Berkeley Economic History Laboratory

BEHL WORKING PAPER SERIES

WP2015-01

If Not Malthusian, Then Why?

Lemin Wu

February 2015

Berkeley Economic History Laboratory (BEHL)
University of California, Berkeley
530 Evans Hall / MC 3880
Berkeley, CA 94602
http://behl.berkeley.edu

Berkeley Economic History Laboratory, University of California, Berkeley, 530 Evans Hall, Berkeley, CA 94720

BEHL Working Paper WP2015-01 | February 2015
http://behl.berkeley.edu/wp/2015-01

If Not Malthusian, Then Why?

Abstract
This paper shows that merely the Malthusian mechanism cannot explain the pre-industrial stagnation of living standards. Technological improvement in luxury production, if faster than improvement in subsistence production, would have kept living standards growing. The Malthusian trap is essentially a puzzle of balanced growth between the luxury sector and the subsistence sector. The author argues that the balanced growth is caused by group selection in the form of biased migration. It is proved that a tiny bit of bias in migration can suppress a strong tendency of growth. The theory reexplains the Malthusian trap and the prosperity of ancient market economies such as Rome and Song. It also suggests a new set of triggering factors of modern economic growth.

Keywords: Malthusian trap, group selection, very long run growth, source-sink migration
JEL classification: N3, O41, B12

Lemin Wu School of Economics, Peking University leminwu@pku.edu.cn

If Not Malthusian, Then Why?

Lemin Wu*

February 2015†

ABSTRACT: This paper shows that merely the Malthusian mechanism cannot explain the pre-industrial stagnation of living standards. Technological improvement in luxury production, if faster than improvement in subsistence production, would have kept living standards growing. The Malthusian trap is essentially a puzzle of balanced growth between the luxury sector and the subsistence sector. The author argues that the balanced growth is caused by group selection in the form of biased migration. It is proved that a tiny bit of bias in migration can suppress a strong tendency of growth. The theory reexplains the Malthusian trap and the prosperity of ancient market economies such as Rome and Song. It also suggests a new set of triggering factors of modern economic growth.

KEYWORDS: Malthusian trap, group selection, very long run growth, source-sink migration
JEL CLASSIFICATION: N3, O41, B12

*School of Economics, Peking University, leminwu@pku.edu.cn. I would like to thank David Levine, Nicholas Papageorge, and Rohan Dutta, who co-author a related paper with me, and George Akerlof, Gerard Roland, Ronald Lee, Bradford DeLong, Pranab Bardhan, Martha Olney, Gregory Clark, Peter Lindert, Jan De Vries, Barry Eichengreen, Yingyi Qian, Yang Xie, Yuriy Gorodnichenko, Jian-Cheng Lai, and the attendants of seminars at Peking University, UC Berkeley, UC Davis, and Tsinghua University for thoughtful suggestions and essential encouragement. I would especially thank Gregory Clark for his insightful and provocative introduction to Malthusian theory in *A Farewell to Alms*, which inspired this work. The usual disclaimers apply.

†First draft: November 2013. An earlier version of this paper, "Does Malthus Really Explain the Malthusian Trap?" is the first chapter of the author's doctoral dissertation.

1 Introduction

> All other arguments are of slight and subordinate consideration in comparison of this. I see no way by which man can escape from the weight of this law which pervades all animated nature.
>
> Thomas Malthus (1809, chap.1)

> But in economics, the admission that mankind need not live at the margin of subsistence [...] meant that, the very long run limit of wages was not physiological subsistence, it was *psychological subsistence*—a much more complicated and difficult matter to formulate exactly.
>
> Lionel Robbins (1998, pg.174)

Life was miserable for most who lived before the Industrial Revolution. "The average person in the world of 1800 was no better off than the average person of 100,000 BC" (Clark, 2008). According to Malthus, poverty lingers because economic progress leads to faster population growth. A larger population depresses average income and brings society back to persistent poverty.

This paper shows that merely Malthus's mechanism cannot explain the Malthusian trap. Technological improvement in luxury production, if faster than improvement in subsistence production, would have kept living standards increasing. To suppress luxury, it takes the force of Darwinian selection in a social context: those cultures and technologies that favor group prosperity at the expense of individual welfare spread faster than the others. So the Malthusian trap is meanwhile a Darwinian trap.

Explaining the Malthusian trap is the core value of Malthusian theory. Despite empirical weakness of the relationship between average income and population growth, most economists hold a Malthusian view of history because Malthus explains the persistent poverty of the pre-industrial world, and no competing theory is ever available. However, Malthus's success of explaining the Malthusian trap is an illusion. The theoretical prediction is right but the mechanism is wrong, analogous to a detective figuring out the true murderer by a false reconstruction of the scene of crime. The judge should not be satisfied with a mere lucky guess.

Malthus's theory relies on a crucial assumption. He assumes away the conflict between individual welfare and group fitness. Whereas in the real world, what promote the individual's welfare do not always expand the group's population. Diamonds, spas, circuses, monuments—be they for sexual attraction, comfort, entertainment or vanity—are immune from the Malthusian force. Increases in these consumption could raise living standards without limits.

Take flower and bread as a metaphor. Flowers attract the mates; bread feeds the mouths. Population increases with the bread but not with the flowers. Choice-theoretically, when flowers are more, people are simply better off. As the average bread is anchored by the Malthusian force, the equilibrium living standards are fully determined by how many flowers the average person consumes, which in turn depends on the ratio of flowers to bread in the economy.

I call flowers the "luxury" and bread the "subsistence". Both contain hedonistic value, but one dollar worth of luxury has a smaller demographic effect than one dollar worth of subsistence (the division is relative: beef is a luxury relative to potato, yet more of a subsistence than diamonds).

The two-sector model allows living standards to vary much more wildly than Malthusianists presume. It allows culture and technology to change equilibrium living standards while Malthusian constraint is still binding. What matters is not the size of economy but the structure of production, not the aggregate demand but the relative preference, i.e. how valuable people regard one thing as compared with another. By affecting the ratio of luxury to subsistence, the variation in production structure and social preference can explain a large portion of fluctuations in living standards that have been misattributed to changes in fertility culture and disease environment when the two-sector model is unavailable.

It follows that the Romans were rich not because technological progress temporarily exceeded population growth—as Malthusianists claim—but because Rome had a business-friendly legal system and an active market economy. Well-functioning courts and marketplaces boost industry more than they boost agriculture. Thus biasing production structure to luxury, they raised the average living standards of the whole society.

Conversely, the Agricultural Revolution left an unfortunate legacy: the hunter-and-gatherer-turned peasants failed to achieve the level of leisure and nutrition their ancestors once enjoyed (Diamond, 1987). Growth was immiserizing because agriculture biased production structure to subsistence. The same tragedy recurred when potato dominated the Irish diet in the late 18$^{\text{th}}$ century.

However, the two-sector model raises a serious puzzle. If the luxury productivity had been growing slightly faster than the subsistence productivity, living standards would have been rising steadily, but this had never happened until the modern era. The lack of trend in the average living standards implies that the luxury sector and the subsistence sector had the same speed of growth. The balance of growth lies at the heart of the question why the Malthusian trap had ever existed, but Malthus was never aware of the puzzle, to say nothing of addressing it.

After rejecting three alternative explanations, I show that the balanced growth is caused by biased migration. Since living standards increase with the ratio of luxury to subsistence, migration is usually from places relatively rich in subsistence to places relatively rich in luxury. Subsistence technologies are thus more likely to be carried around by migrants than luxury technologies. Even if the latter intrinsically grow faster, the advantage of subsistence technologies in spread cancels out the advantage of luxury technologies in growth. The mechanism is robust to varying parameters. A tiny bit of bias in migration (say, if people are extremely reluctant to move and slow to learn) can still suppress a strong tendency of growth.

As migration goes on, the luxury-rich groups have a higher death rate than the birth rate. The under-reproduction is filled up with continuous immigration from the subsistence-rich groups whose birth rate exceed the death rate. The luxury-rich regions thus become a demographic sink that devours the surplus population of the subsistence-rich regions. Since migrants also spread ideas, the source-sink migration disseminates the ways of life that strengthens the group at the expense of individual welfare. Hence I call the Darwinian-style mechanism as group selection of culture and technology.

The paper is naturally composed of two parts. The first part is the two-sector Malthusian model, which raises the balanced growth puzzle (section 3). The second part addresses the puzzle with the group selection theory (section 4).

Malthusian theory is no easy target. To replace it, I (need to) demonstrate three things. First, I show that sectoral division is a salient feature of historical data (section 3.4). Second, I uncover the source-sink pattern in historical migrations (section 4.2). Third, I simulate and mathematically prove that living standards would grow in a Malthusian economy unless selection is introduced, and that a tiny bit of selection can suppress a strong tendency of growth (section 4.3 and 4.4).

In section 5, I discuss the implications of the theory on some major issues of economic history, including the Agricultural Revolution (section 5.1), the rise and fall of ancient market economies (section 5.2), the long-term impacts of wars and migrations (section 5.3), and the Industrial Revolution (section 5.4). Section 6 concludes.

2 Literature review

How sound is the Malthusian fact? Figure 1 shows Maddison's estimates of the world's GDP per capita for the last two millennia. Using Maddison's data, Ashraf and Galor (2011) confirmed that by year 1500, the level of a country's technology explains the country's density of population, but not the income per capita.

However, Maddison's "guesstimates" might be contaminated with a Malthusian presumption. A bunch of revisions in the past few years have largely wiped away the humdrum picture of ancient economic life in the original Maddison series. It was shown that during the pre-modern centuries, while Italy and Spain experienced stagnation and even declines (Malanima, 2011; Álvarez-Nogal and De La Escosura, 2013), the per capita GDP of England doubled between 1270 and 1700, and that of the Netherlands almost tripled between 1000 and 1500. Researchers questioned when sustained growth actually started (Hersh and Voth, 2009; Persson, 2010; Fouquet, 2014), but except Wu et al. (2014), none asked whether the Malthusian trap existed or not. The take-home message of most of the research is that living standards fluctuation was much larger than previously thought—the findings disturb Malthusian theory but not the Malthusian fact of long-term stagnation. Even Wu et al. (2014) agree that the European per capita GDP failed to recover from the collapse of Roman Empire until perhaps the dawn of the Industrial Revolution. Hence it is fair to say that the existence of the Malthusian trap is still a widely-held fact.

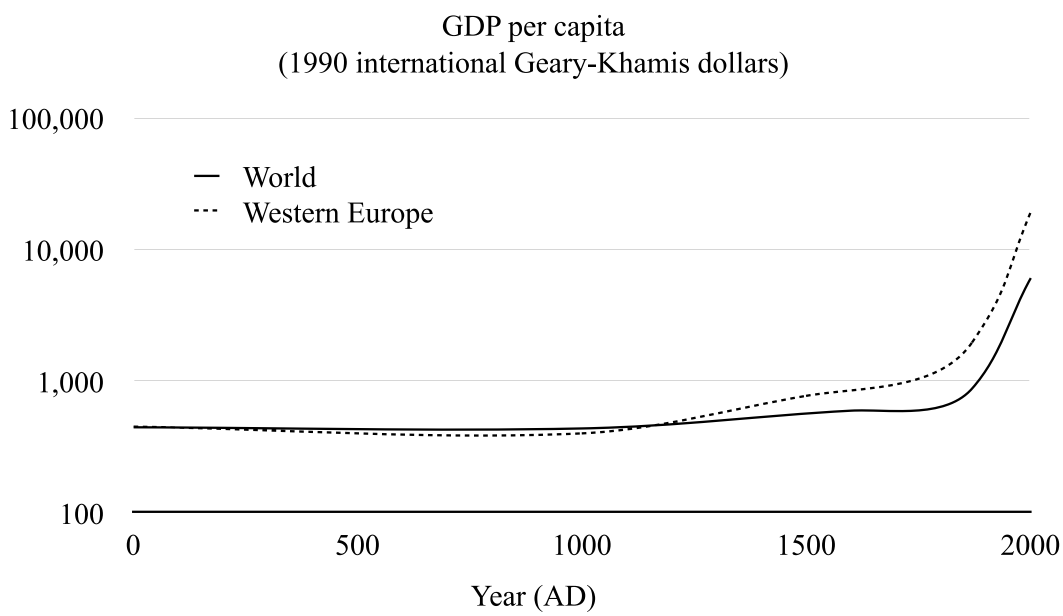

FIGURE 1: World GDP per capita 1-2001 AD. Data source: Maddison (2003)

It has been doubted to what extent Malthusian theory is applicable to explaining short-run changes. As a matter of fact, average income and population growth were poorly correlated in the English data (figure 2). Yet for all its weakness, the Malthusian force is believed to dominate in the long run by its persistence (Lee, 1987, p.452). Allen (2008) worried about the conjecture but mentioned no alternative solution. The solution is given in this paper. The classical Malthusian theory has missed two of the three comparative statics that determine the equilibrium living standards.

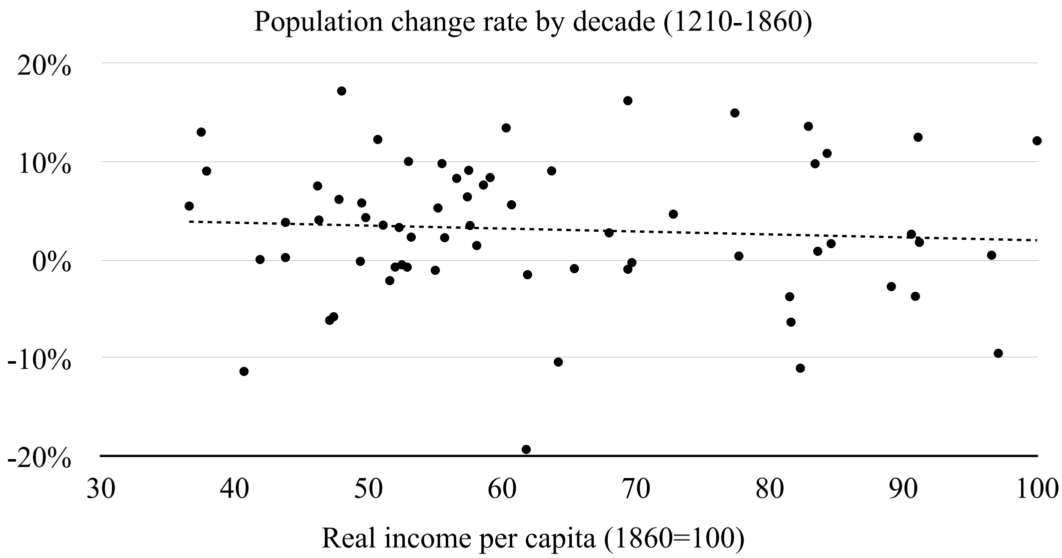

FIGURE 2: Population change rate was weakly correlated with real income per capita in England between 1210 and 1860. Data source: Clark (2010)

Another critique on Malthus targets his failure to predict modern growth. Theorists have endogenized the acceleration of growth to reconcile the Malthusian stagnation with modern economic growth (Simon and Steinmann, 1991; Jones, 2001; Hansen and Prescott, 2002; Galor and Weil, 2000; Galor and Moav, 2002). Presuming the Malthusian force as the cause of stagnation, these researchers described how a Malthusian shackle would have been broken; but they never asked whether the shackle is truly Malthusian or not.

Two-sector Malthusian models have appeared in Restuccia, Yang, and Zhu (2008); Voigtländer and Voth (2013); Yang and Zhu (2013). These researches define sectors production-wise: agriculture uses land; manufacturing does not. I divide sectors consumption-wise: agricultural products and manufactured products have different demographic effects. Steady growth is possible in my model but not in theirs. Rudimentary consumption-wise two-sector models have been built in Davies (1994) and Taylor and Brander (1998). More sophisticated versions appeared in Lipsey, Carlaw, and Bekar (2005, chap.9) and Weisdorf (2008). However, these researchers did not discuss the possibility that a sustained directional change in production structure would disturb the long-term Malthusian fact.

Group selection is a key concept in my theory. The idea was once a taboo but it revived in the past few decades[1]. Among many others, Bowles and Gintis (2002) and Bowles (2006)

[1]The idea was a taboo partly because it reminds one of Nazi eugenics, partly because the old theory

used it to explain how cooperation gets rooted in human nature. In this paper, group selection occurs across the multiple Nash equilibria—it is consistent with individual rationality. What's more, different from Galor and Moav (2002) and Clark (2008), my notion of selection is not genetic but cultural.

Wu et al. (2014) and Levine and Modica (2013) are the closest research to this one. Wu et al. (2014) has a similar two-sector Malthusian model, and comes to the same conclusion that either the Malthusian fact is false, or Malthus's explanation for the fact is wrong. Wu et al. (2014) explores the first possibility; this paper pursues the second. I will sketch on how the two views can be reconciled in the concluding remarks and leave the elaboration to another paper.

Similar to Wu et al. (2014), Levine and Modica (2013) treat the Malthusian trap not as a fact to explain but as a false prediction to challenge. Instead of dividing goods into sectors, they focus on the allocation of resources between the people and the authority—an abstract pair of luxury and subsistence. Their equilibrium is the maximization of authority-controlled resources meant for wars between states. It is a special case of my group selection theory.

3 Why is the trap almost surely not Malthusian?

In this section, I begin with a two-sector Malthusian model that adds two new dimensions of comparative statics to the classical theory. The classical Malthusian theory is shown to be a special case of the two-sector framework that assumes away the conflict between individual welfare and group fitness. Next, I discuss the historical relevance of the multi-sectorality, and its implication for the long-term growth: a persistent gap of growth rates between the luxury and subsistence sectors would bring a trend to living standards growth. That the Malthusian trap existed implies growth had been balanced between the sectors, hence the puzzle of balanced growth.

