NBER WORKING PAPER SERIES

PIKETTY'S BOOK AND MACRO MODELS OF WEALTH INEQUALITY

Mariacristina De Nardi
Giulio Fella
Fang Yang

Working Paper 21730
http://www.nber.org/papers/w21730

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2015

Fella is grateful to UCL for the generous hospitality while working on this paper. We thank Marshall Steinbaum and Emmanuel Saez for comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research, any agency of the Federal Government, nor the Federal Reserve Bank of Chicago. This paper has been written as a chapter for the book "The Global Ramifications of Thomas Piketty's Capital in the 21st Century," edited by Heather Boushey, Bradford DeLong, and Marshall Steinbaum.

Piketty's Book and Macro Models of Wealth Inequality
Mariacristina De Nardi, Giulio Fella, and Fang Yang
NBER Working Paper No. 21730
November 2015
JEL No. D14,E21,H2

ABSTRACT

Piketty's book discusses several factors affecting wealth inequality: rates of return on capital, output growth rates, tax progressivity, top income shares, and heterogeneity in saving rates and inheritances. This paper studies the role of various forces affecting savings in quantitative models of wealth inequality, discusses their success and failures in accounting for the observed facts, and compares these model's implications with Piketty's conclusions.

Mariacristina De Nardi
Federal Reserve Bank of Chicago
230 South LaSalle St.
Chicago, IL 60604
and University College London
and Institute For Fiscal Studies - IFS
and also NBER
denardim@nber.org

Giulio Fella
Mile End Road
London E1 4NS, UK
g.fella@qmul.ac.uk

Fang Yang
Louisiana State University
Department of Economics, 2322
Business Education Complex,
Nicholson Extension
Baton Rouge, LA 70803
fyang@lsu.edu

1 Introduction

Piketty's "Capital in the Twenty-First Century" is, in the author's own words, "...primarily a book about the history of the distribution of income and wealth." It documents the evolution of the distributions of income and wealth since the Industrial Revolution for a significant number of countries and offers a framework to account for the common patterns of the long-run evolution of within-country wealth inequality across a number of developed economies.

This chapter takes stock of the existing literature on models of wealth inequality through the lens of the facts and ideas in Piketty's book and highlights both what we have learned so far and what we still need to learn in order to reach more definitive conclusions on the mechanisms shaping wealth concentration.

The chapter starts by introducing some important stylized facts about the distribution of wealth (Section 2), which are:

1. Wealth is highly concentrated. Its distribution is highly skewed with a long right tail.

2. Overall, there is significant mobility within the wealth distribution, both within an individual lifetime and across generation. Wealth mobility is substantially lower, though, at the top and the bottom of the distribution.

3. Wealth concentration—the share of aggregate wealth in the hands of the richest people—displays a U shape, trending downward for most of the XX century and then increasing from the 1980s onward.

The chapter then turns to a discussion of the main mechanisms identified in Piketty's book as potentially relevant to understand wealth inequality (Section 3) and presents a simple framework to better understand and categorize various mechanisms behind individual wealth accumulation that can account for wealth inequality (Section 4). From there, the chapter then moves onto surveying the existing macroeconomic literature on wealth inequality with an emphasis on the forces that hold better promise to account for the high degree of wealth concentration observed in the data (Sections 5 and 6).

More specifically, in section 5 we discuss the (mostly analytical) literature aiming to account for the observation that the right tail of the wealth distribution is well approximated by a Pareto distribution. This strand of the literature provides the main theoretical underpinning for the mechanism, emphasized in Piketty's book, according to which wealth concentration increases with the difference between the average net rate of return on wealth r and the trend rate of growth of aggregate output g. Multiplicative random shocks to the wealth accumulation process are the main mechanism that generates wealth concentration in this class of models. While Piketty sees the rate of output growth as unambiguously reducing wealth concentration, according to some of these models growth can either reduce or increase wealth concentration depending on the environment.

For reasons of analytic tractability, with the exception of Benhabib, Bisin and Zhu (2015) and Aoki and Nirei (2015), the literature on models with multiplicative shocks abstracts from *endogenous* heterogeneity in saving rates, and *endogenous* rates of return—in the form of entrepreneurial income—as a source of persistence in saving rates. Furthermore, it does not consider life cycle aspects and non-homotheticity in bequest behavior that affect wealth accumulation across generations and are important to explain why rich people have higher saving rates during both their working life and retirement (Dynan, Skinner and Zeldes 2004).

Endogenous heterogeneity in saving against earnings and expenditure shocks (including, possibly, medical and nursing home expenditures during retirement) is instead at the center of the quantitative models that we discuss in Section 6. The comparative advantage of this literature is its emphasis on understanding the forces that shape differences in saving behavior and rates of return and on quantifying the importance of such heterogeneity in accounting for wealth inequality in rich quantitative models. The section argues that previous work has convincingly emphasized that entrepreneurial activity, voluntary bequests, heterogeneity in preferences across families, and compensation risk for top earners can help explain the high degree of wealth concentration. However, it is not clear to what extent each of these forces quantitatively contributes to wealth inequality because, at least so far, most of these forces have been studied in isolation. There is also much work to do in determining to what extent these quantitative frameworks can match the observed large differences in wealth inequality both across countries and over time.

To go from the static understanding of what gives rise to inequality at one point in time, to how inequality changes over time, Section 7 studies the smaller literature that analyzes the transitional dynamics of the wealth distribution.

To quantitatively assess Piketty's conjecture that the changes in the difference between the post-tax rate of return on capital and the rate of output growth may drive the evolution of wealth concentration, Section 8 uses a rich quantitative model to study these questions. It shows that the rate of return on capital and the rate of output growth are not perfect substitutes in their effect on wealth concentration. In fact, an increase in the rate of return on capital raises wealth concentration substantially less than a fall in the the rate of population and output growth by the same amount.

We conclude with a discussion of fruitful areas for future research in Section 9.

2 Stylized facts

It is well established that the cross-sectional wealth distribution is right-skewed, unlike the normal distribution, and that its right tail is well approximated by a Pareto distribution. A Pareto distribution implies a linear relationship between the logarithm of wealth w and the logarithm of the proportion of individuals $P(w)$ with wealth above w.[1] Figure 2 plots this relationship for the top 10 per cent of the wealth distribution in a selected number of

[1]It was this very observation that prompted Pareto to propose his eponymous distribution (Pareto 1897).

countries, with the circles denoting the actual observations and the dashed and solid lines the fitted Pareto model for respectively the 10 and 1 per cent of the distribution.

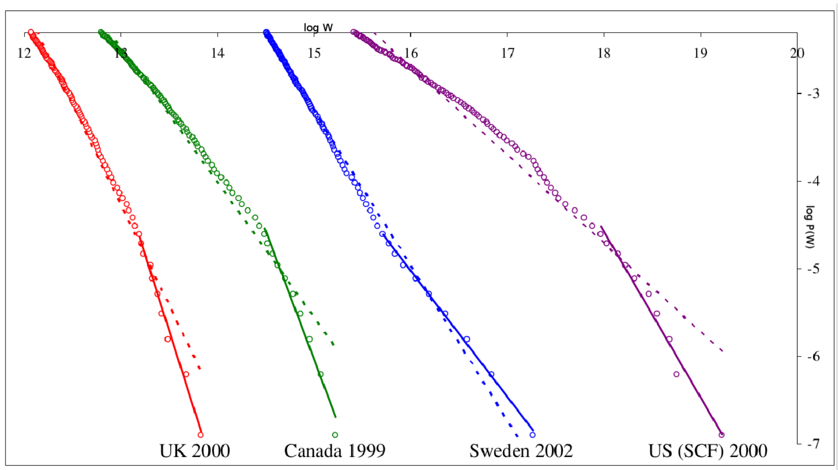

Figure 1: Pareto tails for selected countries: actual values (circles) and average slopes over the top 10 (dashed line) and 1 (solid line) per cent of the distribution (source: Cowell 2011).

The observation that wealth is much more concentrated than labor earnings and income is vast. Wold and Whittle (1957) cite early evidence for the US. More recently, this fact has been documented by Atkinson (1983), Wolff (1998) and Wolff (1992), Kennickell (2003), Díaz-Giménez, Quadrini and Ríos-Rull (1997), and Rodriguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002).

How wealth is distributed at a point in time is important, but equally important is how much churning there is within the distribution, both at the individual/household level and across generations. At the individual level, Hurst, Luoh and Stafford (1998) document significant mobility across the 20-80 percentile range in the U.S. (they use the Panel Study of Income Dynamics (PSID) data over the period 1984-94), but substantial persistence in the top and bottom deciles. For these two groups, the probability of remaining in the same decile ranged between 40 and 60 per cent, depending on the sub-periods.[2] This latter finding implies that 60 per cent of total wealth—the share owned by the top decile in Hurst et al.'s

[2] Klevmarken, Lupton and Stafford (2003) find that *overall* wealth-quintile mobility, though not wealth inequality, in Sweden is comparable to the U.S.

(1998)—is quite persistent.

Concerning the evidence on inter-generational wealth mobility, Mulligan (1997) estimates an elasticity of children's to parents' wealth for the U.S. between 0.32 and 0.43. Charles and Hurst (2003) find a value of 0.37 in the PSID, which falls to 0.17 when controlling for children's age, education and income. Due to data limitations,[3] these estimates are for the inter-generational wealth elasticity for parent-child pairs in which parents are still alive; i.e., *before* the transfer of bequests. For this reason, they possibly underestimate the overall degree of inter-generational wealth persistence. This problem is addressed in the studies by Adermon, Lindahl and Waldenström (2015) for Sweden, Boserup, Kopczuk and Kreiner (2015) for Denmark and Clark and Cummins (2015) for England and Wales, all using wealth data spanning more than one generation. The first two studies use wealth tax data; Adermon et al. (2015) find parent-child rank correlation of 0.3 to 0.4, while Boserup et al. (2015) estimate a wealth elasticity between 0.4 and 0.5. Clark and Cummins (2015) instead exploit a long (1858-2012) panel of families with rare surnames whose wealth is observed at death and find an inter-generational elasticity of 0.4-0.5 for the subsample in which they can match parents and children and of about 0.7 when grouping individuals by surname cohort.

