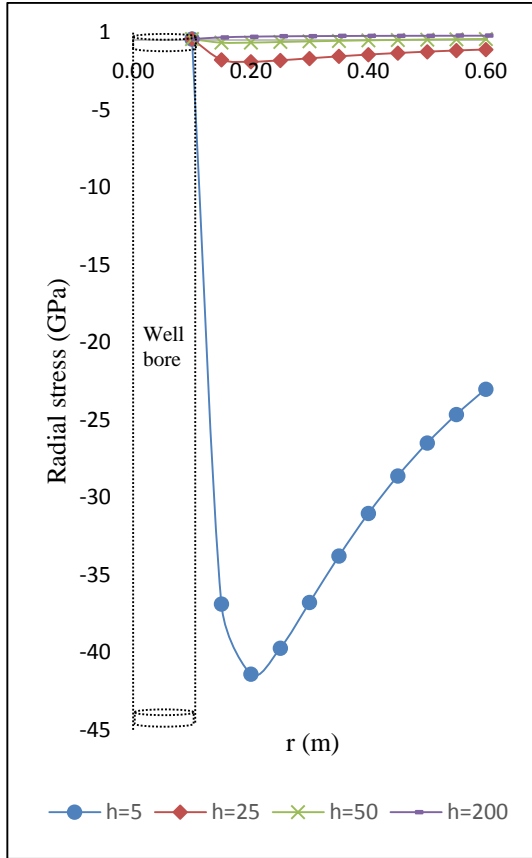


Fig. 9 The effect of pay zone thickness on radial stresses in non-Darcy flow regime

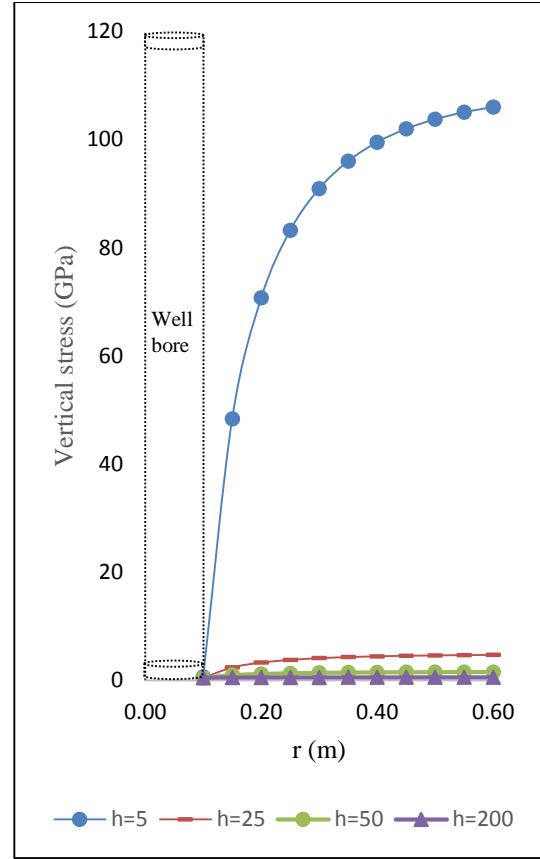


Fig. 11 The effect of pay zone thickness on vertical stresses in non-Darcy flow regime

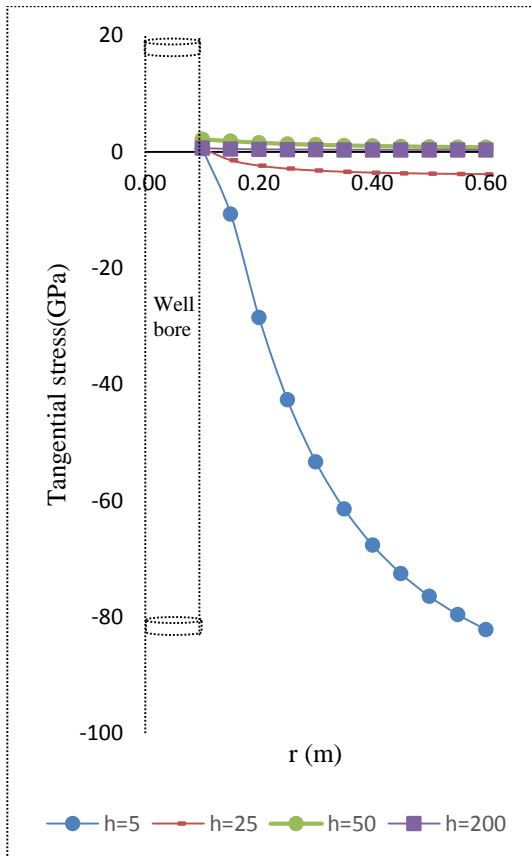


Fig. 10 The effect of pay zone thickness on tangential stresses in non-Darcy flow regime

Appendix A

Drilling induced stresses around a deviated wellbore in a homogenous media subjected to anisotropic horizontal stresses are determined as [3]:

$$\begin{aligned} \sigma_r = & a_1 \left(1 - \frac{R_w^2}{r^2} \right) \\ & + a_2 \left(1 + 3 \frac{R_w^4}{r^4} - 4 \frac{R_w^2}{r^2} \right) \cos 2\theta \\ & + \tau_{xy}^o \left(1 + 3 \frac{R_w^4}{r^4} - 4 \frac{R_w^2}{r^2} \right) \sin 2\theta \\ & + \frac{2\eta}{r^2} \left(\int_{R_w}^r r' \Delta P_f(r') dr' \right. \\ & \left. - \frac{r^2 - R_w^2}{R_0^2 - R_w^2} \int_{R_0}^{R_w} r' \Delta P_f(r') dr' \right) \\ & + P_w \frac{R_w^2}{r^2}, \end{aligned} \quad (A.1)$$

