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Nonfactorizable effects in B to charmonium decays

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Abstract. Nonleptonic B to charmonium decays generally deviate from the factorization predictions. We study rescattering effects mediated by intermediate charmed mesons in this class of decay modes and, in particular, we consider $B^- \to K^- h_c$ with h_c the $J^{PC} = 1^{+-}$ $\bar{c}c$ meson, relating this mode to $B^- \to K^- \chi_{c0}$. We find $\mathcal{B}(B^- \to K^- h_c)$ large enough to be measured at the B factories, hence this process could be used to study the poorly known h_c.

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1 Introduction

Testing the Standard Model description of CP violation in the B sector requires a reduced theoretical uncertainty, a difficult task for nonleptonic decays since a general computational scheme has still to be developed. The amplitude of two-body nonleptonic B decays is given by the matrix element of the effective Hamiltonian for $B \to M_1 M_2$ [1]:

$$A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i c_i(\mu) \langle M_1 M_2 | \mathcal{O}_i(\mu) | B \rangle \,. \quad (1)$$

λ_i are CKM matrix elements, $c_i(\mu)$ Wilson coefficients evaluated at the scale μ and $\mathcal{O}_i$ four-quark operators. The naive factorization ansatz expresses their matrix elements as products of matrix elements of quark currents.

Let us consider $B^- \to K^- M_{\bar{c}c}$, where $M_{\bar{c}c}$ belongs to the charmonium system. Neglecting the annihilation term (suppressed by V_{ub}), the factorized amplitude is:

$$\mathcal{A}_F(B^- \to K^- M_{\bar{c}c}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \Big[a_2(\mu) + \sum_{i=3,5,7,9} a_i(\mu) \Big]$$
$$\langle K^- | (\bar{s}b)_{V-A} | B^- \rangle \times \langle M_{\bar{c}c} | (\bar{c}c)_{V\mp A} | 0 \rangle \quad (2)$$

with $a_2 = c_2 + c_1/N_c$, $a_i = c_i + c_{i+1}/N_c$. Eq.(2) shows the drawbacks of the approach: scale and scheme dependence of the $c_i(\mu)$ are no more compensated by that of the matrix elements; besides, the product of these does not contain any strong phase. An improvement consists in the generalized factorization, with $a_i(\mu)$ replaced by effective (process independent) parameters $a_i^{\rm eff}$ to be fixed using experimental data. Other methods, like QCD-improved factorization [2], PQCD [3], SCET [4], QCD sum rules [5], can only be applied to selected classes of nonleptonic modes.

Generalized factorization indicates nonfactorizable effects in $B \to K^- J/\psi$. The experimental $\mathcal{B}(B \to K^- J/\psi)$ can be fitted with $|a_2^{\rm eff}| = 0.2 - 0.4$ depending on the form factor parameterizing $\langle K^- | (\bar{s}b)_{V-A} | B^- \rangle$; $|a_2^{\rm eff}| = 0.38 \pm 0.05$ is obtained using the form factor in [6]. This must be compared to $a_2 = 0.163(0.126)$ computed in the naive dimensional regularization (or 't Hooft-Veltman) scheme [1]. The difference between $a_2^{\rm eff}$ and a_2 signals nonfactorizable effects. However, the clearest evidence of deviation from factorization is the observation of $B^- \to K^- \chi_{c0}$, with χ_{c0} the lightest $\bar{c}c$ scalar meson. The data:

$$\mathcal{B}(B^- \to K^- \chi_{c0}) = (6.0^{+2.1}_{-1.8} \pm 1.1) \times 10^{-4} \quad (3)$$
$$\mathcal{B}(B^- \to K^- \chi_{c0}) = (2.7 \pm 0.7) \times 10^{-4} \quad (4)$$

from BELLE [7] and BABAR [8], respectively, show that the experimental amplitude is non-zero, while the factorized amplitude vanishes since $\langle \chi_{c0} | (\bar{c}c)_{V\mp A} | 0 \rangle = 0$. Besides, the rate is comparable to $B^- \to K^- J/\psi$ since, for example, $\frac{\mathcal{B}(B^- \to K^- \chi_{c0})}{\mathcal{B}(B^- \to K^- J/\psi)} = (0.60^{+0.21}_{-0.18} \pm 0.05 \pm 0.08)$ [7].

