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Research Article

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Robust dynamic output feedback fault-tolerant control for Takagi-Sugeno fuzzy systems with interval time-varying delay via improved delay partitioning approach

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Abstract: This paper addresses the problem of robust fault-tolerant control design scheme for a class of Takagi-Sugeno fuzzy systems subject to interval time-varying delay and external disturbances. First, by using improved delay partitioning approach, a novel n-steps iterative learning fault estimation observer under H_∞ constraint is constructed to achieve estimation of actuator fault. Then, based on the online estimation information, a fuzzy dynamic output feedback fault-tolerant controller considered interval time delay is designed to compensate for the impact of actuator faults, while guaranteeing that the closed-loop system is asymptotically stable with the prescribed H_∞ performance. Moreover, all the obtained less conservative sufficient conditions for the existence of fault estimation observer and fault-tolerant controller are formulated in terms of linear matrix inequalities. Finally, the numerical examples and simulation results are presented to show the effectiveness and merits of the proposed methods.

Keywords: Fault estimation, T-S fuzzy models, Fault-tolerant control, Actuator fault compensation, Disturbance rejection, Asymptotically stable, Time-varying delay, Linear matrix inequalities

MSC: 34D23, 37B25, 93D09

1 Introduction

In real world, most physical systems are nonlinear and many researchers have been seeking the effective approaches to control nonlinear systems. Among these, there are growing interests in Takagi-Sugeno (T-S) fuzzy model based control [1, 2]. It has been proved that T-S fuzzy models can be used to approximate a wider class of nonlinear system, which are realized by piecewise smoothly connecting a family of local linear models with fuzzy membership functions. This "blending" makes the subsystems of T-S fuzzy model similar to linear systems, and the fruitful results of linear system theories can be directly applied for the stability analysis and synthesis of nonlinear systems [3–7]. A great repercussion of T-S fuzzy models can be verified in many practical fuzzy model based control systems (see, for instance, [8–14] and references therein).

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Due to an increasing demand for higher performances, the random noises and uncertainties commonly exist in practical control systems [15], where various system components such as actuators and sensors may be subjected to unexpected failures during their operation. The effects caused by these failures require appropriate compensation to ensure the system reliability and safety. For this purpose, research into fault estimation (FE) and fault-tolerant control (FTC) for T-S fuzzy systems (TSFSs) has been carried out for many years [16–20]. Since measuring all of the internal states of physical systems may be difficult and costly, and only their outputs are available for control purpose, output feedback T-S fuzzy controllers were preferred. Especially, in [17], an unknown input observer with disturbance-decoupling ability is designed to perform actuator fault detection for discrete-time TSFSs. Bouarar et al. [19] study the problem of FTC by trajectory tracking for uncertain nonlinear system described by Takagi-Sugeno models, where the considered faults are constant, exponential or polynomial. In [20], a robust fault-tolerant controller based on static output-feedback controller design approach is developed to solve the problem of a robust fault estimation and FTC for vehicle lateral dynamics subject to external disturbance and unknown sensor faults. However, the robust FTC based on dynamic output feedback for T-S fuzzy systems has not been studied adequately.

On the other hand, it is well known that time delays are frequently encountered in various engineering and communication systems, and a time delay in dynamical system is often a primary source of instability and performance degradation. Therefore, it is important to develop FE and FTC methods for TSFSs with time delays [21–27]. More recently, in literature [21], a fuzzy descriptor learning observer is constructed to achieve simultaneous reconstruction of system states and actuator faults for T-S fuzzy descriptor systems with time delays. By constructing a virtual tracking model, [22] deals with the output tracking control problem for a class of continuous-time markovian jump systems with time-varying delay and actuator faults. Based on the $(k-1)$th fault estimation information, a k-step fault estimation observer is proposed to estimate the actuator fault of time delay TSFSs in [23]. By utilizing the input delay approach, literature [24] deals with the robust fault tolerant controller design of networked control systems with state delay and stochastic actuator failures. In [25], based on a multiple Lyapunov functions and the slack variables, fault tolerant saturated control problem for discrete-time T-S fuzzy systems with delay is studied. In [27], the adaptive fault estimation problem is studied for a class of T-S fuzzy stochastic markovian jumping systems with time delays and nonlinear parameters. It should be pointed out that the type of time delay considered in all the aforementioned works is constant $\tau(t) = \tau$ or $0 < \tau(t) < \tau$, the lower bound of delay is restricted to 0, which is not more general. The interval time-varying delay, $0 < \tau_1 \leq \tau(t) \leq \tau_2$, has been identified from many practical systems, especially the networked control systems. However, the delay-dependent fault estimation conditions proposed in [21–27] fail to give a feasible solution. Therefore, based on the above analysis and discussion, by using dynamic output feedback control scheme to solve the problem of robust FTC for TSFSs with actuator faults and interval time-varying delays is a meaningful research and motivates our study.

The aim of this paper is to develop a fault-tolerant controller design scheme for a class of TSFSs subject to interval time delays and external disturbances. The basic idea is to construct a n-steps augmented system by taking the fault as auxiliary disturbance vector, and design a fault estimation observer based on improved delay partitioning approach. Then, utilizing the online fault estimation information, a fuzzy dynamic output feedback fault-tolerant controller is designed to compensate for the impact of actuator faults. The main contribution of this paper lies in the following aspects.

1. By using improved delay partitioning approach, a novel n-steps iterative learning fuzzy fault estimation observer under H_∞ performance constraint is constructed to achieve the estimation of actuator faults, and the less conservative sufficient conditions for the existence of observer are explicitly provided.
2. A new type of fuzzy dynamic output feedback fault-tolerant controller considered interval time delay is designed to guarantee that the closed-loop system is asymptotically stable with the prescribed H_∞ performance.
3. All obtained sufficient conditions for the existence of observer and controller are formulated in terms of strictly LMIs. Compared with the existing results, the proposed design schemes are with less conservative and wider application range, simulation examples demonstrate the effectiveness of the proposed approaches.

The rest of this paper is organized as follows. The system description and problem formulations are presented in Section 2. Section 3 presents the main results on robust fault estimation observer and fault-tolerant controller design scheme. In Section 4, simulation results of numerical example are presented to demonstrate the effectiveness and merits of the proposed methods. Finally, Section 5 concludes the paper.

Notations: Throughout the paper, R^n denotes the n-dimensional real Euclidean space; I denotes the identity matrix; the superscripts "T" and "-1" stand for the matrix transpose and inverse, respectively; notation $X > 0 (X \geq 0)$ means that matrix X is real symmetric positive definite (positive semi-definite); $\| \cdot \|$ is the spectral norm. If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. The symbol " $*$ " stands for matrix block induced by symmetry.

2 Problem formulation

Consider a nonlinear system which can be represented by the following extended T-S fuzzy time-delay model with exogenous disturbance and actuator faults simultaneously.
Plant rule i: **IF** $\xi_1(t)$ is M_{i1} and ... and $\xi_p(t)$ is M_{ip} **THEN**

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{\tau i} x(t-\tau(t)) + B_i(u(t) + f(t)) + B_{di} d(t) \\ y(t) = C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_{di} d(t) \\ x(t) = \phi_i(t), \forall t \in [-\tau_2, 0],\ i = 1, 2, \ldots, r. \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in \mathbb{R}^q$ denotes the input vector, $y(t) \in \mathbb{R}^l$ stands for the system output vector. $d(t) \in \mathbb{R}^m$ is the exogenous disturbance input that belongs to $L_2[0, \infty)$, $f(t) \in \mathbb{R}^q$ represents the possible actuator fault. A_i, $A_{\tau i}$, B_i, B_{di}, C_i, $C_{\tau i}$ and D_{di} are constant real matrices of appropriate dimensions. It is assumed that the pairs (A_i, B_i) are controllable, and the pairs (A_i, C_i) are observable, where $i = 1, 2, \ldots, r$. $\xi_1(t), \ldots, \xi_p(t)$ are the premise variables, $M_{ij} (i = 1, 2, \ldots, r,\ j = 1, 2, \ldots, p)$ are fuzzy sets, $\phi_i(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau_2, 0]$. In this paper, it is also assumed that the premise variables do not depend on the input variables $u(t)$, $\tau(t)$ is the time-varying delay and satisfies

$$0 < \tau_1 \leq \tau(t) \leq \tau_2, \quad 0 < \dot{\tau}(t) \leq d \tag{2}$$

where τ_1 and τ_2 are lower and upper bounds of state delay $\tau(t)$, respectively.

Through the use of fuzzy blending, the fuzzy system (1) can be inferred as follows:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_\tau(t)x(t-\tau(t)) + B(t)(u(t) + f(t)) + B_d(t)d(t) \\ y(t) = C(t)x(t) + C_\tau(t)x(t-\tau(t)) + D_d(t)d(t) \\ x(t) = \phi(t),\ t \in [-\tau_2, 0] \end{cases} \tag{3}$$

where

$$A(t) = \sum_{i=1}^{r} \mu_i(\xi(t))A_i, A_\tau(t) = \sum_{i=1}^{r} \mu_i(\xi(t))A_{\tau i}, B(t) = \sum_{i=1}^{r} \mu_i(\xi(t))B_i, D_d(t) = \sum_{i=1}^{r} \mu_i(\xi(t))D_{di}$$

$$C(t) = \sum_{i=1}^{r} \mu_i(\xi(t))C_i, C_\tau(t) = \sum_{i=1}^{r} \mu_i(\xi(t))C_{\tau i}, B_d(t) = \sum_{i=1}^{r} \mu_i(\xi(t))B_{di}, \phi(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\phi_i(t)$$

Fuzzy basis functions are given by $\mu_i(\xi(t)) = \beta_i(\xi(t)) / \sum_{j=1}^{r} \beta_j(\xi(t))$, $\beta_i(\xi(t)) = \prod_{i=1}^{p} M_{ij}(\xi(t))$ where $M_{ij}(\xi_j(t))$ represents the grade of membership of $\xi_j(t)$ in M_{ij}. It is easy to find that

$$\beta_i(\xi(t)) \geq 0, \sum_{j=1}^{r} \beta_j(\xi(t)) > 0\ \forall t,\ \mu_i(\xi(t)) \geq 0, \sum_{j=1}^{r} \mu_j(\xi(t)) = 1\ \forall t$$

Before proceeding further, we will introduce some lemmas to be needed in the development of main results throughout this paper.

