

Self-organization of price fluctuation distribution in evolving markets

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Abstract. – Financial markets can be seen as complex systems in non-equilibrium steady state, one of whose most important properties is the distribution of price fluctuations. Recently, there have been assertions that this distribution is qualitatively different in emerging markets as compared to developed markets. Here we analyse both high-frequency tick-by-tick as well as daily closing price data to show that the price fluctuations in the Indian stock market, one of the largest emerging markets, have a distribution that is identical to that observed for developed markets (e.g., NYSE). In particular, the cumulative distribution has a long tail described by a power law with an exponent $\alpha \approx 3$. Also, we study the historical evolution of this distribution over the period of existence of the National Stock Exchange (NSE) of India, which coincided with the rapid transformation of the Indian economy due to liberalization, and show that this power law tail has been present almost throughout. We conclude that the “inverse cubic law” is a truly universal feature of a financial market, independent of its stage of development or the condition of the underlying economy.

Introduction. – Financial markets are paradigmatic examples of complex systems, comprising a large number of interacting components that are subject to a constant flow of external information [1, 2]. Statistical physicists have studied simple interacting systems which self-organize into non-equilibrium steady states, often characterized by power law scaling [3]. Whether markets also show such behavior can be examined by looking for evidence of scaling functions which are invariant for different markets. The most prominent candidate for such an universal, scale-invariant property is the cumulative distribution of stock price fluctuations. The tails of this distribution has been reported to follow a power law, $P_c(x) > x^{-\alpha}$, with the exponent $\alpha \approx 3$ [4]. This “inverse cubic law” had been reported initially for a small number of stocks from the S&P 100 list [5]. Later, it was established from statistical analysis of stock returns in the German stock exchange [6], as well as for three major US markets, including the New York Stock Exchange (NYSE) [7]. The distribution was shown to be quite robust, retaining the same functional form for time scales of upto several days [7]. Similar behavior has also been seen in the London Stock Exchange [8]. An identical power law tail has also

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been observed for the fluctuation distribution of a number of market indices [9, 10]. This apparent universality of the distribution may indicate that different markets self-organize to an almost identical non-equilibrium steady state. However, as almost all these observations are from developed markets, a question of obvious interest is whether the same distribution holds for developing or emerging financial markets. If the inverse cubic law is a true indicator of self-organization in markets, then observing the price fluctuation distribution as the market evolves will inform us about the process by which this complex system converges to the non-equilibrium steady state characterizing developed markets.

However, when it comes to empirical reports about such emerging markets there seems to be a lack of consensus. The market index fluctuations in Brazil [11] and Korea [10] have been reported to follow an exponential distribution. On the other hand, the distribution for an Indian market index, over approximately the same period, was observed to be heavy tailed [12]. It is hard to conclude about the nature of the fluctuation distribution for individual stock prices from the index data, as the latter is a weighted average of several stocks. Therefore, in principle, the index can show a distribution quite different from that of its constituent stocks if their price movements are not correlated.

Analysis of individual stock price returns for emerging markets have also not resulted in an unambiguous conclusion about whether such markets behave differently from developed markets. A recent study [13] of the fluctuations in the daily price of the 49 largest stocks in an Indian stock exchange has claimed that the distribution has exponentially decaying tails. This implies the presence of a characteristic scale, and the breakdown of universality of the power law tail for the price fluctuation distribution. On the other hand, it has been claimed that this distribution in emerging markets has even more extreme tails than developed markets, with an exponent α that can be less than 2 [14]. More recently, there has been a report of the “inverse cubic law” for the daily return distribution in the Chinese stock markets of Shanghai and Shenzhen [15]. These contradictory reports indicate that a careful analysis of the stock price return distribution for emerging markets is extremely necessary. This will help us to establish definitively whether the “inverse cubic law” is invariant with respect to the stage of economic development of a market.