QCD-improved factorization does not reproduce the measured branching ratios for $B^- \to K^- \chi_{c0}, K^- J/\psi$, giving either small results or producing infrared divergences, a signal of uncontrolled nonperturbative effects [9].

In [10] we investigated if the deviation from factorization in $B \to c\bar{c}$ decays may be ascribed to rescattering of intermediate charm mesons as in the diagrams in fig.1. We found that such effects could be large enough to produce the observed $\mathcal{B}(B^- \to K^- \chi_{c0})$. Other modes with vanishing factorized amplitude can test the rescattering picture, as $B^- \to K^- h_c$ with h_c the lowest lying $J^{PC} = 1^{+-}$ $\bar{c}c$ state. h_c is listed by the PDG among the particles to be confirmed, with $m_{h_c} \simeq 3526\, MeV$ [11]. If $B^- \to K^- h_c$ has a sizeable rate, this decay may be used to study h_c.

To improve the analysis in [10] we introduce an effective lagrangian for the interactions of all the low-lying $\ell = 1$ $c\bar{c}$ states to $D^{(*)}_{(s)}$ mesons, based on the spin symmetry for the heavy quark in the infinite mass limit. This relates all the couplings to a single parameter. The same holds for the couplings of $\ell = 0$ $\bar{c}c$ mesons to $D^{(*)}_{(s)}$.

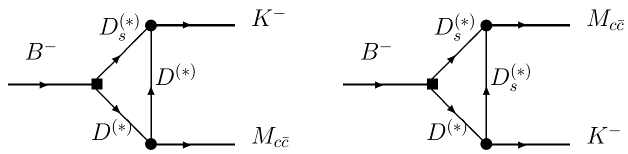

Fig. 1. Typical rescattering diagrams contributing to $B^- \to K^- M_{c\bar{c}}$, with $M_{c\bar{c}}$ belonging to the charmonium system.

2 Calculation of rescattering diagrams

The factorized amplitude $A_F(B^- \to K^- h_c)$ in (2) vanishes since $\langle h_c|(\bar{c}c)_{V\mp A}|0\rangle = 0$ due to conservation of parity and charge conjugation. However, the decay can proceed by rescattering induced by the same $(\bar{b}c)(\bar{c}s)$ effective Hamiltonian. We consider the process $B^- \to X^0_{\bar{u}c} Y^-_{\bar{c}s} \to K^- h_c$. The lowest lying intermediate states $X^0_{\bar{u}c}$, $Y^-_{\bar{c}s}$ are $D_s^{(*)-}$, $D^{(*)0}$ rescattering by exchange of $D^{(*)}_{(s)}$. To compute the diagrams in fig.1 we need the weak vertices $B \to D_s^{(*)} D^{(*)}$ and two strong vertices: the coupling of charmed mesons to kaon, and of the h_c to $D^{(*)}_{(s)}$ mesons. All those vertices are related to few parameters when $m_Q \to \infty$.

Interactions of mesons H_Q with a single heavy quark Q can be described using the Heavy Quark Effective Theory, exploiting the heavy quark spin and flavour symmetries holding for $m_Q \to \infty$. In this limit the heavy quark four velocity v coincides with that of the hadron and is conserved by strong interactions [12]. Because of the invariance under rotations of the heavy quark spin s_Q, states differing only for s^3_Q are degenerate and form a doublet. When the orbital angular momentum of the light degrees of freedom relative to Q is $\ell = 0$, the states in the doublet have $J^P = (0^-, 1^-)$, corresponding to $(D_{(s)}, D^*_{(s)})$, $(B_{(s)}, B^*_{(s)})$. The doublet is represented by the matrix $H_a = P_+[M^\mu_a \gamma_\mu - M_a \gamma_5]$ with $P_\pm = (1 \pm \not{v})/2$. M^μ is the vector state, M the pseudoscalar one (a is a light flavour index). M_a, M^*_a contain a factor $\sqrt{m}$, with m the meson mass.