Lemma 2.1 ([28]). *For any positive semi-definite matrices* $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$, *the following integral inequality holds:*

$$-\int_{t-\tau(t)}^{t} \dot{x}^T(s)X_{33}\dot{x}(s)ds \leq \int_{t-\tau(t)}^{t} [x^T(t)\ x^T(t-\tau(t))\ \dot{x}^T(s)] \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ \dot{x}(s) \end{bmatrix} ds$$

Lemma 2.2 ([29]). *For any constant matrix* $X \in \mathbb{R}^{n\times n}$, $X = X^T > 0$, *scalar* $r > 0$, *and vector function* $\dot{x} : [-r, 0] \rightarrow \mathbb{R}^n$ *such that the following integration is well defined, then*

$$-r\int_{-r}^{0} \dot{x}^T(t+s)X\dot{x}(t+s)ds \leq [x^T(t)\ x^T(t-r)] \begin{bmatrix} -X & X \\ X & -X \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-r) \end{bmatrix}$$

Remark 2.3. *For T-S nonlinear system description* (3)*, we can see that a general system is considered in this paper, including possible state time delay, actuator fault and exogenous disturbance input simultaneously. If there is no state delay, then* (3) *reduces to the existing one in [30]. Moreover, the lower bound of delay is not restricted to 0 as [23], which is even more applicable to networked control systems and other practical systems.*

3 Main results

3.1 Actuator fault estimation

In order to estimate system faults, the n-steps iterative learning fault estimation observer is constructed as follows:

$$\begin{cases} \dot{\hat{x}}_n(t) = A(t)\hat{x}_n(t) + A_\tau(t)\hat{x}_n(t-\tau(t)) + B(t)(u(t) + \hat{f}_n(t)) - L_n(t)(\hat{y}_n(t) - y(t)) \\ \hat{y}_n(t) = C(t)\hat{x}_n(t) + C_\tau(t)\hat{x}_n(t-\tau(t)) \\ \dot{\hat{f}}_n(t) = -F_n(t)(\hat{y}_n(t) - y(t)) + \dot{\hat{f}}_{n-1}(t), \quad n = 1, 2, ..., N \end{cases} \tag{4}$$

where $\hat{x}_n(t) \in \mathbb{R}^n$ is the n-steps observer state, $\hat{y}_n(t) \in \mathbb{R}^l$ is the observer output, and $\hat{f}_n(t) \in \mathbb{R}^q$ is the nth step estimate of fault $f(t)$, $n = 1, 2, ..., N$ is the number of fault estimation steps. Then, the objective of estimating actuator fault by observer (4) is to design the appropriate dimension gain matrices $L_n(t) \in \mathbb{R}^{n\times l}$, $F_n(t) \in \mathbb{R}^{q\times l}$ in the presence of disturbance and state time delay, where $L_n(t) = \sum_{i=1}^{r} \mu_i(\xi(t))L_{ni}$, $F_n(t) = \sum_{i=1}^{r} \mu_i(\xi(t))F_{ni}$.

Let us define $e_{xn}(t) = \hat{x}_n(t) - x(t)$, $e_{yn}(t) = \hat{y}_n(t) - y(t)$, $e_{fn}(t) = \hat{f}_n(t) - f(t)$, and $e_n^T(t) = [e_{xn}^T(t), e_{fn}^T(t)]$, $\omega_n^T(t) = [d^T(t), \dot{f}^T(t) - \dot{\hat{f}}_{n-1}^T(t)]$, then the error dynamic systems is deduced from (3) and (4) as follows:

$$\begin{cases} \dot{e}_n(t) = [\bar{A}(t) - \bar{L}_n(t)\bar{C}(t)]e_n(t) + [\bar{A}_\tau(t) - \bar{L}_n(t)\bar{C}_\tau(t)]e_n(t-\tau(t)) + [\bar{L}_n(t)\bar{D}_d(t) - \bar{B}_d(t)]\omega_n(t) \\ e_{yn}(t) = \bar{C}(t)e_n(t) + \bar{C}_\tau(t)e_n(t-\tau(t)) - \bar{D}_d(t)\omega_n(t) \end{cases} \tag{5}$$

where

$$\bar{A}(t) = \begin{bmatrix} A(t) & B(t) \\ 0 & 0 \end{bmatrix},\ \bar{A}_\tau(t) = \begin{bmatrix} A_\tau(t) & 0 \\ 0 & 0 \end{bmatrix}, \bar{B}_d(t) = \begin{bmatrix} B_d(t) & 0 \\ 0 & I_q \end{bmatrix}, \bar{L}_n(t) = \begin{bmatrix} L_n(t) \\ F_n(t) \end{bmatrix}$$

$$\bar{C}(t) = \begin{bmatrix} C(t) & 0 \end{bmatrix}, \bar{C}_\tau(t) = \begin{bmatrix} C_\tau(t) & 0 \end{bmatrix}, \bar{D}_d(t) = \begin{bmatrix} D_d(t) & 0 \end{bmatrix}$$

For simplicity, we introduce the following vectors:

$$\zeta_1^T(t) = [e_n^T(t)\ e_n^T(t-h)\ \cdots\ e_n^T(t-Nh)\ e_n^T(t-\tau_1-\rho\delta)\ e_n^T(t-\tau(t))\ e_n^T(t-\tau_2)\ \ \omega_n^T(t)]$$

$$\Gamma_1(t) = [\bar{A}(t) - \bar{L}(t)\bar{C}(t)\ \ 0\ \ \cdots\ \ 0\ \ 0\ \ \bar{A}_\tau(t) - \bar{L}(t)\bar{C}_\tau(t)\ \ 0\ \ \bar{L}(t)\bar{D}_d(t) - \bar{B}_d(t)]$$

where $h = \tau_1/n (n = 1, 2, ..., N)$ is the length of each division, N is the number (a positive integer) of divisions of the interval $[-\tau_1, 0]$ and is also the number of fault estimation steps in (4). The delay interval $[\tau_1, \tau_2]$ is divided into two subintervals with an unequal width as $[\tau_1, \tau_1 + \rho\delta]$ and $[\tau_1 + \rho\delta, \tau_2]$, where $\delta = \tau_2 - \tau_1$, $0 < \rho < 1$. Then, the state of error dynamics (5) can be rewritten as $\dot{e}_n(t) = \Gamma_1(t)\zeta_1(t)$. Therefore, the H_∞ fault estimation observer design problem to be addressed in this paper can be formulated as follows:

(i) The error dynamic system (5) with $\omega_n(t) = 0$ is asymptotically stable for any time-delay satisfying (2) when $n = 1, 2, ..., N$;

(ii) For a given scalar γ_n, the following H_∞ performance is satisfied:

$$\int_0^L \|e_{fn}(t)\|^2 dt \le \gamma_n^2 \int_0^L \|\omega_n(t)\|^2 dt, \quad n = 1, 2, ..., N \tag{6}$$

for all $L > 0$ and $\omega_n(t) \in L_2[0, \infty)$ under zero initial conditions.

Theorem 3.1. *For the given scalars τ_1, τ_2, η, γ_n and $0 < \rho < 1$, the error dynamic system (5) is asymptotically stable with $\omega_n(t) = 0$ while satisfying a prescribed H_∞ performance (6), if there exist matrices $P > 0$, $Q_n > 0$, $W_n > 0 (n = 1, 2, ..., N)$, $S_1 > 0$, $S_2 > 0$, $S_3 > 0$, $R_1 > 0$, $R_2 > 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} \ge 0$, $Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12}^T & Z_{22} & Z_{23} \\ Z_{13}^T & Z_{23}^T & Z_{33} \end{bmatrix} \ge 0$ and Y_{ni} $(i = 1, 2, \ldots, r)$, such that the following inequalities hold*

$$\Pi_{ii} < 0 \qquad i = 1, 2, \ldots, r \tag{7}$$

$$\Pi_{ij} + \Pi_{ji} \le 0 \quad 1 \le i < j \le r \tag{8}$$

and

$$R_1 - Y_{33} \ge 0, \quad R_2 - Z_{33} \ge 0 \tag{9}$$

where

$$\Pi_{ij} = \begin{bmatrix} \begin{pmatrix} \Phi_{ij}^1 & \Phi_{ij}^2 \\ * & \Phi_{ij}^3 \end{pmatrix} & \bar{\Gamma}_{1ij} \\ * & -2\eta P + \eta^2(\sum_{n=1}^N h^2 W_n + \rho\delta R_1 + (1-\rho)\delta R_2) \end{bmatrix}$$

with

$$\Phi_{ij}^1 = \begin{bmatrix} \Phi_{11}^{ij} & W_1 & \cdots & 0 \\ * & \Phi_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Phi_{nn} \end{bmatrix}, \Phi_{ij}^2 = \begin{bmatrix} 0 & 0 & \Phi_{1(N+3)}^{ij} & 0 & \Phi_{1(N+5)}^{ij} \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ W_N & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_{ij}^3 = \begin{bmatrix} \Phi_{(N+1)(N+1)} & \Phi_{(N+1)(N+2)} & \Phi_{(N+1)(N+3)} & 0 & 0 \\ * & \Phi_{(N+2)(N+2)} & \Phi_{(N+2)(N+3)} & \Phi_{(N+2)(N+4)} & 0 \\ * & * & \Phi_{(N+3)(N+3)} & \Phi_{(N+3)(N+4)} & 0 \\ * & * & * & \Phi_{(N+4)(N+4)} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\begin{aligned} &\Phi_{11}^{ij} = (P\bar{A}_i - Y_{ni}\bar{C}_j) + (P\bar{A}_i - Y_{ni}\bar{C}_j)^T + Q_1 - W_1 + \bar{I}_q\bar{I}_q^T \\ &\Phi_{1(N+3)}^{ij} = P\bar{A}_{\tau i} - Y_{ni}\bar{C}_{\tau j}, \quad \Phi_{1(N+5)}^{ij} = Y_{ni}\bar{D}_{dj} - P\bar{B}_{di} \\ &\Phi_{nn} = -Q_{n-1} - W_{n-1} + Q_n - W_n,\ n = 2, 3, ..., N,\ \bar{I}_q^T = [0 \quad I_q] \\ &\Phi_{(N+1)(N+1)} = -Q_N - W_N + S_1 + S_2 + \rho\delta Y_{11} + Y_{13} + Y_{13}^T \\ &\Phi_{(N+2)(N+2)} = S_3 - S_2 + \rho\delta Y_{22} - Y_{23} - Y_{23}^T + (1-\rho)\delta Z_{11} + Z_{13} + Z_{13}^T \end{aligned} \tag{10}$$

$$\begin{aligned}
&\Phi_{(N+4)(N+4)} = -S_3 + (1-\rho)\delta Z_{22} - Z_{23} - Z_{23}^T \\
&\bar{\Gamma}_{1ij} = [(P\bar{A}_i - Y_{ni}\bar{C}_j) \;\; 0 \;\; \cdots \;\; 0 \;\; 0 \;\; (P\bar{A}_{\tau i} - Y_{ni}\bar{C}_{\tau j}) \;\; 0 \;\; (Y_{ni}\bar{D}_{dj} - P\bar{B}_{di})]
\end{aligned} \tag{11}$$