3.1 The division of sectors

Suppose there are H identical people living on an isolated island that has M kinds of commodities, $j = 1, 2, ..., M$. The representative agent consumes $E \in \mathbb{R}_+^M$, a bundle she chooses to maximize a utility function that is differentiable and strictly increasing:

$$\max_{E \in C(H)} U(E). \tag{1}$$

of group selection violates individual rationality. But biological research has demonstrated the absurdity of racism based on genetic reasons. Meanwhile, numerous ways have been found for group selection to be compatible with individual rationality. The old label of pseudo-science is gone.

The island is a Malthusian economy. So given resources, her choice set, C, shrinks with population: $\forall H_1 < H_2$, $C(H_1) \supset C(H_2)$. The growth rate of population depends on the average consumption E:

$$\frac{\dot{H}}{H} = n(E). \tag{2}$$

Assume that $n(E)$ is continuous, differentiable and strictly increasing, and there exists a set $\mathbb{S}$ on which population does not change. Call $\mathbb{S}$ the constant population set. Any isolated economy that finds itself on the constant population set is at a Malthusian equilibrium.

If $U(\cdot)$ is not a transformation of $n(\cdot)$, there must exist some bundle of consumption E, at which one commodity is more luxurious than another, i.e., $\exists j_1, j_2 \in \{1, 2, ..., M\}$ such that

$$\frac{\frac{\partial U(E)}{\partial E_{j_1}}}{\frac{\partial n(E)}{\partial E_{j_1}}} > \frac{\frac{\partial U(E)}{\partial E_{j_2}}}{\frac{\partial n(E)}{\partial E_{j_2}}} \tag{3}$$

Compared with j_2, commodity j_1 marginally contributes more to individual utility than to population growth. It makes j_1 a luxury relative to j_2. We can define the "luxuriousness" of each commodity in the following way: $\forall E \in \mathbb{R}^M_+$, $\forall j \in \{1, 2, ..., M\}$, commodity j's luxuriousness at E is

$$\frac{\frac{\partial U(E)}{\partial E_j}}{\frac{\partial n(E)}{\partial E_j}} \tag{4}$$

Order all commodities by their luxuriousness, and we have a spectrum from the most luxurious commodity to the most subsistential commodity. Thus we can always distinguish luxury goods from subsistence goods as long as $U(\cdot)$ is not a transformation of $n(\cdot)$.

3.2 A graphical representation

Figure 3A illustrates the case of two commodities. The innovation is to add a "constant population curve" to the representative agent's consumption space. If the agent's consumption bundle is on the curve, population stays constant; if the bundle lies to the left—consumption is less than reproduction requires—population decreases. If the bundle lies to the right, population increases.

The change of population shifts the agent's production possibility frontier. When population declines, the frontier expands: each person is endowed with a larger choice set. When population rises, the frontier contracts—the economy has diminishing returns to labor. The returns to labor diminish because land is crucial to production and its supply is inelastic.

Assume that the expansion and contraction of the production possibility frontier are shape-preserving, that is, the shape of the production possibility frontier is independent of

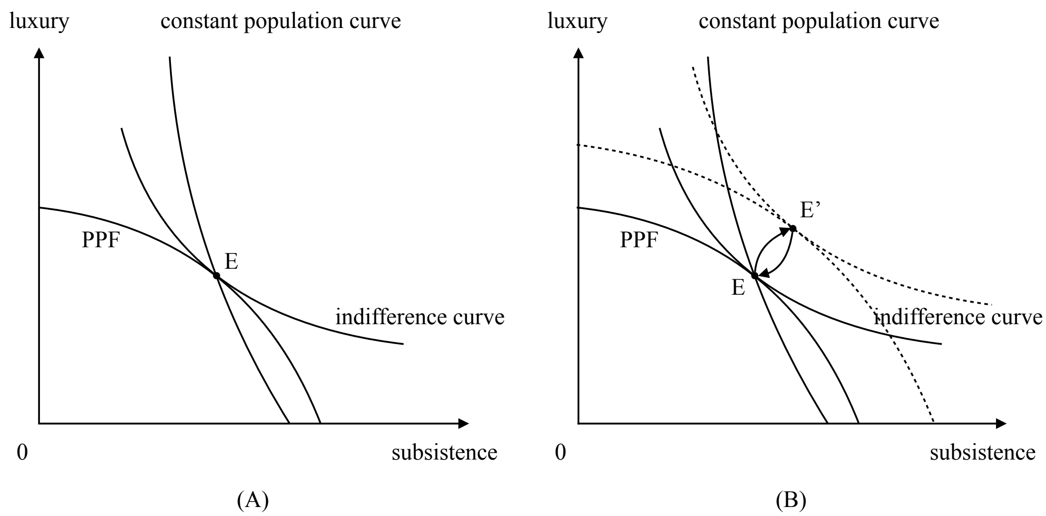

FIGURE 3: The two-sector Malthusian equilibrium

the size of population. It would be an interesting extension to allow subsistence production to be more labor-intensive than luxury production. Relaxing the assumption this way would complicate the model in ways favorable to my hypothesis[2]. Hence I stick to the shape-preserving assumption.

The constant population curve crosses the indifference curve from above because subsistence goods are more important to population growth than luxury goods.[3] But the curve does not have to be vertical. Beef is more luxurious than potato, but beef contains calorie too.

Malthusian equilibria must lie on the constant population curve. As figure 3B shows, if the economy expands, the temporary affluence will raise the density of population. The production possibility frontier will contract until the economy returns to the constant population curve.

Different from the classical Malthusian theory, the two-sector framework allows technological shocks to change equilibrium living standards. A positive shock in luxury technology expands the production possibility frontier vertically (figure 4A). After population adjusts, the economy returns to the constant population curve (figure 4B). The new equilibrium (E'')

[2]One may consult Wu et al. (2014) for the case where the assumption is dropped.

[3]The definition of luxury determines the direction of crossing. If we label the consumption of subsistence as E_A and the consumption of luxury as E_B, the definition of luxury implies

$$\frac{\frac{\partial U(E)}{\partial E_A}}{\frac{\partial n(E)}{\partial E_A}} < \frac{\frac{\partial U(E)}{\partial E_B}}{\frac{\partial n(E)}{\partial E_B}} \Longrightarrow \frac{\frac{\partial U(E)}{\partial E_A}}{\frac{\partial U(E)}{\partial E_B}} < \frac{\frac{\partial n(E)}{\partial E_A}}{\frac{\partial n(E)}{\partial E_B}} \quad \text{i.e., } MRS_U < MRS_n, \tag{5}$$

so the constant population curve is steeper than the indifference curve at E.

is above the old one (E) because the production possibility frontier has become steeper.

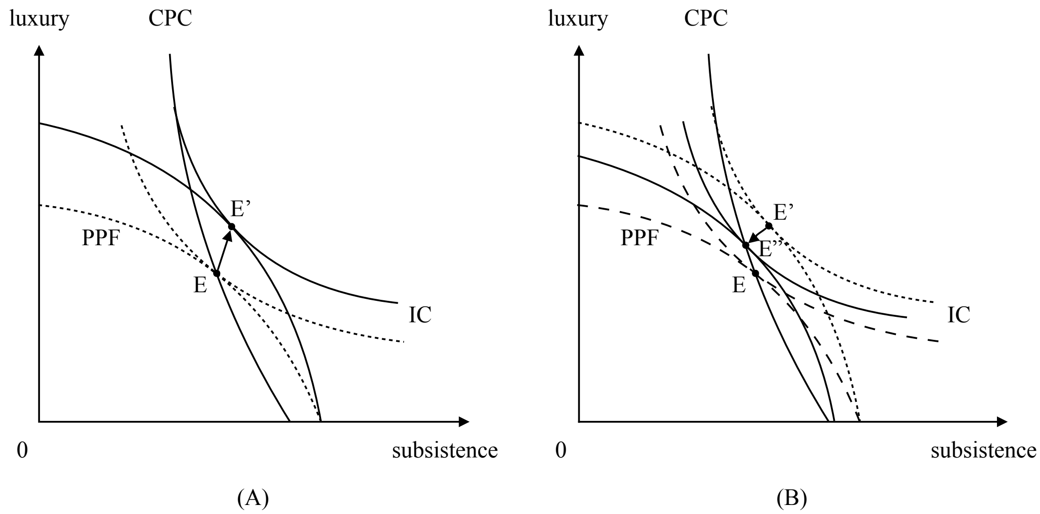

FIGURE 4: Progress of luxury technology improves equilibrium living standards.

However, living standards will decrease if progress occurs in subsistence instead. As figure 5A shows, the production possibility frontier expands horizontally as the subsistence sector expands. The abundance of subsistence goods increases population. After the economy returns to the constant population curve, the new equilibrium stays below the old one, because the production possibility frontier has become flatter (figure 5B). In the long run, what matters for living standards is not the size but the shape of the production possibility frontier.

Luxury expansion characterizes the impact that market economy would have on an ancient economy, such as the Roman Empire and the Song dynasty of China. Market boosts both agriculture and manufacturing, but manufacturing usually reaps more benefit from market than agriculture does, so the equilibrium living standards will rise as a result of the tilted production structure. The Malthusian force would have no way to check the improvement.

Paradoxically, subsistence expansions, such as the Agricultural Revolution, only drag an economy into deeper poverty. Archaeological evidence shows that the ancient peasants lived a worse life than their hunter-gatherer ancestors: leisure time was shortened; diet became less diversified; and harvest failures caused frequent starvation.

Besides technology, the two-sector model also gives culture an important role to play. Suppose there is a cultural shock that arouses desire for more luxury—the indifference curve becomes flatter (figure 6A). The luxury may be a new item of conspicuous consumption, used to signal unobserved income (Moav and Neeman, 2008). Those who reject the cultural norms may have more food to eat but are less attractive on the marriage market. As people

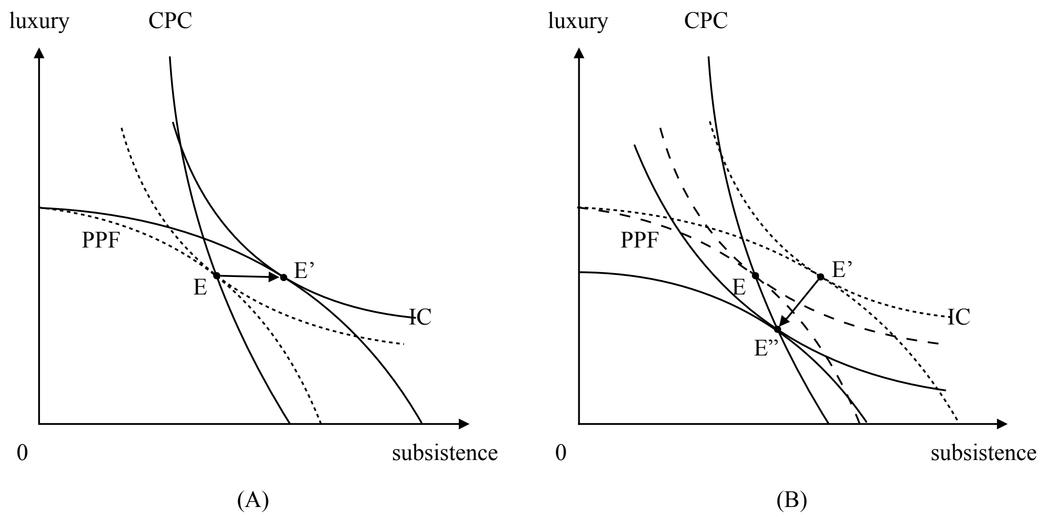

Figure 5: Subsistence technological progress decreases equilibrium living standards.

trade subsistence for luxury, population undergoes a gradual decline. When the adjustment is over, the remaining people enjoy higher equilibrium living standards (figure 6B). Luxury is *socially free*: so long as everyone desires more, more is granted. The extra luxury is paid with lives by those who would have been born and those who would not have died.

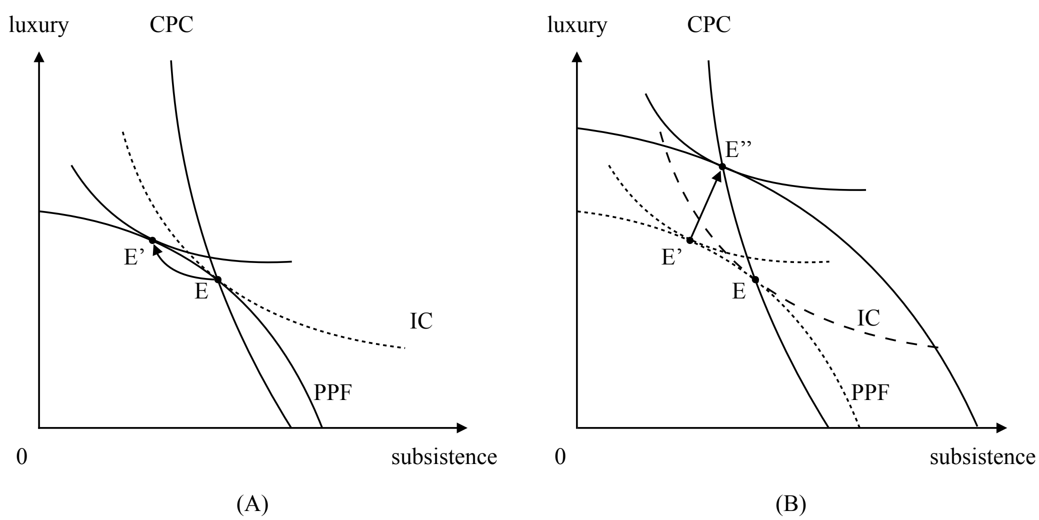

Figure 6: Luxury culture shock increases equilibrium living standards.

The above results seem to suggest that technologies and cultures that are more biased to luxury yield higher living standards. The correlation holds most of the time but exceptions exist. A multiple equilibria problem disturbs the monotonicity. Fortunately, multiple equilibria arise only if subsistence goods are Giffen. To the extent that Giffen goods are rare, we do not have to worry about the exceptions in most cases. In the online appendix, I prove that

Theorem 1 (Production Structure Theorem)**.** *(a) For an economy on a stable equilibrium, a positive luxury technological shock always improves equilibrium living standards. (b) Other things being equal, if subsistence goods are not Giffen, a more luxury-biased production structure always means higher equilibrium living standards.*

Theorem 2 (Free luxury Theorem)**.** *(a) For an economy on a stable equilibrium, a luxury cultural shock always improves equilibrium living standards. (b) Other things being equal, if subsistence goods are not Giffen, a more luxury-biased culture always means higher equilibrium living standards.*

In contrast, the classical Malthusian theory has only one dimension of comparative statics, the one that is associated with the constant population curve. When disease environment worsens, warfare becomes more frequent, or people decide to postpone marriage and have fewer children, the constant population curve shifts rightward: population grows more slowly at each level of consumption (figure 7A). The ensuing decline of population expands the production possibility frontier. People who survive the changes are better off in the new equilibrium than in the old equilibrium (figure 7B). The two-sector model thus preserves the merit of the classical theory.

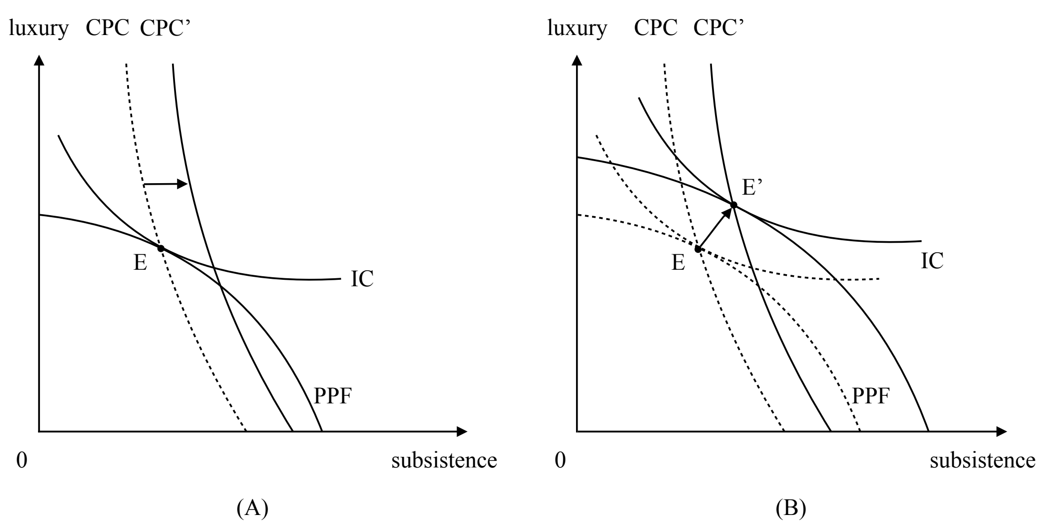

FIGURE 7: Diseases, wars and delayed marriage increase equilibrium living standards.

3.3 Classical Malthusianism as a special case

The classical theory is actually a special case of the two-sector model. When $U(\cdot)$ is a transformation of $n(\cdot)$, all commodities have the same level of luxuriousness, and the constant population curve will coincide with the indifference curve. As the two curves coincide, a luxury technological shock can change the consumption bundle but not the equilibrium living standards. The economy that comes back to the constant population curve will have the same level of utility as in the original equilibrium (figure 8B).

