Two-frequency shell-model calculations for p-shell nuclei

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We have studied p-shell nuclei using a two-frequency shell-model approach with an effective interaction derived from the Bonn-A nucleon-nucleon potential by means of a G-matrix folded-diagram method. Shell-model wave functions of two different oscillator constants , $\hbar\omega_{\rm in}$ and $\hbar\omega_{\rm out}$, are employed, one for the inner $0s$ core orbit and the other for the outer valence orbits, respectively. The binding energies, energy spectra, and electromagnetic properties are calculated and compared with experiment. A quite satisfactory agreement with the experimental data is obtained, which is in some cases even better than that produced by large-basis shell-model calculations.

21.60.Cs; 21.30.Fe; 27.20.+n

I. INTRODUCTION

The p-shell nuclei have long been the subject of theoretical interest. The first shell-model study of these nuclei was performed by Cohen and Kurath in 1965 [1]. In this work, which has been a point of reference for later studies, a successful description of the p-shell nuclei was given by taking ^{4}He as a closed core and letting the valence nucleons occupy the $0p$ shell. The fifteen matrix elements of the two-body interaction and the two single-particle energies were determined by making a least-squares fit to selected observed energy levels. From then on, several other shell-model calculations have been performed in the $0p$ model space employing different kinds of effective interactions [2-5]. It should be mentioned that the calculations of Ref. [2] represent the first attempt to use a realistic effective interaction for these light nuclei.

In recent years, the study of p-shell nuclei has become a subject of special interest owing to the discovery of new aspects of their structure. One main result has been the observation for some neutron-rich nuclei, such as ^{6}He and ^{11}Li, of abnormally large interaction and reaction cross-sections [6,7]. These nuclei have a very small one- and two-neutron separation energy and have been described as having a halo structure [8], namely an extended neutron distribution surrounding a tightly bound inner core.

During the last decade substantial progress in computational techniques has set the stage for more ambitious calculations of the structure of light nuclei. As regards shell-model studies, calculations in a $(0+2)\hbar\omega$ model space have been performed in the early 1990s [9-11]. More recently, larger multi-$\hbar\omega$ spaces have been used [12,13]. In particular, large-basis no-core calculations have been carried out [13] making use of an effective interaction derived from the Reid 93 nucleon-nucleon (NN) potential. Alternatively, there has been a variety of studies in terms of clusters (see Ref. [14] for a comprehensive list of references). In this context, three-body model approaches aimed at describing the structure of halo nuclei have been developed [14,15].

To end this brief review of the various approaches to the study of p-shell nuclei, the quantum Monte Carlo calculations of Refs. [16,17] should be mentioned. Within this approach properties of nuclei with $A \leq 8$ are calculated directly from bare two-nucleon and three-nucleon forces.

A new approach, the two-frequency shell-model (TFSM), has been recently proposed in Refs. [18,19]. Within the TFSM, the model space effective interaction $V_{\rm eff}$ is derived from the free NN potential by way of a G-matrix folded diagram method. Its peculiar feature consists in calculating the G matrix in a space composed of harmonic oscillator wave functions with two different oscillator constants, $\hbar\omega_{\rm in}$ and $\hbar\omega_{\rm out}$, for the core and the valence orbits, respectively (the length parameters $b_{\rm in}$ and $b_{\rm out}$ will be also used from now on, with $b = (\hbar/m\omega)^{1/2}$). Note that $b_{\rm out}$ is chosen substantially larger than $b_{\rm in}$. This idea reflects the fact that the valence nucleons in p-shell nuclei are spatially more extended than those of the core. Actually, these nuclei may be thought of as a ^{4}He nucleus with loosely attached outer nucleons. This feature may be taken into account by including several major shells in the ordinary one-frequency shell model. We shall see that the TFSM, allowing different length parameters for the valence and core orbits, provides a simple and effective alternative. It may be interesting to mention that, albeit in quite different contexts, this idea was also considered in two earlier works [20,21].

The outline of the paper is as follows. In Sec. I the derivation of the effective interaction from a realistic NN potential is described. Our results are presented and compared with the experimental data in Sec. III. Sec. IV contains a summary of our conclusions.

II. DERIVATION OF THE EFFECTIVE INTERACTION

Here, we describe how to derive the effective interaction from a realistic NN potential V_{NN} within the framework of the TFSM.

As usual, one starts from a nuclear many-body problem of the form $H\Psi_\mu = E_\mu \Psi_\mu$ with $H = T + V_{NN}$, where T denotes the kinetic energy. This many-body problem can be formally reduced [22] to a model space (usually referred to as the P-space) problem of the form

$$H_{\rm eff} P\Psi_\mu = E_\mu P\Psi_\mu;\ H_{\rm eff} = H_0 + V_{\rm eff}, \tag{1}$$

where the eigenvalues E_μ are a subset of the eigenvalues of the original Hamiltonian in the full space, $\mu = 1, 2 \cdots d$, with d denoting the dimension of the P space. In Eq. (1) $V_{\rm eff}$ is the model-space effective interaction and $H_0 = T + U$ the unperturbed Hamiltonian, U being an auxiliary potential introduced to define a convenient single-particle (sp) basis. This is chosen to be a harmonic oscillator potential. Note that our P space is defined in terms of the eigenfunctions of H_0.