Case 1: when $\tau_1 \le \tau(t) \le \tau_1 + \rho\delta$

$$\begin{aligned}
&\Phi_{(N+1)(N+3)} = \rho\delta Y_{12} - Y_{13} + Y_{23}^T, \\
&\Phi_{(N+2)(N+3)} = \rho\delta Y_{12}^T - Y_{13}^T + Y_{23} \\
&\Phi_{(N+2)(N+4)} = (1-\rho)\delta Z_{12} - Z_{13} + Z_{23}^T, \\
&\Phi_{(N+3)(N+4)} = \Phi_{(N+1)(N+2)} = 0 \\
&\Phi_{(N+3)(N+3)} = -(1-d)S_1 + \rho\delta Y_{11} + Y_{13} + Y_{13}^T + \rho\delta Y_{22} - Y_{23} - Y_{23}^T,
\end{aligned} \tag{12}$$

Case 2: when $\tau_1 + \rho\delta \le \tau(t) \le \tau_2$

$$\begin{aligned}
&\Phi_{(N+1)(N+2)} = \rho\delta Y_{12} - Y_{13} + Y_{23}^T, \\
&\Phi_{(N+2)(N+4)} = \Phi_{(N+1)(N+3)} = 0 \\
&\Phi_{(N+2)(N+3)} = \Phi_{(N+3)(N+4)} = (1-\rho)\delta Z_{12} - Z_{13} + Z_{23}^T \\
&\Phi_{(N+3)(N+3)} = -(1-d)S_1 + (1-\rho)\delta(Z_{11} + Z_{22}) + Z_{13} + Z_{13}^T - Z_{23} - Z_{23}^T,
\end{aligned} \tag{13}$$

Then the observer gain matrices can be obtained as follows:

$$\bar{L}_{ni} = \begin{bmatrix} L_{ni} \\ F_{ni} \end{bmatrix} = P^{-1}Y_{ni} \quad (n = 1, 2, ..., N)$$

Proof. The following novel Lyapunov-Krasovskii functional candidate is constructed to prove system (5) is asymptotically stable with H_∞ performance.

$$V(x_t, t) = V_1(x_t, t) + V_2(x_t, t) + V_3(x_t, t), \tag{14}$$

where

$$\begin{aligned}
V_1(x_t, t) =& e_n^T(t)Pe_n(t), \\
V_2(x_t, t) =& \sum_{n=1}^{N} \int_{t-nh}^{t-(n-1)h} e_n^T(s)Q_n e_n(s)ds + \int_{t-\tau(t)}^{t-\tau_1} e_n^T(s)S_1 e_n(s)ds \\
&+ \int_{t-\tau_1-\rho\delta}^{t-\tau_1} e_n^T(s)S_2 e_n(s)ds + \int_{t-\tau_2}^{t-\tau_1-\rho\delta} e_n^T(s)S_3 e_n(s)ds, \\
V_3(x_t, t) =& \sum_{n=1}^{N} \int_{-nh}^{-(n-1)h} \int_{t+\theta}^{t} \dot{e}_n^T(s)hW_n\dot{e}_n(s)dsd\theta + \int_{-\tau_1-\rho\delta}^{-\tau_1} \int_{t+\theta}^{t} \dot{e}_n^T(s)R_1\dot{e}_n(s)dsd\theta \\
&+ \int_{-\tau_2}^{-\tau_1-\rho\delta} \int_{t+\theta}^{t} \dot{e}_n^T(s)R_2\dot{e}_n(s)dsd\theta,
\end{aligned}$$

where the unknown matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $S_3 > 0$, $R_1 > 0$, $R_2 > 0$, $Q_n > 0$ and $W_n > 0 (n = 1, 2, ..., N)$ are to be determined.

Then, the time derivatives of $V(x_t,t)$ along the trajectories of the argument systems (5) satisfy

$$\begin{aligned}
\dot V_1(x_t,t) =& e_n^T(t)[P(\bar A(t)-\bar L_n(t)\bar C(t))+(\bar A(t)-\bar L_n(t)\bar C(t))^T P]e_n(t)+2e_n^T(t)P(\bar A_\tau(t)\\
&-\bar L_n(t)\bar C_\tau(t))e_n(t-\tau(t))+2e_n^T(t)P(\bar L_n(t)\bar D_d(t)-\bar B_d(t))\omega_n(t)\\
\dot V_2(x_t,t) =& \sum_{n=1}^{N} e_n^T(t-(n-1)h)Q_n e_n(t-(n-1)h)-\sum_{n=1}^{N} e_n^T(t-nh)Q_n e_n(t-nh)+e_n^T(t-\tau_1)S_1 e_n(t-\tau_1)\\
&-(1-\dot\tau(t))e_n^T(t-\tau(t))S_1 e_n(t-\tau(t))+e_n^T(t-\tau_1)S_2 e_n(t-\tau_1)-e_n^T(t-\tau_1-\rho\delta)S_2 e_n(t-\tau_1-\rho\delta)\\
&+e_n^T(t-\tau_1-\rho\delta)S_3 e_n(t-\tau_1-\rho\delta)-e_n^T(t-\tau_2)S_3 e_n(t-\tau_2)\\
\dot V_3(x_t,t) =& \sum_{n=1}^{N} \dot e_n^T(t)h^2 W_n \dot e_n(t)-\sum_{n=1}^{N}\int_{t-nh}^{t-(n-1)h} \dot e_n^T(s)hW_n\dot e_n(s)ds+\dot e_n^T(t)\rho\delta R_1\dot e_n(t)\\
&-\int_{t-\tau_1-\rho\delta}^{t-\tau_1}\dot e_n^T(s)R_1\dot e_n(s)ds+\dot e_n^T(t)(1-\rho)\delta R_2\dot e_n(t)-\int_{t-\tau_2}^{t-\tau_1-\rho\delta}\dot e_n^T(s)R_2\dot e_n(s)ds\\
=& \dot e_n^T(t)(\sum_{n=1}^{N} h^2W_n+\rho\delta R_1+(1-\rho)\delta R_2)\dot e_n(t)\\
&-\sum_{n=1}^{N}\int_{t-nh}^{t-(n-1)h}\dot e_n^T(s)hW_n\dot e_n(s)ds-\int_{t-\tau_1-\rho\delta}^{t-\tau_1}\dot e_n^T(s)(R_1-Y_{33})\dot e_n(s)ds\\
&-\int_{t-\tau_1-\rho\delta}^{t-\tau_1}\dot e_n^T(s)Y_{33}\dot e_n(s)ds-\int_{t-\tau_2}^{t-\tau_1-\rho\delta}\dot e_n^T(s)(R_2-Z_{33})\dot e_n(s)ds\\
&-\int_{t-\tau_2}^{t-\tau_1-\rho\delta}\dot e_n^T(s)Z_{33}\dot e_n(s)ds
\end{aligned} \tag{15}$$

Case 1. When $\tau_1 \le \tau(t) \le \tau_1+\rho\delta$, the following equations are true:

$$\begin{aligned}
&-\int_{t-\tau_1-\rho\delta}^{t-\tau_1}\dot e_n^T(s)Y_{33}\dot e_n(s)ds-\int_{t-\tau_2}^{t-\tau_1-\rho\delta}\dot e_n^T(s)Z_{33}\dot e_n(s)ds\\
=&-\int_{t-\tau_1-\rho\delta}^{t-\tau(t)}\dot e_n^T(s)Y_{33}\dot e_n(s)ds-\int_{t-\tau(t)}^{t-\tau_1}\dot e_n^T(s)Y_{33}\dot e_n(s)ds-\int_{t-\tau_2}^{t-\tau_1-\rho\delta}\dot e_n^T(s)Z_{33}\dot e_n(s)ds
\end{aligned} \tag{16}$$

Using Lemma 2.1 and the Leibniz-Newton formula, we have

$$\begin{aligned}
&-\int_{t-\tau_1-\rho\delta}^{t-\tau(t)}\dot e_n^T(s)Y_{33}\dot e_n(s)ds\\
\le&\int_{t-\tau_1-\rho\delta}^{t-\tau(t)}[e_n^T(t-\tau(t))\ e_n^T(t-\tau_1-\rho\delta)\ \dot e_n^T(s)]\begin{bmatrix} Y_{11} & Y_{12} & Y_{13}\\ Y_{12}^T & Y_{22} & Y_{23}\\ Y_{13}^T & Y_{23}^T & 0\end{bmatrix}\begin{bmatrix} e_n(t-\tau(t))\\ e_n(t-\tau_1-\rho\delta)\\ \dot e_n(s)\end{bmatrix}ds\\
\le& e_n^T(t-\tau(t))[\rho\delta Y_{11}+Y_{13}+Y_{13}^T]e_n(t-\tau(t))+2e_n^T(t-\tau(t))[\rho\delta Y_{12}-Y_{13}+Y_{23}^T]\\
&\times e_n(t-\tau_1-\rho\delta)+e_n^T(t-\tau_1-\rho\delta)[\rho\delta Y_{22}-Y_{23}-Y_{23}^T]e_n(t-\tau_1-\rho\delta)
\end{aligned} \tag{17}$$

Similarly, we obtain

$$
\begin{aligned}
&-\int_{t-\tau(t)}^{t-\tau_1} \dot{e}_n^T(s)Y_{33}\dot{e}_n(s)ds \\
\leq & e_n^T(t-\tau_1)[\rho\delta Y_{11}+Y_{13}+Y_{13}^T]e_n(t-\tau_1)+2e_n^T(t-\tau_1)[\rho\delta Y_{12}-Y_{13}+Y_{23}^T] \\
&\times e_n(t-\tau(t))+e_n^T(t-\tau(t))[\rho\delta Y_{22}-Y_{23}-Y_{23}^T]e_n(t-\tau(t))-\int_{t-\tau_2}^{t-\tau_1-\rho\delta} \dot{e}_n^T(s)Z_{33}\dot{e}_n(s)ds \\
\leq & e_n^T(t-\tau_1-\rho\delta)[(1-\rho)\delta Z_{11}+Z_{13}+Z_{13}^T]e_n(t-\tau_1-\rho\delta)+2e_n^T(t-\tau_1-\rho\delta) \\
&\times[(1-\rho)\delta Z_{12}-Z_{13}+Z_{23}^T]e_n(t-\tau_2)+e_n^T(t-\tau_2)[(1-\rho)\delta Z_{22}-Z_{23}-Z_{23}^T]e_n(t-\tau_2)
\end{aligned} \tag{18}
$$