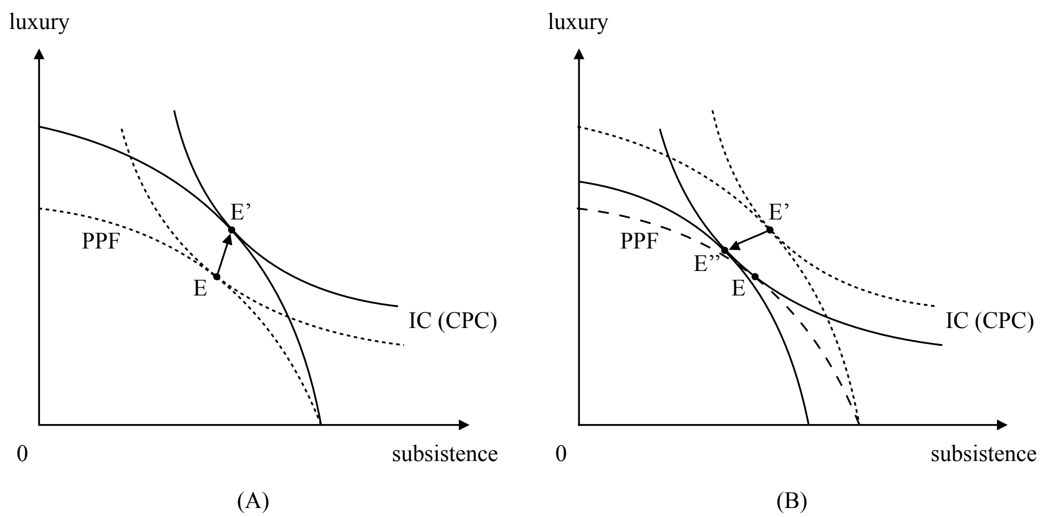

FIGURE 8: Malthusian theory is a special case where population growth and individual utility are fully aligned.

This is why the classical theory predicts stagnation. By the coincidence of curves, two sectors are reduced to one; difference of demographic effects between bread and flowers is ignored; and the conflict of reproductive interest between individual and group is assumed away.

The last point bears emphasizing. The conflict between individual and group is the ultimate reason why the curves cross. The constant population curve is an iso-group-fitness curve, along which population grows at the same rate; the indifference curve is an iso-utility curve, or approximately an iso-individual-fitness curve. Millions of years' natural selection shapes human being's preference system into a maximizer of one's own reproductive success. The conflict of reproductive interest between individual and group prevails in both culture and nature. As sure as the conflict persists, the division of sectors is a perpetual human condition. Assuming the individual's interest to be perfectly aligned with the group's interest is simply unrealistic.

There are still many who believe there is nothing wrong about accepting an unrealistic assumption as long as the theory makes the right prediction, and that among all theories that predict right, the simplest is the best (Friedman, 1953). In their view, Malthusian theory is great. Though the theory fails to predict short-term changes well, *ad hoc* explanations are always available to explain away the inconsistencies. The most important thing is: the theory predicts the Malthusian trap. Then why bother find an alternative?

The above view makes a serious methodological mistake. Unrealistic assumptions are fine only if they are not crucial. If an unrealistic assumption is crucial, that is, if making the assumption more realistic would lead the prediction astray from reality, then the theory is wrong (Solow, 1956). Malthusian theory has an unrealistic assumption—the harmony of reproductive interest between individual and group. If we relax the assumption, turning the one-sector model into a multi-sector one, the multi-sector model will predict a trend of growth in living standards (this part will be shown in section 3.6). Hence the assumption is crucial. We need a new theory to replace Malthusian theory not only because the two-sector model accounts for a larger portion of variation in living standards (it does), but also because Malthusian theory is simply wrong.

3.4 Evidence of multi-sectorality

How relevant is sectoral division in the real history? The short answer is: very much. Whether the division is recognized decides views as important as when the Great Divergence occurred and how real income inequality evolved.

Based on the similarity of wages in terms of calories, Pomeranz (2009) argued that the Yangzi delta of China was on the same level of development as Northwest Europe as late as the end of the 18^{th} century. He ignored that the per capita calories is fixed by the Malthusian force, and that most of the difference in living standards comes from the difference in non-food consumption. As Broadberry and Gupta (2006) showed, if we measure the purchasing power of wage by grams of silver, the "silver wage" of the Yangzi delta was only comparable with the level of the central and eastern parts of Europe (table 1). Regions varied widely in the ratio of silver wage to grain wage. That the ratio is higher in England and the Netherlands than in the other places of the world reflected these two countries' relative advantage in tradable goods production, and it is by this advantage that the Northwest European economies stood out in the pre-modern centuries.

Hoffman et al. (2002) studied the implication of sectoral division on real income inequality. Since the rich spent a larger portion of income on luxury than the poor, the decline of the prices of luxuries relative to the prices of the subsistence goods enlarged the real inequality in Europe between 1500 and 1800.

Table 1: The grain wage and silver wage of different regions, 1750-99

Regions	Grain wage (kg/d.)	Silver wage (g/d.)	Ratio (Sw/Gw)
Southern England	7	8.3	1.2
Antwerp	9.6	6.9	0.7
Vienna	7	3	0.4
India	2.3	1.2	0.5
Yangzi delta, China	3	1.7	0.6

Data source: Broadberry and Gupta (2006).

More direct evidence for multi-sectorality is provided in Wu (2012). In that paper, I did a simple exercise: instead of regressing birth and death rates on real wages, as most people do when testing Malthusian relationship, I regress these demographic variables on "sectoral wages", that is, the purchasing power of wages measured in terms of goods of certain sectors. During the three centuries before 1800, the period conventionally believed to have provided the strongest evidence in support of Malthusianism, multi-sectorality turned out to be a salient feature of the English economy. If pasture goods became more affordable, population growth barely changed; if arable goods were more affordable, population growth rate increased a lot. Within the category of arable goods, the affordability of wheat had little impact on population growth; but the affordability of barley and oats, the poor people's staple food, almost solely explained the impact of real wages on the birth and death rates. However, barley and oats were merely 10% of the English economy, much smaller than the share of wheat. The rest 90%, including wheat, beef, cottons and candles, hardly mattered demographically. Productivity improvement in the 90% sector would surely increase the long-term living standards with more families switching diet from porridge to bread, and starting to call tea, sugar and coffee "necessities" of life. Changes like this might not be reflected in the real wage series historians built, but with the proper data and method, assessing this part of welfare increase is still possible.

Hersh and Voth (2009) estimated that by the end of the 18th century, tea, coffee and sugar had added at least the equivalent of 16% (and possibly as much as 20%) of income to English welfare. Contrast it with two other New World crops, maize and potato. Chen and Kung (2013) estimated that maize accounted for 18% of the population increase in China during 1776-1910, but brought no significant effect on economic growth. Nunn and Qian (2011) estimated that potato accounted for about a quarter of the growth in Old World population between 1700 and 1900. Potato triggered a Malthusian crisis in Ireland in the late 18th century. The explosion of population drove the Irish to extreme poverty that culminated in the Great Famine of 1845. The difference between tea, coffee and sugar on the one hand and maize and potato on the other hand is none other than the difference between luxury and subsistence. While the abundance in the former improved the quality of life, the abundance in the latter increased the population.

3.5 The algebraic two-sector model

This section builds a simple algebraic model that captures all of the three comparative statics. Assume the representative agent maximizes a Cobb-Douglas utility function over her subsistence consumption, x and luxury consumption, y.

$$\max U(x, y) = x^{1-\beta} y^{\beta}. \tag{6}$$

The constant returns to scale makes the magnitude of utility meaningful: utility doubles when consumption doubles.

Specify the subsistence production function as $X = AL_A^{1-\gamma_A} H_A^{\gamma_A}$ and the luxury production function as $Y = BL_B^{1-\gamma_B} H_B^{\gamma_B}$. L_A and L_B are land used in the production of subsistence and luxury respectively. Their sum is the total endowment of land, $L_A + L_B = L$. H_A and H_B are labor employed in the respective sectors, and $H_A + H_B = H$.

Assumption 1. $\gamma_A = \gamma_B \equiv \gamma < 1$.

I assume $\gamma_A = \gamma_B$ so that population growth affects the two sectors in proportion, equivalent to the shape-preserving assumption in the graphical model. γ_A and γ_B are smaller than one because of diminishing returns to labor.

By maximizing the agent's utility under land and labor constraints, we can derive her consumption bundle:

$$x = A(1-\beta)\left(\frac{H}{L}\right)^{\gamma-1} \tag{7}$$

$$y = B\beta\left(\frac{H}{L}\right)^{\gamma-1} \tag{8}$$

Substitute equation 7 and equation 8 into $U = x^{1-\beta} y^{\beta}$. The level of utility is

$$U = A\left(\frac{H}{L}\right)^{\gamma-1}\left(\frac{B}{A}\right)^{\beta}(1-\beta)^{1-\beta}\beta^{\beta}. \tag{9}$$

Since $A(H/L)^{\gamma-1}(1-\beta) = x$ (equation 7), U can be expressed alternatively as

$$U = x\left(\frac{B}{A}\right)^{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta}. \tag{10}$$

The economy converges to equilibrium by population adjustment. In an isolated economy, the *net* growth rate of population, g_H is equal to the *natural* growth rate of population, n. Assume that n depends on the average consumption of subsistence only.

Assumption 2. $g_H \equiv \dot{H}/H = n = \delta(\ln x - \ln \bar{x})$*, and* $\delta > 0$*.*

$\bar{x}$ is the level of average subsistence at which population remains constant. The assumption means a vertical constant population curve—population growth is independent of average luxury consumption. In the equilibrium, $x = \bar{x}$. Therefore,

Proposition 1.

$$U^E = \bar{x}\left(\frac{B}{A}\right)^{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta} \tag{11}$$

The equilibrium level of utility increases with B/A*,* β *and* $\bar{x}$*.*

The equilibrium utility increases with

(a) the relative luxury productivity, $\frac{B}{A}$,

(b) the relative preference for luxury, β,

(c) and the required consumption for population balance, $\bar{x}$.

3.6 The balanced growth puzzle

Proposition 1 implies that living standards will rise steadily if the luxury productivity grows faster than the subsistence productivity. Denote g_A as the growth rate of subsistence productivity and g_B as the growth rate of luxury productivity. Appendix A.1 proves that g_U converges to $\beta(g_B - g_A)$ in the long run.

In the classical Malthusian theory, there is only one sector, therefore $\beta = 0$ and $g_U = 0$. In the two-sector model, β is positive. The equilibrium living standards will have a trend unless $g_B = g_A$.

The implied balanced growth is an extraordinary phenomenon. The world population had grown from several million at the dawn of the agricultural revolution, to three hundred million at the birth of Christ, and to almost one billion on the eve of the industrial revolution—it went up by a factor of at least 1000. To keep up, subsistence production must have grown by 1000 folds (the subsistence technology A must have grown by about 30 folds if $\gamma = 0.5$).

Throughout the 1000-fold growth of subsistence, luxury must have grown in exact proportion. What could keep things balanced over ten thousand years during a 1000-fold growth? In comparison, world population has grown only 6 folds since the industrial revolution. In the meantime the world GDP has grown by a factor of 150—the progress in luxury productivity has been much faster than the progress in subsistence productivity. If the balance of growth is so difficult to keep within such a short period, isn't it extraordinary that growth was once balanced for not decades or centuries, but for millennia and even longer?

Moreover, it is natural to expect luxury production to intrinsically grow faster than subsistence production. There are at least four reasons why we should expect so. First, manufacturing and commerce are usually more labor-intensive than agriculture[4]. Population growth, by increasing labor supply, naturally expand luxury production more than it expands subsistence production. Second, industrial innovations are less constrained by the possibilities of nature than agricultural innovations. Third, the incentives of industrial innovations are better protected than the incentives of agricultural innovations. An ancient farmer that succeeded on a new crop could hardly reap any social benefit spilled out of her own land. Whereas in manufacturing and commerce, keeping trade secrets for monopoly rent was feasible most of the time. Last but not the least, manufacturing allows a larger extent of division of labor. As Adam Smith (1887, Chapter I, Book I) put it,

> "The nature of agriculture, indeed, does not admit of so many subdivisions of labour, nor of so complete a separation of business from another, as manufacture. [...] This impossibility of making so complete and entire a separation of all the different branches of labour employed in agriculture, is perhaps the reason why the improvement of the productive powers of labour, in this art, does not always keep pace with their improvement in manufactures. The most opulent nations, indeed, generally excel all their neighbors in agriculture as well as in manufactures; but they are commonly more distinguished by their superiority in the latter than in the former."

The above reasons combined, the Malthusian trap appears to be a most unlikely coincidence. Two centuries after Malthus, the Malthusian fact is still a fundamental puzzle of history. To explain, I come up with four hypotheses. Three are rejected, reasons given below. The last is explored in detail in the next section.

[4]I assume land plays equal parts in luxury production and subsistence production, i.e. $\gamma_A = \gamma_B$ (a simplifying assumption that is unfavorable to my hypothesis). If I allowed subsistence to rely more on land than luxury does—it would be a very reasonable assumption, for subsistence is mostly about food and basic shelter—then even balanced growth would lead to steady progress of living standards. Nevertheless, I keep the stricter assumption of $\gamma_A = \gamma_B$ for model tractability. That said, it is imprecise to treat balanced growth as the condition for stagnation. This is also an extra reason why the usual balanced growth modeling approach can not be used here. Even if the models produce the result of balanced growth, in order to apply them, we must relax the crucial assumption of $\gamma_A = \gamma_B$. Fortunately, the group selection theory I propose does not depend on balanced growth.

I **Evolutionary adaption**

Long exposure to a luxury good might cause genetic adaption that allows people to use it as a subsistence good. For example, lactose intolerance is relatively rare among the Northwest Europeans, whose ancestors, as a conjecture goes, had a higher reliance on milk as a source of nutrition than people in Asia, who did not develop the gene. The problem is that even if the conjecture is correct, genetic adaption is usually slow, and the mechanism that works for food does not work for manufactured goods.

II **Positional goods**

Diamonds are precious because they are rare. Positional goods become worthless when they are too many: people value how much they own compared with others instead of what they own *per se*. The abundance of a particular luxury drives people away from the luxury. The problem is that the Malthusian fact is never about the lack of desire, but the shortage of goods. The mechanism might explain why being rich does not make one much happier, but it does not explain why physical deprivation lasts.

III **Constant returns to scale**

Solow and Samuelson (1953) showed that, in a dynamic system described as

$$\begin{aligned} A_{t+1} &= F_A(A_t, B_t) \\ B_{t+1} &= F_B(A_t, B_t), \end{aligned}$$

if $F_A(\cdot)$ and $F_B(\cdot)$ have constant returns to scale, then A and B will grow in balance on a stable path. But the theorem only pushes back the question one step further. It is doubtful whether the theorem is applicable to luxury and subsistence growth. Even if it is applicable, we still have to answer why the functions have constant returns to scale. So far, I have seen no reason why they should be so.

IV **Group selection**

I cannot reject the hypothesis. The rest of the paper is devoted to the idea.

4 Then what makes the trap?

Above I have shown that it takes the balanced growth between the luxury and subsistence sectors for living standards to remain constant, and that none of the evolutionary adaptation hypothesis, the positional goods hypothesis and the constant returns to scale hypothesis can explain the balanced growth puzzle. This section elaborates on the last alternative, the group selection mechanism. Selection works through biased migration. I first explain what biased migration is, and discuss historical evidence in support of the migrational pattern. Next, I build a group selection model that allows me to derive the threshold condition of stagnation. Finally, simulations are conducted to compare the paths of global average living standards with and without the selection factor. The models and the simulations both confirm that a Malthusian world without selection would have a trend of growth in average living standards, but when selection is introduced—a tiny bit of selection would suffice—the trend is gone.

4.1 Biased migration

Group selection is related to the phenomenon of "biased migration". This section uses a simple model to explain how certain characters in culture and technology bias the direction of migration.

Suppose there is a sea of identical villages, all at the equilibrium state. Following Tiebout (1956), I assume free migration across villages but forbid trade between them.[5] Bread and flowers are the only commodities. Suddenly, one of the villages discovers a better way to grow flowers. Its production possibility frontier expands vertically. If migration were forbidden, the flower village would end up with higher living standards. But free migration equalizes utility across the villages (figure 9B). With a steeper production possibility frontier tangent with the same indifference curve, the flower village stays to the left of the constant population curve in the migration equilibrium—its death rate is higher than the birth rate. The natural decrease of population does not expand the production possibility frontier because the under-reproduction is filled up with the continuous immigration from the other villages. The flower village becomes a demographic *sink* and the surrounding villages a demographic *source*.

Difference of cultures causes source-sink migration too. Suppose in one of the villages, girls begin to ask for more flowers from their suitors: the indifference curve becomes flatter (figure 10A). If migration were forbidden, the girls would get what they demand for free (remember the free luxury theorem). But in the migration equilibrium, the equality of utility means demographic imbalance. In the beginning, people in the surrounding villages do not move. They will stay put until the population of the flower village decreases enough for the economy to move from E' to E'' (figure 10B). After E'', the continuous immigration will keep the flower village stay to the left of the constant population curve. The source-sink

[5] Trade substitutes migration. If trade is free of cost, different regions will face the same relative price of luxury to subsistence. Then the Malthusian force will equate consumption across regions, and there will be no need to migrate. If trade has a cost, the relative price will differ and migration will emerge. The model applies as long as migration exists. I forbid trade only to simplify the analysis. The assumption is not crucial.