The model-space effective interaction $V_{\rm eff}$ may be written [22] as a folded-diagram series

$$V_{\rm eff} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots, \tag{2}$$

where $\int$ denotes a generalized fold, and $\hat{Q}'$ and $\hat{Q}$ represent the $\hat{Q}$-box, composed of irreducible valence-linked diagrams. $\hat{Q}'$ is obtained from $\hat{Q}$ by removing first-order diagrams. Because of the strong repulsive core contained in all modern NN potentials, as a first step we need to derive the model-space G-matrix corresponding to the chosen V_{NN}, and then calculate the $\hat{Q}$-box from irreducible diagrams with G-matrix vertices. The Brueckner G-matrix is defined by the integral equation [23,24]

$$G(\omega) = V + VQ_2 \frac{1}{\omega - Q_2 T Q_2} Q_2 G(\omega), \tag{3}$$

where ω is an energy variable, T is the two-nucleon kinetic energy and V represents the NN potential. Q_2 is a two-body Pauli exclusion operator, whose complement $P_2 = 1 - Q_2$ defines the space within which the G matrix is calculated. The role of Q_2 in Eq. (3) is to prevent double counting, namely the intermediate states allowed for G must be outside of the P_2 space. Note that our G matrix has orthogonalized plane-wave functions as intermediate states while the operator Q_2 is defined in terms of harmonic oscillator wave functions as

$$Q_2 = \sum_{\rm all\ } Q(ab)|ab\rangle\langle ab|, \tag{4}$$

where $Q(ab) = 0$, if $b \le n_1$, $a \le n_3$, or $b \le n_2$, $a \le n_2$, or $b \le n_3$, $a \le n_1$, and $Q(ab) = 1$ otherwise. The boundary of Q_2 is specified by the three numbers n_1, n_2, and n_3, each representing a sp orbit (the orbits are numbered starting from the bottom of the oscillator well). In particular, n_1 is the number of orbits below the Fermi surface of the doubly magic core, n_2 fixes the orbit above which the passive sp states start, and n_3 denotes the limit of the P_2 space.

It should be noted that in the calculation of G the space of active sp states, i.e. the levels between n_1 and n_2, may be different from the model space within which $V_{\rm eff}$ is defined. Several arguments for choosing the former larger than the latter are given in Ref. [23]. Generally, n_2 is fixed so as to include two major shells above the Fermi surface. In this paper, we consider the p-shell nuclei with ^{4}He as a core, thus we have $n_1 = 1$. Then we take $n_2 = 6$ so as to include all the five orbits of the p and sd shells above the Fermi surface. As regards n_3, it should be infinite, but in practice it is chosen to be a large but finite number. Namely, calculations are performed for increasing values of n_3 until numerical results become stable. For the present case, we have found that a choice of $n_3 = 21$ turns out to be quite adequate.

From the above it is clear that the reaction matrix G depends on the space P_2 and will be different for different choices of this space. In the TFSM approach the P_2 space is defined in terms of harmonic oscillator wave functions with two different length parameters $b_{\rm in}$ and $b_{\rm out}$, the former for the inner core orbits and the latter for the outer valence orbits. As already discussed in the Introduction, this choice, with $b_{\rm out}$ larger than $b_{\rm in}$, allows us to give an appropriate description of the p-shell nuclei.

The presence of the Pauli operator Q_2 adds considerable difficulty to the calculation of the above G-matrix. However, an accurate treatment of it can be carried out using a matrix inversion method [23,25]. With this method, the exact solution of the G-matrix equation (3) reads

$$G = G_F + \Delta G, \tag{5}$$

where the "free" G matrix is

$$G_F(\omega) = V + V \frac{1}{\omega - T} G_F(\omega), \tag{6}$$

and the Pauli correction term ΔG is given by

$$\Delta G(\omega) = \tag{7}$$
$$-G_F(\omega) \frac{1}{e} P_2 \frac{1}{P_2[1/e + (1/e)G_F(\omega)(1/e)]P_2} P_2 \frac{1}{e} G_F(\omega),$$

where $e = \omega - T$.