Overall, the evidence of significant wealth mobility suggests that shocks to economic circumstances are an important determinant of wealth dynamics. This feature lies at the heart of a large literature, surveyed in Section 6, that emphasizes wealth accumulation as a way to smooth consumption in the face of idiosyncratic shocks to income.

A third important feature of the wealth distribution is its evolution over time. Until recently, studies documenting the evolution of wealth inequality over time were few and covering relative short time spans.[4] One important contribution of Piketty's book is to bring together a number of recent studies and document the evolution of the wealth distribution

[3]The age of children in Mulligan's (1997) sample is below 35, while the number of parent-child pairs in which both parents have died is very small in Charles and Hurst's (2003) sample.

[4]Early efforts to document the evolution of the wealth distribution in the XX century are Lampman (1962) for the US and Atkinson and Harrison (1983) for the UK.

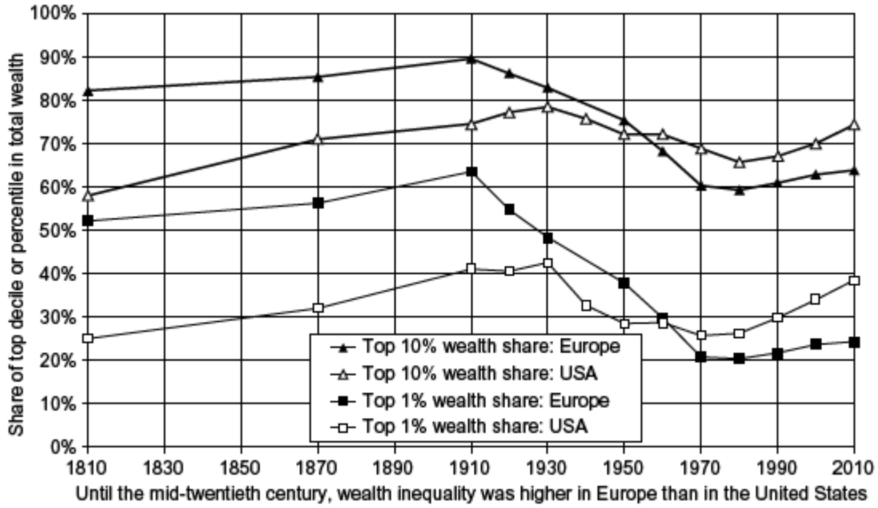

Figure 2: The evolution of wealth inequality in Europe and the United States (source: Piketty and Zucman 2014).

since the Industrial Revolution for a significant number of countries. The main finding, common to the various countries, is that the share of aggregate wealth in the hands of the richest people displays a U-shaped curve starting from very high levels at the beginning the XX century, falling dramatically during the two World Wars to reach its minimum, typically between the the Second World War and the 1970s, and then increasing again from the 1980s onwards (see Figure 2).

There is some debate over the actual magnitude of the increase in the share of wealth held by the top 1% of the distribution since the 80s. Figure 2 reports the estimates from tax data in Saez and Zucman (2015) that imply that the share has increased by about 13 percentage points, effectively reverting to its 1930 peak. Estimates from the SCF imply a substantially smaller increase of about 5 percentage points.[5] Despite the uncertainty around the actual magnitude of the increase, understanding its causes and likely future evolution have become an important research topic. Section 7 discusses the extent to which alternative models of wealth inequality can account for the evolution of top wealth concentration.

[5]Kopczuk (2015) discusses differences among the available estimates and possible explanations for their divergence.

3 Piketty's mechanisms

Piketty's book offers a framework—rooted in the Pareto-tail literature surveyed in Section 5—to explain the evolution of wealth inequality in the course of the XX century. According to this framework, wealth inequality increases with the difference between the net of tax average rate of return on wealth $\bar{r}$ and the trend rate of growth of aggregate output g. This mechanism is consistent with the simultaneous fall in wealth concentration and $(\bar{r} - g)$ over the course of the XX century. Between 1914 and the 1980s the after-tax rate of return of capital fell both as a result of the capital losses stemming from the Great Depression and the two World Wars and as a result of the progressive tax policies that have their roots in the shocks the 1914-1945 period. At the same time, the rate of output growth over the second half of the XX century was dramatically larger that at the beginning of the century.

The intuition behind this "$(\bar{r} - g)$" mechanism is that a higher rate of return $\bar{r}$ increases the rate at which existing wealth is capitalized, amplifying any initial heterogeneity in the wealth distribution. On the other hand, a higher rate of growth g increases the rate of accumulation of "new" wealth through saving out of labor earnings, which tends to reduce inequality. Whether existing wealth has been saved by current generations, or inherited from previous ones, a higher rate of return increases the importance of both relative to current labor income.

Though this mechanism plays an important role in Piketty's book and has drawn a lot of attention, the book mentions other important forces. The first force—related to the previous one in so far as it affects the *inter-generational* rate of of wealth accumulation— is inheritance, both of financial and human wealth, and its interplay with demographics.

A second one is the heterogeneity of rates of return and the fact that wealthy people, who invest large amounts of capital, typically obtain higher returns, for instance because they can take on riskier and less liquid investments, or because they have bigger incentives to hire financial managers and, more generally, to spend time and money to obtain higher returns.

In addition, there is also heterogeneity in saving rates, with people with higher initial wealth saving more.

Another important aspect discussed in the book is the rise of the super-managers—high-level managers— whose share of profits has increased faster than anyone else's, especially in the United States. The book argues that this is an important source underpinning the observed increase inequality in total income. It should be noted that the United States is also the country over which wealth inequality increased the fastest over the same time period.

Finally, the book highlights the importance of government taxes, transfers and more generally government regulations such as the minimum wage, and the importance of market structure in affecting income and wealth inequality across countries and over time for a given country.

4 Accounting for wealth concentration

We now introduce a simple accounting framework, originally due to Meade (1964), to better organize our discussion both of Piketty's $(r - g)$ insight and, in general, of the literature on the determinants of wealth inequality. To this goal, consider an economy in which the aggregate capital stock grows at the exogenous rate g. At birth, individuals are endowed with a, possibly individual-specific, fraction of the contemporaneous, aggregate capital stock. The only source of income in the economy is an idiosyncratic rate of return on individual wealth.

In a given period, individual wealth, normalized by the average capital stock, accrues at the exponential rate $r_t^i - g + s_t^i$, where r_t^i is the realized rate of return for individual i of current age t and s_t^i is the ratio between the individual's flow of (dis)saving[6] and her wealth at the beginning of the period. In this economy there are three main forces that shape the distribution of individual wealth normalized by the average capital stock of capital.

[6]To be precise the flow of saving out of non-capital income which equals minus the flow consumption in the present setup with zero non-capital income.

1. The individual history of saving rates s_t^i, or equivalently the average saving rate over an individual's lifetime. *Coeteris paribus,* individuals with a higher average lifetime saving rate have accumulated wealth at a faster rate.

2. The individual history of the, growth adjusted, rate of return $r_t^i - g$, or equivalently its average rate over the individual's lifetime. *Coeteris paribus* individuals with a larger average lifetime return – i.e. a history of high intra-period returns – have seen their wealth grow faster that individuals with a low average return. Conversely, for a given cross-sectional distribution of individual returns r_t^i, a higher rate of growth g reduces the rate of growth of normalized individual wealth, by reducing the rate at which individual wealth grows relative to the aggregate economy.

3. The distribution of wealth at birth. Even abstracting from individual differences in rates of returns and saving rates, the wealth differential between two individuals within the same cohort born with heterogeneous wealth endowments increases with age at the common rate of accumulation. An increase in the difference the, assumedly, common rate of return r and the growth rate g increases this rate of divergence.

This last effect captures the essence of Piketty's $(r - g)$ insight. For given saving rates, the power of exponential compounding suggests that the persistent changes in the difference $(r - g)$, increase substantially the rate of divergence in the wealth distribution. Therefore, "growth or multiplicative" effects, are crucial to account for why a minority of individuals hold a disproportionate share of aggregate wealth. This basic mechanism lies at the heart of the, mostly analytic, literature on multiplicative models of wealth accumulation surveyed in Section 5.

5 Analytical models generating Pareto tails in the wealth distribution

Since Pareto, this feature of the wealth distribution has been further documented and has motivated a number of studies proposing economic mechanisms generating a wealth distribution with a right Pareto tail.

These mechanisms require multiplicative random shocks to the wealth accumulation process and fall into two main categories. The first type requires individual wealth to grow exponentially at some—positive—average rate until an exponentially distributed stopping time (e.g. death). In the absence of intergenerational transmission of wealth, this class of models[7] implies, counterfactually, that all wealth heterogeneity is between, rather than within, age cohorts. The individuals with higher wealth are the survivors from older cohorts that have accumulated wealth over a longer horizon. Allowing for (stochastic) intergenerational transmission of wealth, as in Benhabib and Bisin (2006), introduces additional heterogeneity: within a cohort, individuals belonging to a dinasty with a longer history of bequests are wealthier. To sum up, the general mechanism implies that the Pareto coefficient, that indexes the thickness of the right tail of the distribution of wealth (relative to the size of the economy), is increasing in the growth rate of individual wealth relative to aggregate wealth, and decreasing in the probability of death and in those forces, such as redistributive estate taxation, that compress the wealth distribution at birth.

The second type of mechanism generating Pareto tails is, to some extent, conceptually opposite to the previous one and requires that the exponential rate of growth of individual wealth follows an appropriate stochastic process with a negative mean. The negative mean growth rate implies that on average individual wealth reverts to the mean of the distribution. Yet, the lucky few with a long history of (above average) positive rates of

[7] Wold and Whittle (1957) is an early example featuring an exogenous rate of wealth accumulation. The latter is endogenized in the optimizing models of Benhabib and Bisin (2006) and Jones (2015).

wealth growth escape this mean reversion force and accumulate large fortunes. Some inflow mechanism—e.g., transfers, positive additive income shocks, precautionary saving and or borrowing constraints—is necessary to provide a reflecting barrier that ensures that the mean of the wealth distribution is bounded away from zero. Models in this class[8] generate both within- and between- cohort inequality. They imply that the Pareto coefficient is larger: (1) the larger the variance of shocks, which increases the probability of long histories of positive shocks; (2) the weaker the offsetting inflow mechanism, which increases the mean of the stationary distribution.