Substituting (16)-(18) into (15), a straightforward computation gives

$$
\begin{aligned}
&\dot{V}(t)+e_{fn}^T(t)e_{fn}(t)-\gamma_n^2\omega_n^T(t)\omega_n(t)=\dot{V}(t)+e_n^T(t)\bar{I}_q\bar{I}_q^T e_n(t)-\gamma_n^2\omega_n^T(t)\omega_n(t) \\
\leq & \zeta_1^T(t)\Big(\Phi(t)+\Gamma_1^T(t)(\sum_{n=1}^{N} h^2W_n+\rho\delta R_1+(1-\rho)\delta R_2)\Gamma_1(t)\Big)\zeta_1(t) \\
&-\int_{t-\tau_1-\rho\delta}^{t-\tau_1} \dot{e}_n^T(s)(R_1-Y_{33})\dot{e}_n(s)ds-\int_{t-\tau_2}^{t-\tau_1-\rho\delta} \dot{e}_n^T(s)(R_2-Z_{33})\dot{e}_n(s)ds
\end{aligned} \tag{19}
$$

When $R_1-Y_{33}\geq 0$, $R_2-Z_{33}\geq 0$, and $\tau_1\leq\tau(t)\leq\tau_1+\rho\delta$, the last two terms in (19) are all less than 0. Then, for any scalar $\eta>0$, it follows from the fact $-PR^{-1}P\leq -2\eta P+\eta^2 R$ and Schur complement theorem, we can see if the following inequalities hold

$$
\Xi_1(t)=\begin{bmatrix}\Phi(t) & \Gamma_1^T(t)P \\ * & -2\eta P+\eta^2(\sum_{n=1}^{N} h^2W_n+\rho\delta R_2+(1-\rho)\delta R_3)\end{bmatrix}<0 \tag{20}
$$

one has $\dot{V}(t)+e_{fn}^T(t)e_{fn}(t)-\gamma_n^2\omega_n^T(t)\omega_n(t)<0$. By noticing $V(L)\geq 0$ and $V(0)=0$ under zero initial conditions, we can conclude that (6) holds for all $L>0$ and any nonzero $\omega_n(t)\in L_2[0,\infty)$. Hence, with the changes of variables as $Y_n(t)=P\bar{L}_n(t)$, we have

$$
\Xi_1(t)=\sum_{i=1}^{r}\mu_i^2(\xi(t))\Pi_{ii}+\sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i(\xi(t))\mu_j(\xi(t))(\Pi_{ij}+\Pi_{ji})<0
$$

which imply that the error dynamics (5) satisfies the prescribed H_∞ performance (6).

In addition, by choosing the same Lyapunov function as (14) and following the similar line in the earlier deduction under conditions (7)-(9), we can easily obtain that the time derivative of $V(x_t,t)$ along the solution of error dynamics (5) with $\omega_n(t)=0$ satisfies $\dot{V}(t)<0$, which indicates the asymptotic stability of systems (5).

Case 2. When $\tau_1+\rho\delta\leq\tau(t)\leq\tau_2$, the following equations are true:

$$
\begin{aligned}
&-\int_{t-\tau_1-\rho\delta}^{t-\tau_1} \dot{e}_n^T(s)Y_{33}\dot{e}_n(s)ds-\int_{t-\tau_2}^{t-\tau_1-\rho\delta} \dot{e}_n^T(s)Z_{33}\dot{e}_n(s)ds \\
=&-\int_{t-\tau_1-\rho\delta}^{t-\tau_1} \dot{e}_n^T(s)Y_{33}\dot{e}_n(s)ds-\int_{t-\tau_2}^{t-\tau(t)} \dot{e}_n^T(s)Z_{33}\dot{e}_n(s)ds-\int_{t-\tau(t)}^{t-\tau_1-\rho\delta} \dot{e}_n^T(s)Z_{33}\dot{e}_n(s)ds
\end{aligned}
$$

The proof can be completed in a similar formulation to Case 1 and is omitted here for simplification. This completes the proof of Theorem 3.1. □

In order to compare our results with the existing ones, based on the improved delay-decomposing approach of Theorem 3.1, we suggest to develop a delay-dependent stability condition for the nominal unforced fuzzy system of (3), which can be written as

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_\tau(t)x(t-\tau(t)) + B_d(t)d(t) \\ y(t) = C(t)x(t) + C_\tau(t)x(t-\tau(t)) + D_d(t)d(t) \\ x(t) = \phi(t),\ t \in [-\tau_2, 0] \end{cases} \tag{21}$$

Corollary 3.2. *For the given scalars τ_1, τ_2, d and $0 < \rho < 1$, the system* (21) *with $d(t) = 0$ is asymptotically stable for any time-varying delay $\tau(t)$ satisfying* (2), *if there exist matrices $P > 0$, $Q_n > 0$, $W_n > 0 (n = 1, 2, ..., N)$, $S_1 > 0$, $S_2 > 0$, $S_3 > 0$, $R_1 > 0$, $R_2 > 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} \geq 0$, $Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12}^T & Z_{22} & Z_{23} \\ Z_{13}^T & Z_{23}^T & Z_{33} \end{bmatrix} \geq 0$, such that the following set of inequalities hold:*

$$\Theta_i = \begin{bmatrix} \Theta_1^i & \Theta_2^i \\ * & \Theta_3^i \end{bmatrix} < 0$$

and

$$R_1 - Y_{33} \geq 0, \quad R_2 - Z_{33} \geq 0$$

where

$$\Theta_1^i = \begin{bmatrix} \Theta_{11}^i & W_1 & \cdots & 0 \\ * & \Phi_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Phi_{nn} \end{bmatrix}, \ \Theta_i = \begin{bmatrix} 0 & 0 & \Theta_{1(N+3)}^i & 0 & \Theta_{1(N+5)}^i \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ W_N & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Theta_i = \begin{bmatrix} \Phi_{(N+1)(N+1)} & \Phi_{(N+1)(N+2)} & \Phi_{(N+1)(N+3)} & 0 & 0 \\ * & \Phi_{(N+2)(N+2)} & \Phi_{(N+2)(N+3)} & \Phi_{(N+2)(N+4)} & 0 \\ * & * & \Phi_{(N+3)(N+3)} & \Phi_{(N+3)(N+4)} & \Theta_{(N+3)(N+5)}^i \\ * & * & * & \Phi_{(N+4)(N+4)} & 0 \\ * & * & * & * & \Theta_{(N+5)(N+5)} \end{bmatrix}$$

$$\Theta_{11}^i = PA_i + A_i^T P + Q_1 - W_1,\ \Theta_{1(N+3)}^i = PA_{\tau i},\ \Theta_{1(N+5)}^i = A_i^T (\sum_{n=1}^{N} h^2 W_n + \rho\delta R_1 + (1-\rho)\delta R_2)$$

$$\Theta_{(N+3)(N+5)}^i = A_{\tau i}^T (\sum_{n=1}^{N} h^2 W_n + \rho\delta R_1 + (1-\rho)\delta R_2),\ \Theta_{(N+5)(N+5)} = -(\sum_{n=1}^{N} h^2 W_n + \rho\delta R_1 + (1-\rho)\delta R_2)$$

and other elements of the matrix Θ_i in Case 1 and 2 are defined in (10)-(13).

Remark 3.3. *To constrain the effect of input disturbance from $\dot{f}(t)$, different from the existing fault estimation observer design result in [14, 21, 26, 27], [30, 31], the information of $f_{n-1}(t)$ is considered in the design scheme of fault estimation observer* (4), *in which $\dot{f}(t)$ is converted to $\dot{f}(t) - \dot{\hat{f}}_{n-1}(t)$. Since $\hat{f}_{n-1}(t)$ is the estimate of $f(t)$, then -$\dot{\hat{f}}_{n-1}(t)$ can increasingly weaken the effect intensity of input disturbance from $\dot{f}(t)$ in the error dynamics. Therefore, when the delay partitioning procedure change from one to N, we can see that FE observer based on the N-steps time delay partitioning approach not only reduces the conservativeness of result, but also better depicts the size and shape of faults with the increase of steps number N. The simulation examples illustrate the effectiveness and merits of the design method.*

Remark 3.4. *Motivated by the delay partitioning approach in [32, 33], we divide the constant part of time-varying delay $[0, \tau_1]$ into N segments, that is, $[0, \frac{1}{N}\tau_1]$, $[\frac{1}{N}\tau_1, \frac{2}{N}\tau_1]$, $\cdots$, $[\frac{N-1}{N}\tau_1, \tau_1]$, in which different energy functions correspond to different segments. Moreover, the interval $[\tau_1, \tau_2]$ is divided into two unequal variable subintervals $[\tau_1, \tau_1 + \rho\delta]$ and $[\tau_1 + \rho\delta, \tau_2]$ $(0 < \rho < 1, \delta = \tau_2 - \tau_1)$ in which ρ is a tunable parameter. It is clear that both the information of delayed state $e_n(t - \frac{n}{N}\tau_1)(n = 1, 2, 3, ...N)$ and $e_n(t - \tau_1 - \rho\delta)(0 < \rho < 1)$ can be taken into* *account, the Lyapunov-Krasovskii functional defined in Theorem 3.1 is more general than the one in [4–7], [34–37]. Therefore, the result of fault estimation of Theorem 3.1 and the stability criterion of Corollary 3.2 can further reduce the analysis and synthesis conservatism.*

Remark 3.5. *Based on such variable decomposition method, the improved FE observer (4) may increase the maximum allowable upper bounds on τ_2 for fixed lower bound τ_1 while giving fault estimation, if one can set a suitable dividing point with relation to ρ. For seeking an appropriate ρ, an algorithm is given as follows:*
Step 1: For given d, choose upper bound on δ satisfying (7)-(9), select this upper bound as initial value δ_0 of δ.
Step 2: Set appropriate step lengths, δ_{step} and ρ_{step} for δ and ρ, respectively. Set k as a counter and choose $k = 1$. Meanwhile, let $\delta = \delta_0 + \delta_{step}$ and the initial value ρ_0 of ρ equals ρ_{step}.
Step 3: Let $\rho = k\rho_{step}$, if (7)-(9) are feasible, go to Step 4; otherwise, go to Step 5.
Step 4: Let $\delta_0 = \delta$, $\rho_0 = \rho$, $k = 1$ and $\delta = \delta_0 + \delta_{step}$, go to Step 3.
Step 5: Let $k = k + 1$, if $k\rho_{step} < 1$, then go to Step 3. otherwise, stop.