All the previous studies of price fluctuations in emerging markets have been done on low-frequency daily data. For the first time, we report analysis done on high-frequency tick-by-tick data, which are corroborated by analysis of daily data over much longer periods. The data set that we have chosen for this purpose is from the National Stock Exchange (NSE) of India, the largest among the 23 exchanges in India, with more than 85% of the total value of transactions for securities in all market segments of the entire Indian financial market in recent times [16]. This data set is of unique importance, as we have access to daily data right from the time the market commenced operations in the equities market in Nov 1994, upto the present when it has become the world’s third largest stock exchange (after NASDAQ and NYSE) in terms of transactions [17]. Over this period, the market has grown rapidly, with the number of transactions having increased by more than three orders of magnitude. Therefore, if markets do show discernible transition in the return distribution during their evolution, the Indian market data is best placed to spot evidence for it, not least because of the rapid transformation of the Indian economy in the liberalized environment since the 1990s.

In this paper, we focus on two important questions: (i) Does an emerging market exhibit a different price fluctuation distribution compared to developed markets, and (ii) if the market is indeed following the inverse cubic law at present, whether this has been converged at starting from an initially different distribution when the market had just begun operation. Both of these questions are answered in the negative in the following analysis.

Data description. – We have looked at two data sets having different temporal resolutions: (i) The high-frequency tick-by-tick data contains information about all transactions carried out in the NSE between Jan 2003 and Mar 2004. This information includes the date and time of trade, the price of the stock during transaction and the volume of shares traded. This database is available in the form of CDs published by NSE. For calculating the price return, we have focused on 489 stocks that were part of the BSE 500 index (a comprehensive indicator for the Indian financial market) during this period. The number of transactions for each company in this set is $\sim 10^6$, on the average. The total number of transactions for the 489 stocks is of the order of 5×10^8 during the period under study. (ii) The daily closing price of all the stocks listed in NSE during its period of existence between Nov 1994 and May 2006. This was obtained from the NSE website [18] and manually corrected for stock splitting. For comparison with US markets, in particular the NYSE, we have considered the 500 stocks listed in S&P 500 during the period Nov 1994 - May 2006, the daily data being obtained from Yahoo! Finance [19].

Results. – To measure the price fluctuations such that the result is independent of the scale of measurement, we calculate the logarithmic return of price. If $P_i(t)$ is the stock price of the i th stock at time t , then the (logarithmic) price return is defined as

$$R_i(t, \Delta t) \equiv \ln P_i(t + \Delta t) - \ln P_i(t). \quad (1)$$

However, the distribution of price returns of different stocks may have different widths, owing to differences in their volatility, defined (for the i -th stock) as $\sigma_i^2 \equiv \langle R_i^2 \rangle - \langle R_i \rangle^2$. To compare the distribution of different stocks, we normalize the returns by dividing them with their volatility. The resulting normalized price return is given by

$$r_i(t, \Delta t) \equiv \frac{R_i - \langle R_i \rangle}{\sigma_i}, \quad (2)$$

where $\langle \dots \rangle$ denotes the time average over the given period.

For analysis of the high-frequency data, we consider the aforementioned 489 stocks. Choosing an appropriate Δt , we obtain the corresponding return by taking the log ratio of consecutive average prices, averaged over a time window of length Δt . Fig. 1 (left) shows the cumulative distribution of the normalized returns r_i with $\Delta t = 5$ mins for five stocks, arbitrarily chosen from the dataset. We observe that the distribution of normalized returns r_i for all the stocks have the same functional form with a long tail that follows a power-law asymptotic behavior. The distribution of the corresponding power law exponent α_i for all the 489 stocks that we have considered is shown in Fig 1 (right).

As all the individual stocks follow very similar distributions, we can merge the data for different stocks to obtain a single distribution for normalized returns. The aggregated return data set with $\Delta t = 5$ mins has 6.5×10^6 data points. The corresponding cumulative distribution is shown in Fig. 2 (left), with the exponents for the positive and negative tails estimated as

$$\alpha = \begin{cases} 2.89 \pm 0.09 & \text{(positive tail)} \\ 2.52 \pm 0.03 & \text{(negative tail)}. \end{cases} \quad (3)$$