In common to the first class of models, they imply that wealth concentration, as measured by the Pareto coefficient, is decreasing in the rate of wealth mean reversion; i.e. it is increasing in the average rate of growth of individual, relative to aggregate, wealth. Such a rate equals the sum, on the one hand, of the difference $(\bar{r} - g)$ between the average, post-tax rate of return $\bar{r}$ and the rate of aggregate growth and, on the other hand, the average ratio s_w between saving out of non-capital income and wealth. The logic of exponential growth implies that, in all models with multiplicative shocks, small variations in $(\bar{r} - g)$ or $\bar{s}_w$ generate large changes in the Pareto coefficient.

Benhabib et al. (2011) build a partial equilibrium, overlapping-generation model with a (homothetic) bequest motive in which individuals are born with idiosyncratic and independently distributed (i.i.d) earnings and rates of return to wealth, which remain constant over an individual's lifetime. They find that in their framework it is rate-of-return, rather than earnings, shocks across individuals that affect the shape of the right tail of the stationary wealth distribution. This is best understood in the case in which individuals have logarithmic preferences and wealth bequeathed equals a common share of wealth at birth but capitalized at the individual-specific rate of returns. Wealthy dynasties have a long history of above average rates of return. This mechanism is isomorphic to the model in Piketty and

[8]Champernowne (1953) is an early, purely statistical, contribution. Benhabib, Bisin and Zhu (2011), Benhabib et al. (2015), Aoki and Nirei (2015), Piketty and Zucman (2014), Gabaix, Lasry, Lions and Moll (2015) exploit the original insight in the context of economic models with optimizing agents.

Zucman (2014) where each generation draws a bequest share and wealthy dynasties have a long history of above average draws. For this reason, capital and bequest taxes can significantly reduce wealth inequality, the former through its effect on the net rates of return, the latter through the share of wealth that is transferred from one generation to the other. The propensity to leave bequests has a similar effect.

Benhabib et al. (2015) and Aoki and Nirei (2015) show that a similar mechanism generates a wealth distribution with an (asympotically) Pareto right tail in the general equilibrium of a Bewley model in which, in addition to the usual additive earnings risk, individuals face idiosyncratic multiplicative rate-of-return risk, in the form of shocks to their backyard production technology and can self-insure by lending and borrowing, up to a limit, using a risk-free asset. The introduction of individual-specific risk introduces a precautionary saving motive which is absent from all the models discussed above. Yet, as wealth gets large enough, the precautionary saving motive goes to zero, as individuals can perfectly insured against the (bounded) earnings risk, and saving is linear in wealth. Therefore, the multiplicative rate-of-return shock tends to dominate the distribution of wealth at high levels.

Because the presence of a borrowing constraint makes it difficult to characterize the Pareto coefficient in closed form, Aoki and Nirei (2015) use numerical simulations to confirm some of the insights from Benhabib et al. (2011) in such a fremework, but also obtain some new results. First, they show that the presence of a precautionary saving motive implies that an increase in additive earnings risk reduces the thickness of the right tail of the wealth distribution by increasing the (precautionary) saving of low-wealth relative to high-wealth individuals. They also find that, contrary to the intuition from partial equilibrium models, an increase in the rate of TFP growth (which is zero in Benhabib et al. (2011) and Benhabib et al. (2015)) has the effect of *increasing*, rather than reducing, wealth inequality. In general equilibrium, a higher rate of TFP growth increases the steady-state capital stock, thus reducing the average return to detrended capital, and reducing inequality. On the other hand, higher TFP growth increases the variability of the idiosyncratic return to the backyard technology, thus increasing inequality. This second effect prevails and implies that higher TFP growth increases inequality, as measured by the Pareto coefficient.

A similar reversal of the partial equilibrium insight on the effect of TFP growth on wealth inequality is also in Jones (2015) who studies a version of Blanchard-Yaari's model with logarithmic preferences and accidental bequests distributed uniformly to newborns. The model generates a Pareto wealth distribution across cohorts through a deterministic, positive growth rate of individual wealth. In general equilibrium, the Pareto coefficient is independent of the rate of TFP growth and is fully determined by the demographic parameters.

The results in Aoki and Nirei (2015) and Jones (2015) suggest that a negative relationship between the rate of growth and wealth concentration is not a robust features of models with multiplicative shocks in general equilibrium.

6 Bewley models

In this framework, precautionary saving against earnings risk is the key force driving wealth concentration. However, the strength of the precautionary motive for saving declines with wealth relative to labor earnings; that is it declines with one's ability to (self-)insure against earnings risk. It follows that if agents are impatient—a necessary condition to ensure stationarity of the wealth distribution—the saving rate is positive below, and turns negative above, the (target) value of net-worth relative to labor earnings for which the precautionary saving motive is exactly offset by impatience. Hence, the saving rate in these models is declining in wealth.

In contrast, Saez and Zucman (2015), among others, find that saving rates tend to rise with wealth, with the bottom 90 per cent of wealth holders saving on average 3 per cent of their income, compared to 15 per cent for the next 9 per cent and to 20-25 per cent for the top 1 per cent of wealth holders. The basic version of the model thus fails to generate the high concentration of wealth in the hands of the richest few—and therefore the emergence and persistence of their very large estates—because it misses the fact that rich people keep saving at high rates.

Saving behavior crucially depends on rates of returns, patience, and earnings risk. High **rates of return** tend to increase savings. However, rates of return are not exogenous, which raises the question of how rates of return are determined, especially at the top of the wealth distribution. For instance, for entrepreneurs they are endogenous as a result of the decision to start a business and of the share of their wealth invested in their own risky activity and, for investors, as a result of portfolio choice.

Patience is not only affected by how much people discount their utility from future consumption, but also by whether or not households care about leaving bequests to others after their own death, and even how long they expect to live.

Taken together, the two points above imply that, if **people differ in their patience and risk tolerance** they might also select into different occupations and portfolio compositions and thus different returns, which will be correlated with their patience and risk attitude. As a result, more patient and less risk averse people will take on riskier positions, and while some of them will fail, some of them will succeed and enjoy very high returns. This implies that there will be a larger fraction of more patient and less risk averse people among the rich, partly because they represent those who got lucky, and partly because they have different preferences and their observed returns depend on their past occupational and saving decisions and on their preferences.

The third element in our list concerns **high and heterogenous earnings risk**. Moderately persistent and skewed earnings shocks have the potential of generating heterogenous savings rate. In fact, Castañeda, Díaz-Giménez and Ríos-Rull (2003) show that a specific form of earnings risk for top earners can generate very large wealth concentration in the hands of the richest few. This relates to the finding discussed by Piketty on the rising importance of the super-managers, especially in the U.S., and the volatility of their total compensation.

In the rest of this section, we discuss these mechanisms that appear to hold better promise to offset the fall of the saving rate as a function of wealth in the standard Bewley model and, therefore, better account for the observed high top wealth share.

6.1 Endogeneity of rates of returns

An important choice generating endogenous rates of return is entrepreneurial activity. Quadrini (2009) provides a nice survey on the factors affecting the decision to become an entrepreneur and the aggregate and distributional implications of entrepreneurship for savings and investment. In addition, Quadrini (2000), Gentry and Hubbard (2004), De Nardi, Doctor and Krane (2007), and Buera (2008) convincingly argue that entrepreneurship is a key element in understanding wealth concentration among the richest households.

Cagetti and De Nardi (2006) show that entrepreneurs constitute a large fraction of rich people in the data. For example, in the 1989 Survey of Consumer Finances, among the richest 1% of people in terms of net worth, 63% were entrepreneurs and they held 68% of the total wealth in the hands of the wealthiest 1% of people. Cagetti and De Nardi (2006) also build a model of entrepreneurship in which altruistic agents care about their children and face uncertainty about their time of death and thus leave both accidental and voluntary bequests. Every period, agents decide whether to run a business or work for a wage and borrowing constrains generate need for collateral and thus increase savings as long as the entrepreneur is constrained.

In Cagetti and De Nardi (2006)'s calibration, the optimal firm size is large and the entrepreneur is borrowing constrained. Thus, even rich entrepreneurs want to keep saving to accumulate collateral to grow their firm and reap higher returns from capital. This is the mechanism that, in this framework, keeps the rich people's saving rate high and generates a high wealth concentration. As a result, their model generates wealth concentration that matches that in the data well, including the right tail of the distribution. In addition, the model implies plausible returns to capital in the range of those found by Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014). Finally, the model generates entry probabilities into the entrepreneurial sector as a function of one's wealth that are consistent with those estimated by Hurst and Lusardi (2004) on micro-level data and also implies that inheritances are a strong predictor of business entry.

Kitao (2008) studies the effect of taxation on entrepreneurial choice in a model with multiple entrepreneurial ability levels.

Among the models studying portfolio choice and wealth inequality, Kacperczyk, Nosal and Stevens (2015) quantitatively evaluate portfolio choice in presence of endogenous information acquisition and heterogenity in investor sophistication and asset riskiness. They show that an increase in aggregate information technology can explain the observed increase in wealth concentration among investors since 1990.

6.2 Earnings risk and the rise of the super-managers

There is large literature studying precautionary savings as a mechanism to self-insure against earnings shocks. Carroll (2007) shows that the marginal propensity to consume out of a permanent income shock is close to, though lower than, one in a precautionary saving model with both transitory and permanent income shocks. This implies that the saving rate out of wealth is hardly affected by permanent earnings shocks. Instead, in the case in which income shocks are purely transitory, consumption smoothing implies that individual will save most of the income change. On the other hand, the transitory nature of the shocks, and therefore of the associated saving response, imply that the effect averages out. Hence these forces do not imply a first-order persistent growth effect on savings and cannot therefore generate substantial wealth concentration.