3.2 Fault tolerant controller design

On the basis of the obtained online fault estimation information, we design a fault tolerant controller considering interval time delay to guarantee stability in the presence of system faults. Since the state $x(t)$ is unmeasurable, we use the fuzzy dynamical output feedback controller scheme [38] as follows:

$$\begin{cases} \dot{x}_c(t) = A_c(t,t)x_c(t) + A_{\tau c}(t,t)x_c(t-\tau(t)) + B_c(t)y(t) \\ u(t) = C_c(t)x_c(t) + C_{\tau c}(t)x_c(t-\tau(t)) + D_c y(t) - \hat{f}(t) \\ x_c(t) = \phi(t),\ \forall t \in [-\tau_2, 0] \end{cases} \tag{22}$$

where $x_c(t)$ is the state vector, $A_c(t,t) \in \mathbb{R}^{n\times n}$, $A_{\tau c}(t,t) \in \mathbb{R}^{n\times n}$, $B_c(t) \in \mathbb{R}^{n\times l}$, $C_c(t) \in \mathbb{R}^{q\times n}$, $C_{\tau c}(t) \in \mathbb{R}^{q\times n}$, $D_c \in \mathbb{R}^{q\times l}$ are the designed controller matrices, and $A_c(t,t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))A_{cij}$, $B_c(t) = \sum_{i=1}^{r}\mu_i(\xi(t))B_{ci}$, $A_{\tau c}(t,t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))A_{\tau cij}$, $C_c(t) = \sum_{i=1}^{r}\mu_i(\xi(t))C_{ci}$, $C_{\tau c}(t) = \sum_{i=1}^{r}\mu_i(\xi(t))C_{\tau ci}$. Denote $\tilde{x}^T(t) = (x^T(t), x_c^T(t))$, $e_f(t) = \hat{f}(t) - f(t)$, $\tilde{\omega}^T(t) = (d^T(t), e_f^T(t))$, then one can obtain the closed-loop systems

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}(t,t)\tilde{x}(t) + \tilde{A}_\tau(t,t)\tilde{x}(t-\tau(t)) + \tilde{B}_\omega(t,t)\tilde{\omega}(t) \\ y(t) = \tilde{C}(t)\tilde{x}(t) + \tilde{C}_\tau(t)\tilde{x}(t-\tau(t)) + \tilde{D}_\omega(t)\tilde{\omega}(t) \end{cases} \tag{23}$$

where

$$\tilde{A}(t,t) = \begin{bmatrix} A(t) + B(t)D_cC(t) & B(t)C_c(t) \\ B_c(t)C(t) & A_c(t,t) \end{bmatrix},\ \tilde{A}_\tau(t,t) = \begin{bmatrix} A_\tau(t) + B(t)D_cC_\tau(t) & B(t)C_{\tau c}(t) \\ B_c(t)C_\tau(t) & A_{\tau c}(t,t) \end{bmatrix},$$
$$\tilde{B}_\omega(t,t) = \begin{bmatrix} B(t)D_cD_d(t) + B_d(t) & -B(t) \\ B_c(t)D_d(t) & 0 \end{bmatrix},\ \tilde{C}(t) = \begin{bmatrix} C(t) & 0 \end{bmatrix}, \tilde{C}_\tau(t) = \begin{bmatrix} C_\tau(t) & 0 \end{bmatrix},\ \tilde{D}_\omega(t) = \begin{bmatrix} D_d(t) & 0 \end{bmatrix}$$

For simplicity, we introduce the following vectors:

$$\zeta_2^T(t) = [\tilde{x}^T(t)\ \tilde{x}^T(t-h)\ \cdots\ \tilde{x}^T(t-Nh)\ \ \tilde{x}^T(t-\tau(t))\ \ \tilde{x}^T(t-\tau_2)\ \ \tilde{\omega}^T(t)]$$

$$\Gamma_2(t,t) = [\tilde{A}(t,t)\ 0\ \cdots\ 0\ 0\ \tilde{A}_\tau(t,t)\ 0\ \tilde{B}_\omega(t,t)],\ \Gamma_3(t) = [\tilde{C}(t)\quad 0\ \cdots\ 0\ 0\ \tilde{C}_\tau(t)\quad 0\ \ \tilde{D}_\omega(t)]$$

Then, the closed-loop argument systems (23) can be rewritten as

$$\begin{cases} \dot{\tilde{x}}(t) = \Gamma_2(t,t)\zeta_2(t) \\ y(t) = \Gamma_3(t)\zeta_2(t) \end{cases} \tag{24}$$

So far, the problem of robust dynamic output feedback control for the closed-loop fuzzy system is to design the gain matrices of (22) such that:

(i) The closed-loop fuzzy system (23) with $\tilde{\omega}(t) = 0$ is asymptotically stable for any time-delay satisfying (2);
(ii) For a given scalar $\gamma_n > 0$, the following H_∞ performance is satisfied:

$$\int_0^L \|y(t)\|^2 dt \le \gamma_n^2 \int_0^L \|\tilde{\omega}(t)\|^2 dt \tag{25}$$

for all $L > 0$ and $\tilde{\omega}(t) \in L_2[0, \infty)$ under zero initial conditions.

In what follows, a useful lemma is needed, which is given here for completeness of the next theorem.

Lemma 3.6. *For the positive scalars τ_1, τ_2 and γ_n, the closed-loop system (23) with $\tilde{\omega}(t) = 0$ is asymptotically stable, and the prescribed H_∞ performance (25) is satisfied under zero initial condition for any nonzero $\tilde{\omega}(t) \in L_2[0, \infty)$, if there exist appropriately dimensional matrices $P > 0$, $Q_n > 0$, $W_n > 0 (n = 1, 2, ..., N)$, $S_1 > 0$, $S_2 > 0$, $R > 0$, $A_c(t, t)$, $A_{\tau c}(t, t)$, $B_c(t)$, $C_c(t)$, $C_{\tau c}(t)$, D_c such that*

$$\Psi(t,t) = \begin{bmatrix} \Psi^{(1)}(t,t) & \Psi^{(2)}(t,t) & \Psi^{(3)}(t,t) & \Psi^{(4)}(t) \\ * & -\sum_{n=1}^{N} W_n & 0 & 0 \\ * & * & -R & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{26}$$

where

$$\Psi^{(1)}(t,t) = \begin{bmatrix} \Psi_{11}(t,t) & W_1 & \cdots & 0 & 0 & P\tilde{A}_\tau(t,t) & 0 & P\tilde{B}_\omega(t,t) \\ * & \Psi_{22} & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & \Psi_{NN} & W_N & 0 & 0 & 0 \\ * & * & \cdots & * & \Psi_{(N+1)(N+1)} & R & 0 & 0 \\ * & * & \cdots & * & * & \Psi_{(N+2)(N+2)} & R & 0 \\ * & * & \cdots & * & * & * & \Psi_{(N+3)(N+3)} & 0 \\ * & * & \cdots & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Psi_{11}(t,t) = P\tilde{A}(t,t) + \tilde{A}^T(t,t)P + Q_1 + S_2 - W_1, \Psi_{nn} = -Q_{n-1} - W_{n-1} + Q_n - W_n,\ n = 2, 3, ..., N$$

$$\Psi_{(N+1)(N+1)} = -Q_N - W_N + S_1 - R, \Psi_{(N+2)(N+2)} = -(1-d)S_1 - 2R,\ \Psi_{(N+3)(N+3)} = -S_2 - R$$

$$\Psi^{(2)}(t,t) = h\Gamma_2^T(t,t)\sum_{n=1}^{N} W_n,\ \Psi^{(3)}(t,t) = (\tau_2 - \tau_1)\Gamma_2^T(t,t)R,\ \Psi^{(4)}(t) = \Gamma_3^T(t)$$

Proof. This proof refers to Theorem 3.1 and is omitted here for simplification. □

Theorem 3.7. *For the positive scalars τ_1, τ_2, γ_n and η, the closed-loop system (23) with $\tilde{\omega}(t) = 0$ is asymptotically stable, and the prescribed H_∞ performance (25) is satisfied under zero initial condition for any nonzero $\tilde{\omega}(t) \in L_2[0, \infty)$, if there exist appropriately dimensional matrices X, Y, M, N, $\hat{Q}_n > 0$, $\hat{W}_n > 0 (n = 1, 2, ..., N)$, $\hat{S}_1 > 0$, $\hat{S}_2 > 0$, $\hat{R} > 0$, $\hat{A}_{ij}$, $\hat{A}_{\tau ij}$, $\hat{B}_i$, $\hat{C}_i$, $\hat{C}_{\tau i}$ and $\hat{D}$ such that*

$$\Xi_{ii} < 0 \qquad i = 1, 2, \ldots, r \tag{27}$$

$$\Xi_{ij} + \Xi_{ji} \le 0 \quad 1 \le i < j \le r \tag{28}$$

where

$$\Xi_{ij} = \begin{bmatrix} \Psi_{ij}^{(1)} & \Psi_{ij}^{(2)} & \Psi_{ij}^{(3)} & \Psi_i^{(4)} \\ * & -2\eta\phi + \eta^2(\sum_{n=1}^{N} \hat{W}_n) & 0 & 0 \\ * & * & -2\eta\phi + \eta^2\hat{R} & 0 \\ * & * & * & -I \end{bmatrix}$$

with

$$\Psi_{ij}^{(1)} = \begin{bmatrix} \tilde{\Psi}_{11}^{ij} & \hat{W}_1 & \cdots & 0 & 0 & \hat{\Psi}_{1(N+2)}^{ij} & 0 & \hat{\Psi}_{1(N+4)}^{ij} \\ * & \hat{\Psi}_{22} & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & \hat{\Psi}_{NN} & \hat{W}_N & 0 & 0 & 0 \\ * & * & \cdots & * & \hat{\Psi}_{(N+1)(N+1)} & \hat{R} & 0 & 0 \\ * & * & \cdots & * & * & \hat{\Psi}_{(N+2)(N+2)} & \hat{R} & 0 \\ * & * & \cdots & * & * & * & \hat{\Psi}_{(N+3)(N+3)} & 0 \\ * & * & \cdots & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\tilde{\Psi}_{11}^{ij} = \hat{\Psi}_{11}^{ij} + (\hat{\Psi}_{11}^{ij})^T + \hat{Q}_1 + \hat{S}_2 + \hat{W}_1,$$