The central ingredient for calculating the above G matrix are the matrix elements of G_F within the P_2 space. As there is no Pauli projection operator for G_F, the calculation of its momentum space (k-space) matrix elements is relatively easy and has been carried out using the standard momentum-space matrix inversion method [23]. Similarly we have calculated the k-space matrix elements of $1/e\ G_F$, $G_F\ 1/e$ and $1/e\ G_F\ 1/e$. For shell model calculations, however, we need the matrix elements of these

operators between oscillator basis wave functions. In our two-frequency approach, sp wave functions of two different length parameters are employed, i.e. our basis consists of both $\phi_n^{\rm in}$ and $\phi_n^{\rm out}$, the oscillator wave functions with length parameters $b_{\rm in}$ and $b_{\rm out}$, respectively. As a consequence, we also have to calculate matrix elements such as $\langle \phi_1^{\rm in}\ \phi_2^{\rm out}|G_F|\phi_3^{\rm out}\ \phi_4^{\rm out}\rangle$.

To calculate matrix elements of the above type, a standard procedure is to first transform the wave functions to the RCM (relative and center of mass) representation. For the above matrix element, the two sp wave functions in the ket $|\phi_3^{\rm out}\ \phi_4^{\rm out}\rangle$ have identical length parameters. While the RCM transformation for this state can be easily carried out using the well-known Moshinsky transformation brackets, this is more complicated for the bra $\langle \phi_1^{\rm in}\ \phi_2^{\rm out}|$, as the two sp wave functions have different length parameters. We have overcome this difficulty by expanding $\phi^{\rm in}$ in terms of $\phi^{\rm out}$ or *vice versa*. By way of illustration, for the above case we have expanded the bra $\langle \phi_1^{\rm in}\ \phi_2^{\rm out}|$ as

$$\langle \phi_1^{\rm in}\ \phi_2^{\rm out}| = \sum_{n=0,N} C_{1,n} \langle \phi_n^{\rm out}\ \phi_2^{\rm out}|. \qquad (8)$$

With this expansion, the above matrix element becomes a linear combination of $\langle \phi_n^{\rm out}\ \phi_2^{\rm out}|G_F|\phi_3^{\rm out}\ \phi_4^{\rm out}\rangle$, which is a one-frequency matrix element and can be readily evaluated. We have found that this expansion can be carried out quite accurately by including only a small number of terms, typically $N \leq 10$, in Eq. (8). Similarly, we have calculated the mixed-frequency matrix elements of $1/e\ G_F$, $G_F\ 1/e$ and $1/e\ G_F\ 1/e$. In this way the G matrix of Eq. (5) is finally obtained.

A problem inherent in the TFSM may be mentioned. We must require the sp wave functions ϕ_n to form an orthonormal basis. This requirement is usually not satisfied by wave functions of different length parameters. For instance, $\phi^{\rm in}_{0p_{3/2}}$ is not orthogonal to $\phi^{\rm out}_{1p_{3/2}}$ when $b_{\rm in}$ is not equal to $b_{\rm out}$. In the present work we consider nuclei with several nucleons in the orbits $0p_{3/2}$ and $0p_{1/2}$ outside the ^{4}He core. We have used a short length parameter $b_{\rm in}$ for the $0s_{1/2}$ orbit and a long length parameter $b_{\rm out}$ for the orbits mentioned above. In the calculation of the Pauli correction terms for the G-matrix and in the derivation of $V_{\rm eff}$, some higher orbits, such as the $1s_{1/2}$ orbit, are also needed. To ensure their orthogonality with the core orbit, we have also used $b_{\rm in}$ for the $1s_{1/2}$ and higher s orbits ($b_{\rm out}$ is used for all the other higher orbits). We shall further discuss this point later.

Using the above G matrix, we can now calculate the $\hat{Q}$-box of Eq. (2). This is done by including the seven first- and second-order irreducibile valenced-linked G-matrix diagrams [26,27], as shown in Fig. 1. After the $\hat{Q}$-box is calculated, $V_{\rm eff}$ is obtained by summing up the folded-diagram series (2) to all orders by means of the Lee-Suzuki iteration method [28,29]. This last step can be performed in an essentially exact way for a given $\hat{Q}$-box. Note that the G matrix is energy dependent in that it depends on the starting energy ω. The folded-diagram effective interaction given by Eq. (2) is, however, energy independent [22].

Before closing this section we should remark that in our derivation of $V_{\rm eff}$ only the calculation of the $\hat{Q}$-box requires certain approximations. In fact, we have neglected its G-matrix diagrams beyond the second-order ones. In Refs. [27] and [30] the role of third-order diagrams was investigated within the framework of standard shell-model calculations. It was shown that for the *sd* nuclei the third-order contributions produce a change of about $10-15\%$ in the effective interaction, which reduces to only 5% or less for heavier nuclei (in this case only the $T=1$ matrix elements were investigated). In the TFSM approach one expects these higher-order diagrams to be even smaller. In fact, the contribution of the D7 diagram of Fig. 1, which is a second-order core-polarization diagram and contributes a significant correction to the G matrix, is rather small when the length parameter $b_{\rm out}$ becomes significantly larger than $b_{\rm in}$. Diagonal matrix elements of this diagram for the states $|(p_{3/2})^2; T=1, J=0\rangle$ and $|p_{3/2}p_{1/2}; T=0, J=1\rangle$ are shown in Fig. 2 as a function of the outer length parameter $b_{\rm out}$. This parameter ranges from 1.45 to 2.50 fm while $b_{\rm in}$ is kept fixed at 1.45 fm. The Bonn-A realistic NN potential [31] is used. We see that the diagram D7 is already largely suppressed when $b_{\rm out}$ becomes nearly 2.0 fm. We have also calculated several typical third-order diagrams and have found that their contribution to the matrix elements of $V_{\rm eff}$ decreases by an order of magnitude as $b_{\rm out}$ goes from 1.45 to 2.0 fm. This is a consequence of the fact that increasing $b_{\rm out}$ corresponds to increasing the average distance between the core and valence nucleons, thus reducing the overlap between their wave functions.