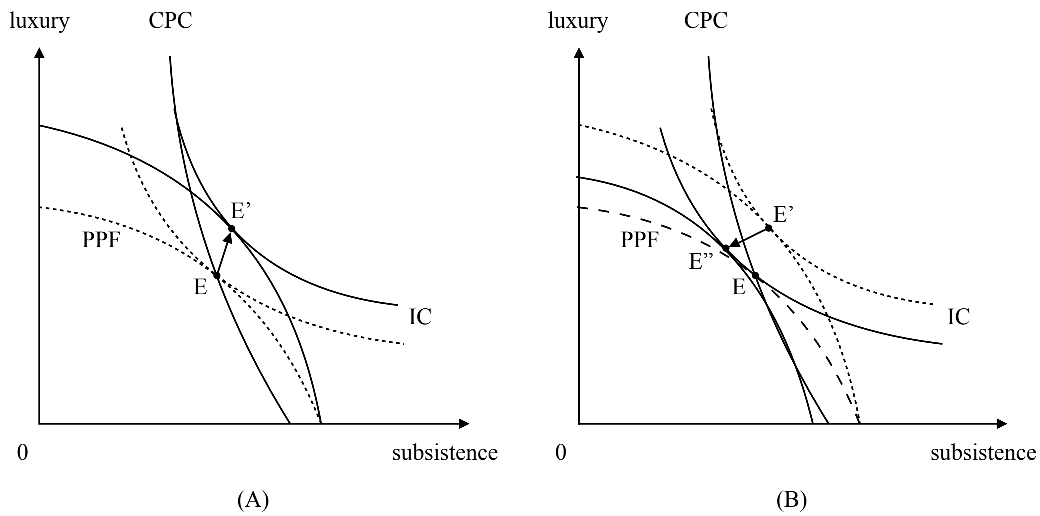

FIGURE 9: Source-sink migration emerges out of difference in production structure.

pattern emerges again.[6]

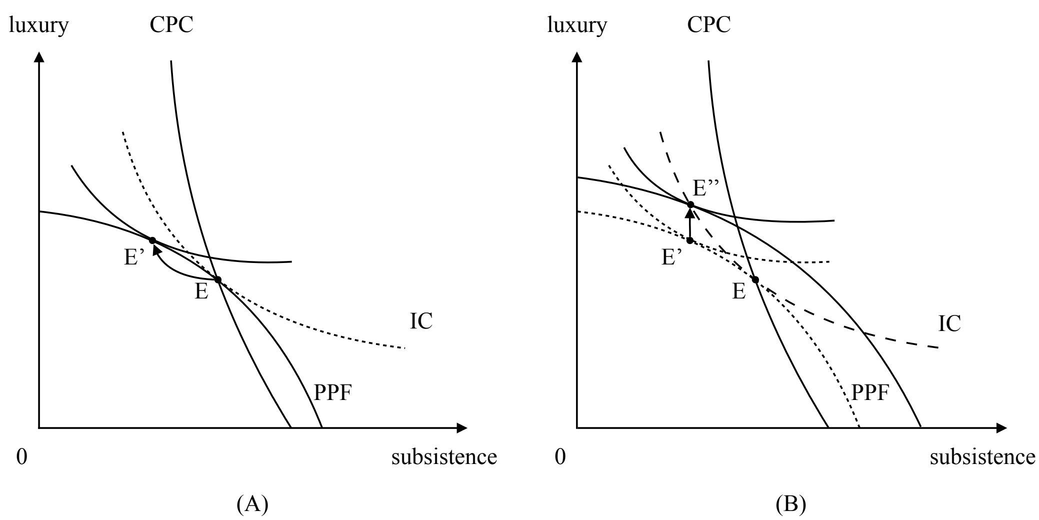

FIGURE 10: Source-sink migration emerges out of difference in social preference.

The craze for flowers will not last forever, because the immigrants who come from the places that do not value flowers as much will be diluting the flower culture. The arms race of conspicuous consumption is constrained not by Malthusian force, but by source-sink migration, and the selection that follows. Selection dissipates luxury cultures and diffuses

[6]Here the migrants are assumed to keep their old preference. If they convert to new cultures, the diagram is slightly different but the source-sink pattern still remains.

subsistence cultures.

4.2 Evidence of biased migration

Source-sink migration is best documented in the context of rural-urban migration, where the phenomenon is called "urban natural decrease": the urban death rate was higher than the birth rate; and the natural decrease coincided with the natural increase in the surrounding rural area.

De Vries (2006) decomposed the net changes of pre-modern European urban population into net immigration and natural growth. As figure 11 shows, during most of the time between 1500 and 1800, urban population had been growing in both Northern and Mediterranean Europe. But despite the *net* increase, urban population had been declining *natually*, that is, the death rate was higher than the birth rate in the cities. During the half century between 1600 and 1650, Northern Europe had an annual growth of 0.32% in its urban population; but meanwhile the urban death rate exceeded the birth rate by 0.33%. A flow of rural migrants that amounted to 0.65% of the size of urban population per year had been replenishing the cities.

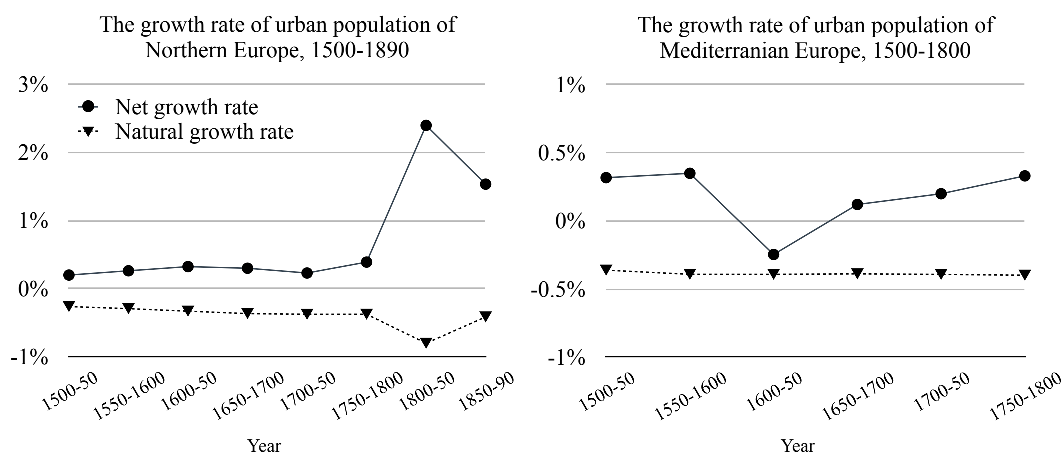

FIGURE 11: The source-sink pattern of migration in pre-modern Europe.
Data source: De Vries (2006, p.203-208).

Malthusian theory is inconsistent with the migration pattern. The period $1800 - 1850$ witnessed a spike in the growth of urban population in Northern Europe. According to the classical theory, the spike suggests a rise in the urban living standards, which would cause a faster natural growth in the urban population. But in fact, the gap between the urban death rate and the urban birth rate only widened in this period.

The anomaly can be easily explained by the biased migration model. After 1800, the growth of manufacturing and commerce accelerated in the urban areas; the growth of agriculture accelerated in the rural areas. The polarization of production structures spurred more migration into the cities than in the previous centuries. The flood of immigrants lowered the average subsistence by so much that the natural growth rate of the urban population dropped further. As proposition 2 will show later, the depth of the demographic sink increases with the distance of production structures.

Another piece of evidence for biased migration comes from Ravenstein (1885), who calculated the gap between a county's population and the number of its natives, enumerated throughout England and Wales in 1881. When the residents were more than the natives, he regarded the county as one of absorption—more people moved in than moved out; otherwise, a county of dispersion. Ravenstein also marked whether a county is "agricultural" or "industrial". He called a county "agricultural" if the county's proportion of agricultural population exceeded the sample average, and "industrial" if otherwise.

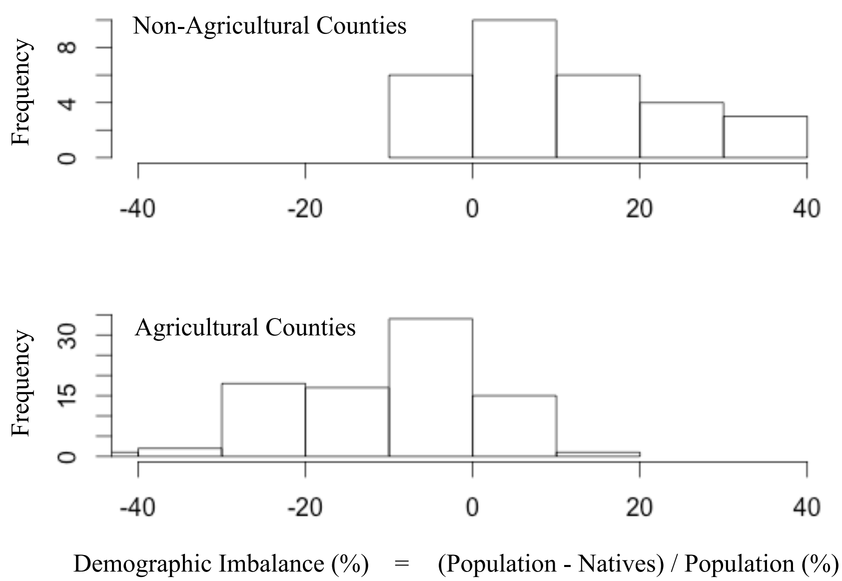

FIGURE 12: The demographic imbalances of non-agricultural and agricultural counties. Data source: Ravenstein (1885, p.185-186).

As figure 12 shows, the overwhelming majority of the non-agricultural counties were counties of absorption, and the overwhelming majority of the agricultural counties were counties of dispersion. The pattern is consistent with the hypothesis that people usually move from subsistence-rich regions to luxury-rich regions.

4.3 Group selection

4.3.1 The spread of ideas through migration

Ideas move with people. The bias of migration naturally gives rise to bias in the spread of ideas. Since people usually move from subsistence-rich regions to luxury-rich regions, a subsistence culture (technology) is easier to spread than a luxury culture (technology), other things being equal.

Today, anyone with electricity and internet can watch lectures taught by the best scholars on the world. Technological spread and migration are largely disentangled from each other. But in the ancient times, when book supply was limited and internet was unheard-of, migration was crucial to the spread of ideas. It took thousands of years for agriculture to spread from the Hilly Flanks to North Europe, and the process coincided with the spread of the original Neolithic groups' genes in both timing and spacial extent (Cavalli-Sforza, Menozzi, and Piazza, 1993). If the hunter-gatherers beyond the frontiers could learn agriculture without immigrants' help, the reproductive advantage of the first farmers would have been quickly lost, and the observed pattern of genetic spread would be impossible. Likewise, the Indo-European family of languages originated in the Caucasian steppe, where the early domestication of horses lent serious military advantage to the herders. If horse-herding had spread without migration, the Caucasians would have found it hard to conquer the neighboring peoples who had learnt the new arts of war, and the proto-indo-european language would have no chance to diffuse at all.

Even in societies with decent literacy levels, migration still played a crucial role in the spread of ideas. In the 15$^{\text{th}}$ century, the learned Byzantine exiles that fled out of the falling Constantinople revived Greek studies in the Renaissance Italy. Considering the potential demand for Greek letters and thoughts, it is extraordinary that the knowledge had not spread earlier to Italy by means other than migration. Another example is the revolutionary impact of Jewish emigres from Nazi Germany on U.S. science. Moser, Voena, and Waldinger (2014) estimated that the arrival of German Jewish emigres brought a 31 percent increase in patenting by U.S. inventors in emigres' research fields after 1933. Even today, complaints about "brain drain" are frequently heard. Again, people are concerned about the spread of technology through migration.

It is fair to say that at least in the ancient times, migration was an important channel of spreading ideas. The bias of migration means a bias in technological diffusion. The question is: can this bias explain the Malthusian trap?

I build two group selection models to answer the question. The first model studies the partial equilibrium of a single village that is surrounded by an infinite number of villages. The relatively simple result highlights the key mechanism at work. The second model studies the general equilibrium of two villages. A threshold condition is derived to tell us when selection dominates growth and when growth overpowers selection. Both models assume β to be constant and equal between the regions. They assume away cultural selection to focus on technological selection only, but the results can be easily extended to cultural selection.

4.3.2 The partial equilibrium

In the beginning, there are an infinite number of identical villages described as in section 3.5. Bread and flower technologies stagnate at A' and B' in all villages except one. Use asterisk to denote the special village. Its subsistence technology $A^* = A'$, but its luxury technology B^* tends to grow at the rate of g. We call it the flower village and the others the bread villages. When B^* exceeds B', there will be continuous migration from the bread villages to the flower village.

Assumption 3. *Trade is forbidden across the villages but migration is free of cost.*

Free migration equalizes the level of utility, $U^* = U'$, which by equation 10 means

$$x^* \left(\frac{B^*}{A^*}\right)^\beta \left(\frac{\beta}{1-\beta}\right)^\beta = x' \left(\frac{B'}{A'}\right)^\beta \left(\frac{\beta}{1-\beta}\right)^\beta$$

where x^* and x' are the average consumption of bread. Rearrange the equation and take logarithm. We get

$$\ln x^* - \ln x' = -\beta \left[\ln\left(\frac{B^*}{A^*}\right) - \ln\left(\frac{B'}{A'}\right)\right]. \tag{12}$$

The net emigration rate from the flower village, m, is equal to the *natural* growth rate of population, n, which in turn depends on the average bread, x^*, that is,

$$m = n = \delta(\ln x^* - \ln \bar{x}). \tag{13}$$

$\bar{x}$ is the level of average bread that keeps population in natural balance. Since $\delta > 0$ and $x^* < \bar{x}$, m is negative: migrants move from the bread villages into the flower village. The emigration has a negligible effect on each bread village, because their number is infinite and migration between them is frictionless. So the bread villages still have $x' = \bar{x}$.

Denote $s^* \equiv \ln(B^*/A^*)$ and $s' \equiv \ln(B'/A')$, the relative luxury productivities. Substituting $x' = \bar{x}$ and equation 12 into equation 13, we get

Proposition 2.

$$m = -\beta\delta(s^* - s') \tag{14}$$

The net emigration rate is proportional to the distance of production structures. Having a higher relative luxury productivity than the neighboring villages causes net immigration.

Migrants spread ideas. Assume that migration affects B^* by displacing hosts' technology with immigrants' technology in proportion to the number of immigrants:

Assumption 4. *From time t to $t + \Delta t$, B^* updates by taking the weighted geometric average of B^* and B' and growing at the rate of g.*

$$B^*(t + \Delta t) = B^*(t)^{1-m\Delta t}(B')^{m\Delta t}(1 + g\Delta t) \tag{15}$$

Divide both sides of equation 15 by A', take logarithms, and calculate the limit as $\Delta t \to 0$. We can rewrite the equation into the motion function of s^*:

$$\dot{s^*} = m(s^* - s') + g \tag{16}$$

Substitute equation 14 into equation 16:

$$\dot{s^*} = -\delta\beta(s^* - s')^2 + g \tag{17}$$

The differential equation has a stable equilibrium:

Proposition 3. *In the long run, even if B^* has an intrinsic tendency to grow at the constant rate g, the flower village's relative productivity, $s^* = \ln(B^*/A^*)$ will stabilize at*

$$s' + \sqrt{\frac{g}{\delta\beta}}$$

Note that g has a level effect but no growth effect on the equilibrium level of s^*.

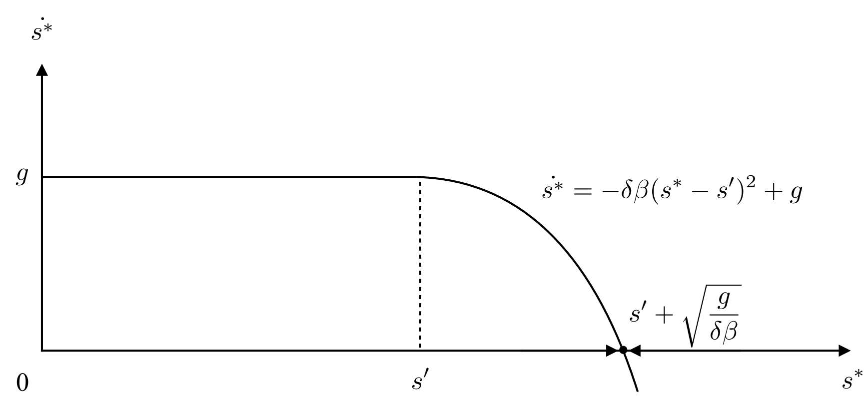

FIGURE 13: The phase diagram of $s^* \equiv \ln(B^*/A^*)$

4.3.3 The general equilibrium

The partial equilibrium model assumes away the influence of the flower village on its infinite number of neighbors. In this section, we study a two-village model to take into account the general equilibrium effect. Suppose village 1 and village 2 start identical. Their bread technologies, A_1 and A_2, grow at the same constant rate g_A, and their flower technologies, B_1 and B_2, drift with noises:

$$d \ln B_i = (g_A + g)dt + \sigma dz_i \tag{18}$$

Here $g > 0$ captures the growth advantage of flower productivity over bread productivity. The error terms z_i's ($i = 1, 2$) are Brownian motions. z_1 and z_2 are independent with each other, and $\mathrm{Var}(\sigma dz) = \sigma^2 dt$. I introduce the stochastic growth of technology as the source of inter-regional variation. Variation is the basis of technological selection as mutation is the basis of natural selection. I fix the growth rates of A_1 and A_2 to keep population equal between the regions. The equality of population makes the model tractable without loss of generality. With $s_i \equiv \ln(B_i/A_i)$, equation 18 can be rewritten as

$$ds_i = gdt + \sigma dz_i. \tag{19}$$

That $g > 0$ allows both villages, if isolated, to grow steadily in living standards. However, selection cancels out growth by adding a "drag" term to the motion of s_i. The drag appears when $s_1 \neq s_2$. Following assumption 4, the drag term is a quadratic of the difference between s_1 and s_2 as in equation 17:

$$ds_i = [g - I_{\{s_i > s_j\}} \beta\delta(s_i - s_j)^2]dt + \sigma dz_i.$$

Here $I_{\{s_i > s_j\}}$ is an indicator function that equals 1 if $s_i > s_j$ and 0 if otherwise. If $s_1 > s_2$, village 1 is relatively rich in flowers. It attracts immigration from village 2, which drags s_1 closer to s_2. If instead $s_1 < s_2$, village 1 is relatively rich in bread. The bread village receives no immigration and selection will not affect its relative luxury productivity.

