AROUND DARMOIS - SKITOVICH AND GHURYE - OLKIN - ZINGER THEOREMS.

IBRAGIMOV I.

1. Consider independent random variables $X_1, \dots X_n$ and two linear statistics

$$L_1 = \sum_1^n a_j X_j, \quad L_2 = \sum_1^n b_j X_j,$$

where $a_j, b_j \neq 0$ are real coefficients.

The Darmois (1953), Skitovich (1953) theorem says that if the statistics L_1, L_2 are independent, the random variables X_j have normal distributions.

The theorem has been extended by Mamai (1960) and Ramachandran (1967) on the case of an infinite number of variables X_j. The Ramachandran theorem says that if the forms L_1, L_2 are independent and both sequences $\{a_j b_j^{-1}\}, \{a_j^{-1} b_j\}$ are bounded, the variables X_j are normal.

Theorem 0.1. *Let $X_1, \dots X_n, \dots$ be independent random variables. If the forms*

$$L_1 = \sum_1^\infty a_j X_j, \quad L_2 = \sum_1^\infty b_j X_j, \quad a_j, b_j \neq 0,$$

are independent and at least one of the sequences $\{a_j b_j^{-1}\}, \{a_j^{-1} b_j\}$ is bounded, the random variables X_j are normal.

2. Ghurye and Olkin (1962) proved a multivariant analogue of the Darmois - Skitovich theorem. They considered linear statistics

$$L_1 = \sum_1^n A_j X_j, \quad L_2 = \sum_1^n B_j X_j,$$

where now X_j are d-dimensional independent random vectors and A_j, B_j are non-singular $d \times d$ real matrices. Ghurye and Olkin proved that the independence of L_1, L_2 implies the normality of the vectors X_j. Later Zinger (1979) proved the following multidimensional analogue of Ramachandran's theorem:

Let $X_1, \dots X_n, \dots$ be independent d-dimensional random vectors. Consider two linear statistics

$$L_1 = \sum_1^\infty A_j X_j, \quad L_2 = \sum_1^\infty B_j X_j,$$

where A_j, B_j are non-singular $d \times d$ matrices. If the forms L_1, L_2 are independent and both sequences $\{A_j^{-1} B_j A_j^{-1}\}, \{A_j B_j^{-1}\}$ are bounded, the random vectors are normal.

This Zinger's result is a solution of a problem from the famous Kagan, Linnik, Rao book.

Theorem 0.2. *Let $X_1, \dots X_n, \dots$ be independent d-dimensional random vectors. Consider two linear statistics*

$$L_1 = \sum_1^\infty A_j X_j, \quad L_2 = \sum_1^\infty B_j X_j,$$

where A_J, B_j are non-singular $d \times d$ matrices. Let the forms L_1, L_2 are independent and let the sequences $\{A_j^{-1} B_j A_j^{-1}\}, \{A_j B_j^{-1}\}$ satisfy the following conditions:

1. at least one of the sequences $\{A_j^{-1} B_j A_j^{-1}\}, \{A_j B_j^{-1}\}$ is bounded;

2. If $\{C_j\}$ is the bounded sequence of the condition 1, the numerical sequence $||C_j|| \cdot ||C_j^{-1}||$ is bounded.

Then the random vectors X_j are normal.