We should like to Recall that to ensure the orthogonality we have used the same length parameter $b_{\rm in}$ for not only the $0s_{1/2}$ but also the $1s_{1/2}$ and other s orbits. Using $b_{\rm in}$ only for the core $0s_{1/2}$ orbit and $b_{\rm out}$ for the other s orbits would of course require an orthogonalization procedure, which is numerically more involved than our present treatment. We are currently examining this point.

III. RESULTS AND COMPARISON WITH EXPERIMENT

Within the framework of the TFSM we have carried out calculations for the p-shell nuclei with $A \leq 9$. Results of this study for $A=8$ nuclei have already been presented in [32,33], together with those obtained in a standard one-frequency shell-model calculation. In these papers comparison between one- and two-frequency calculations has evidenced the merit of the latter approach with respect to the former.

We have assumed that the doubly magic ^{4}He is a closed core and let the valence particles occupy the two orbits

$0p_{3/2}$ and $0p_{1/2}$. As regards the sp spacing between these two levels, we have taken it from the experimental spectrum [34] of ^{5}He, namely $\epsilon_{1/2} - \epsilon_{3/2} = 4.0$ MeV, while we have fixed the sp energy $\epsilon_{3/2}$ at 0.886 MeV, which is the experimental one-neutron separation energy for ^{5}He [35]. It should be noted that the excitation energy of the first $\frac{1}{2}^-$ state in ^{5}He, which is a very broad resonance, has a large error bar (± 1 MeV). The effective interaction has been derived from the Bonn-A free NN potential, as described in Sec. II. All results presented in this paper have been obtained by using the OXBASH shell model code [36].

The $b_{\rm in}$ parameter used for the $0s_{1/2}$ orbit was fixed at 1.45 fm [18], while $b_{\rm out}$ was allowed to vary from 1.45 to 2.50 fm. In Table I we report the experimental ground-state binding energies [35] for nuclei with $6 \leq A \leq 9$ and compare them with the calculated ones for $b_{\rm out} = 1.45$, 1.75, 2.00, 2.25, and 2.50 fm. The theoretical values have been obtained by adding to our calculated ground-state energies the experimental ground-state binding energy [35] of ^{4}He and the Coulomb contributions taken from Ref. [37], where they were determined from a least-squares fit to experimental data.

Table I shows that all calculated binding energies decrease as $b_{\rm out}$ increases. This is an obvious consequence of the fact that most matrix elements of $V_{\rm eff}$ become less attractive when increasing $b_{\rm out}$. As regards the comparison with the experimental data, we see that for the two lowest values of $b_{\rm out}$ all binding energies are significantly overestimated by our calculations. A value of $b_{\rm out} = 2.0$ fm brings the calculated binding energies for Li isotopes and their corresponding mirror nuclei into good agreement with experiment, the discrepancies ranging from 0.3 to 0.6 MeV. As regards the He isotopes (and their mirror nuclei) a larger value of $b_{\rm out}$ (2.25 fm) is needed to reproduce the experimental energies. On the other hand, by increasing $b_{\rm out}$ from 1.75 to 2.0 fm, the calculated binding energies of ^{8,9}Be and ^{9}B are shifted from 1-2 MeV above to 4-5 MeV below the experimental values. This indicates that the optimum value of $b_{\rm out}$ for these nuclei lies between 1.75 and 2.0 fm. It turns out that it is 1.9 fm.

Note that in the above analysis we have not tried to adjust the value of $b_{\rm out}$ for each nucleus, but have been satisfied with discrepancies of a few hundred keV between experiment and theory. We would like to point out that the optimum value of $b_{\rm out}$ is related to the nuclear binding energy (relative to ^{4}He) per valence nucleon. In fact, this quantity is almost constant for nuclei which require the same value of $b_{\rm out}$. More precisely, it is a few hundred keV for the He isotopes, about 2-4 MeV for the Li isotopes, and 6-7 MeV for ^{8,9}Be and ^{9}B. The same situation occurs for all the corresponding mirror nuclei.