Precautionary saving behavior to self-insure against earnings risk, though, can generate top wealth concentration (right skewness) if the stochastic process for labor earnings is appropriately skewed and persistent. Castañeda et al. (2003) were the first to (numerically) generate this result in a model economy with perfectly altruistic agents going though a stochastic life cycle of working age, retirement and death. Their paper calibrates the parameters of the income process to match some features of the U.S. data, including measures of both earnings and wealth inequality. The key force generating large wealth holdings in the hands of the richest is a productivity shock process calibrated so that the highest productivity level is more than 100 times higher than the second highest. Thus, there is a large discrepancy between the highest productivity level and all of the others. Moreover, an agent in the highest productivity state as a roughly 20 per cent chance of becoming more than 100 times less productive during the next period. Intuitively, high-earnings households have very high (precautionary) saving rates for two reasons. First, they face a large downside earning risk and, therefore, they accumulate a large wealth buffer to self-insure against the possibly very large fall in earnings. As a result, they have a large target ratio of wealth relative to earnings. Secondly, a high wealth-to-earnings target corresponds to a very large target wealth *level* for agents with high earnings.

It is important to note that the steady-state fraction of high-earners that drive the top wealth shares in Castañeda et al. is extremely small (of order of 0.04 per cent). This feature is consistent with the following findings by Saez and Zucman (2014). First, that the large increase in wealth inequality in the U.S. over the last thirty years has been mostly driven by the three-fold increase in the wealth share of the top 0.1 per cent of wealth holders. Second, that the main driver of the rapid increase in wealth at the top has been the large increase in the share of earnings earned by top wealth holders.

From a theoretical standpoint, the "economics of superstars" (Rosen (1981)) rationalizes the emergence of a small number of highly compensated individuals and a highly skewed distributions of earnings with very large rewards at the top. Gabaix and Landier (2008) propose a model to rationalize increased CEO compensation between 1980 and 2003 while Lee (2015) develops a model occupational choice for workers, entrepreneurs, and managers, that endogenously generates high managerial wages.

Starting with Piketty and Saez (2003), a series of paper by Thomas Piketty and Emmanuel Saez, together with a number of coauthors, have documented the skewness of the earnings (and income) distribution.[9] Recently, Guvenen, Karahan, Ozkan and Song (2015) have exploited a large panel dataset of earnings histories drawn from U.S. administrative records and documented that earnings shocks display significant negative skewness and that very high earners—individuals in the top fifth percentile of the income distribution—the increase in the (absolute value of) skewness over the lifetime is entirely driven by an increase in the risk of negative shocks, rather than a lower risk of positive ones.

More empirical support for this modeling assumption and calibration is provided by Parker and Vissing-Jørgensen (2009), who find that incomes at the top are highly cyclical because of the labor component and bonuses in particular.

6.3 The importance of intergenerational wealth transmission

Piketty's book also stresses the importance of inherited wealth. Intergenerational transfers account for at least 50-60% of total wealth accumulation (Gale and Scholz (1994)) in the U.S. and potentially an important transmission channel of wealth inequality across generations. Furthermore, a luxury-good-type bequest motive can help to explain why rich households save at much higher rates than the rest (Dynan et al. (2004), Carroll (2000)), why the portfolio of the rich are skewed towards risky assets (Carroll (2002)) and, possibly in conjuction with medical expenses, the low rates of dissaving of the (rich) elderly (De Nardi, French and Jones (2010)).

[9] The complete set of studies is collected in Atkinson, Piketty and Saez (2010). The dataset is available at Alvaredo, Atkinson, Piketty and Saez (2015).

De Nardi (2004) introduces two types of intergenerational links in the overlapping-generation, life-cycle model used by Huggett (1996): voluntary bequests and transmission of human capital. She models the utility from bequests as providing a "warm glow". In this framework, parents and their children are thus linked by voluntary and accidental bequests and by the transmission of earnings ability. The households thus save to self-insure against labor earnings shocks and life-span risk, for retirement, and possibly to leave bequests to their children. In De Nardi's model, therefore, voluntary and accidental bequests coexist and their relative size and importance are determined by the calibration. The calibration adopted implies that bequests are a luxury good, generates a realistic distribution of estates and is also quantitatively consistent with the elasticity of the savings of the elderly to permanent income that has been estimated from microeconomic data by Altonji and Villanueva (2002).

Her work shows that voluntary bequests can explain the emergence of large estates, which are often accumulated in more than one generation and characterize the upper tail of the wealth distribution in the data. The calibration implies that a much stronger bequest motive to save for the richest households, who, even when very old, keep some assets to leave to their children. The rich leave more wealth to their offspring, who, in turn, tend to do the same. This behavior generates some large estates that are transmitted across generations because of the voluntary bequests. Transmission of ability between parents and children also helps generating a concentrated wealth distribution. More productive parents accumulate larger estates and leave larger bequests to their children, who, in turn, are more productive than average in the workplace. The presence of a bequest motive also generates lifetime saving profiles that imply slower wealth decumulation in old age for richer people, consistent with the facts documented by De Nardi et al. (2010), using micro-level data from the Health and Retirement Survey.[10] Yet, although modeling explicitly intergenerational links helps

[10] De Nardi et al. (2010) suggest that medical expenses are another important mechanism that can generate this kind of slow decumulation. Lockwood (2012) argues that both medical expenses and a luxury bequest motive are necessary to account for both the low rate of asset decumulation *and* the low rate of insurance

explain the savings of the richest, the model by De Nardi is not capable of matching the wealth concentration of the richest 1% without adding complementary forces generating a high wealth concentration for the rich.[11]

Thus, De Nardi and Yang (2015) merge a version of the model with intergenerational links with the high earnings risk for the top earners mechanism proposed by Castañeda et al. (2003) (discussed in Section 6.2) and find that these two forces together match important features of the data well. Interestingly, they distinguish between the contribution to wealth inequality of the stochastic earnings process and that of bequests. They show that bequests account for about 10 percentage points in the share of wealth held by individuals in the top twenty percentiles.

As Piketty's book also point out, wealth inequality is large also within various age and demographic groups. Venti and Wise (1988) and Bernheim, Skinner and Weimberg (2001), for instance, show that wealth is highly dispersed at retirement even for people with similar lifetime incomes and argue that these differences cannot be explained only by events such as family status, health, and inheritances, nor by portfolio choice. Hendricks (2004*a*) focuses on the ability of a basic overlapping-generations model to match the cross-sectional wealth inequality at retirement age. He shows that the model overstates wealth differences at retirement between earnings-rich and earnings-poor, while it understates the amount of wealth inequality conditional on similar lifetime earnings. Instead De Nardi and Yang (2014) show that an overlapping-generations model augmented with voluntary bequests and intergenerational transmission of earnings also matches the observed cross-sectional differences in wealth at retirement and their correlation with lifetime incomes quite well.

against medical expenses.

[11] Nishiyama (2002) obtains similar results in an overlapping-generation model with bequests and inter-vivos transfers in which households in the same family line behave strategically.

6.4 Preference heterogeneity

An additional plausible avenue to help explain the vastly different amounts of wealth held by individuals is exogenous heterogeneity in saving behavior. The source of this heterogeneity in saving behavior is an important issue. There is enough micro-level empirical evidence of preference heterogeneity to suggest that preference heterogenity might be a plausible avenue to help explain the vastly different amounts of wealth held by people. For example, Lawrance (1991) and Cagetti (2003) find large heterogenity in preferences across people.

Krusell and Smith (1998) study the impact of preference heterogeneity, in the form of persistent (with an average persistence of one generation) shocks to the time-preference rate in a infinite horizon model with idiosyncratic, transitory earnings shocks.[12] They find that a small amount of preference heterogeneity dramatically improves the model's ability to match the variance of the cross-sectional distribution of wealth.[13] However, while capturing the variance of the wealth distribution, their model and calibration fail to match the extreme degree of concentration of wealth in the hands of the richest few.

Hendricks (2004*b*) studies the effects of preference heterogeneity in a life-cycle framework with persistent earnings shocks and only accidental bequests. He shows that time preference heterogeneity makes a modest contribution in accounting for high wealth concentration if the heterogeneity in discount factors is chosen to generate realistic patterns of consumption and wealth inequality as cohorts age.

In sum, previous work indicates that preference heterogeneity, and especially patience heterogenity, can generate increased wealth dispersion. It would be interesting to deepen the previous analysis by both studying richer processes for patience and allowing for richer formulations of the utility function in which, for instance, risk aversion and intertemporal substitution do not have to coincide (see Wang, Wang and Yang (2015) for some interesting

[12] Though the model also allows for aggregate shocks, they do not have quantitatively important implications for the wealth distribution.

[13] They also find that heterogeneity in risk aversion does not affect the results much (however, Cagetti (2001) shows that this result is sensitive to chosen utility parameter values).

findings on this).

7 Transitional dynamics of the wealth distribution

An important contributions of Piketty's book is to document the evolution of wealth inequality over a long span of time.

In the absence of (big) shocks, one may expect the concept of stationary state to provide a useful reference to describe where an economy will settle down in the long-run. In fact, the whole line of research discussed above studies the impact of alternative mechanisms on the shape of the *stationary* wealth distribution and mostly abstracts from both stochastic and deterministic aggregate changes in the economy's fundamentals in driving either the stochastic steady state, or the transition of the economy as some force such as government policy, for instance, changes over time.

Yet, one question that naturally arises when observing, for example, the evolution of the top wealth shares for Europe and the US reported in Figure 2 is the extent to which there were no big shocks, nor other deterministic changes, in the fundamentals driving inequality during that long time period. For instance, the top 1 and 10 per cent wealth shares in Figure 2 display an upward trend up to 1910 for Europe and up to 1930 for the US, and an upward trend for both areas after 1970. In addition, during the pre-1910 period the transitional dynamics seem to be quite slow while the post-1970 US experience point to rapid changes in the concentration of wealth in its upper tail. Two recent contributions tackle the question of whether existing models can account for the fast increase in US top wealth inequality after 1970.