From this figure we confirm that the distribution does indeed follow a power law decay, albeit with different exponents for the positive and negative return tails. Such a difference between the positive and negative tails have also been observed in the case of stocks listed in the NYSE [7]. To further verify that the tails are indeed consistent with a power law form, we perform an alternative measurement of α using the Hill estimator [20,21]. We order the returns

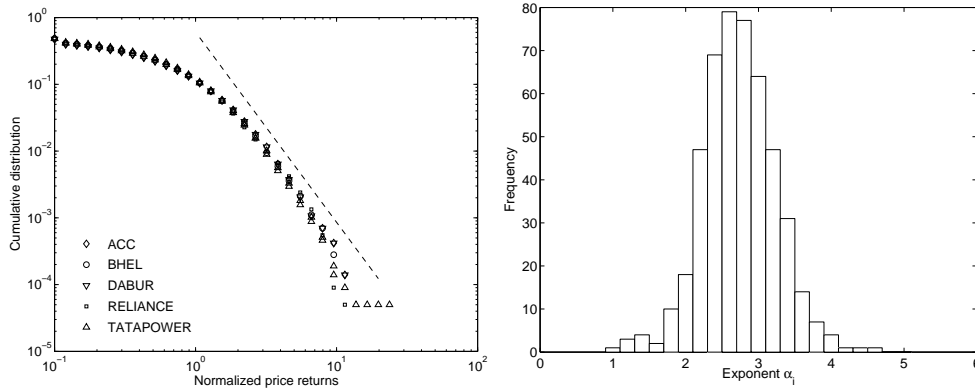


Fig. 1 – (Left) Cumulative distribution of the positive tails of the normalized 5-min returns distribution of 5 stocks chosen arbitrarily from those listed in the NSE for the period Jan 2003 to March 2004. The broken line indicates a power law with exponent $\alpha = 3$. (Right) The histogram of the power-law exponents obtained by regression fit for the positive tail of individual cumulative return distributions of 489 stocks. The median of the exponent values is 2.85.

in a decreasing order such that $r_1 > \dots > r_n$ and then obtain the Hill estimator (based on the largest $k + 1$ values) as $H_{k,n} = \frac{1}{k} \sum_i^k \log \frac{r_i}{r_{k+1}}$, for $k = 1, \dots, n - 1$. The estimator $H_{k,n} \rightarrow \alpha^{-1}$ when $k_n \rightarrow \infty$ and $k_n/n \rightarrow 0$. For our data, this procedure gives $\alpha = 2.97$ and 2.56 for the positive and the negative tail respectively, which are consistent with (3).

Next, we extend this analysis for longer time scales, to observe how the nature of the distribution changes with increasing Δt . As has been previously reported for US markets, the distribution is found to decay faster as Δt becomes large. However, upto $\Delta t = 1$ day, i.e., the

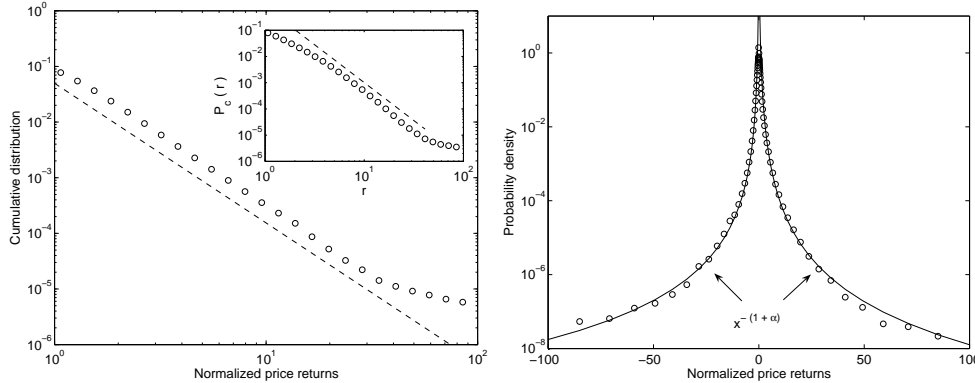


Fig. 2 – (left) Cumulative distribution of the negative and (inset) positive tails of the normalized returns for the aggregated data of 489 stocks in the NSE for the period Jan 2003 to Mar 2004. The broken line for visual guidance indicates the power law asymptotic form. (Right) Probability density function of the normalized returns. The solid curve is a power-law fit in the region 1 – 40. We find that the corresponding cumulative distribution exponent, $\alpha = 2.89$ for the positive tail and $\alpha = 2.52$ for the negative tail.