Based on these findings, we have found it appropriate to calculate the spectra and electromagnetic properties of the various nuclei reported in Table I by using the values of $b_{\rm out}$ derived from the above analysis. We have verified that use of values of $b_{\rm out}$ different from the adopted ones leads to an overall worse agreement between experimental and calculated spectra. However, states with $T > T_z$ require a separate discussion, which will be given at the end of this Section.

Here we focus attention on $^{6-8}$Li and their corresponding mirror nuclei. In Figs. 3-5 we compare the experimental spectra with the calculated ones ($b_{\rm out} = 2.0$ fm). While the observed spectra of ^{7}Li and ^{7}Be are quite similar (the only significant difference is the absence of a second $\frac{7}{2}^-$ state in the latter one), the experimental information for ^{8}B is very scanty. For this reason, the following discussion will only concern Li isotopes.

As a general remark, we see that in the considered energy regions our calculations give rise to all the observed levels for each of the three nuclei. However, while for ^{6}Li and ^{7}Li no more levels than the observed ones are predicted by the theory, for ^{8}Li we find several states without an experimental counterpart.

Let us now make some more specific comments on each Li isotope separately. The ground state of ^{6}Li is stable while the first excited state with $(J^\pi;T) = (3^+;0)$ is just above the threshold for breakup into $\alpha + d$ and has a narrow width of 24 keV. The other two $T = 0$ states have, instead, fairly large widths ($\Gamma > 1000$ keV). The 0^+ state at 3.6 MeV is the isobaric analog of the ground state in ^{6}He and in ^{6}Be, while the 2^+ state at 5.4 MeV is the analog of the first excited state. From Fig. 3 we see that the first excited state is very well reproduced by the theory. As regards the other two $T = 0$ states, our calculation overestimates the experimental excitation energies by more than 1 MeV, while the $(0^+;1)$ and $(2^+;1)$ states are underestimated by about 1.2 and 0.3 MeV, respectively.

The spectrum of ^{7}Li contains the stable ground state with $(J^\pi;T) = (\frac{3}{2}^-;\frac{1}{2})$ and the $(\frac{1}{2}^-;\frac{1}{2})$ first excited state, which decays by γ emission. All other excited states lie above the threshold for breakup into $\alpha + t$, but only the $(\frac{3}{2}^-;\frac{1}{2})$ at 9.8 MeV is a broad resonance with $\Gamma \gg 1200$ keV. The $T = \frac{3}{2}$ state at 11.2 MeV with a width of 260 keV is the $T_z = \frac{1}{2}$ member of an isobaric quartet. The analog states with $|T_z| = \frac{3}{2}$ are the ground states of ^{7}He and ^{7}B, while the member with $T_z = -\frac{1}{2}$ is the state at 11.0 MeV in ^{7}Be. The quantitative agreement between calculated and experimental excitation energies is very satisfactory for all the levels, the only exceptions being the second $(\frac{3}{2}^-;\frac{1}{2})$ state and the $(\frac{3}{2}^-;\frac{3}{2})$ state. In fact, the discrepancies are about 1 and 3 MeV for the former and the latter states, respectively, while they are less than few a hundred keV for all the other states.

Turning to ^{8}Li, the ground and first excited state are very stable against the breakup, the former decaying by β^- emission. The second excited state lies just above the threshold for breakup into $^7{\rm Li} + n$ and is fairly narrow with a width of 33 keV. A number of higher ex-

cited states have been identified, some of them with large widths. In particular, the $(1^+;1)$ state at 3.2 MeV excitation energy and the $((3);1)$ state at 6.1 MeV have widths $\Gamma \gg 1000$ keV. The $(0^+;2)$ isobaric analog of the ^{8}He ground state occurs at 10.8 MeV with a width less than 12 keV. From Fig. 5 we see that not only the first four calculated levels are in the right order but also the excitation energies are in very good agreement with experiment. Above these levels and up to 6 MeV our calculation predicts four states, three of them without an experimental counterpart. More precisely, we have two $(2^+;1)$ states and two states with $(J^\pi;T) = (0^+;1)$ and $(1^+;1)$, respectively, while only one experimental level with spin equal to 0 or 1 is available in this energy region. Our calculation suggests that this state, which lies at 5.4 MeV, has $J^\pi = 1^+$. Between 6 and 8 MeV three levels have been observed, and the same number is predicted by our calculation. Among them only one has a firm spin-parity assignment and can be safely identified with the calculated $(4^+;1)$ state, whose excitation energy is only 80 keV larger than the experimental value. As regards the $((3);1)$ level and that at 7.1 MeV with unknown spin and parity, we propose the assignment $(3^+;1)$ and $(1^+;1)$, respectively. In this case, the excitation energy of the latter state is almost exactly reproduced while that of the former one is overestimated by about 1 MeV. Finally, we see that the calculated $(0^+;2)$ level lies about 3 MeV below the experimental one.