$$\begin{aligned}
 \sigma_{\theta} &= a_1 \left(1 + \frac{R_w^2}{r^2} \right) \\
 &- a_2 \left(1 + 3 \frac{R_w^4}{r^4} \right) \cos 2\theta \\
 &- \tau_{xy}^{\circ} \left(1 + 3 \frac{R_w^4}{r^4} \right) \sin 2\theta \\
 &- \frac{2\eta}{r^2} \left(\int_{R_w}^r r' \Delta P_f(r') dr' - r^2 \Delta P_f(r) \right) \\
 &+ \frac{r^2 + R_w^2}{R_0^2 - R_w^2} \int_{R_w}^{R_0} r' \Delta P_f(r') dr' \\
 &- P_w \frac{R_w^2}{r^2}, \quad (A.2)
 \end{aligned}$$

$$\begin{aligned}
 \sigma_z &= \sigma_z^{\circ} \\
 &- 4\nu \left(a_2 \frac{R_w^2}{r^2} \cos 2\theta + \tau_{xy}^{\circ} \frac{R_w^2}{r^2} \sin 2\theta \right) \\
 &+ 2\eta \Delta P_f(r) \quad (A.3) \\
 &- 4\eta \frac{\nu}{R_0^2 - R_w^2} \int_{R_w}^{R_0} r' \Delta P_f(r') dr'
 \end{aligned}$$

In equations A.1 to A.3:

$$\begin{aligned}
 a_1 &= \left(\frac{\sigma_x^{\circ} + \sigma_y^{\circ}}{2} \right), a_2 = \left(\frac{\sigma_x^{\circ} - \sigma_y^{\circ}}{2} \right) \\
 \eta &= \frac{1 - 2\nu}{2(1 - \nu)} \alpha
 \end{aligned}$$

$$\Delta P_f(r) = P_f(r) - P_{f0},$$

$$\sigma_x^{\circ} = I_{xx}^2 \sigma_H + I_{xy}^2 \sigma_h + I_{xz}^2 \sigma_v,$$

$$\sigma_y^{\circ} = I_{yx}^2 \sigma_H + I_{yy}^2 \sigma_h + I_{yz}^2 \sigma_v,$$

$$\sigma_z^{\circ} = I_{zx}^2 \sigma_H + I_{zy}^2 \sigma_h + I_{zz}^2 \sigma_v,$$

$$\tau_{xy}^{\circ} = I_{xx} I_{yx} \sigma_H + I_{xy} I_{yy} \sigma_h + I_{xz} I_{yz} \sigma_v,$$

where

$$I_{xx'} = \cos a \times \cos i; I_{yx'} = -\sin a; I_{zx'} = \cos a \times \sin i,$$

$$I_{xy'} = \sin a \times \cos i; I_{yy'} = \cos a; I_{zy'} = \sin a \times \sin i,$$

$$I_{xz'} = -\sin i; I_{yz'} = 0; I_{zz'} = \cos i,$$

In equations A.1 to A.2, σ_H and σ_h are the maximum and minimum horizontal stresses, R_w is the radius of the wellbore, R_0 is the drainage radius, P_f is the pore pressure in the distance r from the wellbore center, P_{f0} is the reservoir pressure, ν is formation's Poisson's ratio and α is Biot coefficient. As depicted In Figure (A.1), the angles a and i are used for transformation of stresses from a vertical to a deviated wellbore[30].

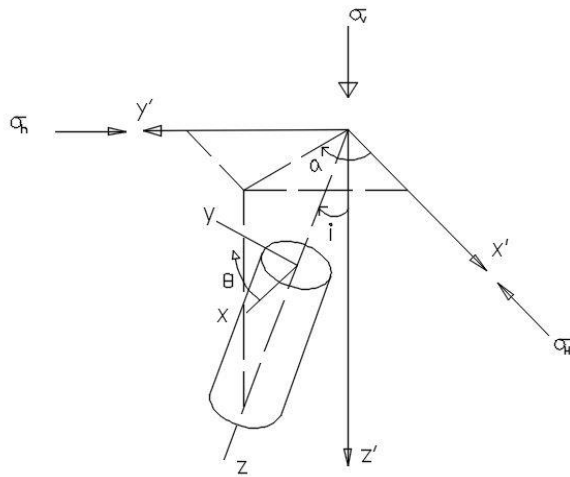


Fig A.1. Angles α and i are used for transformation of stresses from a vertical to a deviated wellbore [31].

For solving equations A.1 to A.3 simplified assumptions have been made by researchers; the graphical presentation of equations A.1 to A.3, when $\frac{\partial p}{\partial r} = 0$ is shown in figure (A.2)[3].

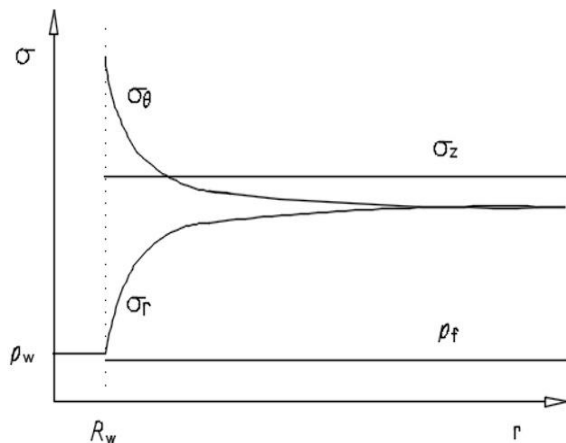


Fig A.2. Analytical solution of equations A.1 for $P_f = \text{constant}$ (Elastic stress solution).

Biographies

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