Since utility depends on s, the most interesting variables are the global average of s_i's, $\mu = \frac{1}{2}(s_1 + s_2)$, and the inter-regional variation, $\nu = \frac{1}{2}(s_1 - s_2)^2$.[7].

[7] ν is the sample variance: $\nu \equiv \frac{1}{2}(s_1 - s_2)^2 = [s_1 - \frac{1}{2}(s_1 + s_2)]^2 + [s_2 - \frac{1}{2}(s_1 + s_2)]^2$

Applying Itō's lemma, we get

$$d\mu = (g - \beta\delta\nu)dt + \frac{\sqrt{2}}{2}\sigma dz \tag{20}$$

$$d\nu = (\sigma^2 - 2\sqrt{2}\beta\delta\nu^{\frac{3}{2}})dt + 2\sqrt{\nu}\sigma dz \tag{21}$$

where z is a Brownian motion.

Taking long-term expectation of both sides of equation 20, we have

$$\mathop{\mathbb{E}}_{t\to+\infty}\left(\frac{d\mu}{dt}\right) = g - \beta\delta \mathop{\mathbb{E}}_{t\to+\infty}(\nu) \tag{22}$$

Denote $S \equiv \beta\delta\,\mathbb{E}_{t\to+\infty}(\nu)$, the force of selection. $(g - S)$ captures the race between growth and selection.

Appendix A.2 proves that the variation ν will always converge to a finite value, and

$$S = \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}, \tag{23}$$

where $k \equiv \left[3^{\frac{1}{3}}\,\mathrm{Gamma}\left(\frac{4}{3}\right)\right]^{-1} \approx 0.78$ is a constant. Comparing g and S, we get the threshold condition:

Proposition 4. *Growth overcomes selection, if*

$$g > \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}. \tag{24}$$

Otherwise, selection dominates growth.

When $g < S$, $\mathbb{E}_{t\to+\infty}\left(\frac{d\mu}{dt}\right) < 0$. The possibility that μ could decline seems to contradict Malthusian stagnation. But the decline of μ has a natural limit. Luxury consumption cannot decrease further when it reaches zero. In this sense, the model explains why the average luxury consumption was almost nil in the ancient times, and gives the condition for this to happen. That said, if one feels uncomfortable with zero utility under Cobb-Douglas utility function, a simple remedy is to assume, not unreasonably, that luxury productivity growth accelerates if average luxury is close to zero (demand is huge when luxury is rare). Then the equilibrium will have a positive amount of average luxury. The simulations will use the method.

As proposition 4 indicates, two sets of variables determine how strong selection is. The first is the variance of technological growth σ^2, which provides the necessary heterogeneity for selection to work on. The second is the product of two exogenous variables, $\beta\delta$. Denote

$\lambda \equiv \beta\delta$, and call it the intensity of selection. In a richer setting, λ would further incorporate the migrants' willingness to move and the hosts' susceptibility to migrants' influence. A little calibration can help gauge the relative strength of selection. If $\beta = 0.5$, $\delta = 0.1$ and $\sigma = 0.02$, the threshold is 0.78‰. The world population had been growing at about 1‰ per year. Since appendix A.1 proves that g_A converges to $(1-\gamma)g_H$, if $\gamma = 0.5$, g_A is roughly 0.5‰. Therefore, if $g_B \leq 1.28$‰, or less than about 2.5 times the level of g_A, the global average living standards will have no trend of growth.

4.4 Simulations

4.4.1 The baseline simulation

The general equilibrium model has three limitations. First, the two-region setup may fail to capture the real-world intense competition among hundreds of regimes. Second, the model assumes migration is free of cost and finishes instantly, so there is no difference of utility across the regions. Third, the model assumes immigrants' technologies to be able to displace natives' technologies, no matter whose technologies are better. The assumption is unreasonable in times of peace.

In this section, I relax all of the three assumptions. The simulations have a hundred regions instead of two. Migration is gradual and its speed increases with the utility gap across regions. Besides the baseline case where technologies are indiscriminately substituted, I also study the case where learning occurs only if the immigrants have a better technology.

The simulated world is a chess board of 10×10 grids. Each grid represents a region that has the same population dynamics and the same production and utility functions as the baseline model specifies. Time is discrete. At period t, the state of grid (i, j) is characterized by $\{A_{ijt}, B_{ijt}, H_{ijt}\}$, the subsistence technology, the luxury technology and the level of population.

Assume A_{ij} and B_{ij} evolve the following way:

$$A_{ij}(t+1) = A_{ij}(t)(1 + g_{Aij} + \sigma_A \epsilon_{Aij}) + \text{selection effect} \tag{25}$$

$$B_{ij}(t+1) = B_{ij}(t)(1 + g_{Bij} + \sigma_B \epsilon_{Bij}) + \text{selection effect} \tag{26}$$

The error terms ϵ_A and ϵ_B have normal distributions, $\epsilon_A, \epsilon_B \sim N(0, 1)$, i.i.d. g_{Aij} is the same across all grids: $g_{Aij} = g_A$, but g_{Bij} increases with the relative rarity of luxury:

$$g_{Bij} = g_B \left[1 + \left(\frac{B_{ij}}{A_{ij}} \right)^{\alpha} \right]. \tag{27}$$

The additional term in the bracket is meant to prevent the downward trend as discussed in the last section. α is arbitrarily set to be a large negative number to minimize its impact when $B_{ij}/A_{ij} > 1$ ($\alpha = -10$). Though appearing *ad hoc*, adding the term increases the growth rate of luxury, which is unfavorable to my hypothesis and only makes the theory even more robust.

At each period, residents of each grid decide whether they should move to a neighboring grid for higher living standards. For two grids next to each other, if grid 1 has a higher utility than grid 2, some residents of grid 2 will move to grid 1, and the migration rate is proportional to the difference of utility:

$$\frac{\text{Migrants}}{\text{Population of the Origin}} = \theta(\ln U_1 - \ln U_2) \tag{28}$$

Unlike the previous model, here θ is finite.

I simulate two scenarios. In the first scenario, the immigrants' technologies displace the natives' technologies, no matter whose technologies are better. I call this case "indiscriminate substitution". In the second scenario, people only learn from those who do better: if the immigrants are better at producing things, the natives will update their technology in the same way as "indiscriminate substitution"; but if the immigrants' technologies are inferior, the natives will keep their old ways to produce, and the immigrants will convert to the natives' technologies. I call the scenario "selective learning".

The force of selection is weaker under selective learning, yet it still favors the spread of subsistence technologies. To see this, suppose there are two identical regions. If one has a positive shock in subsistence productivity, people will emigrate to the other to spread the improved subsistence technology. But if it is the luxury technology that has improved, no emigration will occur and the luxury technology has to remain local (contrast it with the indiscriminate substitution scenario where selection happens either way).

Indiscriminate substitution is a feature of the many barbarians' invasions that destroyed complex social orders that alway took a civilization centuries to build. Unlike them, peaceful migrants on the wagons usually do not lower technology levels of the host society. Selective learning is then the dominant way of knowledge spread. But between the scenarios, the simulated histories have one thing common: over a vast range of parameters, there is no trend of growth.

To save computer time, I treat each simulated period as a decade. As table 3 in appendix C summarizes, I parameterize $g_A = 0.5\%$ and $g_B = 1\%$ (per decade) for the baseline case. The size of g_A guarantees a growth rate of population close to historical rates. g_B is twice as big as g_A, imposing a strong tendency of luxury growth. The (subsistence) income elasticity of population, $\delta = 0.2$, matches the estimation from the English demographic and price data. The other crucial parameters include the standard deviation of the growth errors, $\sigma = 5\%$, and the migration propensity, $\theta = 0.1$. At first approximation, $\theta = 0.1$ means that if there is an opportunity to move to a twice richer place next to home, only 1% of people will take the opportunity in a typical year.

Figure 14(A) presents the key result of the simulation. It compares the global average utility, weighted by regional population[8], with and without biased migration. Merely the Malthusian assumption fails to deliver the Malthusian result. Over ten thousand years, the global average utility increases about tenfold if without migration (the result is the same if migration is allowed but migrants are assumed to carry no technologies). But when the knowledge-spreading migration is introduced, the trend is gone.

The lack of growth under selection is not a result of technological stagnation. Rather, technologies grow faster when migrants are allowed to carry them around. This is how the "Malthusian + migration" case achieves a faster population growth than the purely Malthusian case in figure 14(B).

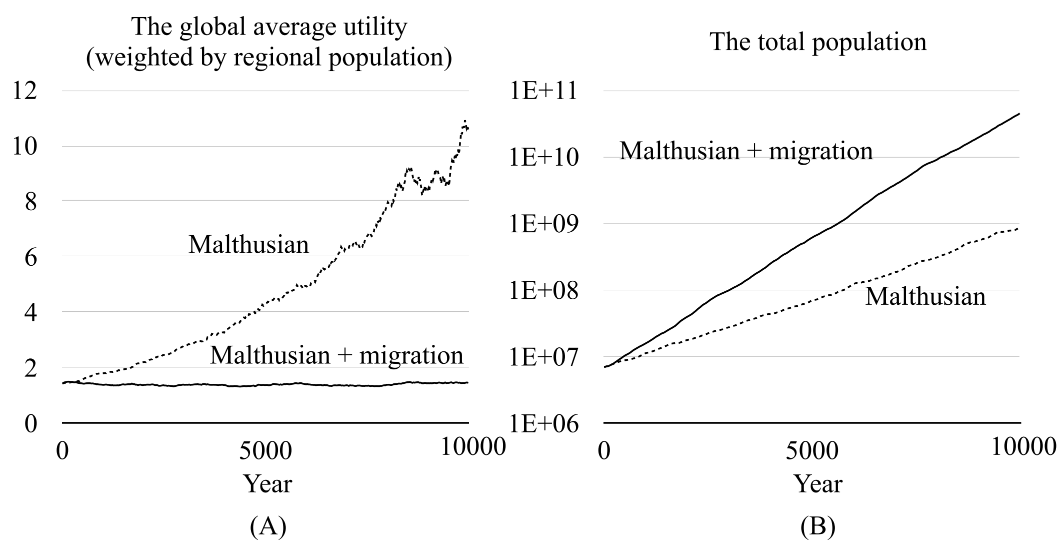

FIGURE 14: A purely Malthusian simulation does not produce the stagnation of living standards. To ensure stagnation, the Malthusian mechanism has to be combined with biased migration.

As is predicted, the distribution of regional utility under biased migration is stationary (figure 20 in appendix B). The richest region's utility never exceeds twice the poorest region's

[8]The weighted global average utility is the "true" average that assigns equal weight to each person of the world. If I drop the weighting and use the average of regional utility instead, the path of global utility will only be more stable, as utility is negatively correlated with population. I will stick to the weighted average, which is unfavorable to my hypothesis, throughout all simulations.

utility. Selection keeps all regions interlocked. In contrast, if there is only Malthusian force but no selection, the variation will be enormous and divergent.

Figure 19 in appendix B further traces the utility of three representative regions—one at the corner of the world, one on the side, and one in the middle. Despite cycles spaning thousands of years, there is no trend of growth in any single region.

Figure 15 is the selective learning scenario. The weakened selection still dominates growth. The average utility climbs up slowly before it stabilizes at a plateau. In the long run, there is no trend of growth either.

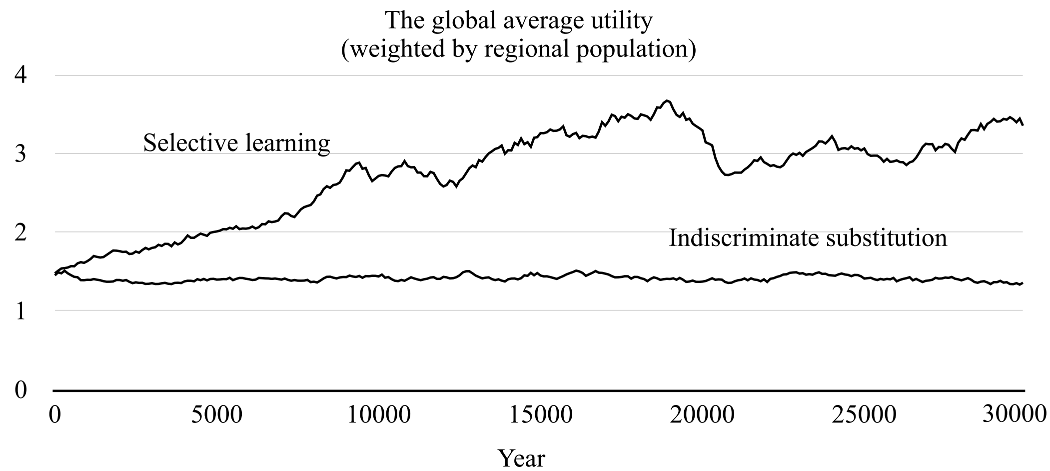

FIGURE 15: If learning is selective, the global average utility will stabilize at a higher level than if technology is indiscriminately substituted.

4.4.2 Robustness checks

In this section, I show that the dominance of selection over growth is robust to variation in (a) the standard deviation of growth errors, σ, (b) the side length of the simulated world, w and l, and (c) the migration propensity, θ.

First, I vary σ from 0% to 15% with each step equal to 1%, and g_B from 0% to 2% with each step equal to 0.1%, keeping all the other parameters the same as in the baseline case. For each pair of σ and g_B, I run simulation five times. I adopt a stringent criterion of stagnation. If the global average utility grows more than 25% from the 300^{th} period to the 600^{th} period—over a length of 3000 years—I treat it as a trend of growth, and if there are more than one simulations (excluding one) having trend, I mark the pair of parameters as "progressive"; otherwise, "stagnant".

I conduct the robustness check for both scenarios of knowledge spread. The result is figure 21 in the appendix. Selection gets stronger with a larger σ: under indiscriminate substitution, when $\sigma = 3\%$, selection dominates if $g_B - g_A \leq 0.6\%$; when $\sigma = 5\%$, selection dominates if $g_B - g_A \leq 1\%$. As expected, selection is weaker under selective learning, but there is a caveat. A simulated history treated as progressive does not necessarily have a trend. The path of selective learning in figure 15 keeps rising until stabilized at about the 20,000$^{\text{th}}$ year. Applying the above criterion, I would treat the history as progressive but it actually has no trend in the long run.

To verify that the force of selection is robust to various sizes of the square world, I experiment with every integer value of side length from 3 to 20, running five simulations for each. Figure 22 in appendix B shows the cumulative growth from the 300$^{\text{th}}$ period to the 600$^{\text{th}}$ period of these experiments. The variation is bigger when the world is smaller, for the results are then more likely to be driven by the idiosyncrasies of individual grids. Nevertheless, there is hardly any difference between a world of 100 grids and a world of 400 grids. The baseline simulation, which assumes a 10×10 world, is representative in this respect.

The results are also robust to variation in θ. To verify this, I run 10 experiments under each scenario for each value of θ from 0 to 0.2 with the step equal to 0.01. The power of small θ's is extraordinary. The baseline simulation assumes $\theta = 0.1$. As mentioned before, it has already been a conservative estimate of people's willingness to move. But as figure 16 shows, even if θ is as small as 0.01—only 0.1% of people would move each year to a neighboring region that is twice as rich—selection still dominates. This by no means suggests that migration is unimportant. If $\theta = 0$ (the pure Malthusian case), the cumulative growth is way larger than if $\theta = 0.01$. Growth precipitates as θ slightly deviates from zero. A tiny bit of migration is strong enough to dominate a strong tendency of growth. Why this is the case I will leave to section 5.4 where I discuss the Industrial Revolution.

4.4.3 Duet dance

Selection ensures the equality of long-run average growth rates between the sectors. But the mere equality is not sufficient for the stagnation of living standards. World population growth had changed speed several times (figure 17). Behind each change is the acceleration of subsistence technology growth. At these moments, for living standards to keep constant, the progress of luxury technology must accelerate to exactly the same speed—a perfect duet dance.