$$\hat{\Psi}_{11}^{ij} = \begin{bmatrix} A_i X + B_i \hat{C}_j & A_i + B_i \hat{D} C_j \\ \hat{A}_{ij} & Y A_i + \hat{B}_j C_i \end{bmatrix}, \hat{\Psi}_{1(N+2)}^{ij} = \begin{bmatrix} A_{\tau i} X + B_i \hat{C}_{\tau j} & A_{\tau i} + B_i \hat{D} C_{\tau j} \\ \hat{A}_{\tau ij} & Y A_{\tau i} + \hat{B}_j C_{\tau i} \end{bmatrix},$$

$$\hat{\Psi}_{1(N+4)}^{ij} = \begin{bmatrix} B_i \hat{D} D_{dj} + B_{di} & -B_i \\ \hat{B}_j D_{di} + Y B_{di} & -Y B_i \end{bmatrix}, \hat{\Psi}_{nn} = -\hat{Q}_{n-1} - \hat{W}_{n-1} + \hat{Q}_n - \hat{W}_n,\ n = 2, 3, ..., N$$

$$\hat{\Psi}_{(N+1)(N+1)} = -\hat{Q}_N - \hat{W}_N + \hat{S}_1 - \hat{R}, \hat{\Psi}_{(N+2)(N+2)} = -(1-d)\hat{S}_1 - 2\hat{R},$$

$$\hat{\Psi}_{(N+3)(N+3)} = -\hat{S}_2 - \hat{R},\ \Psi_{ij}^{(2)} = h\hat{\Gamma}_{2ij}^T,\ \Psi_{ij}^{(3)} = (\tau_2 - \tau_1)\hat{\Gamma}_{2ij}^T,$$

$$\hat{\Gamma}_{2ij} = \begin{bmatrix} \hat{\Psi}_{11}^{ij} & 0 \cdots 0 & \hat{\Psi}_{1(N+2)}^{ij} & 0 & \hat{\Psi}_{1(N+4)}^{ij} \end{bmatrix}, \phi = \begin{bmatrix} X & I \\ I & Y \end{bmatrix},$$

$$\Psi_i^{(4)} = \begin{bmatrix} [C_i X \ \ C_i] & 0 & \cdots & 0 & [C_{\tau i} X \ \ C_{\tau i}] & 0 & [D_{di} \ \ 0] \end{bmatrix}^T$$

Then, the gain matrices of the dynamic output feedback fault tolerant controller are given by

$$\begin{aligned} D_c &= \hat{D},\ B_{ci} = N^{-1}(\hat{B}_i - Y B_i \hat{D}),\ C_{ci} = (\hat{C}_i - \hat{D} C_i X) M^{-T},\ C_{\tau ci} = (\hat{C}_{\tau i} - \hat{D} C_{\tau i} X) M^{-T}, \\ A_{cij} &= N^{-1}(\hat{A}_{ij} - (Y A_i - \hat{B}_i C_j) X) M^{-T} + N^{-1} Y B_i C_{cj}, \\ A_{\tau cij} &= N^{-1}(\hat{A}_{\tau ij} - (Y A_{\tau i} - \hat{B}_i C_{\tau j}) X) M^{-T} + N^{-1} Y B_i C_{\tau cj}, \end{aligned}$$

where M, N satisfy $MN^T = I - XY$.

Proof. For any scalar $\eta > 0$, by the Schur complement theorem and the fact $(\eta R - P)R^{-1}(\eta R - P)$ that $-PR^{-1}P \leq -2\eta P + \eta^2 R$, we can conclude that (26) holds if the following inequality hold:

$$\Xi_2(t,t) = \begin{bmatrix} \Psi^{(1)}(t,t) & h\Gamma_2^T(t,t)P & (\tau_2 - \tau_1)\Gamma_2^T(t,t)P & \Gamma_3^T(t) \\ * & -2\eta P + \eta^2(\sum_{n=1}^N W_n) & 0 & 0 \\ * & * & -2\eta P + \eta^2 R & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{29}$$

Then, we partition the symmetric positive definite matrix P and its inverse matrix P^{-1} into components

$$P = \begin{bmatrix} Y & N \\ * & W \end{bmatrix}, P^{-1} = \begin{bmatrix} X & M \\ * & Z \end{bmatrix}$$

Since $PP^{-1} = I$, where $N^T X + W M^T = 0,\ YM + NZ = 0$, we denote $F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}$ then it follows that $PF_1 = F_2$.

Let

$\Upsilon_1^T = \text{diag}\{\overbrace{F_1^T, F_1^T, ..., F_1^T}^{(N+4)}, F_1^T, F_1^T, F_1^T, I\}$, then, pre and post multiplying (29) by $diag\{\Upsilon_1^T, F_1^T, F_1^T, I\}$.

(29) can be expressed as

$$\Xi_2(t,t) = \begin{bmatrix} \hat{\Psi}^{(1)} & h\hat{\Gamma}_2^T(t,t) & (\tau_2-\tau_1)\hat{\Gamma}_2^T(t,t) & \hat{\Gamma}_3^T(t) \\ * & -2\eta\phi + \eta^2(\sum_{n=1}^N \hat{W}_n) & 0 & 0 \\ * & * & -2\eta\phi + \eta^2 \hat{R} & 0 \\ * & * & * & -I \end{bmatrix} < 0$$

where

$$\hat{\Psi}^{(1)} = \begin{bmatrix} \tilde{\Psi}_{11}(t,t) & \hat{W}_1 & \cdots & 0 & 0 & \hat{\Psi}_{1(N+2)}(t,t) & 0 & \hat{\Psi}_{1(N+4)}(t,t) \\ * & \hat{\Psi}_{22} & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & \hat{\Psi}_{NN} & \hat{W}_N & 0 & 0 & 0 \\ * & * & \cdots & * & \hat{\Psi}_{(N+1)(N+1)} & \hat{R} & 0 & 0 \\ * & * & \cdots & * & * & \hat{\Psi}_{(N+2)(N+2)} & \hat{R} & 0 \\ * & * & \cdots & * & * & * & \hat{\Psi}_{(N+3)(N+3)} & 0 \\ * & * & \cdots & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$\tilde{\Psi}_{11}(t,t) = \hat{\Psi}_{11}(t,t) + \hat{\Psi}_{11}^T(t,t) + \hat{Q}_1 + \hat{S}_2 + \hat{W}_1, \hat{\Psi}_{nn} = -\hat{Q}_{n-1} - \hat{W}_{n-1} + \hat{Q}_n - \hat{W}_n,\ n = 2,3,...,N$

$\hat{\Psi}_{(N+1)(N+1)} = -\hat{Q}_N - \hat{W}_N + \hat{S}_1 - \hat{R}, \hat{\Psi}_{(N+2)(N+2)} = -(1-d)\hat{S}_1 - 2\hat{R}, \hat{\Psi}_{(N+3)(N+3)} = -\hat{S}_2 - \hat{R}$

$\hat{\Gamma}_2(t,t) = \left[\hat{\Psi}_{11}(t,t)\ \ 0 \cdots 0\ \hat{\Psi}_{1(N+2)}(t,t)\ 0\ \hat{\Psi}_{1(N+4)}(t,t)\right],$

$\hat{\Gamma}_3(t) = \left[[C(t)X\ \ C(t)]\ \ 0\ \cdots\ 0\ \ [C_\tau(t)X\ \ C_\tau(t)]\ 0\ [D_d(t)\ \ 0]\right]$

with

$$\hat{\Psi}_{11}(t,t) = \begin{bmatrix} A(t)X + B(t)\hat{C}(t) & A(t) + B(t)\hat{D}C(t) \\ \hat{A}(t,t) & YA(t) + \hat{B}C(t) \end{bmatrix},$$

$$\hat{\Psi}_{1(N+2)}(t,t) = \begin{bmatrix} A_\tau(t)X + B(t)\hat{C}_\tau(t) & A_\tau(t) + B(t)\hat{D}C_\tau(t) \\ \hat{A}_\tau(t,t) & YA_\tau(t) + \hat{B}C_\tau(t) \end{bmatrix},$$

$$\hat{\Psi}_{1(N+4)}(t,t) = \begin{bmatrix} B(t)\hat{D}D_d(t) + B_d(t) & -B(t) \\ \hat{B}(t)D_d(t) + YB_d(t) & -YB(t) \end{bmatrix}.$$

Then, $\Xi_2(t,t) < 0$ can be rewritten as

$$\Xi_2(t,t) = \sum_{i=1}^r \mu_i^2(\xi(t))\Xi_{ii} + \sum_{i=1}^r \sum_{i<j}^r \mu_i(\xi(t))\mu_j(\xi(t))(\Xi_{ij} + \Xi_{ji}) < 0$$

Therefore, by Lemma 3.6, the closed-loop fuzzy system (23) with time-varying state delay is asymptotically stable (with $\tilde{\omega}(t) = 0$) while satisfying a prescribed H_∞ performance (25). This completes the proof. □

Remark 3.8. *As in [30], from $\phi = F_1^T P F_1 > 0$, we can obtain $Y > 0$ and $X - Y^{-1} < 0$ which imply that $I - XY$ is nonsingular. Therefore, we can always find nonsingular matrices M and N satisfying $MN^T = I - XY$, and they can be calculated by the QR function of Matlab toolbox.*

Remark 3.9. *Note that* (27)-(28) *are LMIs. This indicates that it can be included as an optimization variable, which can be exploited to reduce the attenuation level bound. Then, the minimum attenuation level of H_∞ performance can be obtained by solving a convex optimization problem **P**: min ϑ subject to* (27)-(28) *with $\vartheta = \gamma^2$. Different from the existing results, the main advantage of the proposed design method is the reduction of conservatism by presenting a delay-dependent result. Also, an interval time-varying delay has been considered in the design scheme.*

4 Numerical example

In this section, three examples are provided to demonstrate the effectiveness of the proposed approaches.