Gabaix et al. (2015) study this question in a partial equilibrium model with multiplicative and idiosyncratic rate-of-return shocks that give rise to a wealth distribution with a Pareto right tail, of the type discussed in Section 5. They find that, without additional amplifying mechanisms, the model implies a transitional dynamics of wealth inequality in response to a one-off shock, for example a change in the tax rate on capital, that is too slow compared to the post-1980 rise in US top wealth inequality as documented in the Survey of Consumer Finances, let alone compared to the much faster rate of increase documented by Saez and Zucman (2015). On the basis of their findings they conjecture that a positive correlation between wealth and either saving rates or rates of return is necessary to account for the observed speed of the change in wealth inequality in terms of an increase in post-tax rates of returns or a fall in the aggregate rate of growth. As we have seen in sections 6.1 and 6.3, both models of entrepreneurship and a non-homothetic bequest motive can generate this type of correlations.

Kaymak and Poschke (2015) study the transitional dynamics associated with the changes in the US tax and transfer system over the last 50 years within a Bewley economy with an income process à la Castañeda et al. (2003) calibrated to match the income and wealth (including the top) distributions in the 1960s. They find that the increase in wealth inequality over the period can be accounted for by the increase in wage inequality, the changes in the tax system and the expansion of Social Security and Medicare. More specifically, the increase in wage inequality accounts for more than half of the increase in top wealth inequality, as the increase in downside risk for workers in the top income states substantially boosts their precautionary saving. The remaining share of the increase in top wealth inequality is due to the fall in taxes—that increases the net return to saving— and the expansion of Social Security and Medical—that reduces precautionary saving by poorer households. The latter effect increases the equilibrium interest rate and wealth accumulation by the wealth rich. They also show, that assuming no further shocks after 2010, the top 1% wealth share would take roughly 50 years to increase by about 10 percentage points towards its new steady state value of about 50%.

One lesson to draw from comparing Kaymak and Poschke's (2015) results with those by Gabaix et al. (2015) and our findings in Section 8 is that an important advantage of a quantitative framework is the ability to model in a more realistic way the evolution of important determinants of wealth inequality. For example, on the fiscal side the *whole* set of changes to the progressivity of the tax and Social Security modelled by Kaymak and Poschke (2015) accounts for a substantially larger change in wealth inequality than the stylized experiments conducted in our Section 8 and in Gabaix et al. (2015).

8 A simulation exercise

This section uses the quantitative model of wealth inequality in De Nardi and Yang (2015) to test some of the predictions in Piketty's book. For convenience, we discuss the model in the next subsection. The model is calibrated to the U.S. economy (see Appendix for a discussion of the calibration choices) and the crucial features that allow it to match the U.S. wealth distribution are the combination of a voluntary bequest motive and a stochastic earnings process similar to that in Castañeda et al. (2003) (See Section 6.2 for more discussion on this).

8.1 The Model

The model is a discrete-time, incomplete-markets, overlapping-generations economy with an infinitely lived government.

8.1.1 The Government

The government taxes capital at rate τ_a, labor income and Social Security pay-outs at rate τ_l, and estates at rate τ_b above the exemption level x_b to finance government spending G. Social Security benefits, $P(\widetilde{y})$, are linked to one's realized average annual earnings $\widetilde{y}$, up to a Social Security cap $\widetilde{y}_c$, and are financed through a labor income tax τ_s. The two government budget constraints, one for Social Security and the other one for government spending, are balanced during each period.

8.1.2 Firm and Technology

There is one representative firm producing goods according to the aggregate production function $F(K; L) = K^{\alpha}L^{1-\alpha}$, where K is the aggregate capital stock and L is the aggregate labor input. The final goods can either be consumed or invested in physical capital, which depreciates at rate δ.

8.1.3 Demographics and Labor Earnings

Each model period lasts five years. Agents start their economic life at the age of 20 ($t = 1$). By age 35 ($t = 4$), the agents' children are born. The agents retire at age 65 ($t = 10$). From that period on, each household faces a positive probability of dying, given by $(1 - p_t)$, which only depends on age.[14] The maximum life span is age 90 ($T = 14$), and the population grows at a constant rate n. The online appendix (on Science Direct) graphically illustrates the demographic structure of our overlapping generations model.

Total labor productivity of worker i at age t is given by $y_t^i = e^{z_t^i + \epsilon_t}$, in which ϵ_t is the deterministic age-efficiency profile. The process for the stochastic earnings shock z_t^i is: $z_t^i = \rho_z z_{t-1}^i + \mu_t^i,\ \mu_t^i \sim N(0, \sigma_\mu^2)$.

To capture the intergenerational correlation of earnings, we assume that the productivity of worker i at age 55 is transmitted to children j at age 20 as follows: $z_1^j = \rho_h z_8^i + \nu^j,\ \nu^j \sim N(0, \sigma_h^2)$, as parents are 35 years (seven model periods) older than their children.

8.1.4 Preferences

Preferences are time separable, with a constant discount factor β. The period utility function from consumption is given by $U(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$.

People derive utility from holding onto assets because they turn into bequests upon death. This form of 'impure' bequest motive implies that an individual cares about total bequests

[14] We make the assumption that people do not die before age 65 to reduce computational time. This assumption does not affect the results since in the U.S., the number of adults dying before age 65 is small.

left to his/her children, but not about the consumption of his/her children.

The utility from bequests b is denoted by

$$\phi(b) = \phi_1 \left[(b + \phi_2)^{1-\gamma} - 1 \right].$$

The term ϕ_1 measures the strength of bequest motives, while ϕ_2 reflects the extent to which bequests are luxury goods. If $\phi_2 > 0$, the marginal utility of small bequests is bounded, while the marginal utility of large bequests declines more slowly than the marginal utility of consumption. In the benchmark model, we set b as bequest net of estate tax, b_n. We also consider the case in which gross bequests, b_g, enter the utility function. In that case, we set $b = b_g$. Our formulation is thus more flexible than in De Nardi (2004), Yang (2013), and De Nardi and Yang (2014) because we allow for two kinds of bequest motives. In the first one, parents care about bequests net of taxes. In the second one, parents care about bequests gross of taxes. A more altruistic parent would take into account that some of the estate is taxed away, but parents might just care about what assets they leave, rather than how much their offspring receive.

8.1.5 The Household's Recursive Problem

We assume that children have full information about their parents' state variables and infer the size of the bequests that they are likely to receive based on this information. The potential set of a household's state variables is given by $x = (t, a, z, \widetilde{y}, S_p)$, where t is household age (notice that in the presence of a fixed age gap, one's age is also informative about one's parents' age), a denotes the agent's financial assets carried from the previous period, z is the current earnings shock, and $\widetilde{y}$ stands for annual accumulated earnings, up to a Social Security cap $\widetilde{y_c}$, which are used to compute Social Security payments. The term S_p stands for parental state variables other than age and, more precisely, is given by $S_p = (a_p, z_p, \widetilde{y_p})$. It thus includes parental assets, current earnings, and accumulated earnings. When one's parent retires, z_p, or current parental earnings, becomes irrelevant and we set it to zero with no loss of generality.

From 20 to 60 years of age ($t = 1$ to $t = 9$), the agent works and survives for sure to next period. Let $V_w(t, a, z, \widetilde{y}, S_p)$ and $V_w^I(t, a, z, \widetilde{y})$ denote the value functions of a working-age person whose parent is alive and dead, respectively, where I stands for "inherited." In the former case, the household's parent is still alive and might die with probability p_{t+7}, in which case the value function for the orphan household applies, and assets are augmented by inheritances in per-capita terms. That is,

$$
\begin{aligned}
(1) \qquad V_w(t, a, z, \widetilde{y}, S_p) \; &= \; \max_{c,a'} \Big\{ U(c) + \beta p_{t+7} E\big[V_w(t+1, a', z', \widetilde{y}', S_p')\big) \\
&\quad + \beta(1 - p_{t+7}) E\big[V_w^I(t+1, a' + b_n/N, z', \widetilde{y}')\big] \Big\},
\end{aligned}
$$

subject to

$$
\begin{aligned}
(2) \qquad c + a' \; &= \; (1 - \tau_l)wy - \tau_s \min(wy, 5\widetilde{y}_c) + [1 + r(1 - \tau_a)]a, \\
(3) \qquad a' \; &\geq \; 0, \\
(4) \qquad \widetilde{y}' \; &= \; \big[(t-1)\widetilde{y} + \min(wy/5, \widetilde{y}_c)\big]/t, \\
(5) \qquad \widetilde{y}_p' \; &= \; \left\{ \begin{array}{ll} \big[(t+6)\widetilde{y}_p + \min((wy_p/5, \widetilde{y}_c)\big]/(t+7) & \text{if } t < 3 \\ \widetilde{y}_p & \text{otherwise} \end{array} \right\} \\
(6) \qquad b_n \; &= \; b_n(S_p),
\end{aligned}
$$

where N is the average number of children determined by the growth rate of the population. The expected values of the value functions are taken with respect to (z', z_p'), conditional on (z, z_p). The agent's resources depend on labor endowment y and asset holdings a.

Average yearly earnings for children and parents evolve according to equations (4) and (5), respectively. Since current income y refers to a five-year period, current income is divided by five when the yearly lifetime average labor income ($\widetilde{y}$) is updated. Equation (6) is the law of motion of bequest for the parents, which uses their optimal decision rule.

The value function of an agent who is still working but whose parent is dead is

$$V_w^I(t,a,z,\widetilde{y}) = \max_{c,a'}\left\{U(c)+\beta E\big[V_w^I(t+1,a',z',\widetilde{y}')\big]\right\}, \tag{7}$$

subject to (2), (3), and (4).

From 65 to 85 years of age ($t = 10$ to $t = 14$), the agent is retired and receives Social Security benefits and his parent is already deceased. He faces a positive probability of dying, in which case he derives utility from bequeathing the remaining assets.

$$V_r(t,a,\widetilde{y}) = \max_{c,a'}\left\{U(c)+\beta p_t V_r(t+1,a',\widetilde{y})+(1-p_t)\phi(b)\right\}, \tag{8}$$

subject to (3),

$$c+a' \;=\; [1+r(1-\tau_a)]a+(1-\tau_l)P(\widetilde{y}), \tag{9}$$

$$b_n \;=\; \left\{\begin{array}{ll} a' & \text{if } a'<x_b, \\ (1-\tau_b)(a'-x_b)+x_b & \text{otherwise,} \end{array}\right\} \tag{10}$$

and, in the case of net bequest motives,

$$b = b_n, \tag{11}$$

while in the case of gross bequest motives,

$$b = b_g = a', \tag{12}$$

regardless of the structure of the estate tax.