To test the duet dance hypothesis, I fix g_B at 1%, and have g_A jump from 0.25% to 0.75% at the 1001$^{\text{st}}$ period of the simulation. If the duet dance exists, the global luxury technology growth will speed up to the same rate as the subsistence technology growth immediately after the juncture. This is exactly what happens in the simulation, as figure 17 shows. I

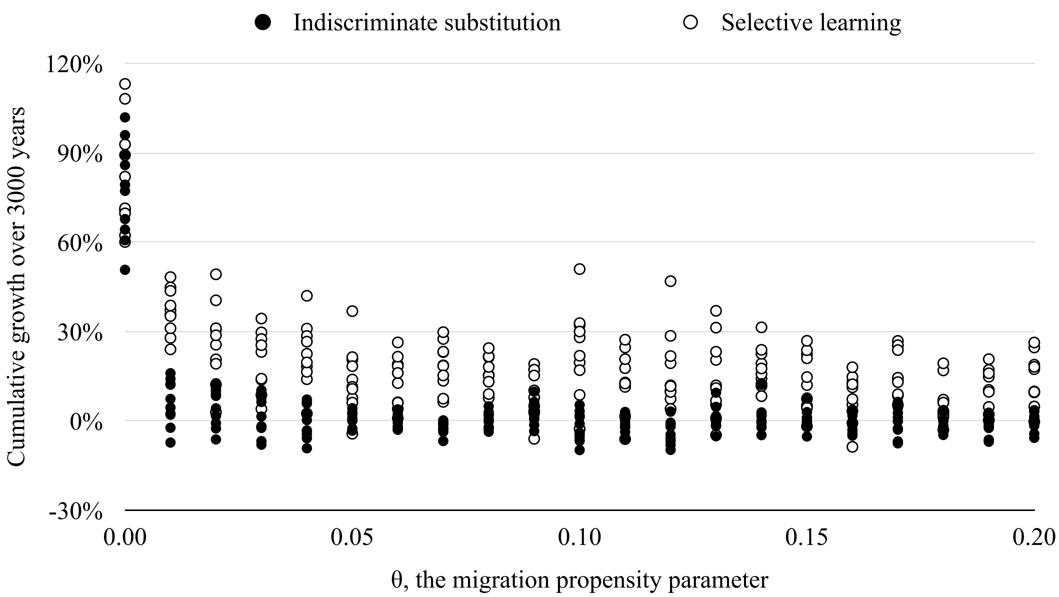

FIGURE 16: The cumulative growth of global average utility over 3000 years. Notes: each point represents an experiment at the corresponding level of migration propensity, θ.

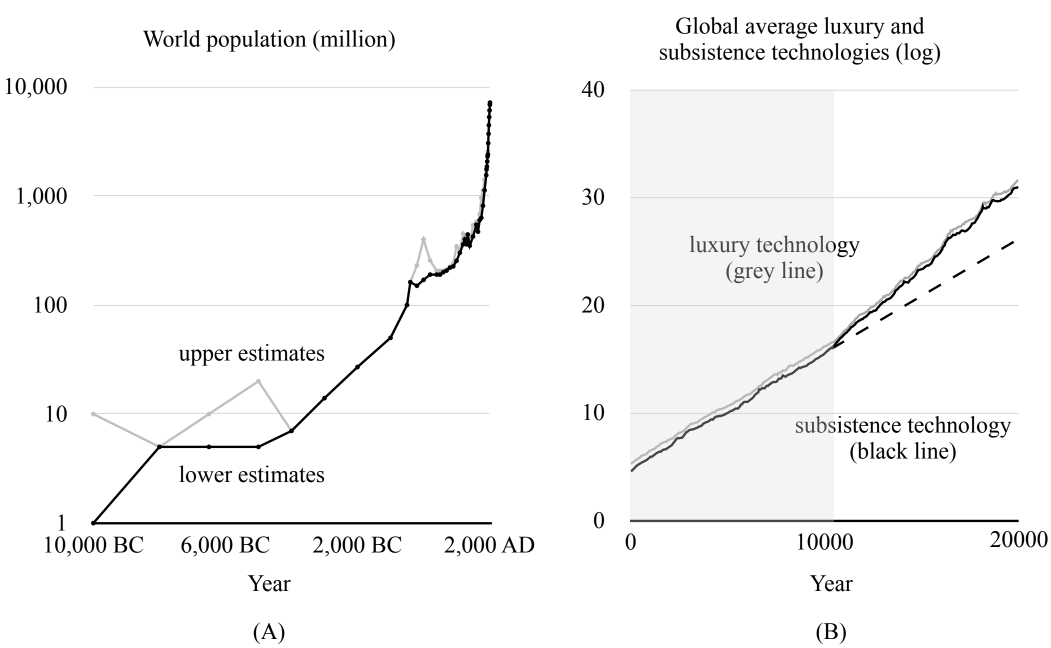

FIGURE 17: (A) The world historical population estimates. Data source: US census bureau. (B) When subsistence technology growth accelerates, luxury technology growth will accelerate to the same speed.

further conduct a Chow test:

$$\Delta \log(\text{luxury technology}) = \underset{(1e^{-3})}{5e^{-3}} + \underset{(0.6e^{-3})}{10e^{-3}} \times \text{break dummy}_{t=1001} + \epsilon \tag{29}$$

With p-value as low as 10^{-6}, the test rejects the null hypothesis that there is no kink in luxury technology growth at the $1,001^{\text{st}}$ decade. The estimated coefficient of the break dummy, $10e^{-3}$ is exactly twice as large as the constant term, $5e^{-3}$. It means that when the growth rate of subsistence productivity triples from 0.25% to 0.75%, the growth rate of luxury technology triples from 0.25% to 0.75% too. Despite that g_B is fixed, luxury growth catches up fast and fully. Selection ensures balanced growth not only trend-wise but also point-wise.

5 Rethinking major events of economic history

The combination of the two-sector model and the group selection model paint a new picture of economic history. In what follows, I will discuss the implications of the theory to four issues, namely, the Agricultural Revolution, the ancient market economies, the welfare consequences of wars and migrations, and the Industrial Revolution.

5.1 Why farm?

The early farmers were worse off than their hunter-gatherer ancestors. They had less leisure, worse nutrition and larger inequality between sexes and across castes. The paradox of immiserizing growth can be explained by the fact that agriculture is a subsistence technology. By tilting production structure towards subsistence, it caused living standards to decline in the long run. Yet if agriculture was so bad, "Why [did people] farm? Why work harder, for food less nutritious and a supply more capricious? Why invite famine, plague, pestilence and crowded living conditions (Harlan, 1975)?"

Farmers are faced with a prisoner's dilemma. People choosing what is best for themselves are hardly concerned with the prospect of the whole group's misery. That the farmers as a whole would end up worse off would not bother one who saw agriculture as the dominant strategy to maximize her own chance of survival and reproduction. But even if there is a group of altruistic visionaries that coordinated to keep the hunting-gathering lifestyle, the group could not compete with one that had switched to agriculture. The latter was relatively richer in subsistence. The higher density of population, the consequent impoverished life, and the greed for new lands would drive the agricultural group to invade the hunting-gathering group instead of the other way around. Selection would wipe away whoever refused to farm (Cavalli-Sforza, Menozzi, and Piazza, 1993).

5.2 The rise and fall of the wealth of nations

First published in 1776, *The Wealth of Nations* declared the birth of modern economics. However, Gregory Clark (2008, chap.2, pg.35) commented, "[I]n 1776, when the Malthusian economy still governed human welfare in England, the calls of Adam Smith for restraint in government taxation and unproductive expenditure were largely pointless [... while] those scourges of failed modern states—war, violence, disorder, harvest failures, collapsed public infrastructures, bad sanitation—were the friends of mankind before 1800."

Provocative as he sounded, Gregory Clark was only making explicit the natural conclusion of the classical Malthusian theory. Without the two-sector model, no matter how uncomfortable one may feel about the remark, there is no way to refute it.

But in fact, Smith is right, though in a way he was never aware of. The policies he suggested can improve living standards, not only in the short run but also in the long run, not only in Solow's time but also in his own time, and much earlier times as well. Laissez-faire, light tax and the division of labor, if applied to economic policies, raise productivity in all sectors, but manufacturing and commerce benefit more from them than agriculture does. The rise of the ratio of luxury to subsistence lead to higher equilibrium living standards.

This explains why the average Romans and Song Chinese were richer than the other peoples in history. According to Lo Cascio and Malanima (2009)'s estimation, the per capita GDP of Roman Italy reached $1400 in US 1990 dollars in 150 AD, and the per capita GDP of the whole Roman Empire was as high as $1000. Among the many mentioned estimates, Temin (2013) regards this set of numbers closest to reality. To put the estimates into perspective, consider that Maddison (2003) estimated the per capita GDP of most ancient societies at slightly above or around $450. $1400 per capita is what the Netherlands achieved at late as 1700. The reason why the Romans were rich is very similar to the reason why people living in modern developed countries are rich. As Temin (2013) shows, Rome had a functioning legal system, an active financial market, and a broad market network. The security of property rights stimulated investment; the scale of the market facilitated labor division; the standardized mass production improved the quality of consumer goods to a high level. All these were meaningless in old Malthusian view of history, but in light of the new theory, they were as crucial to ancient living standards as they are to modern life.

On the contrary, the "friends of mankind"—wars, violence, disorder, collapsed infrastructures—often destroy more luxury than subsistence and decrease living standards in the long run.

5.3 A short history of the long war against luxury

Fatal clashes on the group level has been a persistent human condition since primitive society. Of the fourteen groups studied in Mae Enga, a modern hunter-gatherer society in Papua New Guinea, five went extinct in tribal clashes over a 50-year period. In place of the extinct groups, new groups formed out of the old groups that survived and expanded (Soltis, Boyd, and Richerson, 1995). A group that spent too much on luxuries would hardly survive.

The domestication of animals and plants divided the world into nomadic zones and arable zones. Until the mass use of gunpowder, clashes between the two had disrupted growth over and over again. The three pre-modern peaks in Ian Morris (2011)'s social development index all ended in "barbarian" invasions. The sea peoples raided Anatolia, Levant and Egypt; the Huns and Goths ruined the Western Roman Empire; the Jurchens and Mongols conquered the Song Dynasty of China. A brief review of the three events can help us appreciate the crucial role migration plays in suppressing the trend of luxury growth.

Around 1000 BC, the sea peoples, arguably the nomads from the hinterland of Europe, destroyed a number of highly developed kingdoms the Hittites, Minoans, and Mycenaeans had built. Urban centers, artistic representation, elaborate writing system, large-scale trading, shipping and construction vanished; civilizations were reduced to impoverished, illiterate, technically backward and violent small communities. The population of the largest city in the West declined from 80,000 (Babylon, Thebes) in 1200 BC to 25,000 (Susa) in 1000 BC. "The invasions were not merely military operations, but involved the movements of large populations, by land and sea, seeking new lands to settle (Bryce, 1999)".

The collapse of Rome was even more dramatic than the collapse of the Hittite kingdom. In the post-Roman Europe, production shrank to meet only local needs again. World-wide copper pollution plummeted to a seventh of the Roman peak level (Hong et al., 1996). Elites found it hard to afford the tiled roofs that once even the lowest class of Roman peasants had for their houses (Ward-Perkins, 2005). It is of course unfair to blame all of the loss and decline on the invaders. There was evidence of mild recession in the third and fourth centuries, arguably caused by civil wars and epidemics. But the invasions had certainly done most of the devastation.

Observe how this view contradicts Malthusianists' version of Roman history. In their view, the average Roman lived beyond the verge of subsistence only because the Roman population had not caught up with technology for a short while; when it finally did, prosperity was gone (Temin, 2013). The problem is: if the Romans had lived a pinched life under population pressure, why would Emperor Valens have bothered to recruit armies from the "barbarian" immigrants, allowing the Gothic refugees from the Huns to reside within Roman territory at the first place? After the collapse of Rome and the decline of population that followed, why did average living standards not rise as Malthusian theory predicts but plunged? My answer is: the Romans were rich because their economic system encouraged luxury production and consumption; the post-Roman Europeans were poor because the new rulers—not so unlike bandits—adopted policies hurting commerce and industry. Europe later turned into a feudal society where obligation replaced profit as the guiding principle of economic life. In many parts of the continent, money transactions disappeared. Individuals' freedom gave way to group survival. It was then, by the contraction of commerce and industry, that Europe became a true "subsistence economy". The next time Europe recovered, it was a thousand years later when another round of commercial revolution began in the Italian cities.

The same catastrophe befell Song China. Broadberry, Guan, and Li (2014) estimated that the per capita GDP of Song was about \$1500 in US 1900 dollars in the eleventh century. Manufacturing and commerce were so developed that they contributed two thirds of the government's tax revenue (Liu, 2015). Song's textile machine was comparable with European designs in the eighteenth century. Its furnaces outputted as much iron as the whole of Europe would produce in 1700. Song's coal mines were large enough for hundreds of workers to work at the same time. However, while the combination of textile, iron and coal sent England onto the track of the Industrial Revolution, Song failed prematurely.

Unlike England, Song had little geographical barrier to protect itself from invasions. Its collapse is best viewed as one of several waves of group selection that surged in East Asia in the 12th and 13th centuries. Before the Jurchens, Song's rival was Liao, a country the nomadic Khitans built. After twenty five years' war, Song and Liao signed a peace treaty on the condition that Song should pay an annual tribute to Liao. The peace lasted more than 120 years, bringing prosperity to both sides. Occupying part of China proper that included today's Beijing, Liao turned from a backward pasture economy into a civilized country that had a highly developed manufacturing sector. But the Khitans, now civilized, ended up an easy prey of the Jurchen barbarians. Two years after Liao fell, the Jurchens further conquered Kaifeng (Song's capital) and annexed the northern half of China. But ironically, merely a century later, the civilized Jurchens were in turn wiped out by the barbarian Mongols.

Though the Mongols inherited enormous wealth from the Jurchens and the Song Chinese, they threw away many of the institutions and policies that had made the wealth possible. The Mongols divided subjects into four castes with institutionalized discrimination between them. For cheap and stable supply of labor force, the government forbid workers and their offspring from changing jobs. The system was later inherited by the Hong-Wu Emperor of the Ming dynasty, who concluded from the hyperinflation under the Mongols' rule that money was a dangerous thing, and that the best way to organize economy was to fix people at preassigned places and jobs, discouraging movement of goods and people. The result is: Ming China became a predominantly agrarian economy. While Song collected two thirds of tax from commerce and industry, agriculture provided 84% of Ming's tax revenue. Even so, Ming's total agricultural tax was still smaller than Song's total agricultural tax. The living standards that the Song Chinese once achieved was never reached again in China until perhaps Deng Xiaoping's reform.

Above, we have seen how invasions from less developed regions destroyed some of the greatest civilizations the world had ever seen. But that is only one aspect of wars' impact on luxury growth. In response to wars, groups often intentionally cut down luxuries, and that might have been an even stronger force undermining luxury growth.

For example, during the Warring Period (476-221BC) of China, the restraint on luxury was the theme of a series of political and economic reforms[9]. In face of constant nomadic harassment, King Wu-Ling of Zhao (340-295 BC) commanded his subordinates to take off the wide sleeves and long robes[10] and switch to nomadic uniform—pants, belts and boots—in order to fight as cavalry. Half a century before King Wu-Ling, *Shang Yang*'s reform swept another kingdom, Qin. The reformer punished commerce, rewarded cultivation, forbid migration and restricted entertainment. In a word, he cut down luxury and directed as much resource as possible to subsistence. The subjects were deprived, but the Qin kingdom, over the next century, defeated all of the six rival kingdoms and united China for the first time in history. A contemporary philosopher commented, "Qin is different from all the other kingdoms. The people are poor and the government is cruel. Whoever hopes for a better life can do nothing but combat hard. This makes Qin army the strongest of all."[11]

Qin's idea of governance had a lasting impact on the later Chinese dynasties. Part of the influence is reflected in the mainstream of ancient Chinese economic thoughts, which emphasized the restraint on luxury and commerce. The ancient thinkers thought differently than Adam Smith not because they were blind to the benefit of commerce but because they cared about the country's survival more than about the subjects' welfare. In a country where tax-fed mercenary has not been the backbone of military strength, the government would better sacrifice commercial gains to ensure the ease of conscription. Adam Smith is certainly great, but he is unique not because he discovered something no one had thought of, but because he lived on the eve of modern era, when individual welfare was finally reconciled with group survival and expansion—it was the richer Europeans that had moved to America,

[9]To name a few, Li Hui's reform in Wei, Wu Qi's in Chu, Shen Buhai's in Han, Shang Yang's in Qin and King Wu-Ling's in Zhao.

[10]Veblen (1899) pointed out that the inconvenience in the clothing style is an equilibrium of conspicuous consumption.

[11]*Xun Zi*, chapter *Yi Bing* (On Wars).

Africa and India, instead of the other way around. Smith prophesied the day at dawn.

5.4 Luxury explosion and the Industrial Revolution

Thanks to the Industrial Revolution, we have escaped the Malthusian trap. Understanding why the Malthusian trap had existed is basic to explaining how we escaped it. The classical Malthusian theory predicts instability when birth rates decrease with income. So most researchers have used multiple equilibria (demographic transition) to make sense of the Industrial Revolution. The conventional interpretation is *not* incompatible with the two-sector model. But besides the conventional interpretation, the current theory points to a new set of triggering factors of modern economic growth (table 2).

TABLE 2: The "revolutionary" factors in the old and new theories

Model feature	Triggering event	Result
Classical Malthusian theory		
Birth rates decrease with income	Income rises	Switch to the higher equilibrium
Group selection theory		
Trade replaces migration	Trade cost drops	Selection slows down
Migrants spread knowledge	Printing Tech. develops	Tech. & migration disentangled
Threshold: $g > \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}$	g increases	Balance is tipped
Threshold: $g > \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}$	σ decreases	Growth dominates selection
Literacy was a luxury	It becomes subsistence	Literacy spurs growth

The first factor is trade. Trade substitutes for migration. A decline in the cost of trade, combined with a rise in the cost of migration (political barriers to migration increased in the modern era), can slow down the selection that draws living standards downward.

The second factor is books. It is crucial to the group selection theory that migration should be a major channel to spread knowledge. The assumption no longer held after printing presses spread.

The next two factors appear in the threshold condition of growth, $g > \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}$. Since 1500 till 1800, Northwest Europe had experienced a steady decline in the relative price of luxuries over staple food and fuels (Hoffman et al., 2002). It implies that the gap of growth rates, g, had become larger. What is more, Fouquet (2014) shows that the variance of GDP growth rates of European economies decreased in the 19^{th} century. The increase in g and the decrease in σ^2 might reverse the inequality relationship.

The last factor may be regarded as a theory by itself, the luxury explosion theory. The theory holds that a technology—akin to culture—that turns from a luxury technology into a subsistence technology spreads in an explosive way.