From the above we can conclude that, as regards the binding and excitation energies, the overall agreement between theory and experiment may be considered quite satisfactory. In fact, significant discrepancies occur only for the excitation energies of states with fairly large widths or with $T >\mid T_z \mid$. As regards these latter states some comments are in order. The $(0^+;1)$, $(\frac{3}{2}^-;\frac{3}{2})$, and $(0^+;2)$ states in ^{6}Li, ^{7}Li, and ^{8}Li, respectively, are isobaric analogs of the ground states of ^{6}He, ^{7}He, and ^{8}He. The $(2^+;1)$ in ^{6}Li is a member of the isospin triplet which is comprised of the first excited state in ^{6}He and in ^{6}Be. At the beginning of this Section, we have shown that for the He isotopes a larger value of $b_{\rm out}$ is required as compared to that adopted for the Li isotopes. We have then found it appropriate to calculate the energies of the $T >\mid T_z \mid$ states in Li isotopes by making use of $b_{\rm out} = 2.25$ fm. It has turned out that all the new calculated excitation energies (relative to the ground-state energies obtained with $b_{\rm out} = 2.0$ fm) go in the right direction largely reducing the discrepancies with the experimental data.

Let us now come to the electromagnetic observables. In Table II the measured moments [39] together with the $E2$ and $M1$ transition rates [34] for $^{6-8}$Li and ^{8}B are compared with the calculated values. In our calculations no effective charge has been attributed to the proton and neutron, and use has been made of free gyromagnetic factors. We have also calculated electric and magnetic effective operators including only diagrams first order in G [38]. We have found that the results do not significantly differ from those obtained with bare operators. This is not surprising, as our effective operators take essentially into account the core-polaritation effects, which, as pointed out in Sec. II, are largely suppressed for $b_{\rm out}$ significantly larger than $b_{\rm in}$.

From Table II we see that the experimental magnetic moments and the $B(M1)$ values are very well reproduced by our calculations. As regards the electric observables, the agreement is not of the same quality. However, while our calculations underestimate the $E2$ transition rates as well as the quadrupole moments, they reproduce the signs of the latter quantities (the sign of the the quadrupole moment of ^{8}B has not been measured).

IV. SUMMARY

In this paper, we have described how to calculate, for a chosen free nucleon-nucleon potential, the Brueckner G matrix in a space composed of harmonic oscillator wave functions of two different length parameters $b_{\rm in}$ and $b_{\rm out}$, one for the inner core orbits and the other for the outer valence orbits. Using this G matrix the model-space effective interaction $V_{\rm eff}$ is then derived within the framework of the folded-diagram method. Starting from the Bonn-A potential we have constructed an effective interaction for the $0p$ shell with a G matrix corresponding to the space specified by $b_{\rm in} = 1.45$ fm for the $0s$ core orbit and a longer length parameter $b_{\rm out}$ for all the valence orbits (see Sec. II). The second-order core polarization contribution to the effective interaction turns out to be largely suppressed when $b_{\rm out}$ is sufficently larger than $b_{\rm in}$. We have also calculated some typical third-order diagrams and we have found that, in this situation, they are very small. This shows that the effective interaction can be derived in a very accurate way using the first- and second-order G-matrix diagrams. Similar suppression of core polarization effects was also observed in our TFSM calculation of electromagnetic observables.

By employing this effective interaction we have performed a shell-model study of nuclei with $6 \le A \le 9$. To start with, we have analyzed the dependence of the ground-state binding energies on the value of $b_{\rm out}$. It turned out that the binding energies for all the considered nuclei can be quite satisfactorily reproduced by using three values of $b_{\rm out}$. In particular, we have found that nuclei having about the same nuclear binding energy (relative to ^{4}He) per nucleon require the same value of $b_{\rm out}$. We have then focused attention on the spectra of Li isotopes and their mirror nuclei, which were calculated by using $b_{\rm out} = 2.0$ fm. A good overall agreement between theory and experiment is obtained, significant discrepancies existing only for the energies of resonant states with fairly large widths and for states with $T > T_z$. As regards the latter, we have shown that they can be better described by making use of a larger value of $b_{\rm out}$ (see dis-

cussion in Sec. III). Finally, the electromagnetic observables, calculated using bare operators, were compared with experiment. While the dipole moments and the $M1$ transition rates are in remarkably good agreement with the measured values, the experimental electric observables are all underestimated by our calculations. Note that the theoretical values may be brought into agreement with experiment by using an effective proton charge $e_p^{\rm eff} = 1.5e$.