We focus on a stationary equilibrium concept in which factor prices and age-wealth distribution are constant over time. Due to space constraints, the definition of a stationary equilibrium for our economy is in the online appendix in De Nardi and Yang (2015).

		Percentile (%)					
	Gini	1	5	20	40	60	80
Data (SCF 1998)	0.63	14.8	31.1	61.4	84.7	97.2	100.00
Benchmark	0.62	14.7	31.3	63.0	85.0	93.4	100.00

Table 1: Percentage of earnings in the top percentiles.

8.2 Some results

Table 1 reports the earnings distribution at selected percentiles in the SCF data (reported by Castañeda et al. 2003) and that generated by the model. Comparing the first two lines in the table reveals that benchmark calibration matches the earnings distribution very well.

Table 3 reports the wealth distribution at selected percentiles in the SCF data and the various versions of the model studied. The comparison of the first two lines in the table shows that the benchmark calibration matches the wealth distribution in the SCF well. This is especially true for the share of wealth held by the top percentile. The model also matches, by construction, a bequest flow-GDP ratio of 2.8 per cent[15] and the 90th percentile of the bequest distribution normalized by income. Thus, the model is capable of accounting for a number of data moments that should be informative on the contribution of the various motives for saving to individual wealth accumulation and to the cross-sectional wealth distribution

We use the model to conduct two sets of experiments to discuss the $(\bar{r} - g)$ mechanism highlighted in Piketty's book. In particular, we compare the effect of a fall in the rate of population growth n in experiment (1) and of an increase the post-tax rate of return to capital $\bar{r}$. For the latter case, we study three different scenarios. In experiment (2), the increase in the post-tax rate is due to a partial-equilibrium increase in the pre-tax rate of return. Experiment (3), is the general-equilibrium counterpart of (2), with the increase in the rate of return on capital being a consequence of an increase in the capital income share, which implies that both the rate of return on capital, and the capital-income ratio

[15]In the data, this fraction increases to 3.8 per cent if inter-vivos transfers and college expenses are also included.

and the capital income share all increase. Finally, in the partial equilibrium experiment (4) the increase in the post-tax rate of return is due to the elimination of capital taxation at unchanged gross return.

In experiments (1), (2), and (3) we keep the difference between the post-tax rate of return and the rate of population growth constant to explore Piketty's conjecture that it is the difference between the average net rate of return on capital and the rate of GDP growth that matters for wealth inequality. The model economy does not feature TFP growth, so the steady state rate of GDP growth coincides with the rate of growth of the population n.

In all experiments, the proportional rate of contribution to social security adjusts to balance the Social Security budget while the proportional labor income tax adjusts to balance the rest of the government budget. The calibrated value of the coefficient of relative risk aversion is 1.5 while the subjective discount factor $\beta = 0.945$. Table 2 reports the values of the social security tax τ_{SS}, the proportial labor tax rate τ_l, output Y and the ratios of the aggregate stock of assets A and flow of bequests B to output, as well as factor prices, in the benchmark and the experiments. In what follows r denotes the gross (pre-tax) rate of return on capital and $\bar{r} = (1 - \tau_a)r$ the rate of return net of the proportional capital income tax $\tau_a = 0.2$. The value of τ_a is 0.2 in the benchmark calibration.

8.3 Lower rate of population growth

In experiment (1), we consider the effect of reducing the rate of population growth n from 1.2 per cent a year to zero. We conduct the experiment in partial equilibrium for ease of comparison with the, partial equilibrium experiments (2) and (4). The implications for the wealth distribution, which is our focus here, are very similar, but marginally amplified, in general equilibrium.

Comparing the first two rows in Table 3 reveals that a fall in the population growth rate marginally increases overall wealth inequality as measured by the Gini coefficient and

	τ_{SS}	τ_l	Y	A/Y	B/Y	$\bar{r}-g$	$\bar{r}$	wage
Benchmark	0.12	0.19	1.0	3.1	2.8%	3.3	4.5	0.49
(1) $n=0$, PE	0.17	0.18	-	4.1	4.5%	4.5	4.5	0.49
(2) Higher r, PE	0.12	0.14	-	4.7	5.1%	4.5	5.7	0.49
(3) α=0.45	0.12	0.22	0.9	3.3	3.4%	4.5	5.7	0.39
(4) $\tau_a=0$, PE	0.12	0.25	-	3.5	3.5%	3.3	4.5	0.49

Table 2: Aggregate effects, adjusting the labor income tax. Aggregate labor is expressed as ratio to that in the benchmark. Aggregate output is expressed as ratio to that in the benchmark.

		Percentile (%)					
	Gini	1	5	20	40	60	80
Data (SCF 1998)	0.80	34.7	57.8	69.1	81.7	93.9	98.9
Benchmark	0.80	35.7	52.0	65.9	82.8	95.4	99.5
(1) $n=0$, PE	0.81	40.3	54.8	67.4	83.3	95.7	99.4
(2) Higher r, PE	0.79	35.9	51.2	64.1	80.2	94.1	98.9
(3) $\alpha=0.45$	0.79	36.3	51.4	64.4	80.4	94.2	98.9
(4) $\tau_a=0$, PE	0.80	36.5	51.7	64.7	80.9	94.5	99.0

Table 3: Percentage of total wealth held by households in the top percentiles.

substantially increase the share of aggregate wealth accruing to the top 20 percentile of the wealth distribution. The effect is particularly pronounced for the top percentile whose share increases by about 5 percentage points. The fall in the rate of population growth increases the average age of the labor force and, since the share of super-rich increases with age, the top shares of earnings and the wealth-GDP ratio. In addition, the higher ratio of deaths to births increases the the aggregate flow of bequests to GDP (cf. the first two rows in Table 2) and the average bequest size. Since the calibration implies that bequests are a luxury good, this last effect increases wealth concentration at the top.

8.4 Higher return on capital

In experiments (2) and (3), we increase that yearly post-tax rate of return on capital by 1.2 percentage points, so that the difference between yearly post-tax rate of return on capital and the rate of population growth increases by the same amount as in the previous experiment. Experiment (2) is conducted in partial equilibrium. In the general equilibrium experiment

(3) the increase in the interest rate is the result of an increase in the capital income share α from 0.36 to 0.45 [16]. In experiments (2) and (3) the increase in the interest rate is thus associated with a higher wealth-income ratio *and* share of capital relative to labor income. This is exactly the kind of scenario discussed in Piketty's book.

Starting with experiment (2), comparing rows 3 and 4 in Table 2 reveals that the increase in the return to capital has a even larger effect on the aggregate stock of wealth and the flow of bequest than the fall in population growth. Conversely, and contrary to the conjecture that it is the difference between the two rates that matters for inequality, the higher interest rate increases the share of wealth owned by the top 1 percent only marginally and actually *reduces* wealth concentration in the top 20th percentile.

Intuitively, the increase in the rate of return on capital reduces impatience by raising $\beta(1+\bar{r})$, and thus increases precautionary saving by individuals with low wealth relative to earnings. The negative wealth effect associated with the higher rate at which future earnings are discounted also boosts saving for these individuals. This increases the average wealth holding and reduces inequality. Conversely, in the case of wealthy savers for which capital is the main source of income, the precautionary saving and wealth effects are small and the income and substitution effect are roughly offsetting. As a consequence, the ratio between consumption and wealth is not much affected and the higher interest rate translates in a higher rate of capital accumulation.

Turning to experiment (3), the response of wealth concentration to a general equilibrium increase in the capital return in response to a higher capital income share is very similar to that in the counterpart partial equilibrium experiment (2). The main difference is that share of wealth accruing to the top 1 per cent increase marginally more than in the partial equilibrium experiment, as aggregate labour income falls in general equilibrium, due to the fall in wages and the increase in the labor tax required to balance the budget (cf. rows

[16] Due to our choice of Cobb-Douglas production function, a higher interest rate implies a lower wealth-income ratio and is associated to unchanged factor income shares in general equilibrium.

3 and 4 in Table 2). Furthermore, the combination of lower wages and higher labor tax rate significantly reduces the variance of labor earnings and therefore precautionary saving and the capital-income and bequest-income ratios relative to experiment (2). It would be interesting to explore whether these findings are sensitive to relaxing the Cobb-Douglas assumption for the aggregate production function.

Finally, in the partial equilibrium experiment (4) the increase in the post-tax interest rate is obtained by reducing the proportional tax on capital income τ_a from 0.2 to zero at unchanged gross interest rate. As a result of the fall in revenue from capital taxation the labor tax rate increases substantially to balance the budget and the distributional implications of the policy change are similar to those in experiment (3). In fact, also the allocation and welfare implications of the policy change are similar, as can be seen from comparing rows 4 and 5 in Tables 2 and 3.

8.5 What we have learnt

To sum up, our findings confirm Piketty's insight that, qualitatively, an increase in the rate of return on capital or a fall in the rate of growth of output due to a demographic change both increase wealth concentration. Contrary to his conjecture, though, we find that the rate of return on capital and the rate of population growth are not perfect substitutes in their effect on wealth concentration. For the same change in the difference between the two rates, an increase in the rate of return has a much smaller effect on wealth concentration than a fall in the rate of population growth by the same amount. The intuition is that a lower rate of population growth is associated with a higher ratio of deaths to births and, as a consequence, a higher average bequest size. To the extent that bequests are a luxury good, this last effect has a significant impact on wealth concentration at the top.

9 Where do we go (in terms of modelling) and what data do we need

In addition to providing many important facts and ideas, Piketty's book has revitalized interest in inequality, and especially wealth inequality, and in understanding the determinants of savings across all levels of the wealth and earnings distributions. This raises the question of where we go from here both in terms of modelling and in terms of data needed to better discipline these models.