Example 4.1. *Consider the following time-delayed nonlinear system:*

$$\begin{cases} \dot{x}_1(t) = 0.5(1 - sin^2(\theta(t)))x_2(t) - x_1(t-\tau(t)) - (1 + sin^2(\theta(t)))x_1(t) \\ \dot{x}_2(t) = sgn(|\theta(t)| - \dfrac{\pi}{2})(0.9cos^2(\theta(t)) - 1)x_1(t-\tau(t)) - x_2(t-\tau(t)) \\ \qquad - (0.9 + 0.1cos^2(\theta(t)))x_2(t) \end{cases}$$

which can be exactly expressed as a nominal T-S delayed system with the following rules [4–7], [37]:

$$Rule1: if\,\theta(t) is \pm\frac{\pi}{2}, then\ \dot{x}(t) = A_1 x(t) + A_{\tau 1} x(t-\tau(t))$$
$$Rule2: if\,\theta(t) is\ 0, then\ \dot{x}(t) = A_2 x(t) + A_{\tau 2} x(t-\tau(t))$$

The membership functions for above rules 1, 2 are $\mu_1(x_1(t)) = sin^2(x_1(t))$, $\mu_2(x_1(t)) = cos^2(x_1(t))$, *where*

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_{\tau 1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},\ A_{\tau 2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}$$

The purpose here is to find the allowable maximum time-delay value τ_2 *under which the fuzzy system is stable. Considering interval time-varying delay, the upper delay bounds* τ_2 *derived from [4–7], [37] and the method proposed in this paper are tabulated in Table 1 under different values of* τ_1. *It is seen from Table 1 that the results obtained from Corollary 3.2 (*d *is unknown, set* $S_1 = 0$*) of this paper are significantly better than those obtained from the other methods.*

Table 1. Example 4.1 – maximum allowable delay bounds τ_2 under different values of τ_1 with d unknown

Methods \ τ_1	0	0.4	0.8	1.0	1.2
[4] Corollary 3.2	-	1.2647	1.3032	1.3528	1.4214
[5] Theorem 3.1	1.2780	1.3030	1.3160	1.3610	1.4250
[7] Corollary4	-	1.2836	1.3394	1.4009	1.4815
[6] Theorem 3.1	1.3800	1.3900	1.4300	-	1.5700
[37] Theorem4	-	1.5274	1.5361	1.5762	1.6340
Corollary 3.2 (N=1,ρ=0.7)	1.6057	1.9012	2.1775	2.3142	2.4522

Example 4.2. *Consider a two rule T-S fuzzy system borrowed from [4, 7], [34–36]:*

$$Rule1: if\ x_1\ is\ W_1, then\ \dot{x}(t) = A_1 x(t) + A_{\tau 1} x(t-\tau(t))$$
$$Rule2: if\ x_1\ is\ W_2, then\ \dot{x}(t) = A_2 x(t) + A_{\tau 2} x(t-\tau(t))$$

where

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.5 & 1 \\ 0 & -0.75 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} -1 & 0 \\ 1 & -0.85 \end{bmatrix}$$

To compare with the existing results, the improvement of this paper is shown in Table 2. It can be concluded that the obtained results in this paper are less conservative than those of [4, 7], [34–36]. Moreover, if we assume time-varying delay $\tau(t)$ *satisfies* (2), *the delay-dependent fault estimation conditions proposed in [21–25], [27] fail to give a feasible solution. However, by using LMI toolbox in matlab, a feasible solution of Theorem 3.1 can be obtained for* $\tau(t) = 1 + 0.1sint(\tau_1 = 0.9, \tau_2 = 1.1)$ *and other cases. In order to further illustrate the effectiveness of proposed approach, the problem of FTC for T-S fuzzy systems with interval time delay is considered in the next example.*

Table 2. Example 4.2 – maximum allowable delay bounds τ_2 under different values of τ_1 with d unknown

Methods \ τ_1	0.2	0.4	0.6	0.8
[34]	0.6870	0.8500	0.9460	1.0480
[35] Corollary 3.2	0.7945	0.8487	0.9316	1.0325
[36] Corollary5	0.9119	0.9793	1.0639	1.1662
[4] Corollary 3.2	1.1410	1.1500	1.1720	1.2090
[7] Corollary4	1.1639	1.1734	1.1994	1.2532
Corollary 3.2 (N=1,ρ=0.7)	1.7126	1.8638	2.0020	2.1330

Example 4.3. *We apply the above analysis technique to design robust fault estimation observer and dynamic output feedback fault-tolerant controller for a computer simulated truck-trailer system borrowed from [39]. The time delay model with actuator fault $f(t)$ and disturbance $d(t)$ is given by T-S fuzzy systems as follows*

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))[A_i x(t) + A_{\tau i} x(t-\tau(t)) + B_i(u(t) + f(t)) + B_{di} d(t)] \\ y(t) = \sum_{i=1}^{2} \mu_i(\xi(t))[C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_{di} d(t)] \\ x(t) = \sum_{i=1}^{2} \mu_i(\xi(t))\phi_i(t), t \in [-\tau, 0] \end{cases}$$

where $x(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$ and

$$A_1 = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \ A_{\tau 1} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, C_1 = C_2 = [-2 \quad 0.05 \quad -0.15]$$

$$A_2 = \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{dv^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix}, \ A_{\tau 2} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, C_{\tau 1} = (1-a)C_1, \ C_{\tau 2} = (1-a)C_2$$

where $x_1(t)$ is the angle difference between the truck and the trailer, $x_2(t)$ is the angle of the trailer, $x_3(t)$ is the vertical position of the rear end of the trailer, $u(t)$ is the steering angle. The constant a is the retarded coefficient, which satisfies the conditions: $a \in [0,1]$. The limits 1 and 0 correspond to no delay term and to a completed delay term, respectively. In this example, the model parameters is given as $a = 0.7$, $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$.

Here, it is supposed that the disturbance distribution matrices are $B_{d1} = B_{d2} = [0.01\ \ 0.01\ \ 0.01]^T$, $D_{d1} = D_{d2} = 0.001$ and the delay $\tau(t)$ satisfies (2), where $\tau(t) = 1 + 0.1sin(t)$ (thus $\tau_1 = 0.9$, $\tau_2 = 1.1$, $d = 0.1$). Meanwhile, in order to facilitate simulation, we choose membership functions for Rules 1 and 2 are $\mu_1(\xi(t)) = 1/(1 + exp(x_1(t) + 0.5))$, $\mu_2(\xi(t)) = 1 - \mu_1(\xi(t))$ with initial condition $[0.5\pi\ 0.75\pi\ \ -5]^T$, $d = 10 * t_0/\pi$. It is also assumed that $d(t)$ is band-limited white noise with power 0.1 and sampling time 0.01s. When using the inequality $-PR^{-1}P \leq -2\eta P + \eta^2 R$ to give main results, we can consider different η to find the minimum index γ. Then, setting $\eta = 2$, $\rho = 0.7$ and computing matrix inequalities (7)-(9) in Theorem 3.1 based on the mincx function of Matlab toolbox, one obtains the feasible solution in Table 3.

Then, using the obtained observer gain matrix, a constant fault and a time-varying fault are respectively created

$$f_1(t) = \begin{cases} 0 & 0 \leq t < 5 \\ 30(1 - e^{-0.5(t-5)}) & 5 \leq t \leq 20 \end{cases}, \ f_2(t) = \begin{cases} 0 & 0 \leq t < 5 \\ 30sin(t/2) & 5 \leq t \leq 20 \end{cases}$$

Fig. 1-2 illustrates the simulation result of the improved n-steps fault estimation with $N = 1,2,3$. Therein, the actual fault is depicted by dashed line, and the fault estimation is represented by the solid one. As shown in Fig.

Table 3. Example 4.3 – the feasible solution of fault estimation observer gain matrix with $d = 0.1$

	The gain matrix $L(t)$	The gain matrix $F(t)$
$N = 1$	$L_1 = \begin{bmatrix} -140.7143 \\ 7.8890 \\ 62.7727 \end{bmatrix}, L_2 = \begin{bmatrix} -141.2645 \\ 7.9583 \\ 60.0092 \end{bmatrix}$	$F_1 = 150.8112, F_2 = 151.2707$
$N = 2$	$L_1 = \begin{bmatrix} -136.7264 \\ 8.1540 \\ 59.9273 \end{bmatrix}, L_2 = \begin{bmatrix} -137.3991 \\ 8.2402 \\ 57.1751 \end{bmatrix}$	$F_1 = 145.9467, F_2 = 146.5154$
$N = 3$	$L_1 = \begin{bmatrix} -134.8046 \\ 8.1321 \\ 58.7985 \end{bmatrix}, L_2 = \begin{bmatrix} -135.5113 \\ 8.2239 \\ 56.0464 \end{bmatrix}$	$F_1 = 143.4600, F_2 = 144.0539$

1-2, it's obvious that the robust fault estimation observer is insensitive to the exogenous disturbance and has a good performance to estimate the constant and time-varying fault $f(t)$ when the positive integers N change from one to three. Meanwhile, it follows from Fig. 3 that the error states are also stable.

Fig. 1. Fault estimation result using robust FE observer in $f_1(t)$

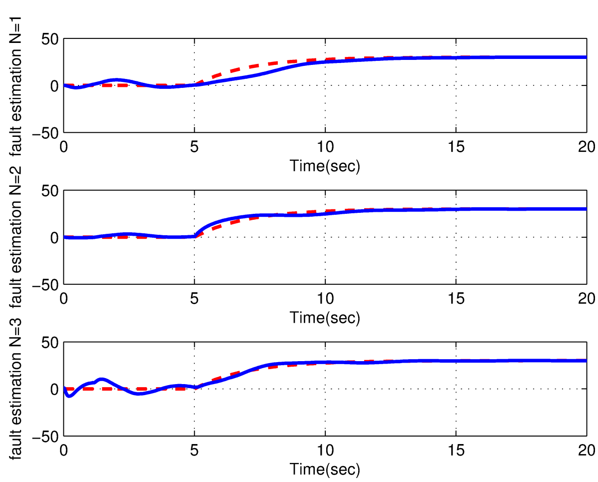


Fig. 2. Fault estimation result using robust FE observer in $f_2(t)$

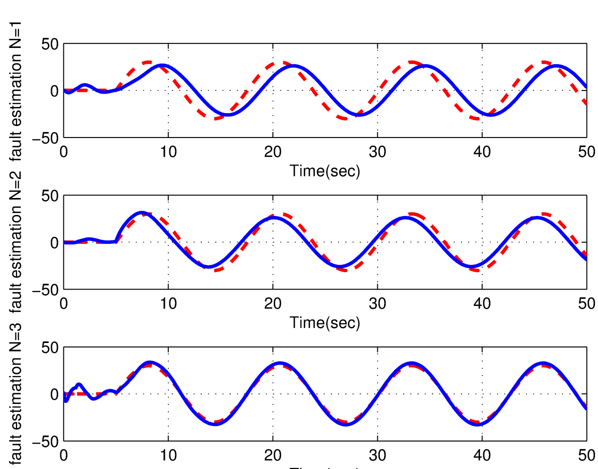
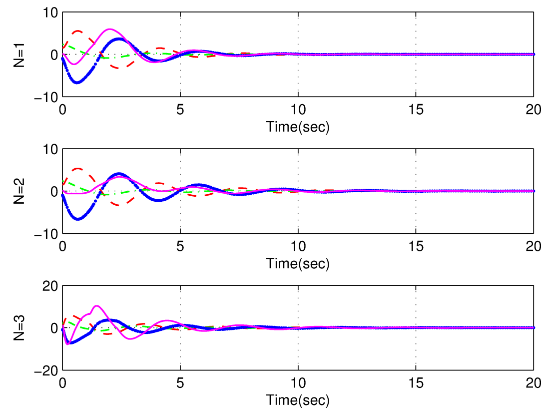
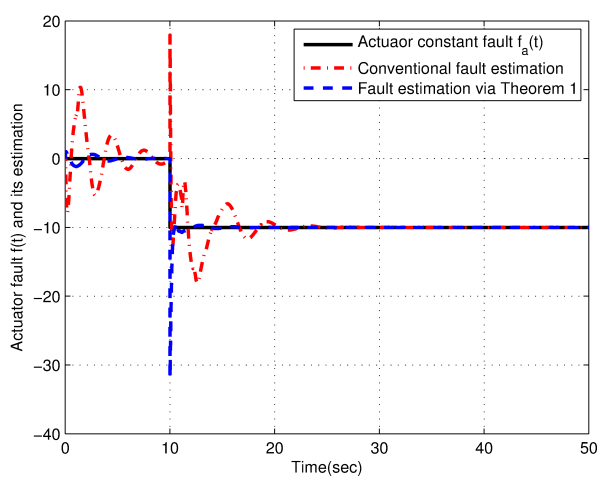