To conclude, we have shown that most properties of the p-shell nuclei can be satisfactorily explained making use of a realistic effective interaction within the framework of the TFSM. As already mentioned in the Introduction, several $0\hbar\omega$ shell-model calculations have been performed for these nuclei since the mid 1960s, the most popular one being that of Cohen and Kurath [1]. For all the nuclei considered in the present paper, the agreement with experiment is overall better that that obtained in Ref. [1]. More gratifying, however, is the fact that our study yields results which are comparable to, and in same cases even better than, those obtained from large-basis shell-model calculations. In fact, on the one hand we have obtained an agreement with experiment which is quite similar to that of Ref. [10], where a complete $(0+2)\hbar\omega$ and an empirical effective interaction were used. On the other hand, our calculations give a more satisfactory description of the p-shell nuclei than that provided by the large-basis no-core shell-model calculations of Ref. [13], which make use of effective interactions derived from a modern NN potential. This indicates that in the TFSM approach most of the effects which are not explicitly taken into account in the model space are included in the effective interaction.

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[1] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).
[2] E. C. Halbert, Y. E. Kim, and T. T. S. Kuo, Phys. Lett. **20**, 657 (1966).
[3] P. S. Hauge and S. Maripuu, Phys. Rev. C **8**, 1609 (1973).
[4] L. D. Skouras and J. C. Varvitsiotis, Nucl. Phys. **A513**, 264 (1990).
[5] J. M. G. Gómez, J. C. Pérez Cerdán, and C. Prieto, Nucl. Phys. **A551**, 451 (1993).
[6] I. Tanihata *et al.*, Phys. Rev. Lett. **55**, 2676 (1985).
[7] I. Tanihata *et al.*, Phys. Lett. B **160**, 380 (1985).
[8] P. G. Hansen and B. Jonson, Europhys. Lett. **4**, 409 (1987).
[9] T. Hoshimo, H. Sagawa, and A. Arima, Nucl. Phys. **A506**, 271 (1990).
[10] A. A. Wolters, A. G. M. van Hees, and P. W. M. Glaudemans, Phys. Rev. C **42**, 2062 (1990).
[11] H. Nakada and T. Otsuka, Phys. Rev. C **49**, 886 (1994).
[12] S. Karataglidis, B. A. Brown, K. Amos, and P. J. Dortmans, Phys. Rev. C **55**, 2826 (1997).
[13] P. Navrátil and B. R. Barrett, Phys. Rev. C **57**, 3119 (1998), and references therein.
[14] V. I. Kukulin, V. N. Pomerantsev, Kh. D. Razikov, V. T. Voronchev, and G. G. Ryzhikh, Nucl. Phys. **A586**, 151 (1995), and references therein.
[15] I. J. Thompson, B. V. Danilin, V. D. Efros, J. S. Vaagen, J. M. Bang, and M. V. Zhukov, Phys. Rev. C **61**, 024318 (2000), and references therein.
[16] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. C **56**, 1720 (1997).
[17] R. B. Wiringa, S. C. Pieper, J. Carlson, and V. R. Pandharipande, Phys. Rev. C **62**, 014001 (2000).
[18] T. T. S. Kuo, H. Müther, and K. Amir Azimi-Nili, Nucl. Phys. **A606**, 15 (1996).
[19] T. T. S. Kuo, F. Krmpotić, and Y. Tzeng, Phys. Rev. Lett. **78**, 2708 (1997).
[20] L. R. B. Elton, *Nuclear Sizes*, (Oxford University Press, London, 1961).
[21] M. Bouten and M. C. Bouten, in *Shell Model and Nuclear Structure: where do we stand?*, Proceedings of the Second International Spring Seminar on Nuclear Structure, Capri, 1998, edited by A. Covello (World Scientific, Singapore, 1989), p. 581.
[22] T. T. S. Kuo and E. Osnes, *Lecture Notes in Physics*, Vol. 364 (Springer-Verlag, Berlin, 1990), p. 1.
[23] E. M. Krenciglowa, C. L. Kung, T. T. S. Kuo, and E. Osnes, Ann. Phys. (N.Y.) **101**, 154 (1976).
[24] H. Müther and P. Sauer, in *Computational Nuclear Physics 2*, ed. by K. Langanke, J.A. Maruhn and S. Koonin, Springer-Verlag 1992, p. 30.
[25] S. F. Tsai and T.T.S. Kuo, Phys. Lett. B **39**, 427 (1972).
[26] M. F. Jiang, R. Machleidt, D. B. Stout, and T. T. S. Kuo, Phys. Rev. C **46**, 910 (1992).
[27] M. Hjorth-Jensen, T. T. S. Kuo, and E. Osnes, Phys. Rep. **261**, 15 (1995).
[28] S. Y. Lee and K. Suzuki, Phys. Lett. B **91**, 173(1980).
[29] K. Suzuki and S. Y. Lee, Prog. Theor. Phys. **64**, 2091(1980).
[30] M. Hjorth-Jensen, H. Müther, E. Osnes, and A. Polls, J. Phys. G **22**, 321 (1996).
[31] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[32] T.T.S. Kuo, L. Coraggio, A. Covello, and A. Gargano, in *Highlights of Modern Nuclear Structure*, Proceedings of the Sixth International Spring Seminar on Nuclear Structure, S. Agata sui due Golfi, 1998, edited by A. Covello (World Scientific, Singapore, 1999), p. 105.
[33] A. Gargano, L. Coraggio, A. Covello, N. Itaco, and T. T. S. Kuo, in *Perspectives on Theoretical Nuclear Physics*, Proceedings of the VIII Convegno su Problemi di Fisica Nucleare Teorica, Cortona, 2000, edited by G. Pisent, S. Boffi, L. Canton, A. Covello, A. Fabrocini, and S. Rosati

