Mathematical Model of Coulomb Field Transport in Non-relativistic Case

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One of the motives for expanding classical Maxwell electrodynamics is the absence of Coulomb field transport, this factor leads to the appearance of long-range action in electrodynamics. Coulomb field transport should be realized by means of longitudinal wave, but, as is known, classical Maxwell equations contain only transverse wave solutions. In the framework of quantum electrodynamics, an approach is developed for describing the Coulomb interaction of charged particles by way of a longitudinal photons (more precisely - "time" photons)[3], although the latter are to be considered non-physical (virtual), otherwise, the quantum theory faces a number of unsurmountable difficulties [4, 5]. However, the methods for excluding non-physical photons elaborated in the framework of quantum electrodynamics (see review [5]) exclude also the wave transport of the Coulomb field in this theory. The number of works devoted to expansion of the classical Maxwell electrodynamics grows in recent years [6, 7, 8, 9].

In works [1, 2] a non-relativistic theory of Coulomb field transport was proposed. Our approach, from a view point of continued physics, is based on the existing analogy between electrodynamics and linear elasticity theory, which allows one to consider electromagnetic vacuum as a compressible medium. From the viewpoint of the quantum-field concept, the considered expansion of Maxwell electrodynamics is based on the assumption that the Coulomb field is a superposition of scalar photons or superposition of massless scalar particles with a zero spin, rather than a superposition of vector photons with a zero spin projection [1]. At the same time, it is assumed that the introduced scalar particles realize physical conditions of the field, in other words, they appear as observables. Constructively, this assumption is expressed in that along with the 4-vector potential A^μ, there exists a 4-scalar massless field λ. In the non-relativistic case, the equations for potentials take the form [1]:

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \Delta \mathbf{A} = -\frac{4\pi}{c}\mathbf{j}, \tag{1}$$

$$-\frac{1}{c^2}\frac{\partial^2 \lambda}{\partial t^2} + \Delta \lambda = -4\pi\rho. \tag{2}$$

Here, c is the velocity of light in vacuum, ρ and $\mathbf{j}$ are, respectively, the density of charges and density of transfer current, and $\mathbf{A}$ is the spatial component of the 4-vector potential A^μ (the time component φ of the 4-potential can be excluded via gauge transformations [10]), with $\mathbf{A}$ satisfying the Coulomb condition: $\mathrm{div}\mathbf{A} = 0$. By introducing the following field definitions

$$\mathbf{E}_\perp = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{H} = \mathbf{rot}\ \mathbf{A}, \tag{3}$$
$$\mathbf{E}_{||} = \nabla\lambda, \qquad W = -\frac{1}{c}\frac{\partial \lambda}{\partial t},$$

one obtains two systems of equations from (1). The first system is for the fields $\mathbf{E}_\perp$ and $\mathbf{H}$:

$$-\frac{1}{c}\frac{\partial \mathbf{E}_\perp}{\partial t} + \mathbf{rot}\ \mathbf{H} = \frac{4\pi}{c}\mathbf{j},$$
$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} + \mathbf{rot}\ \mathbf{E}_\perp = 0, \tag{4}$$

while the second one is for the fields $\mathbf{E}_{||}$ and W, being henceforth referred to as a system of electroscalar fields:

$$\frac{1}{c}\frac{\partial \mathbf{E}_{||}}{\partial t} + \nabla W = 0,$$
$$\frac{1}{c}\frac{\partial W}{\partial t} + \mathrm{div}\mathbf{E}_{||} = -4\pi\rho. \tag{5}$$

It is seen from these systems that the electrostatics equation $\mathrm{div}\mathbf{E}_{||} = -4\pi\rho$ follows from the system (5) and the magnetostatics one $\mathbf{rot}\ \mathbf{H} = 4\pi\mathbf{j}/c$ follows from (4). System (4) describes propagation of strictly transverse waves (proof for this can be found, for example, in [10]), and system (5), in contrast, describes propagation of longitudinal waves. Now, let us consider the vacuum solution of system (5) in the form of plane waves $\mathbf{E}_{||} = \mathbf{E}_0 \exp[i(\omega t + \mathbf{kr})], W = W_0 \exp[i(\omega t + \mathbf{kr})]$, where $\mathbf{E}_0$ and W_0 are the amplitudes, $\mathbf{k}$ is the wave vector defining the direction of propagation of a wave with the frequency ω. Then, we obtain from (5):

$$\frac{\omega}{c}\mathbf{E}_0 + \mathbf{k}W_0 = 0,$$
$$\frac{\omega}{c}W_0 + (\mathbf{kE}_0) = 0, \quad \mathbf{k}\times\mathbf{E}_0 = 0,$$

in other words, the vector $\mathbf{E}_{||}$ oscillates along the direction of wave propagation. The dispersion relationship for the given wave takes the normal form $\omega = kc$.