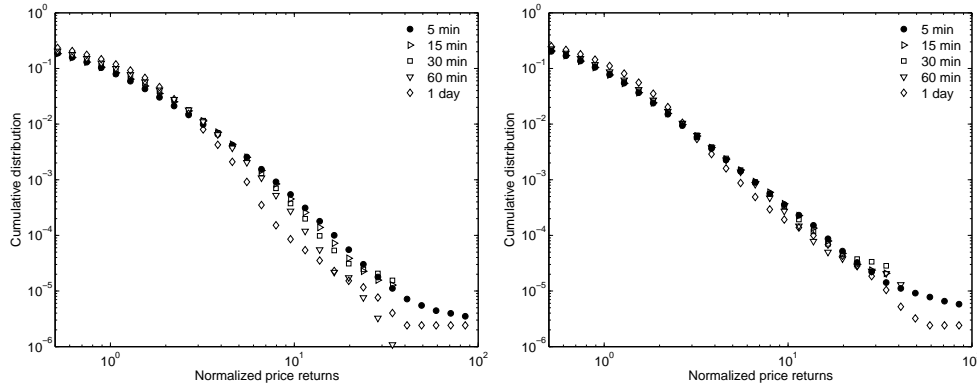


Fig. 3 – Cumulative distribution of the positive (left) and negative (right) tails of the normalized returns distribution for different time scales ($\Delta t \leq 1$ day).

daily closing returns, the distribution clearly shows a power-law tail (Fig. 3). The deviation is because of the decreasing size of the data set with increase in Δt . Note that, while for $\Delta t < 1$ day we have used the high-frequency data, for $\Delta t = 1$ day we have considered the longer data set of closing price returns for all stocks traded in NSE between Nov 1994 to May 2006.

To compare the distribution of returns in this emerging market with that observed in more advanced markets, we have considered the daily return data for the 500 stocks from NYSE listed in S&P 500 over the same period. As seen in Fig. 4, the distributions for NSE and NYSE are almost identical, implying that the price fluctuation distribution of emerging markets cannot be distinguished from that of developed markets, contrary to what has been claimed recently [13].

We now turn to the second question, and check whether it is possible to see any discernible change in the price fluctuation distribution as the stock market evolved over time. For this

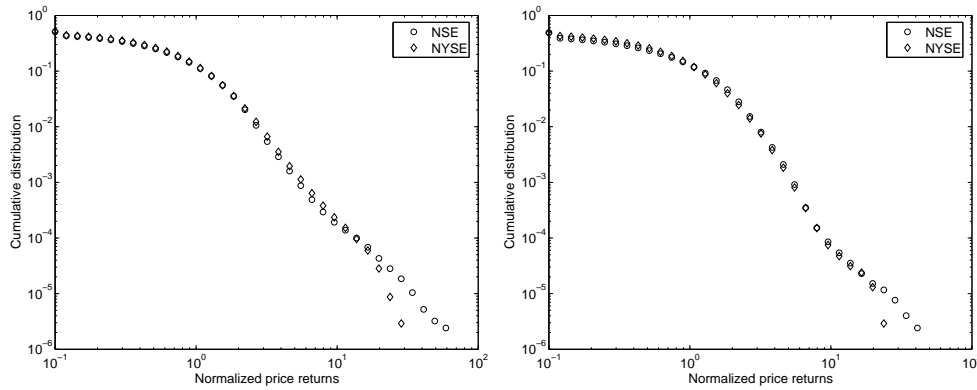


Fig. 4 – Comparison of the (left) positive and (right) negative tails of the cumulative normalized daily returns distribution for all stocks traded at NSE (o) and 500 stocks traded at NYSE (◇) during the period Nov 1994 to May 2006.