(World Scientific, Singapore, 2001), in press.
[34] Data extracted from the ENSDF data base revision of 15-Feb-2001, using the NNDC Online Data Service.
[35] G. Audi and A.H. Wapstra, Nucl. Phys. **A595**, 409 (1995).
[36] B. A. Brown, A. Etchegoyen, and W. D. M. Rae, The computer code OXBASH, MSU-NSCL Report Number 524.
[37] A. A. Wolters, A. G. M. van Hees, and P. W. M. Glaudemans, Phys. Rev. C **42**, 2053 (1990).
[38] L. Coraggio, A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Phys. Rev. C **60**, 064306 (1999), and references therein.
[39] N. J. Stone, preprint, 2001. Information extracted from the NNDC Online Data Service.

FIG. 1. First- and second-order $\hat{Q}$-box diagrams.

FIG. 2. Dependence of the second-order core-polarization diagram G_{3p1h} on $b_{\rm out}$.

FIG. 3. Experimental and calculated levels of ^{6}Li.

FIG. 4. Experimental and calculated levels of ^{7}Li and ^{7}Be.

FIG. 5. Experimental and calculated levels of ^{8}Li and ^{8}B.

TABLE I. Experimental and calculated ground-state binding energies (MeV). See text for details.

^{A}Z	Expt	TFSM(1.45)	TFSM(1.75)	TFSM(2.00)	TFSM(2.25)	TFSM(2.50)
^{6}He	29.27	32.84	31.53	30.58	29.76	29.13
^{6}Be	26.92	30.48	29.17	28.22	27.40	26.67
^{6}Li	31.99	35.93	33.70	32.25	30.97	29.93
^{7}He	28.82	33.04	31.20	30.16	29.26	28.56
^{7}B	24.72	28.78	26.94	25.90	25.00	24.30
^{7}Li	39.24	46.57	41.78	38.65	35.95	33.83
^{7}Be	37.60	44.91	40.12	36.99	34.29	32.17
^{8}He	31.41	38.85	35.51	33.53	31.79	30.43
^{8}C	24.78	31.89	28.55	26.57	24.83	23.47
^{8}Li	41.28	51.04	44.80	40.96	37.71	35.16
^{8}B	37.74	47.48	41.24	37.40	34.15	31.60
^{8}Be	56.50	67.41	57.62	51.17	45.57	41.13
^{9}He	30.26	37.31	34.20	32.13	30.03	28.26
^{9}Li	45.34	57.57	49.97	45.27	41.15	37.89
^{9}C	39.03	51.31	43.71	39.01	34.89	31.63
^{9}Be	58.16	72.39	60.98	53.88	46.73	42.88
^{9}B	56.31	70.40	59.08	51.98	44.83	40.98

TABLE II. Experimental and calculated $B(E2)$ and $B(M1)$ values (W.u.), Q moments ($e$mb), and μ moments (nm) in $^{6-8}$Li and ^{8}B.

Nucleus	Quantity	TFSM	Expt.
^{6}Li	$B(E2; 3^+_1 \to 1^+_1)$	4.8	16.5 ± 1.3
	$B(E2; 2^+_1 \to 1^+_1)$	4.5	6.8 ± 3.5
	$Q(1^+_1)$	-0.60	-0.83 ± 0.08
	$\mu(1^+_1)$	+0.87	$+0.82 \pm 0.00$
^{7}Li	$B(E2; \frac{1}{2}^-_1 \to \frac{3}{2}^-_1)$	5.9	19.7 ± 1.2
	$B(E2; \frac{7}{2}^-_1 \to \frac{3}{2}^-_1)$	2.5	4.3
	$B(M1; \frac{1}{2}^-_1 \to \frac{3}{2}^-_1)$	2.50	2.75 ± 0.14
	$Q(\frac{3}{2}^-_1)$	-24.4	-40.0 ± 0.3
	$\mu(\frac{3}{2}^-_1)$	+3.81	$+3.26 \pm 0.00$
^{8}Li	$B(M1; 1^+_1 \to 2^+_1)$	2.7	2.8 ± 0.9
	$B(M1; 3^+_1 \to 2^+_1)$	0.21	0.29 ± 0.13
	$Q(2^+_1)$	+24	$+32.7 \pm 0.6$
	$\mu(2^+_1)$	+1.52	$+1.65 \pm 0.00$
^{8}B	$Q(2^+_1)$	+44	64.6 ± 1.5
	$\mu(2^+_1)$	+1.15	$+1.04 \pm 0.00$