Fig. 3. Response curves of error dynamic states $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_f(t)$



As comparison, fault estimation using the conventional adaptive fault estimation (CAFE) algorithm in [26] are depicted in Figs. 4 and 5. It is observed that, compared with CAFE, although there is initial estimation error, the n-steps estimation approach not only provides better rapidity of fault estimation but also achieves more accurate estimation of actuator fault by increasing steps N. Moreover, improved delay partitioning approach is employed to deal with interval time delay for increasing the maximum allowable delay bounds, while the approach in [21, 23, 27], [26] fail to give a feasible estimation. All of these make it meaningful for the approach to be implemented in practice.

Fig. 4. Fault estimation result of a constant fault using conventional fault estimation approach and Theorem 3.1 $(N = 3)$



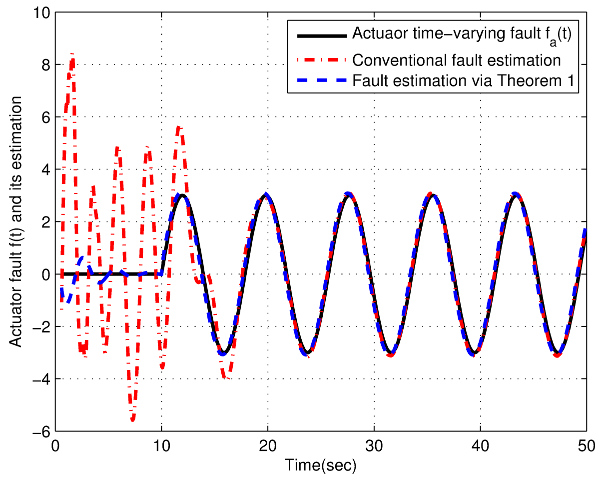
Then, by solving the conditions in Theorem 3.7, we design a fault-tolerant controller ($N = 1$) with the minimum attenuation value $\gamma = 0.1419$ on the basis of the obtained fault-estimation as follows:

$$A_{c11} = \begin{bmatrix} -3.9738 & -0.7761 & -1.2933 \\ 209.8643 & -7.0311 & 17.3469 \\ -359.8673 & 11.9056 & -29.4078 \end{bmatrix}, A_{\tau c11} = \begin{bmatrix} -0.6069 & -0.1076 & -0.0132 \\ 47.4608 & -6.9120 & -2.6111 \\ -82.9240 & 11.7618 & 4.3041 \end{bmatrix}$$

$$A_{c12} = \begin{bmatrix} -6.6191 & -0.9376 & -1.3707 \\ 338.2784 & 1.8191 & 21.6169 \\ -573.4481 & -2.7994 & -36.5021 \end{bmatrix}, A_{\tau c12} = \begin{bmatrix} -1.9678 & -0.3040 & -0.2188 \\ 106.2766 & -0.3122 & 4.9951 \\ -180.8551 & 0.7413 & -8.3818 \end{bmatrix}$$

$$A_{c21} = \begin{bmatrix} -4.2682 & -0.7884 & -1.2054 \\ 215.8110 & -11.9498 & 13.5066 \\ -371.6297 & 19.3324 & -24.6910 \end{bmatrix}, A_{\tau c21} = \begin{bmatrix} -0.6259 & -0.1096 & -0.0249 \\ 47.1583 & -8.9702 & -3.2539 \\ -83.6416 & 14.5442 & 4.9061 \end{bmatrix}$$

Fig. 5. Fault estimation result of a time-varying fault using conventional fault estimation approach and Theorem 3.1 $(N = 3)$



$$A_{c22} = \begin{bmatrix} -6.5989 & -0.7909 & -1.2771 \\ 338.1133 & 0.6354 & 20.8635 \\ -573.2894 & -1.6714 & -35.7853 \end{bmatrix}, A_{\tau c22} = \begin{bmatrix} -1.5443 & -0.1430 & -0.0802 \\ 102.9605 & -1.5701 & 3.9112 \\ -177.7806 & 1.9050 & -7.3781 \end{bmatrix}$$

$$C_{c1} = \begin{bmatrix} 440.2710 & -113.5283 & -120.0370 \end{bmatrix}, \quad C_{\tau c1} = \begin{bmatrix} 165.0503 & -29.0606 & 1.4027 \end{bmatrix}$$

$$C_{c2} = \begin{bmatrix} 442.8730 & -110.1644 & -118.3330 \end{bmatrix}, \quad C_{\tau c2} = \begin{bmatrix} 143.3467 & -38.1933 & -5.9422 \end{bmatrix}$$

$$B_{c1}^T = \begin{bmatrix} -0.0574 & -13.0855 & 22.7614 \end{bmatrix}, \; B_{c2}^T = \begin{bmatrix} 0.1346 & -22.3452 & 38.1632 \end{bmatrix}, \; D_c = -68.0204$$

Simulation results for the stability of the closed-loop systems and the systems output response are shown in Fig. 6 and Fig. 7-8. It can be seen that although the open-loop systems are unstable, the proposed design still achieves the performance under actuator faults, and the stability of closed-loop systems is guaranteed while satisfying the prescribed H_∞ performance. As indicated by the simulation result graph, we can see that whether the interval time delay fuzzy systems are considered with constant fault or time-varying fault, the fuzzy n-steps fault estimation observer can almost realize accurate fault estimation, and the fuzzy dynamic output feedback control strategy can effective accommodate the effect of actuator faults on system performance.

Fig. 6. State response curves of the closed-loop argument system with fuzzy dynamic output feedback fault-tolerant control

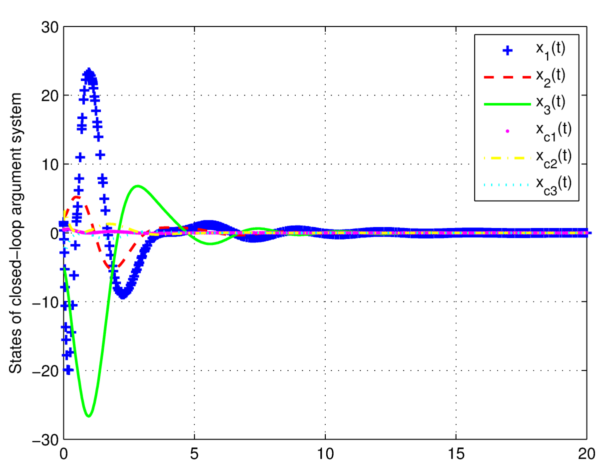


Fig. 7. System output response $y(t)$ with fuzzy dynamic output feedback fault-tolerant control using fault information shown in Fig. 1

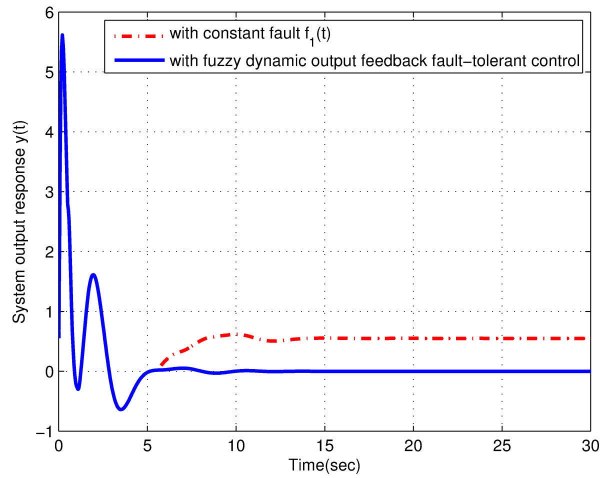
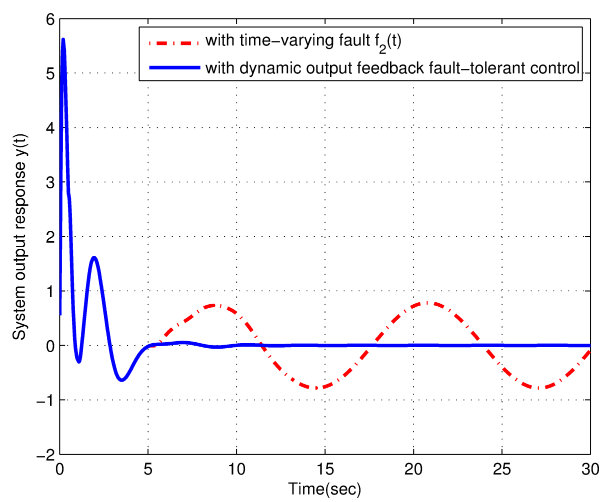


Fig. 8. System output response $y(t)$ with fuzzy dynamic output feedback fault-tolerant control using fault information shown in Fig. 2



5 Conclusion

In this paper, by using improved delay partitioning approach, a novel n-steps robust fault estimation observer has been constructed for a class of T-S fuzzy model with interval time-varying delay and external disturbances. Then, utilizing realtime information on estimated faults, a fuzzy dynamic output feedback fault-tolerant controller considering interval time delay is proposed to accommodate the effect of actuator faults while satisfying the prescribed H_∞ performance. An advantage of the proposed approach is that with the increase of steps n, not only the approach can give a better performance of FE and FTC, but also the maximum allowable delay bounds of fuzzy systems is increased. Finally, some examples have clearly verified the effectiveness of the proposed method for FE and FTC. This paper focus on robust FTC for T-S fuzzy systems with actuator fault and does not consider sensor fault. The consideration of the system with actuator and sensor fault simultaneously will be studied in our future work.

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