Let us consider $B^- \to D_s^{(*)-} D^{(*)0}$, for which factorization empirically works [13]. The factorized amplitude is:

$$\langle D_s^{(*)-} D^{(*)0}|H_W|B^-\rangle = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} a_1$$
$$\langle D^{(*)0}|(V-A)^\mu|B^-\rangle \times \langle D_s^{(*)-}|(V-A)_\mu|0\rangle \qquad (5)$$

with $a_1 = c_1 + c_2/N_c$. In the heavy quark limit, the matrix elements in (5) can be written in terms of the Isgur-Wise function ξ, and a single leptonic constant $\hat{F}$ [12].

The $D_s^{(*)} D^{(*)} K$ couplings, in the soft kaon limit, are related to a single parameter g through the effective lagrangian describing strong interactions of charmed mesons with the octet of the light pseudoscalar mesons [14]: $\mathcal{L}_I = i\, g\, Tr[H_b \gamma_\mu \gamma_5 A^\mu_{ba} \bar{H}_a]$. In $\mathcal{L}_I$: $A_{\mu ba} = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right)_{ba}$, $\bar{H}_a = \gamma^0 H^\dagger_a \gamma^0$ and $\xi = e^{\frac{i\mathcal{M}}{f}}$, with $f \simeq f_\pi = 131$ MeV and $\mathcal{M}$ a 3×3 matrix with the light pseudoscalar meson fields.

We now consider the strong vertex involving the h_c. For $Q_1\bar{Q}_2$ mesons heavy quark flavour symmetry does not hold any more, but degeneracy is expected under rotations of the two heavy quark spins. This allows us to build up multiplets for each value of ℓ. For $\ell = 0$ one has a $J^P = (0^-, 1^-)$ doublet. The corresponding matrix is [15]: $R^{(Q_1\bar{Q}_2)} = P_+[L^\mu \gamma_\mu - L\gamma_5]P_-$ with $L^\mu = J/\psi$, $L = \eta_c$ for $\bar{c}c$. For $\ell = 1$, the multiplet is $P^{(Q_1\bar{Q}_2)\mu} = P_+\left(\chi_2^{\mu\alpha}\gamma_\alpha + \frac{1}{\sqrt{2}}\epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta \chi_{1\gamma} + \frac{1}{\sqrt{3}}(\gamma^\mu - v^\mu)\chi_0 + h_1^\mu \gamma_5\right)P_-$. In the case of $\bar{c}c$, $\chi_2 = \chi_{c2}$, $\chi_1 = \chi_{c1}$, $\chi_0 = \chi_{c0}$ correspond to the spin triplet, $h_1 = h_c$ to the spin singlet [16].

For $\ell = 1$ $Q_1\bar{Q}_2$ states, the most general lagrangian describing the coupling to two heavy-light mesons $Q_1\bar{q}_a$ and $q_a\bar{Q}_2$ can be written as follows [17]:

$$\mathcal{L}_1 = i\frac{g_1}{2} Tr\left[P^{(Q_1\bar{Q}_2)\mu} \bar{H}_{2a} \gamma_\mu \bar{H}_{1a}\right] + h.c. + (Q_1 \leftrightarrow Q_2)\,,$$

where $H_{2a} = [M'^\mu_a \gamma_\mu - M'_a \gamma_5]P_-$ describes $q_a\bar{Q}_2$ mesons. $\mathcal{L}_1$ is invariant under independent rotations of the spin of the heavy quarks. g_1 describes the interaction of heavy-light mesons with the three χ_c states and with h_c, relating the couplings in absolute value and in sign as well. This allows a proper analysis of the diagrams in fig.1.