In order to derive the equations of energy balance for the fields $\mathbf{E}_{||}$ and W, let us multiply the first equation (5) scalarwise by $\mathbf{E}_{||}$ and the second equation by W, and obtain after volume integration:

$$\frac{\partial}{\partial t}\int \epsilon_{EW} dV + \oint \mathbf{s}_{EW}\mathbf{d\sigma} = -c\int (\rho W) dV \tag{6}$$

where $\mathbf{d}\sigma$ is the surface area element for the volume V and the following definitions are introduced for the energy density ϵ_{EW} and for the energy flux vector $\mathbf{s}_{EW}$ of the electroscalar field:

$$\epsilon_{EW} = \frac{\mathbf{E}_{||}^2 + W^2}{8\pi}, \tag{7}$$

$$\mathbf{s}_{EW} = \frac{c}{4\pi}\mathbf{E}_{||}W \tag{8}$$

From the definition of the vector $\mathbf{s}_{EW}$ it follows that the energy flux in the above plane wave is collinear to the vector of the field $\mathbf{E}_{||}$. Of particular interest is the integrand ρW which enters into the right-hand side (6). Using the definition $W = -\partial\lambda/\partial(ct)$ and continuity equation $\partial\rho/\partial t + \mathrm{div}\mathbf{j} = 0$, this expression can be transformed in the following manner:

$$\begin{aligned} \rho W = -\rho\frac{1}{c}\frac{\partial\lambda}{\partial t} = \\ -\frac{\partial}{\partial t}\left(\frac{1}{c}\lambda\rho\right) + \frac{1}{c}\lambda\frac{\partial\rho}{\partial t} = \\ -\frac{\partial}{\partial t}\left(\frac{1}{c}\lambda\rho\right) - \mathrm{div}\left(\frac{1}{c}\lambda\mathbf{j}\right) + \frac{1}{c}\mathbf{j}\nabla\lambda. \end{aligned} \tag{9}$$

By virtue of the definition $\mathbf{E}_{||} = \nabla\lambda$, (6) can be presented in the form

$$\frac{\partial}{\partial t}\int(\epsilon_{EW} - \rho\lambda)dV + \oint(\mathbf{s}_{EW} - \lambda\mathbf{j})\mathbf{d}\sigma = -\int(\mathbf{j}\mathbf{E}_{||})dV$$

The first integral in this formula represents the total energy of the system of fields $\mathbf{E}_{||}$ and W interacting with charges. The summand $\lambda\mathbf{j}$, which can be interpreted as interaction energy transfer at the expense of charge motion, has been added into the surface integral, and taking into consideration that $\mathbf{j} = \rho\mathbf{v}$, one obtains $\lambda\mathbf{j} = (\rho\lambda)\mathbf{v}$. In other words, the total flux consists of two components. The first one is responsible for the energy transfer by way of electroscalar radiation, and the second one is associated with the mechanical outflow of charges from the area limited by the integration surface. The system of equations (5) can be presented in the form of a system of wave equations for the fields $\mathbf{E}_{||}$ and W:

$$-\frac{1}{c^2}\frac{\partial^2\mathbf{E}_{||}}{\partial t^2} + \Delta\mathbf{E}_{||} = -4\pi\nabla\rho, \quad \mathbf{rot}\ \mathbf{E}_{||} = 0,$$

$$-\frac{1}{c^2}\frac{\partial^2 W}{\partial t^2} + \Delta W = \frac{4\pi}{c}\frac{\partial\rho}{\partial t} \tag{10}$$

It is seen from these equations that time-dependent non-homogeneous charge density serves as a source for the longitudinal waves, while the radial oscillations of the electrons of a spherically symmetric metal particle, mentioned in the introduction, can serve as an example of a source for longitudinal electroscalar waves. Radial oscillations of the electron gas of a spherically symmetric metallic particle, which are sketched out in Fig. 1, can serve as a possible macroscopic source for such a longitudinal wave.

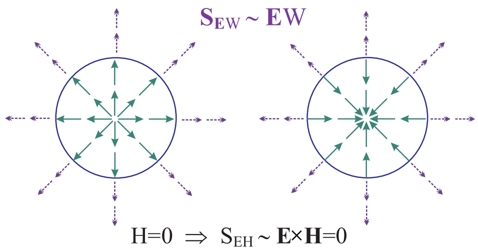

Figure 1: Radial electron current in a spherical metallic particle. The arrows show the direction of transfer of the electron current flowing strictly along the radius

Owing to the spherical symmetry, the magnetic field of the fluctuating radial transfer current equals zero and, thus, radiation losses of this system can be realized only at the expense of radiation of a longitudinal wave in which the electric-field vector is collinear to the wave energy flux vector.

Thus, the developed formalism allows one to describe the wave transport of the Coulomb field using a longitudinal electroscalar wave.

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