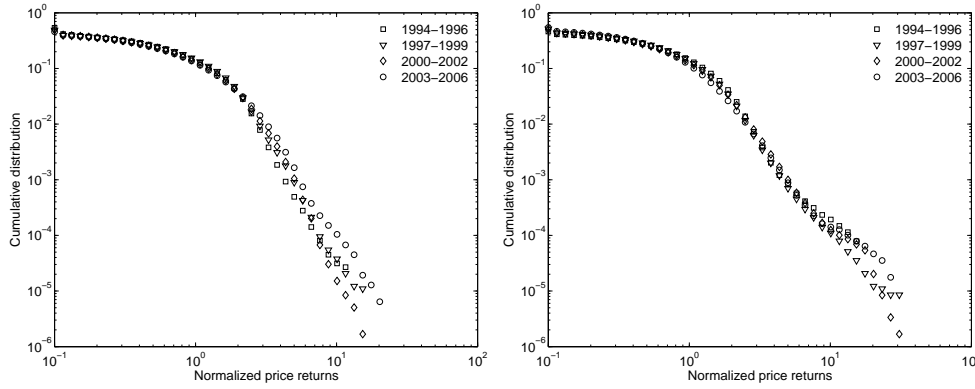


Fig. 5 – The (left) positive and (right) negative tails of the cumulative normalized daily returns distribution for all NSE stocks traded during the periods 1994-1996 (\square), 1997-1999 (∇), 2000-2002 (\diamond) and 2003-2005 (\circ).

we focus on the daily return distribution for *all* stocks that were traded during the entire period of existence of NSE. This period is divided into four intervals (a) 1994-1996, (b) 1997-1999, (c) 2000-2002, and (d) 2003-2006 [22], each corresponding to increase in the number of transactions by an order of magnitude. Fig. 5 shows that the return distribution at all four periods are similar, the negative tail even more so than the positive one. While the numerical value of the tail exponent may appear to have changed somewhat over the period that the NSE has operated, the power law nature of the tail is apparent at even the earliest period of its existence. We therefore conclude that the convergence of the return distribution to a power law functional form is extremely rapid, indicating that a market is effectively always at the non-equilibrium steady state characterized by the inverse cubic law.

We have also verified that stocks in the Bombay Stock Exchange (BSE), the second largest in India after NSE, follow a similar distribution [23]. Moreover, the return distribution of several Indian market indices (e.g., the NSE Nifty) also exhibit power law decay, with exponents very close to 3 [24]. As the index is a composite of several stocks, this behavior can be understood as a consequence of the power law decay for the tails of individual stock price returns, provided the movement of these stocks are correlated [7, 23]. Even though the Indian market microstructure has been refined and modernized significantly in the period under study as a result of the reforms and initiatives taken by the government, the nature of the return distribution has remained invariant, indicating that the nature of price fluctuations in financial markets is most probably independent of the level of economic development.

Discussion and Conclusion. – Most of the previous studies on emerging markets had focussed on either stock indices or a small number of stocks. In addition, all these studies were done with low-frequency daily data. This means that the number of data points used for calculating the return distribution were orders of magnitude smaller compared to ours. Indeed, the paucity of data can result in missing the long tail of a power law distribution and falsely identifying it to be an exponential distribution. Matia et al [13] claimed that differences in the daily return distribution for Indian and US markets were apparent even if one looks at only 49 stocks from each market. However, we found that this statement is critically dependent upon the choice of stocks. Indeed, when we made an arbitrary choice of 50 stocks in both Indian and US markets, and compared their distributions, we found them

to be indistinguishable. Therefore, the results of analysis done on such small data sets can hardly be considered stable, with the conclusions varying depending on the particular sample of stocks.

In this study, we have shown conclusively that the inverse cubic law for price fluctuations holds even in emerging markets. It is indeed surprising that the nature of price fluctuations is invariant with respect to large changes in the number of stocks, trading volume and number of transactions that have all increased significantly at NSE during the period under study. The robustness of the distribution implies that it should be possible to explain it independent of the particular features of different markets, or the various economic factors underlying them.

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