As we have argued, quantitative Bewley models can generate realistic wealth inequality both in steady state and along the transition path. In addition, they offer the possibility of exploring quantitatively the contribution of various competing mechanisms shaping the dynamics of individual wealth accumulation, as well as of modelling detailed features of the institutional environment. In this framework, entrepreneurship, intergenerational links, earnings risk, medical expenses, and heterogenous preferences have been shown to be important to understand saving behavior and wealth inequality. They should be studied more in depth to better understand their workings, and jointly to better understand how they interact and what is their relative importance.

Thinking more about modelling entrepreneurial heterogeneity is both empirically reasonable and potentially important. Campbell and De Nardi (2009) find, for instance, that aspirations about the size of the firm that one would like to run are different for men and women, and that many people who are trying to start a business also work for an employer and thus work very long hours in total. It would be interesting to generalize the model to allow, for instance, for heterogeneity in entrepreneurial total factor productivity and optimal firm size (or decreasing-returns-to-scale parameters), and to convincingly take the model to data to estimate those additional parameters. Given the data on time allocation, it would also be interesting to think more about the time allocation decision between working for an employer, starting and running one's firm, working on home production, and enjoying leisure.

More work is also warranted to evaluate the role of intergenerational links. How should we model bequests? How important are inter-vivos transfers, and do they provide an important dimension of wealth heterogeneity early on in life that is then amplified by shocks and individual saving behavior later on?

Nardi, French and Jones (2010) have shown that medical expenses have large effects on old age savings throughout the income distribution. How do lifespan risk and out-of-pocket medical expense interact and to what extent heterogeneity in out-of-pocket medical expenses—which rise quickly with age and income during retirement—coupled with heterogeneous lifespan risks, contribute to wealth inequality?

Is the type of top-earnings risk necessary to account for the observed degree of top wealth concentration on the basis of precautionary behavior consistent with the empirical, micro-level evidence from earnings data. This question has been notoriously difficult to address, given that the usual survey datasets of individual earnings are either top coded or do not oversample the rich. Comprehensive administrative data on earnings that have recently become available provide a way to address this problem. De Nardi, Fella and Paz Pardo (2015) provide a first attempt to tackle this question by studying the implications for wealth inequality of an income process which is consistent with the skewness and kurtosis documented by recent studies (Guvenen et al. (2015)) relying on U.S. Social Security administrative data.

Finally, to what extent preference heterogeneity amplifies and interacts with the mechanisms above? How much preference heterogeneity is needed to understand the data once other observable factors, such as entrepreneurial choice, for instance, are accounted for and properly calibrated or estimated?

References

Adermon, Adrian, Lindahl, Mikael and Waldenström, Daniel (2015), Intergenerational wealth mobility and the role of inheritance: Evidence from multiple generations. Mimeo.

Altonji, Joseph G. and Villanueva, Ernesto (2002), The effect of parental income on wealth and bequests. NBER Working Paper 9811.

Alvaredo, Facundo, Atkinson, Anthony B., Piketty, Thomas and Saez, Emmanuel (2015), 'The word top incomes database', `http://topincomes.g-mond.parisschoolofeconomics.eu/`.

Aoki, Shuhei and Nirei, Makoto (2015), Pareto distribution in Bewley models with capital income risk. mimeo, Hitotsubashi University.

Atkinson, Anthony B. (1983), *The Economics of Inequality*, Clarendon Press, Oxford.

Atkinson, Anthony B. and Harrison, Allan J. (1983), *Distribution of Personal Wealth in Britain, 1923-1972*, Cambridge University Press, Cambridge.

Atkinson, Anthony B., Piketty, Thomas and Saez, Emmanuel (2010), 'Top incomes in the long run of history', *Journal of Economic Literature* **49**, 3–71.

Benhabib, Jess and Bisin, Alberto (2006), The distribution of wealth and redistributive policies. mimeo, NYU.

Benhabib, Jess, Bisin, Alberto and Zhu, Shenghao (2011), 'The distribution of wealth and fiscal policy in economics with finitely lived agents', *Econometrica* **79**, 123–157.

Benhabib, Jess, Bisin, Alberto and Zhu, Shenghao (2015), 'The wealth distribution in Bewley models with capital income risk', *Journal of Economic Theory* **159**, 459–515.

Bernheim, B. Douglas, Skinner, Jonathan and Weimberg, Steven (2001), 'What accounts for the variation in retirement wealth among u.s. households?', *The American Economic Review* **91**(5), 832–857.

Boserup, Simon Halphen, Kopczuk, Wojciech and Kreiner, Claus Thustrup (2015), Stability and persistence of intergenerational wealth formation: Evidence from danish wealth records of three generations. Mimeo.

Bosworth, Barry P. and Anders, Sarah (2011), Saving and wealth accumulation in the psid, 1984-2005. NBER Working Paper 17689.

Buera, Francisco (2008), Persistency of poverty, financial frictions, and entrepreneurship. Working paper, Northwestern University.

Cagetti, Marco (2001), 'Interest elasticity in a life-cycle model with precautionary savings', *American Economic Review* **91**(2), 418–421.

Cagetti, Marco (2003), 'Wealth accumulation over the life cycle and precautionary savings', *Journal of Business and Economic Statistics* **21**(3), 339–353.

Cagetti, Marco and De Nardi, Mariacristina (2006), 'Entrepreneurship, frictions and wealth', *Journal of Political Economy* **114**(5), 835–870.

Campbell, Jeffrey R. and De Nardi, Mariacristina (2009), 'A conversation with 590 entrepreneurs', *Annuals of Finance* **5**, 313–327.

Carroll, Christopher D. (2000), Why do the rich save so much?, *in* Joel B.Slemrod, ed., 'Does Atlas Shrug? The Economic Consequences of Taxing the Rich', Harvard University Press.

Carroll, Christopher D. (2002), Portfolios of the rich, *in* LuigiGuiso, MichaelHaliassos and TullioJappelli, eds, 'Household Portfolios: Theory and Evidence', MIT Press, Cambrdige MA.

Carroll, Christopher D. (2007), 'Precautionary saving and the marginal propensity to consume out of permanent income', *Journal of Monetary Economics* **56**, 780–790.

Castañeda, Ana, Díaz-Giménez, Javier and Ríos-Rull, José-Víctor (2003), 'Accounting for u.s. earnings and wealth inequality', *Journal of Political Economy* **111**(4), 818–857.

Champernowne, D. G. (1953), 'A model of income distribution', *Economics Journal* **63**, 318–351.

Charles, Kerwin Kofi and Hurst, Erik (2003), 'The correlation of wealth across generations', *Journal of Political Economy* **111**(6).

Clark, Gregory and Cummins, Neil (2015), 'Intergenerational wealth mobility in england, 18582012: Surnames and social mobility', *The Economic Journal* **125**(582), 61–85.

Cowell, Frank K. (2011), Inequality among the wealthy. CASE Working Paper No. 150.

De Nardi, Mariacristina, Fella, Giulio and Paz Pardo, Gonzalo (2015), Fat tails in life-cycle earnings and wealth inequality. Mimeo, UCL.

DeBacker, Jason, Panousi, Vasia and Ramnath, Shanthi (2012), The properties of income risk in privately held businesses. Federal Reserve Board Working Paper 2012-69.

Díaz-Giménez, Javier, Quadrini, Vincenzo and Ríos-Rull, José-Victor (1997), 'Dimensions of inequality: Facts on the U.S. distributions of earnings, income and wealth', *Federal Reserve Bank of Minneapolis Quarterly Review* **21**(2), 3–21.

De Nardi, Mariacristina (2004), 'Wealth inequality and intergenerational links', *Review of Economic Studies* **71**(3), 743–768.

De Nardi, Mariacristina and Yang, Fang (2014), 'Bequests and heterogeneity in retirement wealth', *European Economic Review* **72**, 182–196.

De Nardi, Mariacristina and Yang, Fang (2015), Wealth inequality, family background, and estate taxation. Working Paper 21047, National Bureau of Economic Research.

De Nardi, Mariacristina, Doctor, Phil and Krane, Spencer D. (2007), 'Evidence on entrepreneurs in the United States: Data from the 1989-2004 Survey of Consumer Finances', *Economic Perspectives* **4**, 18–36.

De Nardi, Mariacristina, French, Eric and Jones, John B. (2010), 'Why do the elderly save? The role of medical expenses', *Journal of Political Economy* **118**(1), 39–75.

Dynan, Karen E., Skinner, Jonathan and Zeldes, Stephen P. (2004), 'Do the rich save more?', *Journal of Political Economy* **112**(2), 397–444.

Gabaix, Xavier and Landier, Augustin (2008), 'Why has CEO pay increased so much?', *"Quarterly Journal of Economics"* **123**(1), 49–100.

Gabaix, Xavier, Lasry, Jean-Michel, Lions, Pierre-Louis and Moll, Benjamin (2015), The dynamics of inequality. NBER Working Paper No. 21363.

Gale, William G. and Scholz, John Karl (1994), 'Intergenerational transfers and the accumulation of wealth', *Journal of Economic Perspectives* **8**(4), 145–160.

Gentry, William M. and Hubbard, R. Glenn (2004), 'Entrepreneurship and household savings', *Berkely Economic Journal: Advances in Macroeconomics* **4**(1). Article 1.

Guvenen, Fatih, Karahan, Fatih, Ozkan, Serdar and Song, Jae (2015), What do data on millions of u.s. workers reveal about life-cycle earnings risk? Mimeo.

Hendricks, Lutz (2004*a*), Accounting for patterns of wealth inequality. Mimeo. Iowa State University.

Hendricks, Lutz (2004*b*), How important is preference heterogeneity for wealth inequality? Mimeo. Iowa State University.

Huggett, Mark (1996), 'Wealth distribution in life-cycle economies', *Journal of Monetary Economics* **38**(3), 469–494.

Hurst, Erik and Lusardi, Annamaria (2004), 'Liquidity constraints, wealth accumulation and entrepreneurship', *Journal of Political Economy* **112**(2), 319–347.