The interactions of $\ell = 0$ states $R^{(Q_1\bar{Q}_2)}$ with the heavy-light $J^P = (0^-, 1^-)$ mesons proceed in P-wave and can be described by a lagrangian with a derivative:

$$\mathcal{L}_2 = \frac{g_2}{2} Tr\left[R^{(Q_1\bar{Q}_2)} \bar{H}_{2a} \overleftrightarrow{\not{\partial}}\, \bar{H}_{1a}\right] + h.c. + (Q_1 \leftrightarrow Q_2)$$

again invariant under heavy quark spin rotations.

g_1 and g_2 can be estimated using vector meson dominance (VMD). One can consider $\langle D(v')|\bar{c}c|D(v)\rangle$, assuming the dominance in the t-channel of the 0^+ $\bar{c}c$ state, and using the normalization $\xi(1) = 1$. One obtains $g_{DD\chi_{c0}} = 2m_D m_{\chi_{c0}}/f_{\chi_{c0}}$, where $f_{\chi_{c0}}$ is defined by $\langle 0|\bar{c}c|\chi_{c0}(q)\rangle = f_{\chi_{c0}} m_{\chi_{c0}}$. This relation gives: $g_1 = -\sqrt{m_{\chi_{c0}}}/(\sqrt{3} f_{\chi_{c0}})$. The same argument gives $g_2 = \sqrt{m_\psi}/(2m_D f_\psi)$, where f_ψ is defined by $\langle 0|\bar{c}\gamma^\mu c|J/\psi(p,\epsilon)\rangle = f_\psi m_\psi \epsilon^\mu$. To account for the off-shell effect of the exchanged particles, the virtuality of which can be large, we write: $g_i(t) = g_{i0}\, F_i(t)$, with g_{i0} the on-shell couplings and: $F_i(t) = (\Lambda_i^2 - m^2_{D^{(*)}})/(\Lambda_i^2 - t)$. The parameters Λ_i represent a source of uncertainty.

In the numerical analysis we exploit the heavy quark limit, putting $f_{D^*_s} = f_{D_s}$ and use $f_{D_s} = 240$ MeV [5]. For the Isgur-Wise function, we use $\xi(y) = (2/(1+y))^2$, compatible with data from semileptonic $B \to D^{(*)}$ decays. As for g, CLEO Collaboration obtained: $g = 0.59 \pm 0.01 \pm 0.07$ by measuring $\Gamma(D^*)$ and the D^* branching fraction to $D\pi$ [18]. This value should be compared to theoretical estimates ranging from $g \simeq 0.3$ up to $g \simeq 0.77$ [19]. Since the rescattering amplitudes always depend on $g \cdot F_i(t)$, we put $\Lambda_i = \Lambda$ for all i and use $g = 0.59$, leaving to Λ the task of spanning the range of possible variation of g.

For g_1 and g_2 we use the VMD relations, together with the QCD sum rule result $f_{\chi_{c0}} = 510 \pm 40$ MeV [10] and the experimental value $f_{J/\psi} = 405 \pm 14$ MeV.

We computed the imaginary part of the rescattering diagrams; the determination of the real part is more uncertain. Using a dispersive representation, we obtained for

$B^- \to K^- \chi_{c0}(J/\psi)$ that $\mathrm{Re}\mathcal{A}_i \simeq \mathrm{Im}\mathcal{A}_i$ [10]. Hence we account for the real part of the amplitudes considering them as fractions of the imaginary part varying from 0 to 100%. Such an uncertainty will affect the final result.

We constrain Λ considering rescattering contributions to $B^- \to K^- J/\psi$, where $\mathcal{A}(B^- \to K^- J/\psi) = \mathcal{A}_{fact} + \mathcal{A}_{resc}$ is bounded by experimental data. For $\Lambda \simeq 2.6 - 3$ GeV, the sum does not exceed the experimental bound. Moreover, we repeat the analysis in [10] for $B^- \to K^- \chi_{c0}$, using now the relations stemming from the lagrangian $\mathcal{L}_1$. We get a branching ratio compatible with the experimental result from BABAR if Λ is varied around 3.0 GeV.