Go back to the group selection model. With a little abuse of notation, denote a negative selection, the case where a trait is selected against (luxury), as $\lambda < 0$, and a positive selection, the case where a trait is selected for (subsistence), as $\lambda > 0$. Following a similar derivation as in section 4.3.3, the relationship between the force of selection and the intensity of selection is still

$$S = \Phi \lambda^{\frac{1}{3}}, \tag{30}$$

except that the sign of λ now indicates whether the trait is selected for or selected against.

As figure 18 shows, the "S" shape of $S(\lambda)$ means that even a tiny λ can produce a large force of selection. The "S" shape results from a subtle *variation effect*: when selection is less intense—because, say, people are more reluctant to move—regions tend to deviate farther away from each other in terms of production structure and the level of utility. The increased gap of utility motivates people to move notwithstanding the inertia; and the enlarged difference of lifestyle means migrants have more surprise to offer to the host culture. Overall, the greater variation compensates the loss of interest in migration. It makes weak selections still have a strong impact[12].

The derivative of $S(\lambda)$ is infinite at 0. When environmental changes make a luxury trait less luxurious, little will change if the trait remains to be luxury. But if the trait turns into subsistence hereby, even if the environmental change is extremely tiny, it will trigger a big change in S—a luxury explosion.

The luxury explosion has a profound implication to the triggering of modern growth. Consider literacy. The tradeoff between quantity and quality of children is a classic example of the choice between subsistence and luxury. To spend more on quality might increase the number of grandchildren (Galor and Klemp, 2014), but if all households do so, the density of population will decline in most cases. The quality of children is the pivot of transition in a bunch of unified growth theories (Galor and Moav, 2002; Galor and Weil, 2000; Clark, 2008; Galor, 2011). Transitions in these models are mostly driven by multiple equilibria of fertility choice. Here selection provides a new mechanism of transition: literacy, which was meant for reading the Bible at the onset of the Religious Reformation, unintentionally equipped the mass with scientific knowledge, engineering knowhow and nationalist enthusiasm, by which Europe colonized the other parts of the world. What used to be a luxury turned into a

[12]A similar mechanism can explain why hardly intermarried peoples are still genetically close to each other. Pinker (2003, p.143) notes that "Rare genes can offer immunity to endemic diseases, so they get sucked into one group from a neighboring group like ink on a blotter, even if members of one group mate with members of the other infrequently. That is why Jews, for example, tend to be genetically similar to their non-Jewish neighbors all over the world, even though until recently they tended to marry other Jews. As little as one conversion, affair, or rape involving a gentile in every generation can be enough to blur genetic boundaries over time."

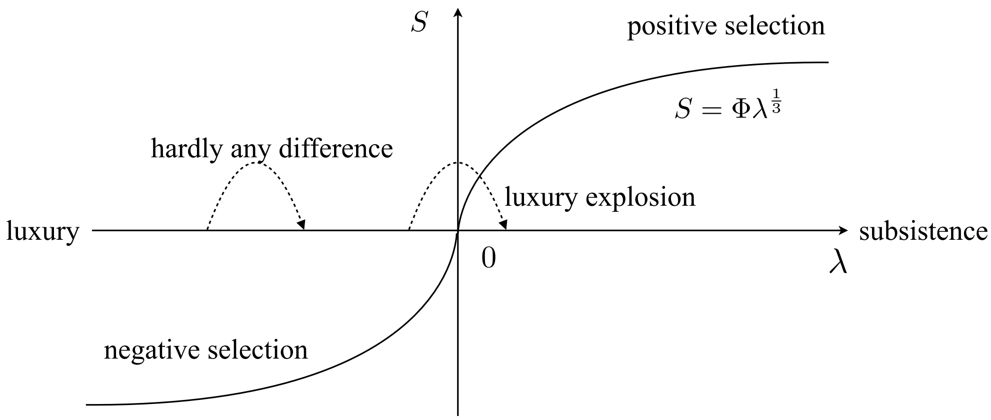

FIGURE 18: The relationship between the force of selection S and the intensity of selection λ. Here $\lambda < 0$ means the commodity is luxury, and it is subject to negative selection; $\lambda > 0$ means the commodity is subsistence, and it is subject to positive selection.

subsistence. The luxury explosion then made a revolution.

The Industrial Revolution is unique because human capital is a very special luxury. Most other luxuries, like diamonds and yachts, cannot switch into subsistence. Weapons switch but they do not improve living standards. Human capital not only enriches the individual but also strengthens a country. If an immigrant wants to benefit from the "luxury", she has to learn it; whereas a learned emigrant can apply his knowledge away from home. Hardly any other luxury combines these wonderful features.

There have been tons of explanations for the Industrial Revolution. It is unlikely that any single factor can account for the whole transition experience. However, the current status of the Industrial Revolution research is that most existing ideas are anecdotal. The idea that receives the most rigorous modeling is demographic transition (there are many versions of it, but the underlying mechanism is always demographic transition). The disproportionate popularity of that single explanation is rooted in the widely-held presumption that the Malthusian trap is caused by the Malthusian mechanism. Now that the presumption is shown wrong, the Industrial Revolution is open to a new set of interpretations: institution, trade, social insurance, Renaissance, Scientific Revolution. As shown above, to each of these the group selection theory has insight to offer; and more importantly, the theory provides a benchmark that makes rigorous modeling of these factors possible.

6 Concluding remarks

For more than two centuries, scholars have taken Malthus's explanation for the Malthusian trap for granted. The conventional wisdom is wrong.

Different from the Malthusian view of history, this paper suggests the following basic story. Imagine a world where people live on two things: bread and flowers. Population increases with bread, hence the average consumption of bread is fixed in the long run by the Malthusian force. But population hardly responds to flowers. If the flower sector grows faster than the bread sector, people will live better and better by having more and more flowers each. Such had never happened until the Industrial Revolution. Throughout the thousands of years before that time, flower productivity had somehow grown at the same rate as bread productivity.

The cause of the balanced growth is group selection. People organize themselves into competing groups. When a group has comparative advantage at making bread, its average member will have fewer flowers than their neighbors do. Greed drives them to move abroad. As they move, they spread the technology of their hometown to the other places. The consequence is that the bread technology tends to spread faster than the flower technology. Even if the flower sector intrinsically grows faster than the bread sector, a tiny bit of spread advantage of the bread sector can offset a large growth advantage of the flower sector. With the whole world interlocked in a network of migration, living standards were stagnant almost everywhere. Thus the Malthusian trap is also a Darwinian trap in the mean time.

For all its novelty, the group selection theory is a tautology, a tautology that makes the theory irrefutably robust. Here is how: what the theory is set to explain is why the average pre-industrial person had so little luxury (flower)[13]. The theory ascribes this Malthusian fact to group competition. By definition, luxury contributes to individual utility at the expense of group fitness. Fitness matters only in the corresponding context of competition. Therefore, luxury must be constrained by group competition, the context in which group fitness ever matters. So the way I define luxury has already ensured that group competition is the main suppressor of luxury.

This paper has five contributions:

I It replaces Malthus's explanation of the Malthusian trap.

II It explains why the Malthusian relationship between average income and population growth is empirically weak: the classical theory misses two of the three determinants of long-run equilibrium, i.e. the social preference and the production structure.

[13]Daily calorie intake per person has hardly changed since the Industrial Revolution. The improvement of life is mostly reflected in the diversity and abundance of luxury.

III It explains why living standards declined after the mankind took up agriculture and why agriculture spread despite its negative effect on living standards.

IV It explains the prosperity of ancient market economies such as Rome and Song.

V It suggests a new set of triggering factors of modern economic growth.

Malthusian theory is fundamental to our understanding of history. Replacing it opens numerous possibilities for economic history research. Here I discuss two of them.

First, the two-sector model liberates living standards researchers from the Malthusian presumption. The presumption is evident in Maddison's series, where both Rome's and Song's per capita GDP were estimated at $450, that is, only $50 above the lowest number in the data. The presumption is also evident in the dubious methodology of many empirical researchers, who ignored non-agricultural output when estimating income, for they believe, as Baumol (1990) put it, "[i]n a period in which agriculture probably occupied some 90 percent of the population, the expansion of industry [...] could not by itself have created a major upheaval in living standards." The presumption is even evident in many skeptics' work. Too often we have seen researchers who provide strong evidence of high living standards in certain historical episodes concluding with an apologetic tone that the prosperity must be a temporary phenomenon that is doomed to disappear when population catches up with technology. Temin (2013, p.193), for example, said, "[i]t reveals even Malthusian economies can have economic growth, that is, can have rising standards of living. This can go on for a long time, even centuries, *even though without industrialization, it is doomed to end*." Now, with the two-sector model available, large swings of living standards becomes a serious theoretical possibility. Researchers no longer have to hide or distort facts to fit any theory. Hopefully, the two-sector model will also direct more researchers' attention to the non-agricultural sectors. In the classical model, commerce and industry are unimportant for living standards; but in the two-sector model, they become crucial.

Second, the group selection theory calls on the profession to embrace a "macro" view of the Industrial Revolution. What I mean is this: the classical Malthusian theory has led most economists to believe that the key mechanism of the Industrial Revolution lies in the demographic transition. Guided by the belief, researchers spent most of their energies on changes of fertility behavior in pre-modern Europe—how fertility varied with income, status, education, and etc. Fertility is a micro decision, made on the household level. Fertility is important. But it is only one of the three comparative statics in the two-sector model. The other two, the social preference and the production structure, are no less important than fertility. What determine these two? They are determined by politics, policies, institutions, wars, trade, migration, cultures, and geography. Incorporating social preference and production structure into analysis means we need to rethink the roles these "macro" factors play in a society. When it comes to the Industrial Revolution, previous researchers asked why households changed minds about children, now the added question is: what had the prince done that made his country stand out?

Finally, I would like to address the difference between Wu, Dutta, Levine, and Papageorge (2014), henceforth WDLP, and this paper. Most related studies treat the Malthusian trap as a fact, but WDLP is an exception. They argue that because manufacturing and commerce usually grow faster than agriculture, the income per capita had a slow yet still significant trend of growth before the Industrial Revolution.

As stated previously, the two-sector Malthusian theory leads to two possibilities. One is that the Malthusian fact is right, but requires a new explanation; the other is that the Malthusian trap did not exist at all. This paper explores the first possibility, while WDLP studies the second. The reality must lie in between. The two theories can actually be reconciled. I will leave the details of the reconciliation to another paper. Here I only sketch the idea.

Many luxuries are culture-specific. They are desired within a culture but not without. Group selection has no way to eliminate the growth of such luxuries because migration never responds to the difference of consumption in these items. Therefore, a distinction should be made between "universal luxury" and "provincial luxury". Universal luxuries are desired by all human beings; provincial luxuries only a group of people. Group selection suppresses universal luxuries, but leaves provincial luxuries free to grow. This explains why culture was so diverse across pre-industrial societies despite the monotony of life (measured by universal luxury). This is also why most economic historians accept the Malthusian fact while WDLP come to a different conclusion. The profession has focused on universal luxury only, but WDLP are concerned with all types of luxuries. The current paper explains the Malthusian trap in its usual sense, that is, why the average consumption of universal luxury had been constantly low throughout the pre-industrial era.

References

Allen, Robert C. 2008. "A Review of Gregory Clark's a Farewell to Alms: A Brief Economic History of the World." *Journal of Economic Literature* 46 (4):946–973.

Álvarez-Nogal, Carlos and Leandro Prados De La Escosura. 2013. "The Rise and Fall of Spain (1270–1850)." *The Economic History Review* 66 (1):1–37.

Ashraf, Quamrul and Oded Galor. 2011. "Dynamics and Stagnation in the Malthusian Epoch." *The American Economic Review* :2003–2041.

Baumol, William J. 1990. "Entrepreneurship: Productive, Unproductive, and Destructive." *Journal of Political Economy* :893–921.

Bowles, Samuel. 2006. "Group Competition, Reproductive Leveling, and the Evolution of Human Altruism." *Science* 314 (5805):1569–1572.

Bowles, Samuel and Herbert Gintis. 2002. "Behavioural Science: Homo Reciprocans." *Nature* 415 (6868):125–128.

Broadberry, Stephen, Hanhui Guan, and David D. Li. 2014. "China, Europe and the Great Divergence: A Study in Historical National Accounting, 950-1850." Economic History Department Paper, London School of Economics.

Broadberry, Stephen and Bishnupriya Gupta. 2006. "The Early Modern Great Divergence: Wages, Prices and Economic Development in Europe and Asia, 1500–1801." *The Economic History Review* 59 (1):2–31.

Bryce, Trevor. 1999. *The Kingdom of the Hittites*. Oxford University Press.

Cavalli-Sforza, Luigi L, Paolo Menozzi, and Alberto Piazza. 1993. "Demic Expansions and Human Evolution." *Science* 259 (5095):639–646.

Chen, Shuo and James Kai-sing Kung. 2013. "Of Maize and Men: The Effect of a New World Crop on Population and Economic Growth in China." *Available at SSRN 2102295* .

Clark, Gregory. 2008. *A Farewell to Alms: A Brief Economic History of the World*. Princeton University Press.

———. 2010. "The Macroeconomic Aggregates for England, 1209–2008." *Research In Economic History* 27:51–140.

Davies, John E. 1994. “Giffen Goods, the Survival Imperative, and the Irish Potato Culture.” *Journal of Political Economy* :547–565.

De Vries, Jan. 2006. *European Urbanisation, 1500-1800*, vol. 4. Routledge.

Diamond, Jared. 1987. “The worst mistake in the history of the human race.” *Discover* 8 (5):64–66.

Fouquet, Roger. 2014. “Seven Centuries of Economic Growth and Decline.” Presented at the AEA/ASSA Annual Meeting 2015.

Friedman, Milton. 1953. “The Methodology of Positive Economics.” *Essays In Positive Economics* 3 (8).

Galor, Oded. 2011. *Unified Growth Theory.* Princeton University Press.

Galor, Oded and Marc Klemp. 2014. “The Biocultural Origins of Human Capital Formation.” Tech. rep., University Library of Munich, Germany.

Galor, Oded and Omer Moav. 2002. “Natural Selection and the Origin of Economic Growth.” *The Quarterly Journal of Economics* 117 (4):1133–1191.

Galor, Oded and David N Weil. 2000. “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond.” *American Economic Review* 90 (4):806–828.

Hansen, Gary D and Edward C Prescott. 2002. “Malthus to Solow.” *American Economic Review* :1205–1217.

Harlan, Jack Rodney. 1975. *Crops and Man.* American Society of Agronomy.

Hersh, Jonathan and Joachim Voth. 2009. “Sweet Diversity: Colonial Goods and the Rise of European Living Standards after 1492.” .

Hoffman, Philip T, David S Jacks, Patricia A Levin, and Peter H Lindert. 2002. “Real Inequality in Europe since 1500.” *The Journal of Economic History* 62 (02):322–355.

Hong, Sungmin, Jean-Pierre Candelone, Clair C Patterson, and Claude F Boutron. 1996. “History of Ancient Copper Smelting Pollution during Roman and Medieval Times Recorded in Greenland Ice.” *Science* 272 (5259):246–249.

Jones, Charles I. 2001. “Was an Industrial Revolution Inevitable? Economic Growth over the Very Long Run.” *Advances In Macroeconomics* 1 (2).

Lee, Ronald. 1987. "Population Dynamics of Humans and Other Animals." *Demography* 24 (4):443–465.

Levine, David and Salvatore Modica. 2013. "Anti-Malthus: Conflict and the Evolution of Societies." *Research In Economics* .

Lipsey, Richard G, Kenneth I Carlaw, and Clifford T Bekar. 2005. *Economic Transformations: General Purpose Technologies and Long-Term Economic Growth*. Oxford University Press.

Liu, William Guanglin. 2015. "The Making of a Fiscal State in Song China, 960–1279." *Economic History Review* 1 (68):48–78.

Lo Cascio, Elio and Paolo Malanima. 2009. "Ancient and Pre-Modern Economies: GDP in the Roman Empire and Early Modern Europe." Presented at the conference on "Long-Term Quantification in Mediterranean Ancient History," Brussels.

Maddison, Angus. 2003. *The World Economy: Historical Statistics*. OECD Publishing.

Malanima, Paolo. 2011. "The Long Decline of a Leading Economy: GDP in Central and Northern Italy, 1300-1913." *European Review of Economic History* 15 (02):169–219.

Malthus, Thomas Robert. 1809. *An Essay on the Principle of Population, as It Affects the Future Improvement of Society*, vol. 2.

Moav, Omer and Zvika Neeman. 2008. "Conspicuous Consumption, Human Capital, and Poverty." *Human Capital and Poverty* .

Morris, Ian. 2011. *Why the West Rules - For Now: The Patterns of History, and What They Reveal about the Future*. Random House LLC.

Moser, Petra, Alessandra Voena, and Fabian Waldinger. 2014. "German-Jewish Emigrés and US Invention." Tech. rep., National Bureau of Economic Research.

Nunn, Nathan and Nancy Qian. 2011. "The Potato's Contribution to Population and Urbanization: Evidence From a Historical Experiment." *The Quarterly Journal of Economics* 126 (2):593–650.

Persson, Karl Gunnar. 2010. "The End of the Malthusian Stagnation Thesis." Tech. rep., Mimeo (University Of Copenhagen).

Pinker, Steven. 2003. *The Blank Slate: the Modern Denial of Human Nature*. Penguin.

Pomeranz, Kenneth. 2009. *The Great Divergence: China, Europe, and the Making of the Modern World Economy*. Princeton University Press.

Ravenstein, Ernest George. 1885. "The Laws of Migration." *Journal of the Statistical Society of London* :167–235.