Hurst, Erik, Luoh, Ming Ching and Stafford, Frank P. (1998), 'The Wealth Dynamics of American Families, 1984-94', *Brookings Papers on Economic Activity* **29**(1), 267–338.

Jones, Charles I. (2015), 'Pareto and Piketty: The macroeconomics of top income and wealth inequality', *Journal of Economic Perspectives* **29**, 29–46.

Kacperczyk, Marcin, Nosal, Jaromir and Stevens, Luminita (2015), Investor sophistication and capital income inequality. Mimeo Boston College.

Kartashova, Katya (2014), 'The returns to entrepreneurial investment: A private equity premium puzzle?', *American Economic Review* **104**(10), 3297–3394.

Kaymak, Bari s and Poschke, Markus (2015), The evolution of wealth inequality over half a century: the role of taxes, transfers and technology. Mimeo, McGill University.

Kennickell, Arthur B. (2003), A rolling tide: Changes in the distribution of wealth in the U.S., 1989-2001. Mimeo.

Kitao, Sagiri (2008), 'Entrepreneurship, taxation, and capital investment', *Review of Economic Dynamics* **11**(1), 44–69.

Klevmarken, N. Anders, Lupton, Joseph P. and Stafford, Frank P. (2003), 'Wealth dynamics in the 1980s and 1990s: Sweden and the united states', *The Journal of Human Resources* **38**(2), 322–353.

Kopczuk, Wojciech (2015), 'What do we know about the evolution of top wealth shares in the united states?', *Journal of Economic Perspectives* **29**(1), 47–66.

Krusell, Per and Smith, Jr., Anthony (1998), 'Income and wealth heterogeneity in the macroeconomy', *Journal of Political Economy* **106**(5), 867–896.

Lampman, Robert J. (1962), *The Share of Top Wealth-Holders in National Wealth 1922-1956*, Princeton University Press, Princeton.

Lawrance, Emily (1991), 'Poverty and the rate of time preference: Evidence from panel data', *Journal of Political Economy* **99**, 54–77.

Lee, Sang Yoon (2015), Entrepreneurs, managers and inequality. Mimeo.

Lockwood, Lee (2012), Incidental bequests: Bequest motives and the choice to self-insure late-life risks. Mimeo.

Meade, James E. (1964), *Efficiency, Equality and the Ownership of Property*, Allen and Unwin, London.

Moskowitz, Tobias J. and Vissing-Jørgensen, Annette (2002), 'The returns to entrepreneurial investment: A private equity premium puzzle?', *American Economic Review* **92**(4), 745–778.

Mulligan, Casey B. (1997), *Parental Priorities and Economic Inequality*, Chicago: University of Chicago Press.

Nardi, Mariacristina De, French, Eric and Jones, John B. (2010), 'Why do the elderly save? the role of medical expenses', *Journal of Political Economy* **118**, 39–75.

Nishiyama, Shinichi (2002), 'Bequests, inter vivos transfers, and wealth distribution', *Review of Economic Dynamics* **5**(4), 892–931.

Pareto, Vilfredo (1897), *Cours d'Economie Politique*, Vol. II, F. Rouge, Lausanne.

Parker, Jonathan A. and Vissing-Jørgensen, Annette (2009), 'Who bears aggregate fluctuations and how?', *American Economic Review* **99**(2), 399–405.

Piketty, Thomas and Saez, Emmanuel (2003), 'Income inequality in the united states, 1913-1998', *The Quarterly Journal of Economics* **118**, 1–39.

Piketty, Thomas and Zucman, Gabriel (2014), Wealth and inheritance in the long run, *in* Anthony J.Atkinson and FrancoisBourguignon, eds, 'Handbook of Income Distribution', Vol. 2B, Elsevier B. V., chapter 15, pp. 1303–1368.

Quadrini, Vincenzo (2000), 'Entrepreneurship, saving and social mobility', *Review of Economic Dynamics* **3**(1), 1–40.

Quadrini, Vincenzo (2009), 'Entrepreneurship in macroeconomics', *Annals of Finance* **5**(3), 295–311.

Rodriguez, Santiago Budria, Díaz-Giménez, Javier, Quadrini, Vincenzo and Ríos-Rull, José-Victor (2002), 'Updated facts on the U.S. distributions of earnings, income, and wealth', *Federal Reserve Bank of Minneapolis Quarterly Review* **26**(3), 2–35.

Rosen, Sherwin (1981), 'The economics of superstars', *American Economic Review* **71**(5), 845–858.

Saez, Emmanuel and Zucman, Gabriel (2015), Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. Mimeo, LSE.

Tauchen, George (1986), 'Finite state markov-chain approximations to univariate and vector autoregressions', *Economic Letters* **20**(2), 177–181.

Venti, Steven F. and Wise, David A. (1988), 'The cause of wealth dispersion at retirement: Choice or chance?', *American Economic Review* **88**(2), 185–191.

Wang, Chong, Wang, Neng and Yang, Jinqiang (2015), 'Optimal consumption and savings with stochastic income and recursive utility', Mimeo.

Wold, Herman O. and Whittle, Peter (1957), 'A model explaining the pareto distribution of wealth', *Econometrica* pp. 591–595.

Wolff, Edward N. (1992), 'Changing inequality of wealth', *American Economic Review* **82**(2), 552–558.

Wolff, Edward N. (1998), 'Recent trends in the size distribution of household wealth', *Journal of Economic Perspectives* **12**(3), 131–150.

Yang, Fang (2013), 'Social security reform with impure intergenerational altruism', *Journal of Economic Dynamics and Control* **37**, 52–67.

A Appendix: Calibration for the simulations in Section 8

The model is the one used by De Nardi and Yang (2015). The calibration table 4 summarizes the parameters that are either taken from other studies or can be solved independently of the endogenous outcomes of the model. See De Nardi and Yang (2015) for a discussion of these choices. Our calibration of labor earnings process is based on the observation that the Panel Study of Income Dynamics (PSID) provides excellent data on the earnings dynamics for much of the population, but not for those of the richest households (see, for instance, Bosworth and Anders (2011)). To match the earnings dynamics of all the population, we thus proceed as follows.

1. We assume four possible earnings states: low, middle, high, and super-high. We take the support of the earnings shocks from Castañeda et al. Castañeda et al. (2003). The resulting grid points for ψ are [1, 3.15, 9.78, 1,061].

	Parameters		Value
Demographics	n	annual population growth	1.2%
	p_t	survival probability	see text
Preferences	γ	risk aversion coefficient	1.5
Labor earnings	ϵ_t	age-efficiency profile	see text
	ψ	labor earnings levels	see text
	Q_y	labor earnings transition matrix	see text
	ρ_h	AR(1) coef. of prod. inheritance process	0.50
	σ_h^2	innovation of prod. inheritance process	0.37
Production	α	capital income share	0.36
	δ	depreciation	6.0%
Government policy	τ_a	capital income tax	20%
	$P(\widetilde{y})$	Social Security benefit	see text
	τ_s	Social Security tax	12.0%

Table 4: Exogenous parameters used in the benchmark model.

2. We take the persistence ρ_h of the earnings inheritance process to be 0.5 (**need reference**) and the variance σ_h^2 from De Nardi (2004). We then discretize the earnings inheritance process as proposed by Tauchen (1986).

3. We take PSID estimates on the persistence (0.92) and variance (0.38) over five-year periods from Table A.1 in appendix A in De Nardi De Nardi (2004); and we discretize this process for the lowest three grid points using Tauchen (1986) to make sure that our process accurately represents the estimated earnings dynamics for much of the population. This gives us a three by three transition matrix.

4. We pick the remaining six elements of our four by four transition matrix to match the following aspects of the earnings distribution: The Gini coefficient and the share of total earnings earned, respectively, by the top 1%, 5%, 20%, 40%, 60%, and an earnings persistence at the top of 80%. The latter is consistent with work by DeBacker, Panousi and Ramnath (2012), which reports that the persistence of both labor and business income at the top of labor and business income distributions is high and that, in particular, the probability of staying there, both after one year and five years (the

latter results are available from the authors on request), is around 80%. We also impose adding-up restrictions.

The transition matrix for Q_y is:

$$\begin{bmatrix} 0.8239 & 0.1733 & 0.0027 & 0.000112 \\ 0.2171 & 0.6399 & 0.1428 & 0.000200 \\ 0.0067 & 0.2599 & 0.7334 & 0.000000 \\ 0.0720 & 0.0000 & 0.1252 & 0.802779 \end{bmatrix}.$$

The transition matrix for Q_{yh} in the benchmark model is

$$\begin{bmatrix} 0.8272 & 0.1704 & 0.0024 & 0.0000000000 \\ 0.5000 & 0.4696 & 0.0304 & 0.0000000000 \\ 0.1759 & 0.6513 & 0.1728 & 0.0000000051 \\ 0.0000 & 0.0018 & 0.9678 & 0.0304357624 \end{bmatrix}.$$

The transition matrices induce an initial distribution of earnings with probability masses over the respective earnings levels, given by [59.89%,35.88%,4.24%,0.00154845%].

Table 5 lists the parameters we use to calibrate the model, see De Nardi and Yang (2015) for a discussion of these targets and implied parameter values.

Moment		Data	Benchmark	No Bequest Motives
Wealth-output ratio		3.10	3.10%	3.10%
Bequest-wealth ratio		0.88-1.18%	0.87%	0.56%
90th perc. bequest distribution		4.34	4.36	4.53
Fraction of estates paying taxes		2.0%	1.85%	1.89%
Revenue from estate tax/output		0.33%	0.33	0.11%
Government spending/output		18%	17.99%	17.76%
Parameters				
β	discount factor		0.9453	0.9513
ϕ_1	bequest utility		-5.3225	0.0000
ϕ_2	bequest utility shifter (in \$ 2000)		1116K	0.0000
τ_b	tax on estates		21.52%	21.52%
x_b	estate exemption level (in \$ 2000)		782K	782K
τ_l	tax on labor income		19.19%	19.19%

Table 5: Parameters calibration for the benchmark model and the model with no voluntary bequests.