Using such constraints we analyze $B^- \to K^- h_c$. In fig.2 we plot the branching ratio versus Λ. We find [17]:

$$\mathcal{B}(B^- \to K^- h_c) = (2 - 12) \times 10^{-4} \ , \qquad (6)$$

where we accounted for the uncertainty on the real part of the amplitudes and on the variation of Λ. This result suggests $\mathcal{B}(B^- \to K^- h_c)$ large enough to be measured at the B-factories. Moreover, this mode represents a sizeable fraction of the inclusive $B^- \to X h_c$ decay, for which, considering the production of the $c\bar{c}$ pair in h_c in the color-octet state, $\mathcal{B}(B^- \to h_c X) = (13 - 34) \times 10^{-4}$ is predicted [20].

The main uncertainty affecting our results is due to cancellations between different amplitudes, which are of similar size. Another uncertainty is due to the neglect of contributions of higher states, even though a minor role can be presumed for higher resonances.

Bearing such uncertainties in mind we can conclude that rescattering terms may contribute to the nonfactorizable effects observed in $B \to$ charmonium transitions.

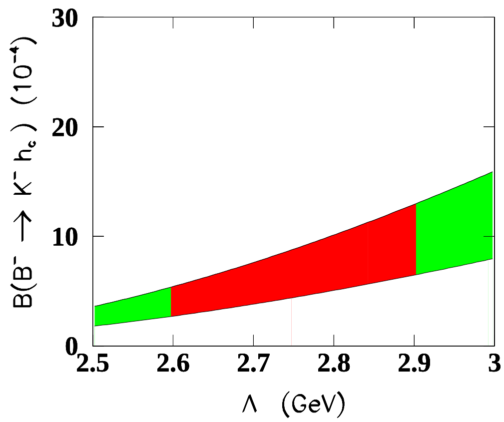

Fig. 2. Branching ratio $\mathcal{B}(B^- \to K^- h_c)$ versus Λ. The lowest curve corresponds to $\mathrm{Re}\mathcal{A}_i = 0$, the highest one to $\mathrm{Re}\mathcal{A}_i = \mathrm{Im}\mathcal{A}_i$. The dark region corresponds to the result (6).

3 Conclusions

The h_c was observed in $p\bar{p}$ annihilation and in $p -$ Li interactions [11]. In B decays, one could access h_c looking either at its hadronic modes: $h_c \to J/\psi \pi^0$, $\rho^0 \pi^0$, etc. , or at its radiative modes: $h_c \to \eta_c \gamma$, $\chi_{c0}\gamma$, etc. The channel $h_c \to \eta_c \gamma$ seems promising, as noticed by Suzuki who estimated: $\mathcal{B}(h_c \to \eta_c \gamma) \simeq 0.50 \pm 0.11$ [22]. A similar result: $\mathcal{B}(h_c \to \eta_c \gamma) = 0.377$ is obtained in [23]. These two predictions, together with the experimental $\mathcal{B}(\eta_c \to K \bar{K} \pi)$ and our result (6), allow us to predict: $\mathcal{B}(B^- \to K^- h_c \to K^- \eta_c \gamma \to K^- K \bar{K} \pi \gamma) = (4 - 26) 10^{-6}$, within the reach of current experiments.

As for rescattering effects in $B \to$ charmonium decays, we found that they can produce a branching ratio for $B^- \to K^- h_c$ comparable with that of $B^- \to K^- \chi_{c0}$. The same holds for $B^- \to K^- \psi(3770)$ which, because of the smallness of $f_{\psi(3770)}$, is predicted by factorization with a tiny branching ratio. The experimental measure $\mathcal{B}(B^- \to K^- \psi(3770)) = (0.48 \pm 0.11 \pm 0.12) \times 10^{-3}$ [24] is a further evidence of large nonfactorizable effects. In our approach, using $g_{DD\psi(3770)} = 14.94 \pm 0.86$ obtained from the width of $\psi(3770)$, we get $\mathcal{B}(B^- \to K^- \psi(3770)) = (0.9 - 4) \times 10^{-4}$. Similar conclusion applies to $B^- \to K^- \chi_{c2}$ with χ_{c2} the $J^{PC} = 2^{++}$ $c\bar{c}$ state, the factorized amplitude of which also vanishes. The observation of this process with branching ratio comparable to $\mathcal{B}(B^- \to K^- \chi_{c0})$, $\mathcal{B}(B^- \to K^- h_c)$ would support the rescattering picture.

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