Restuccia, Diego, Dennis Tao Yang, and Xiaodong Zhu. 2008. "Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis." *Journal of Monetary Economics* 55 (2):234–250.

Robbins, Lionel. 1998. *A History of Economic Thought: the LSE Lectures*. Wiley Online Library.

Simon, Julian Lincoln and Gunter Steinmann. 1991. "Population Growth, Farmland, and the Long-Run Standard of Living." *Journal of Population Economics* 4 (1):37–51.

Smith, Adam. 1887. *An Inquiry into the Nature and Causes of the Wealth of Nations*. T. Nelson and Sons.

Solow, Robert M. 1956. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics* 70 (1):65–94.

Solow, Robert M and Paul A Samuelson. 1953. "Balanced Growth under Constant Returns to Scale." *Econometrica* :412–424.

Soltis, Joseph, Robert Boyd, and Peter J Richerson. 1995. "Can Group-Functional Behaviors Evolve by Cultural Group Selection? An Empirical Test." *Current Anthropology* :473–494.

Taylor, M Scott and James A Brander. 1998. "The Simple Economics of Easter Island: A Ricardo-Malthus Model of Renewable Resource Use." *The American Economic Review* 88 (1):119–138.

Temin, Peter. 2013. *The Roman Market Economy*. Princeton University Press.

Tiebout, Charles M. 1956. "A Pure Theory of Local Expenditures." *The Journal of Political Economy* :416–424.

Veblen, Thorstein. 1899. *The Theory of the Leisure Class: an Economic Study of Institutions*. Macmillan.

Voigtländer, Nico and Hans-Joachim Voth. 2013. "The Three Horsemen of Riches: Plague, War, and Urbanization in Early Modern Europe." *The Review of Economic Studies* 80 (2):774–811.

Ward-Perkins, Bryan. 2005. *The Fall of Rome: and the End of Civilization.* Oxford University Press.

Weisdorf, Jacob L. 2008. "Malthus Revisited: Fertility Decision Making Based on Quasi-Linear Preferences." *Economics Letters* 99 (1):127–130.

Wu, Lemin. 2012. "Does Malthus Really Explain the Constancy of Living Standards?" *Available at SSRN 2187113* .

Wu, Lemin, Rohan Dutta, David K Levine, and Nicholas W Papageorge. 2014. "Entertaining Malthus: Bread, Circuses and Economic Growth." Tech. rep., UCLA Department Of Economics.

Yang, Dennis Tao and Xiaodong Zhu. 2013. "Modernization of Agriculture and Long-Term Growth." *Journal of Monetary Economics* 60 (3):367–382.

Appendices

A Proofs

A.1 Prove that g converges to $\beta(g_B - g_A)$ in the long run.

First, I prove the following lemma.

Lemma 5. *If an isolated economy has constant growth rates of technology g_A and g_B, then $g_A - (1-\gamma)g_H$ converges to 0.*

Proof:

Population evolves in the following way:

$$g_H = \delta(\ln x - \ln \bar{x})$$

Since $x = A(1-\beta)^\gamma H^{\gamma-1}$ (equation 7),

$$g_H = \delta[\ln A + \gamma \ln(1-\beta) + (\gamma - 1)\ln H - \ln \bar{x}]$$

Denote $M \equiv \ln A + (\gamma - 1)\ln H$, then

$$g_H = \delta[M + \gamma \ln(1-\beta) - \ln \bar{x}]$$

The motion of M follows

$$\begin{aligned} dM &= g_A + (\gamma - 1)g_H \\ &= g_A + (\gamma - 1)\delta[M + \gamma \ln(1-\beta) - \ln \bar{x}] \end{aligned}$$

Since $(\gamma - 1)\delta < 0$, M will stabilize at

$$M^* = \frac{g_A}{(1-\gamma)\delta)} - \gamma \ln(1-\beta) + \ln \bar{x}$$

Hence $dM = g_A - (1-\gamma)g_H$ converges to 0. □

Proposition 6. *g_U converges to $\beta(g_B - g_A)$.*

Proof: Start by expressing U as a function of A and B. We can not use the formula of equilibrium utility (equation 11) because the continuous progress of technology will pull the economy slightly away from the equilibrium state. So I turn to equation 9, which applies to dynamic scenario as well.

Suppose land is fixed. By log-linearizing equation 9, we get

$$g_U = \beta(g_B - g_A) + g_A - (1-\gamma)g_H.$$

Lemma 5 holds that $g_A - (1-\gamma)g_H$ converges to 0. Therefore, g_U converges to $\beta(g_B - g_A)$. □

A.2 Prove that $S \equiv \beta\delta\,\mathbb{E}_{t\to+\infty}(\nu) = \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}$.

Proof: By Ito's lemma,

$$d\nu^x = \left[\sigma^2 x(2x-1)\nu^{x-1} - 2\sqrt{2}|\lambda|x\nu^{x+\frac{1}{2}}\right]dt + 2\sigma x\nu^{x-\frac{1}{2}}dz$$

Since $\mathbb{E}_{t\to+\infty}(d\nu^x) \to 0$, the long-run expectation of the drift term

$$\sigma^2 x(2x-1)\mathop{\mathbb{E}}_{t\to+\infty}(\nu^{x-1}) - 2\sqrt{2}|\lambda|x\mathop{\mathbb{E}}_{t\to+\infty}(\nu^{x+\frac{1}{2}}) = 0.$$

Let $f(x) \equiv \mathbb{E}_{t\to+\infty}(\nu^x)$ and denote $\frac{\sigma^2}{2\sqrt{2}|\lambda|}$ as a, then the above equation can be rewritten as a general term formula:

$$f\left(x+\frac{3}{2}\right) = a(2x+1)f(x)$$

with $f(0) = \mathbb{E}_{t\to+\infty}(\nu^0) = 1$.

The general solution is

$$f(x) = \frac{1}{3}(3a)^{\frac{2}{3}x}\,\text{Pochhammer}\left(\frac{4}{3}, \frac{2}{3}x - 1\right).$$

Let $x = 1$, then

$$f(1) = \frac{a^{\frac{2}{3}}}{3^{\frac{1}{3}}\,\text{Gamma}\left(\frac{4}{3}\right)}.$$

Denote $k \equiv \left[3^{\frac{1}{3}}\,\text{Gamma}\left(\frac{4}{3}\right)\right]^{-1} \approx 0.78$, then $f(1)$ can be written as $ka^{\frac{2}{3}}$.

By definition,

$$\mathop{\mathbb{E}}_{t\to+\infty}(\nu) = f(1) = k\left[\frac{\sigma^2}{2\sqrt{2}|\lambda|}\right]^{\frac{2}{3}} = k\nu^*$$

Substituting $\mathbb{E}_{t\to+\infty}(\nu) = k\nu^*$ into $S \equiv \beta\delta\,\mathbb{E}_{t\to+\infty}(\nu)$, we get

$$S = \lambda k \nu^* = \frac{k}{2}(\beta\delta)^{\frac{1}{3}}(\sigma^2)^{\frac{2}{3}}.$$

□

B Figures

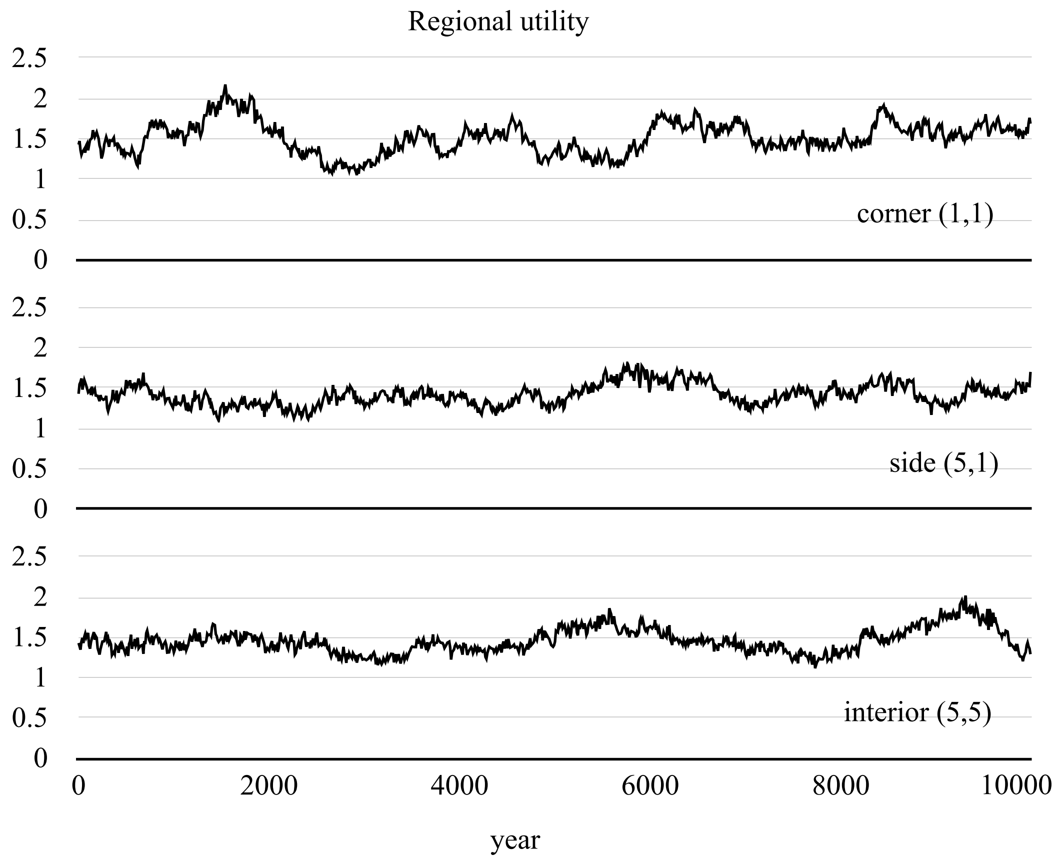

FIGURE 19: The regional utility fluctuates wildly but has no trend. Here is the history of regional utility of three representative regions: a corner region $(1, 1)$, a side region $(5, 1)$, and an interior region $(5, 5)$.

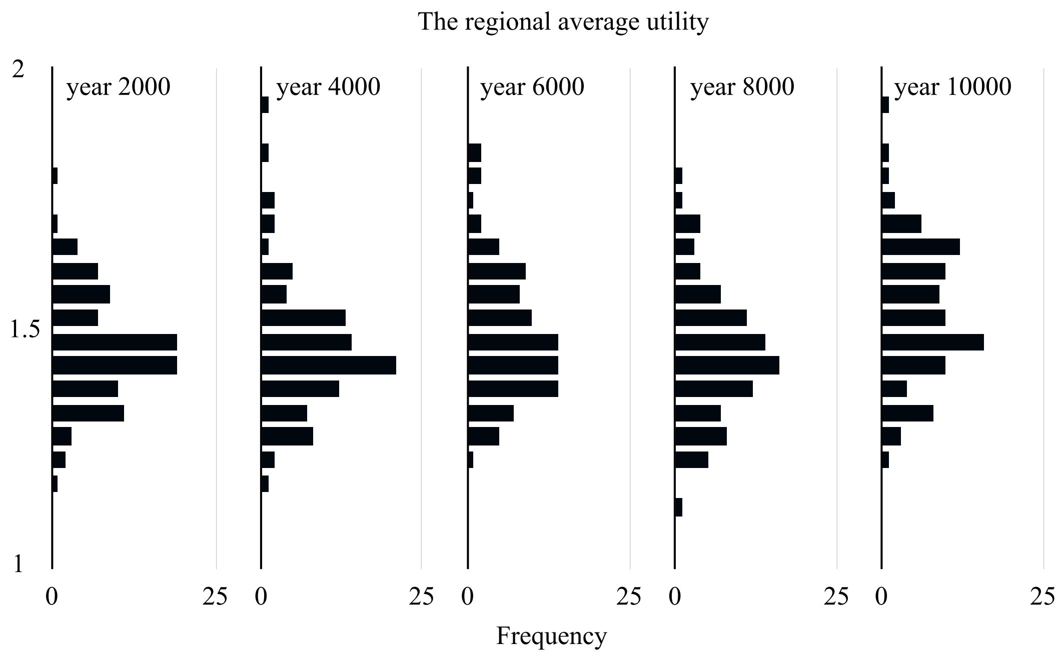

FIGURE 20: The distribution of regional average utility is stable over time.

C Tables

TABLE 3: Parameterization of the baseline simulation

Parameter	Value	Interpretation
g_A	0.5%	Subsistence growth rate
g_B	1%	Luxury growth rate
σ_A	5%	Std. of subsistence growth
σ_B	5%	Std. of luxury growth
δ	0.2	$n = \delta(\ln x - \ln \bar{x})$
γ	0.5	$X = AL_A^{1-\gamma}H_A^{\gamma}$, $Y = BL_B^{1-\gamma}H_B^{\gamma}$
$\bar{x}$	1	$n = \delta(\ln x - \ln \bar{x})$
θ	0.1	migrational rate
β	0.5	$U = x^{1-\beta}y^{\beta}$
α	-10	$g_{Bij} = g_B[1 + (B_{ij}/A_{ij})^{\alpha}]$

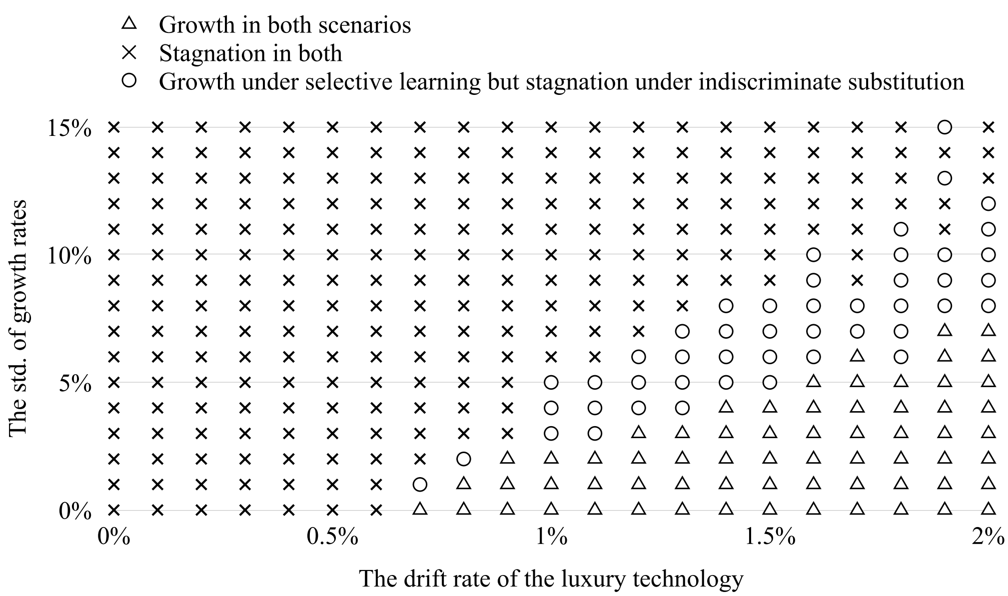

FIGURE 21: The progressive and stagnant areas on the parameter space. Notes: a point is counted as a "growth point" only if the global average utility grows more than 25% over 3000 years. There are three areas in the parameter space. The upper left area that is marked with crosses is where the global average utility stagnates in both the indiscriminate substitution case and the selective learning case. The middle area marked with circles is where the utility grows under selective learning but not under indiscriminate substitution. The lower right area marked with triangles is where growth occurs under both scenarios.

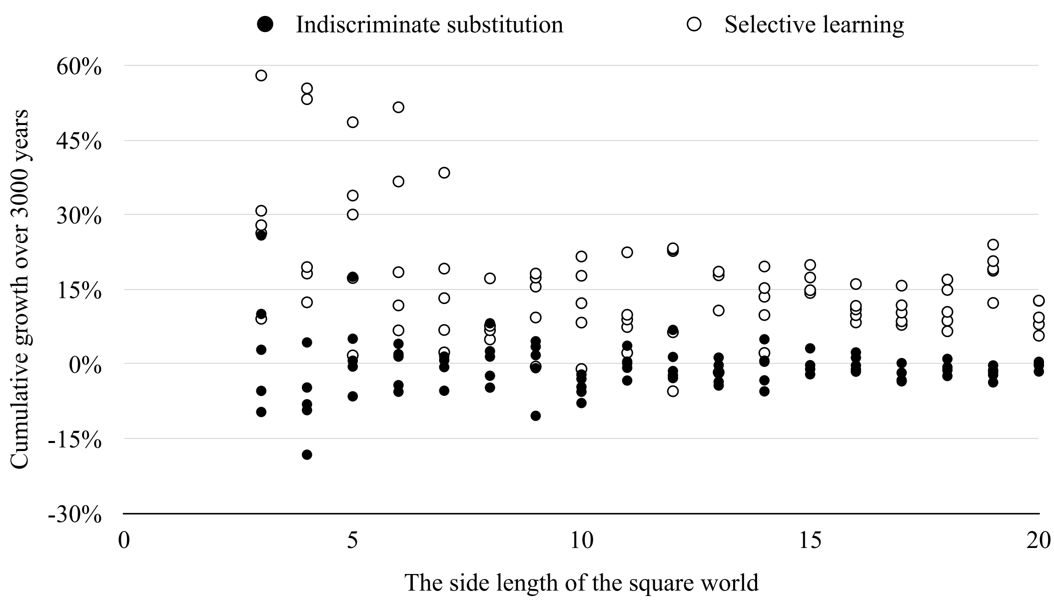

FIGURE 22: The cumulative growth of global average utility over 3000 years. Notes: each point represents an experiment at the corresponding side length of the world.